Electron-nucleus interaction in laser fields: The laser-assisted internal conversion process

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We present a general formalism for an efficient treatment of a broad range of electron-nucleus laser processes. The interaction with the laser beam is taken into account by transforming the system into an oscillating frame, called the Henneberger picture. General expressions for the transition probability per unit time are given in the weak laser field and in the n photonic cases, and connection to previous methods is demonstrated in the appropriate limits. As an example, the transition probability per unit time of laser-induced internal conversion (IC) processes is presented. The conservation of angular momentum in the multiphoton process can be traced well in our calculation. Numerical values of the IC coefficient of the energetically forbidden IC process in case of ^{99m}Tc ignited by the absorption of up to three soft-x-ray laser photons are also given. The increase in the rate of IC decay is found comparable to or greater than the natural decay rate of the isomer in case of appropriate intensity and photon energy of the laser. Hard UV laser-induced internal conversion coefficients (ICCs) are also calculated for energetically forbidden shells of ^{107}Ag^m (K shell, $E3$, 25.47 keV), ^{90}Nb^m (L_2 shell, $M2+E3$, 2.3 keV), ^{183}W^m (N_1 shell, $E1$, 544 eV and M_5 shell, $E2$, 1.79 keV), ^{188}Re^m (M_2 shell, $M3+E4$, 2.63 keV), ^{205}Pb^m (M_5 shell, $E2$, 2.4 keV), and ^{235}U^m (O_4 and O_5 shells, $E3$, 73.5 eV). Measurable induced ICCs are found in case of available intensities and photon energies of the laser beam for the above isomers. Experiments, that may demonstrate the effect and may be tools for determination of nuclear transition energies, are also suggested.

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I. INTRODUCTION

Intense laser fields can exert an influence on atomic, as well as on nuclear processes. Such atomic processes are, e.g., multiphoton ionization and higher harmonics generation [1,2]. After lasers with high intensity had become available, the investigation of nuclear processes in the presence of intense laser fields was boosted and it has been concluded that direct effects of the intense lasers on the nucleus can be considered negligible [3]. Moreover, the effect of electronic shielding hinders the possibility of the direct interaction of the laser and the nucleus [4]. However, internal conversion (IC) and electronic-bridge processes have been investigated steadily, since lasers may cause significant modification in the rate of nuclear transitions in case of these electron-nucleus laser combined processes [5]. In the precursor studies of the topic Volkov and Coulomb-Volkov solutions were used to describe the laser modified free-electron states in the calculations [6] and, on the other hand, calculations were carried out employing dressed bound electronic solutions, too [7]. However, the treatment of nucleus-electron laser combined processes in Refs. [6,7] does not let us see clearly what kind of mechanism actually occurs as the laser modifies the nucleus-electron processes. It is shown here that all these processes, i.e., both atomic and nuclear processes, can be traced back to laser field induced modification of the electromagnetic interaction between the electron and nucleons.

One aim of this paper is to give a rather general way for the calculation of the rate of several electron-nucleus laser combined processes. The weak-field limits and the results for n photonic processes of the rates are determined. Furthermore, it is shown that our treatment results in the well-known, laser modified Coulomb potential of the nucleus felt by the electron in the presence of the laser field. As an example for the laser-assisted electron-nucleus combined processes we give the transition probability per unit time of laser-induced internal conversion (IC) process and the soft-x-ray laser-assisted IC coefficient of the energetically forbidden IC process in the case of ^{99m}Tc. Moreover, hard UV laser-induced IC coefficients of a few energetically forbidden processes have been also calculated.

II. ELECTRON-NUCLEUS INTERACTION IN THE PRESENCE OF A LASER FIELD

Our model consists of an electron and A nucleons bound in the nucleus. In the laser-free case the electron-nucleon interaction can be described by the electromagnetic four-vector potential $A^\mu(x)$ felt by the electron and created by the nucleon transition current $j^\mu_{\beta\alpha}(y_N)$ in the transition $\alpha\rightarrow\beta$ as

$$A^\mu(x)=\int d^4y_N D_F(x-y_N) j^\mu_{\beta\alpha}(y_N), \tag{1}$$

where

$$D_F(x-y_N)=\frac{1}{(2\pi)^4}\int \frac{-4\pi e^{-iq(x-y_N)}}{q^2+i\varepsilon} d^4q \tag{2}$$

is the causal (Feynman) Green function. The notations x and y_N are the four coordinate of the electron and one of the nucleons, respectively.

The first-order scattering amplitude of the electron on this potential is

$$S_{fi}=-\frac{i}{\hbar c}\int d^4x\int d^4y_N J_\mu^{21}(x)A^\mu(x), \tag{3}$$

where J_μ^{21} is the four-transition current of the electron in the transition $1\rightarrow 2$ [8].

The modification of the above process due to the presence of the laser field can be traced in the following way. The laser field is described by a classical field of the form $\vec{A}_{cl}(t)=\vec{\varepsilon}A_0\cos(\omega_L t)$, where we have introduced the angular frequency ω_L and the unit vector $\vec{\varepsilon}$ for the state of polarization of the laser. As the atomic radii are much smaller than the wavelength of the laser, the dipole approximation is justified. We use nonrelativistic approximation in the laser-electron interaction Hamiltonian

$$H_{Ie}=-e_e\frac{\vec{p}}{\kappa c}\cdot\vec{A}_{cl}(t)+\frac{e_e^2}{2\kappa c^2}\vec{A}_{cl}^2(t), \tag{4}$$

where κ is the rest mass of the electron, $\vec{p}$ is its momentum, c is the velocity of light, and $e_e=-e$ is the electronic charge (e is the elementary charge). The $\vec{A}_{cl}^2$ term in H_{Ie} disappears after transforming the system by the unitary operator $U=\exp[-ie_e^2/(2\hbar\kappa c^2)\int\vec{A}_{cl}^2dt]$.

Now we define another unitary operator

$$B=\exp\left(\frac{i}{\hbar}\frac{e_e}{\kappa}\vec{p}\cdot\vec{Z}(t)\right), \tag{5}$$

where

$$\vec{Z}(t)=-\frac{1}{c}\int\vec{A}_{cl}(t)dt. \tag{6}$$

The operator B acts only on the operators containing the electron coordinates $\vec{x}$. Applying this unitary transformation the remainder of H_{Ie} disappears and it can be shown [9] that $\vec{x}$ is transformed to $\vec{x}_B$ as

$$\vec{x}_B=B\vec{x}B^\dagger=\vec{x}+\frac{e_e}{\kappa}\vec{Z}, \tag{7}$$

and, similarly,

$$Bf(\vec{x})B^\dagger=f\left(\vec{x}+\frac{e_e}{\kappa}\vec{Z}\right). \tag{8}$$

This transformation of the system is called Henneberger transformation [10]. We can say that the effect of the laser appears in $A^\mu(x)$ [and similarly in $D_F(x-y_N)$] only as a vibrating term that is added to the electron coordinate. So, after the Henneberger transformation $\exp[i(\vec{x}+e_e/\kappa\vec{Z})\vec{q}]$ appears in the Green-function D_F.

Now we can give the nucleus-electron scattering amplitude in the presence of the laser field. As a consequence of the Henneberger transformation we have to substitute $\vec{x}_B$ for $\vec{x}$ in the Green function and with this substitution, the S-matrix element of the process is given by

$$S_{fi}=-\frac{i}{\hbar c}\sum_N\int d^4x\int d^4y_N J_\mu^{21}(x)D_F(x'-y_N)j_{\beta\alpha}^\mu(y_N), \tag{9}$$

where

$$D_F(x'-y_N)=\frac{1}{(2\pi)^4}\int\frac{-4\pi e^{-iq(x'-y_N)}}{q^2+i\varepsilon}d^4q. \tag{10}$$

Here, the four-vector $q=\{q_0,\vec{q}\}$; x' represents the Henneberger transformed electron four coordinate; ($x'=\{x_0,\vec{x}_B\}$), and $y_N=\{y_0,\vec{y}_N\}$ is the four coordinate of one of the nucleons. $J_\mu^{21}(x)$ is the four-transition current density of the electron, $j_{\beta\alpha}^\mu(y_N)$ is the four-transition current density of a nucleon

$$J_\mu^{21}(x)=e_e\overline{\psi^2}(\vec{x})\gamma_\mu\psi^1(\vec{x})e^{ik_{21}x_0}, \tag{11}$$

$$j_{\beta\alpha}^\mu(y_N)=e^{ik_{\beta\alpha}y_0}\begin{cases}j_{\beta\alpha}^{\mu,p}(\vec{y}_N), & \text{if } N=1,\ldots,Z\\ j_{\beta\alpha}^{\mu,n}(\vec{y}_N), & \text{if } N=Z+1,\ldots,A,\end{cases} \tag{12}$$

where

$$\begin{aligned}j_{\beta\alpha}^{\mu,p}(\vec{y}_N)=\int\cdots\int e_p\overline{\psi}_\beta^p(\vec{y}_1,\ldots,\vec{y}_N,\ldots,\vec{y}_Z)\gamma^\mu\\ \times\psi_\alpha^p(\vec{y}_1,\ldots,\vec{y}_N,\ldots\vec{y}_Z)d^3y_1\cdots d^3y_{N-1}\\ \times d^3y_{N+1}\cdots d^3y_Z\end{aligned} \tag{13}$$

is the proton part of the nuclear four-current density and

$$\begin{aligned}j_{\beta\alpha}^{\mu,n}(\vec{y}_N)=\int\cdots\int e_n\overline{\psi}_\beta^n(\vec{y}_{Z+1},\ldots,\vec{y}_N,\ldots,\vec{y}_A)\gamma^\mu\\ \times\psi_\alpha^n(\vec{y}_{Z+1},\ldots,\vec{y}_N\ldots\vec{y}_A)d^3y_{Z+1}\cdots d^3y_{N-1}\\ \times d^3y_{N+1}\cdots d^3y_A\end{aligned} \tag{14}$$

is the neutron part of the nuclear four-current density, and the notations e_p and e_n refer to the charges of a proton and neutron bound in the nucleus [11], and Z is the proton number. γ^μ ($\mu=0,1,2,3$) are the four γ matrixes. ψ^2 and ψ^1 stand for the electron bispinors in the final and initial states of the process, and ψ_β^p, ψ_α^p and ψ_β^n, ψ_α^n are the final and initial states of Slater determinant type, constructed from one-particle bispinors, of the bound protons and neutrons, respectively. We have introduced $k_{21}=(E_2-E_1)/(\hbar c)$ where E_2 and E_1 are the energy eigenvalues of the final and initial electron states, respectively, and $k_{\beta\alpha}=(E_\beta-E_\alpha)/(\hbar c)$ where E_β and E_α stand for the energy eigenvalues of the final and initial nuclear states.

III. THE WEAK-FIELD LIMIT

First we deal with the weak-field case, i.e., with the lowest-order process in the laser field. In D_F we can write

$$e^{-iqx'} \simeq \left(1+\frac{e_e}{\kappa}\vec{Z}\cdot\vec{\nabla}_x\right)e^{-iqx}, \tag{15}$$

where

$$\vec{Z} = -i\vec{Z}_0(e^{ik_Lx_0}-e^{-ik_Lx_0}), \tag{16}$$

with $\vec{Z}_0 = -[A_0/(2c\omega_L)]\vec{\varepsilon}$ and $k_L=\omega_L/c$. The contribution that originates from the first term on the right-hand side of Eq. (15), the term that equals to 1, gives the *S*-matrix element without laser field. The other term represents the first-order contribution of the laser field.

The integration over x_0 results in

$$\pm\int_{-\infty}^{\infty} e^{i(k_{21}-q_0\pm k_L)x_0}dx_0 = \pm 2\pi\delta(k_{21}-q_0\pm k_L). \tag{17}$$

Performing the integral over y_0 leads, similarily, to a Dirac-delta $2\pi\delta(k_{\beta\alpha}+q_0)$. This way it is possible to perform the integration over q_0, yielding

$$\pm\int_{-\infty}^{\infty}\delta(k_{21}-q_0\pm k_L)\,\delta(k_{\beta\alpha}+q_0)\frac{-4\pi}{q_0^2-\vec{q}^2+i\varepsilon}dq_0$$

$$= \pm\frac{-4\pi}{k_{\beta\alpha}^2-\vec{q}^2+i\varepsilon}\delta(k_{21}+k_{\beta\alpha}\pm k_L). \tag{18}$$

Integrating over the $\vec{q}$ space we carry out first the integration over the solid angle Ω_q,

$$\int d\Omega_q e^{i\vec{q}(\vec{x}-\vec{y}_N)} = \frac{2\pi}{iqR_N}(e^{iqR_N}-e^{-iqR_N}), \tag{19}$$

where $R_N=|\vec{x}-\vec{y}_N|$. Then we carry out the integration over q using contour integration techniques. The two poles are at $q=|k_{\beta\alpha}|+i\varepsilon$ and $q=-|k_{\beta\alpha}|-i\varepsilon$. Finally, we obtain

$$S_{fi}^{(\pm)} = K_{fi}^{(\pm)}2\pi\delta(k_{21}+k_{\beta\alpha}\pm k_L), \tag{20}$$

with

$$K_{fi}^{(\pm)} = \mp\frac{1}{\hbar c}\sum_N\int\int d^3x d^3y_N J_\mu^{21}(\vec{x})j_{\beta\alpha}^\mu(\vec{y}_N)$$

$$\times\frac{e_e}{\kappa}\vec{Z}_0\cdot\vec{\nabla}_x\left(\frac{e^{i|k_{\beta\alpha}|R_N}}{R_N}\right). \tag{21}$$

The transition probability per unit time is given by

$$W_{fi} = 2\pi\sum_s |K_{fi}^{(s)}|^2c^2\delta(\omega_{21}+\omega_{\beta\alpha}+s\omega_L), \tag{22}$$

where $s=\pm1$, and $\omega_{21}=ck_{21}$, $\omega_{\beta\alpha}=ck_{\beta\alpha}$, and $\omega_L=ck_L$.

A formally similar result was obtained for the inverse electronic-bridge process using pure Coulomb interaction $(1/R_N)$ between the electron and the nucleus, instead of $\exp(i|k_{\beta\alpha}|R_N)/R_N$ [12]. Moreover, in Ref. [12] the laser field was quantized and the current-current interaction term between the electron and the nucleus was neglected. The special result of Ref. [12] was then employed to calculate laser initiated energetically forbidden internal conversion coefficients [13]. The question of direct γ- or electronic-bridge decay modes of the ^{229m}Th isomeric state was discussed by extending the model of [12] to include the magnetic dipole-dipole interaction term [14].

IV. GENERAL CASE AND DISCUSSION

Next, we extend our calculation to include higher-order terms in the laser intensity, i.e., we deal with n photonic processes. Instead of relying on the approximation of Eq. (15), we now carry out the calculation in the following way. We expand the term $e^{-qx'}$ in the Green-function D_F, Eq. (10), as $\exp(-iqx')=\exp(-iq_0x_0+i\vec{q}\cdot\vec{x}_B)=\exp(-iqx)\exp[i(e_e/\kappa)\vec{q}\cdot\vec{Z}]=\exp(-iqx)\exp[i\vec{q}\cdot\vec{Z}_0(2e_e/\kappa)\sin(k_Lx_0)]$, and we employ the Jacobi-Anger formula $e^{iz\sin(\theta)}=\sum_{n=-\infty}^{\infty}e^{in\theta}J_n(z)$, where J_n represents the Bessel function of the first kind of order n. Thus, we obtain

$$D_F(x'-y_N) = \frac{1}{(2\pi)^4}\int\frac{-4\pi e^{-iq(x-y_N)}}{q^2+i\varepsilon}$$

$$\times\sum_n e^{ink_Lx_0}J_n\left(\frac{2e_e}{\kappa}\vec{q}\cdot\vec{Z}_0\right)d^4q. \tag{23}$$

Integrating over y_0 and over q_0:

$$\int dq_0\int dy_0 e^{ik_{\beta\alpha}y_0}D_F(x'-y_N)$$

$$= \sum_n e^{ik_Lnx_0}V_n(\vec{x},\vec{y}_N;k_{\beta\alpha}), \tag{24}$$

where

$$V_n(\vec{x},\vec{y}_N;k_{\beta\alpha}) = \frac{-1}{2\pi^2}\int\frac{J_n(\xi\vec{\varepsilon}_j\cdot\vec{q})}{k_{\beta\alpha}^2-\vec{q}^2+i\varepsilon}e^{i\vec{q}\cdot(\vec{x}-\vec{y}_N)}d^3q. \tag{25}$$

Here, we have introduced $\xi=e_eA_0/(\kappa c\omega_L)$. Carrying out the integration over x_0 finally results in

$$S_{fi}^{(n)} = K^{(n)}2\pi\delta(k_{21}+k_{\beta\alpha}+nk_L) \tag{26}$$

with

$$K_{fi}^{(n)} = -\frac{i}{\hbar c}\sum_N\int d^3x d^3y_N J_\mu^{21}(\vec{x})j_{\beta\alpha}^\mu(\vec{y}_N)V_n(\vec{x},\vec{y}_N;k_{\beta\alpha}), \tag{27}$$

and the transition probability per unit time is given by

$$W_{fi} = 2\pi\sum_n |K_{fi}^{(n)}|^2c^2\delta(\omega_{21}+\omega_{\beta\alpha}+n\omega_L). \tag{28}$$

If the intensity I_L of the laser vanishes then $V_n\to0,(n\neq0)$ since $J_n(x)\simeq x^n$ for small x, and $\vec{Z}_0\propto\sqrt{I_L}$. As $\lim_{x\to0}J_0(x)=1$ we obtain $V_0(k_{\beta\alpha})=e^{i|k_{\beta\alpha}|R_N}/R_N$ with I_L

$\rightarrow 0$. Thus, in the case of vanishing intensity we reobtain the well-known formula describing the transition probability per unit time for the pure interaction between the electron and the nucleus [11]. Note that our general result, Eq. (28), also describes those processes where the nucleus does not change its state and only assists the process. This is the case when $|\beta\rangle = |\alpha\rangle$. It gives $V_0(0) = 1/|\vec{x} - \vec{y}_N|$ in the absence of the laser. Summing $V_0(0)$ for all the nucleons we can write the electrostatic potential in the case of the pointlike nucleus as

$$\sum_N \frac{e_N}{|\vec{x} - \vec{y}_N|} \simeq \frac{Ze}{|\vec{x}|} = \overline{V}_0, \tag{29}$$

where we have introduced the average electrostatic potential of the nucleus $\overline{V}_0$. We also note that we have not utilized the specific form of the electronic and the nuclear eigenfunctions so far. Our derivation, however, tells us the way in which the electronic part of the free Hamiltonian H_e should be modified, viz., by adding $\overline{V}_0 = Ze/|\vec{x}|$ to our original H_e, which results in the usual bound atomic eigenfunctions ψ of the electron. Now, with these solutions we can calculate the scattering amplitude by substituting $A^0(x) - \overline{V}_0$ in Eq. (3). The transition probability per unit time for the laser-free process is determined by Eq. (28) in case of $|\beta\rangle \neq |\alpha\rangle$ with $n = 0$ and $I_L = 0$.

Similarly, if the laser field is present and we still assume $|\beta\rangle = |\alpha\rangle$ then, using the point-like nucleus approximation, the $j^0_{\beta\alpha}$ term produces the average, laser intensity-dependent electrostatic potential [15]

$$\overline{V}_L = \frac{Ze}{|\vec{x} + e_e \vec{Z}(t)/\kappa|}, \tag{30}$$

experienced by the electron. Using this average potential, the nonrelativistic electronic eigenfunctions can be obtained [16] from the time-dependent Schrödinger equation

$$i\hbar \frac{\partial \psi_L}{\partial t} = \left[\frac{\vec{p}^2}{2\kappa} + e_e \overline{V}_L \right] \psi_L . \tag{31}$$

Now we have to use $A^0(x') - \overline{V}_L$ in the calculation of the scattering amplitude while Eq. (28) remains valid in case of $|\beta\rangle \neq |\alpha\rangle$.

It can be shown in the high-freqency limit that, as a lowest-order approximation, one can use the unperturbed (laser-free) wave-function ψ instead of ψ_L for the description of the bound electron in the oscillating frame [16]. Therefore, it can be used in Eq. (28) for the calculation of the transition probability per unit time of the process under consideration.

V. LASER-ASSISTED INTERNAL CONVERSION PROCESS

The laser can modify electron-nucleus processes more effectively if its angular frequency lies near one of the characteristic frequencies of the system. Therefore, it is expected that lasers in the UV, soft- and hard-x-ray regime will be better suited for this purpose. Our description leads to a simpler calculation in these cases since the requirements of the high frequency limit of the electronic solutions of Eq. (31) are better fulfilled. So, laser-assisted internal conversion processes are expected to be well described in this manner. In other words, if $|\beta\rangle \neq |\alpha\rangle$, $k_{\beta\alpha} < 0$ then Eq. (28) gives quite accurately the rate of the laser-assisted internal conversion processes in the high-frequency approximation.

We continue with our general formulas Eqs. (25)–(27), and we expand the exponent in V_n as

$$e^{i\vec{q}(\vec{x} - \vec{y}_N)} = (4\pi)^2 \left[\sum_{l=0}^{\infty} \sum_{m=-l}^{l} i^l j_l(qx) Y^*_{lm}(\hat{q}) Y_{lm}(\hat{x}) \right] \times \left[\sum_{L=0}^{\infty} \sum_{M=-L}^{L} (-i)^L j_L(qy_N) Y_{LM}(\hat{q}) Y^*_{LM}(\hat{y}_N) \right], \tag{32}$$

where j_l and j_L are the spherical Bessel functions of order l and L, respectively, and Y_{lm} and Y_{LM} denote the spherical harmonics; $\hat{x}$, $\hat{y}_N$, and $\hat{q}$ represent the unit vectors pointing in the directions $\vec{x}$, $\vec{y}_N$, and $\vec{q}$, respectively. We choose the frame of reference as $\vec{q}_z \| \vec{\varepsilon}$ (the laser is polarized in the z direction). Then we can write $d^3q = -q^2 dq d\phi_q du$, where $u = \cos\theta_q$ and ϕ_q, θ_q are the polar angles of $\vec{q}$. The spherical harmonics can be expanded as

$$Y_{lm}(\hat{q}) = \frac{1}{\sqrt{2\pi}} P^m_l(u) \sqrt{\frac{(2l+1)(l-m)!}{2(l+m)!}} e^{im\phi_q}, \tag{33}$$

where P^m_l is the associated Legendre polinom of the first kind. Carrying out the integration over the whole solid angle in Eq. (25), the integration over ϕ_q results in $\delta_{m,M}$. Thus, we obtain

$$V_n(\vec{x}, \vec{y}_N; k_{\alpha\beta}) = \sum_{l,L,m,M} (-8) i^l (-i)^L Y_{lm}(\hat{x}) Y^*_{LM}(\hat{y}_N) \times \int_0^{\infty} \int_{-1}^{1} I_n(u,q) \varphi_{l,m,L,M}(u) \delta_{m,M} du dq, \tag{34}$$

where $k_{\alpha\beta} = -k_{\beta\alpha}$, and we have introduced

$$\varphi_{l,m,L,M}(u) = P^m_l(u) P^M_L(u) \sqrt{\frac{(2l+1)(l-m)!}{2(l+m)!}} \times \sqrt{\frac{(2L+1)(L-M)!}{2(L+M)!}}, \tag{35}$$

and

$$I_n(u,q) = \frac{J_n(-\xi q u) j_l(qx) j_L(qy_N) q^2}{k^2_{\alpha\beta} - \vec{q}^2 + i\varepsilon}. \tag{36}$$

Expanding the expression $J_n(u) P^M_l(u) P^M_L(u)$ into power series of u it yields a sum of terms of the form of $u^{n+2k}(1-u^2)^M u^{l+L-2M} F(a_1, b_1; c_1; 1/u^2)$

$F(a_2,b_2;c_2;1/u^2)$, where k is the index of summation in the series and F is the Gauss's hypergeometric function [17,18]. It can be seen that the power series is an even function of u if $n+l+L$ is even. In this case, the integration over u can be written as $\int_{-1}^{1}du=2\int_{0}^{1}du$. On the other hand, if $n+l+L$ is odd, the integration results in $\int_{-1}^{1}J_nP_l^MP_L^Mdu=0$, which also gives $W_{fi}=0$ after all. Therefore, we have to evaluate the rate of the process for $l+L+n=2p$ cases only. Similarly, expanding the Bessel function and the spherical Bessel functions into power series of their arguments [18] the q dependence of the numerator of $I_n(u,q)$ is a power series of $q^{l+L+n+2k}$, where k is the index of summation in the series again. If $l+L+n$ is even, then we can change the integration as $\int_0^\infty dq=1/2\int_{-\infty}^{\infty}dq$.

So we can carry out the integration over q using contour integration techniques. We suppose that $\xi+y_N\ll x$, i.e., the oscillation in the electron coordinate caused by the laser is smaller than the characteristic size of the bound-electron shell incorporated in the process, furthermore the nucleus is pointlike. Now we can employ the asymptotic formula of the Bessel function for large arguments [19] and the expansion of the spherical Bessel functions [20]. From the three exponents emerging from the three Bessel functions the one originated from $j_l(qx)$ determines that the contour of integration is closed in the upper or in the lower half of the complex q plane. Carrying out the integration we obtain

$$\int_{-\infty}^{\infty}I_n(u,q)dq=I_n(u)=\pi ik_{\alpha\beta}J_n(-\xi k_{\alpha\beta}u)\times j_L(k_{\alpha\beta}y_N)h_l^{(1)}(k_{\alpha\beta}x), \tag{37}$$

where $h_l^{(1)}(k_{\alpha\beta}x)$ is the spherical Henkel function of the first kind of order l. We note that if the laser field ceases, i.e., if we insert $J_0(0)=1$ [and $J_n(0)=0$ if $n\neq 0$] in the upper formula, it yields the well-known potential for V_0 that is usually used to describe the IC process in pointlike nucleus approximation [21].

In the following, we are going to calculate the transition probability per unit time (W_{fi}) of the laser-assisted IC process. We presume that the metastable state of the nucleus decays mainly by an electric multipole decay mode of order L (denoted as EL). Furthermore, nonrelativistic wave functions for the electronic states are used, and the current-current interaction between the nucleus and the electron is not considered here. The nuclear transition is described by the multipole transition operator of the nucleus, which is defined as $\mathcal{M}(EL,M)=\Sigma_N e_N y_N^L Y_{LM}(\hat{y}_N)$; here, e_N is the charge of the Nth nucleon. Its matrix element can be written as

$$\langle I_i,M_i|\mathcal{M}(EL,M)|I_f,M_f\rangle=(-1)^{I_i-M_i}\begin{pmatrix} I_i & L & I_f \\ -M_i & M & M_f\end{pmatrix}\langle I_i\|\mathcal{M}(EL)\|I_f\rangle, \tag{38}$$

where $\langle I_i\|\mathcal{M}(EL)\|I_f\rangle$ is the reduced matrix element of $\mathcal{M}(EL,M)$, I_i and I_f are the angular momentum quantum numbers of the initial and final nuclear states, respectively, while M_i and M_f denote the magnetic quantum numbers of the initial and final nuclear states, furthermore we have applied the usual notation of $3j$ symbols. We use the reduced transition probability $B(EL,I_i\to I_f)=|\langle I_i\|\mathcal{M}(EL)\|I_f\rangle|^2/(2I_i+1)$ for describing the nuclear transition [22].

Imposing the condition $\xi k_{\alpha\beta}\ll 1$, which can be fulfilled (see later), we can approximate $J_n(-\xi k_{\alpha\beta}u)\simeq(-\xi k_{\alpha\beta})^n u^n/(2^n n!)$, and similarly $j_L(k_{\alpha\beta}y_N)\simeq(k_{\alpha\beta}y_N)^L/(2L+1)!!$. After summing up for all the magnetic quantum numbers of the final states (m_f,M_f) and averaging for all the initial ones (m_i,M_i), m_i, m_f are the magnetic quantum numbers of the initial and final electronic states, respectively, we obtain for W_{fi} by following the usual train of thought of such calculations

$$\begin{aligned}W_{fi}&=\int\sum_{m_i,m_f}\sum_{M_i,M_f}\sum_n 2\pi\frac{|K_{fi}^{(n)}(EL)|^2}{(2I_i+1)(2l_i+1)}\\&\quad\times c^2\delta(\omega_{21}-\omega_{\alpha\beta}+n\omega)\rho(\epsilon_2)d\epsilon_2\\&=\int\sum_{l,n}\frac{8\pi^2e^2}{\hbar^2}B(EL,I_i\to I_f)\\&\quad\times\frac{k_{\alpha\beta}^{2L+2+2n}\xi^{2n}}{2^{2n}(n!)^2}\frac{(2l_f+1)(2l+1)}{(2L+1)[(2L+1)!!]^2}\\&\quad\times\begin{pmatrix} l_i & l & l_f\\ 0&0&0\end{pmatrix}^2 S_{lL}^n|R_l^{fi}|^2\delta(\omega_{21}-\omega_{\alpha\beta}+n\omega)\rho(\epsilon_2)d\epsilon_2,\end{aligned} \tag{39}$$

where we have defined the quantities

$$S_{lL}^n=\sum_m\left[\int_{-1}^{1}\varphi_{l,m,L,m}(u)u^n du\right]^2 \tag{40}$$

and

$$R_l^{fi}=\int_0^\infty R_f h_l^{(1)}(k_{\alpha\beta}x)R_i x^2dx. \tag{41}$$

Here, R_f and R_i are the radial parts of the nonrelativistic wave functions in the final and initial electronic states, respectively; l_i and l_f are the angular momentum quantum numbers of the IC electron in the initial and final states, respectively. $\rho(\epsilon_2)$ denotes the line-shape function of the final electronic state if it is a bound state, as it stands for the phase-space density if the outgoing IC electron is in a free state, $\epsilon_2=E_2/\hbar$.

We introduce the IC coefficient (ICC) α of the laser-assisted process which can be calculated as $\alpha=W_{fi}/W_\gamma$, where $W_\gamma=8\pi(L+1)k_{\alpha\beta}^{2L+1}B(EL,I_i\to I_f)/[L((2L+1)!!)^2\hbar]$ is the rate of direct γ decay [22]. Thus, in the case of bound-bound electronic transitions

TABLE I. Computed values of S_{l3}^n for different electronic transitions of multipolarity l and numbers of laser photons n incorporated in the electron-nucleus process. (Here the multipolarity of the nuclear transition is $L=3$.) Remember that $S_{l3}^n=S_{l3}^{-n}$.

	$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$	$l=6$	$l=7$
$n=0$	0	0	0	7.0	0	0	0	0
$n=1$	0	0	1.0	0	1.3333	0	0	0
$n=2$	0	0.16	0	0.9437	0	0.2963	0	0
$n=3$	0.0229	0	0.3905	0	0.5174	0	0.0693	0
$n=4$	0	0.1253	0	0.4093	0	0.2266	0	0.0166

$$\alpha=\sum_{l,n}\frac{\pi\alpha_f(k_{\alpha\beta}\xi)^{2n}\omega_{\alpha\beta}}{2^{2n}(n!)^2}\frac{(2l_f+1)(2l+1)L}{(2L+1)(L+1)}\times\begin{pmatrix} l_i & l & l_f\\ 0 & 0 & 0\end{pmatrix}^2 S_{lL}^n|R_l^{fi}|^2\rho(\epsilon_2'). \tag{42}$$

Here, α_f is the fine-structure constant, and $\epsilon_2'=\omega_1+\omega_{\alpha\beta}-n\omega$, $\rho(\epsilon_2')=\gamma/(2\pi)[(\epsilon_{20}-\epsilon_2')^2+\gamma^2/4]^{-1}$, where γ is the width of the final electronic state and $\hbar\epsilon_{20}$ is the central energy of the distribution. In the case of a free final electronic state with energy $\epsilon_2\to 0$, i.e., near threshold, one has to substitute $|\tilde{R}_l^{fi}|^2\hbar/(2\mathcal{R})$ instead of $|R_l^{fi}|^2\rho(\epsilon_2')$ in Eq. (42), where $\tilde{R}_l^{fi}=\sqrt{4\pi}\int\tilde{R}_i(y)h_l^{(1)}(k_{\alpha\beta}a_0y)J_{2l_f+1}(\sqrt{8Z_{eff}^i y})y^{3/2}dy$, which is a dimensionless quantity; $\tilde{R}_i(y)$ is the dimensionless radial part of the wave function of the initial $2p$ state, $y=x/a_0$ and $\tilde{R}_i(y)=a_0^{3/2}R_i(x)$, where R_i is the radial part of the initial wave function. $\mathcal{R}$ is the Rydberg energy, a_0 is the Bohr radius, Z_{eff}^i is the effective charge of the initial state (see later), and for the radial part of the free electronic state the $R_{l_f}=\sqrt{4\pi/x}J_{2l_f+1}(\sqrt{8Z_{eff}^i x/a_0})$ approximation is used (valid for $\epsilon_2\to 0$).

The conservation of angular momentum in laser-assisted IC process is ensured by the $3j$ symbols and the quantity S_{lL}^n ($S_{lL}^n=S_{lL}^{-n}$). We show the properties of S_{l3}^n in Table I. It can be seen, e.g., that in the case of an $E3$ nuclear transition IC process containing a $p\to p$ electronic transition ($l_i=l_f=1, l=0$) is allowed if $|n|=3$ of laser photons take part in the process. Although our restriction ($l+3+n$ must be even) allows the process with $|n|=1$ if $l=0$, the $S_{03}^1=0$ value forbids this possibility. Similarly, if $l>4$ the IC process becomes forbidden for low $|n|$ values. It can be seen from Table I that S_{l3}^n gives zero in any case that is forbidden by momentum conservation rule. Here, it is tacitly assumed that n corresponds to the number of absorbed or emitted photons and each photon carries unit angular momentum. (The $n<0$ values give the absorbtion branch and the $n>0$ values correspond to laser photon emission.) In such a way this model is able to give an account of a significant aspect of the n-photonic laser interaction, namely, angular momentum conservation.

VI. NUMERICAL RESULTS FOR ENERGETICALLY FORBIDDEN IC PROCESS OF ^{99}Tc ISOMER IGNITED BY SOFT COHERENT X-RAY FIELD

Next, we apply our results for a realistic situation, viz., we investigate the laser-assisted IC process in case of ^{99m}Tc. ^{99m}Tc has a metastable state with transition energy $E_{\alpha\beta}=2.1726$ keV and it decays via an $E3$ transition. The energy of the nuclear transition is smaller than the binding energy of the electrons of the K and L shells, therefore the IC process from these shells is energetically forbidden. However, IC may be ignited on these inner shells if an external radiation field assists the process. So laser ignited ICC (α) is added to the total, laser-free ICC (α_{tot}) in this case. Such a situation was examined in a more simple model in a previous work by us [13], where the possibility of acceleration of the decay of ^{99m}Tc was discussed by switching on the $2p$ shells into the IC process calculating with one-photon absorption. Our considerations presented here make it possible to discuss the possibility of the ignition of the energetically forbidden IC process in the case of many-photons absorption, too.

Here, we examine the possibility of switching on the $2p_{3/2}$ electronic shell into the IC process numerically. This shell has $\hbar\omega_1=-2676.9$ eV binding energy [23]. First, we consider that the final electronic state of the process is a bound state of the Tc atom. It is hard to predict its exact binding energy due to the fact that, e.g., the energy levels of these electronic shells are altered by the chemical surroundings, therefore we have calculated the process by choosing $E_2=-8$ eV according to [13]. Moreover, we have taken $\hbar\gamma=1.91$ eV for the width of the final bound state which is the width of the IC created vacancy (see Ref. [24]). To calculate the matrix element R_l^{fi} we have used the radial parts of nonrelativistic hydrogenlike bound wave functions. The interaction with the other electrons has been taken into account by introducing the effective charge Z_{eff} as $E_b=\mathcal{R}\,(Z_{eff}/n_p)^2$, where E_b is the binding energy, $\mathcal{R}$ is the Rydberg-energy, and n_p denotes the principal quantum number of the shell. Calculating R_l^{fi} for allowed electronic transitions we have obtained that those channels will dominate at which $l=l_f+l_i$, and any other R_l^{fi} has orders of magnitude smaller value. The results of our calculation are given in the first half of Table II, where we have given α and the critical intensity $I_{crit}^{(1)}$ (in parentheses) of the laser in case of different n, l, and electronic transitions, sorted out by different R_l^{fi} values. $I_{crit}^{(1)}$ is the laser intensity at which α becomes unity.

TABLE II. Computed values of laser induced IC coefficients (α) of ^{99m}Tc in the case of bound-bound (BB) and bound-free (BF) electronic transitions, for different atomic transitions of different multipolarities (l) and photon numbers (n). I denotes the laser intensity given in W/cm^2 units. Under the values of α in the parentheses the critical intensity $I^{(1)}_{crit}$ has been tabulated, which is defined by the condition $\alpha=1$.

BB	$l=1$ $2p\rightarrow 6s$	$l=2$ $2p\rightarrow 5p$	$l=3$ $2p\rightarrow 4d$	$l=4$ $2p\rightarrow 4f$	$l=5$ $2p\rightarrow 5g$	$l=6$ $2p\rightarrow 6h$
$n=-1$	0	$5.40\times10^{-19}I$ (1.9×10^{18})	0	$5.83\times10^{-17}I$ (1.7×10^{16})	0	0
$n=-2$	$4.94\times10^{-43}I^2$ (1.4×10^{21})	0	$1.94\times10^{-38}I^2$ (7.2×10^{18})	0	$3.95\times10^{-36}I^2$ (1.8×10^{18})	0
$n=-3$	0	$1.54\times10^{-60}I^3$ (8.7×10^{19})	0	$1.66\times10^{-58}I^3$ (1.8×10^{19})	0	$5.96\times10^{-56}I^3$ (2.6×10^{18})
BF	$2p\rightarrow l_f=2$	$2p\rightarrow l_f=1$	$2p\rightarrow l_f=2$	$2p\rightarrow l_f=3$	$2p\rightarrow l_f=4$	$2p\rightarrow l_f=5$
$n=-1$	0	$8.94\times10^{-17}I$ (1.1×10^{16})	0	$1.66\times10^{-10}I$ (6.0×10^{9})	0	0
$n=-2$	$1.20\times10^{-42}I^2$ (9.1×10^{20})	0	$2.88\times10^{-34}I^2$ (5.9×10^{16})	0	$1.29\times10^{-29}I^2$ (2.8×10^{14})	
$n=-3$	0	$2.55\times10^{-58}I^3$ (1.6×10^{19})	0	$4.73\times10^{-52}I^3$ (1.3×10^{17})	0	$2.16\times10^{-48}I^3$ (7.7×10^{15})

On the other hand, we have also calculated the IC process in the case of free final electronic states. We have carried out the calculation for small kinetic energy of the outgoing IC electron, viz., we give the threshold values of the IC coefficients of the process. Our results are tabulated in the second half of Table II.

Comparing the bound-bound and bound-free cases we can see that the bound-free transitions always dominate, and the most leading channel is the $l=4,n=-1$ one.

Our model, however, is valid only if the $\xi k_{\alpha\beta}\ll 1$ and the $Z^i_{eff}\xi/(a_0 n_i)\ll 1$ conditions are fulfilled; here, $Z^i_{eff}=28.06$ is the effective charge number, $n_i=2$ is the principal quantum number of the initial electronic state. We have calculated the critical intensities $I^{(2)}_{crit}$ with $\xi k_{\alpha\beta}=1$ and $I^{(3)}_{crit}$ with $Z^i_{eff}\xi/(a_0 n_i)=1$. These values versus the photon number n are tabulated in Table III. The applied laser intensity (I) has to fullfill the conditions $I\ll I^{(2)}_{crit}$ and $I\ll I^{(2)}_{crit}$. It is important to mention that at rising $|n|$ (and with rising l) the values of $I^{(1)}_{crit}$ also rise according to a flattening out function, while the values $I^{(3)}_{crit}$ decrease rapidly. Because of this limitation, significant α is expected in $n=-1,-2$ cases only (as the ratio $I^{(1)}_{crit}/I^{(3)}_{crit}<1$ at higher number of absorbed photons). This is the reason why we have tabulated our results in Table II for the $|n|<4$ cases only.

TABLE III. Values of photon energies required to ignite the IC process on the $2p$ electron shell of ^{99m}Tc are tabulated for the cases of different photon numbers (n) incorporated in the process. The validity of the calculation is limited by the intensities $I^{(2)}_{crit}$ and $I^{(3)}_{crit}$. $I^{(2)}_{crit}$ is defined by the condition $(\xi k_{\alpha\beta})^2=1$, while $I^{(3)}_{crit}$ is defined by $(Z^i_{eff}\xi)^2/(a_0 n_i)^2=1$. The applied laser intensity (I) has to fulfill the conditions $I\ll I^{(2)}_{crit}$ and $I\ll I^{(2)}_{crit}$.

	$n=-1$	$n=-2$	$n=-3$
$\hbar\omega$ (eV)	496	248	165
$I^{(2)}_{crit}$ (W/cm^2)	1.1×10^{22}	7.1×10^{20}	1.4×10^{20}
$I^{(3)}_{crit}$ (W/cm^2)	7.9×10^{19}	4.9×10^{18}	9.6×10^{17}

Our results show that in the case of bound-free electronic transition, a significant effect can be achieved, applying a laser with appropriately high intensity. The laser-ignited ICC $\alpha=\alpha_{tot}$ can be induced at $I=10^{17}$ W/cm^2 in the case of $n=-1$ ($\hbar\omega_L=496$ eV). α_{tot} is the total laser-free IC coefficient $\alpha_{tot}\simeq 1.6\times10^7$ [25], which incorporates all the enabled IC decay channels of the isomer. So at $I=10^{17}$ W/cm^2 the halflife of the sample is expected to be halved.

The intensity required to reach the α_{tot} value (in case of bound-free transition and with $|n|=1$) is available nowadays using, e.g., free-electron lasers (FEL), which operate in the x-ray region [26]. The ignition of the IC process on the $2p$ shell may be detected with measuring the x-ray signal emitted by Tc when the vacancy in the $2p$ state is filled up. After the deexcitation of the metastable nuclear state by induced IC process, the nucleus decays into its ground state via γ decay, emitting a γ photon of energy $E_\gamma=149.5$ keV (and of half life $\tau=0.19$ ns). This character of the decay of ^{99}Tc makes it an ideal species for measurements, as the induced IC may be detected measuring both the outgoing x-ray and γ signals in delayed coincidence. On the other hand, it is hard to make precise predictions for α and for the frequency of the laser that is required, as the irradiation at such intensities makes the Tc atom highly ionized changing the binding energies of the participating electrons. Fortunately, FEL provides a broad bandwidth of radiation at appropriately high intensity (brightness), making it an ideal candidate for such experiments. However, stripping the Tc atom can cause the hindering of the rate of laser free total IC decay (decreasing α_{tot} as well), and therefore smaller intensity will be needed

TABLE IV. Computed values of β (in cm^2/W units) that give laser-induced IC coefficients (α) as $\alpha=\beta I$, where I denotes the laser intensity given in W/cm^2 units. In the table the transition energies ($\hbar\omega_{\alpha\beta}$) [23], the multipolarity of the nuclear electric transition with the symbols of the electronic shells that take part in the IC process, the binding energies (E_B) of the electron on the shell in question [23], the photon energies ($\hbar\omega_L$) of the applied laser beam ($\hbar\omega_L=\Delta$ with $\Delta=E_B-\hbar\omega_{\alpha\beta}$), and the total laser free IC coefficients (α_{tot}) [28] (a) [29] (b) of the transitions investigated are listed. I_{max} denotes the highest applicable laser intensity (in W/cm^2 units).

Isomer	$\hbar\omega_{\alpha\beta}$ (keV)	$EL/shell$	E_B (keV)	$\hbar\omega_L$ (eV)	β (cm^2/W)	α_{tot}	I_{max}
$^{105}Ag^m$	25.47	$E3/K$	25.514	44	7.6×10^{-14}	$3.6\times10^{4}(a)$	1.3×10^{13}
$^{90}Nb^m$	2.3	$E3/L_2$	2.368	68	1.9×10^{-7}	$1.1\times10^{10}(b)$	8×10^{14}
$^{183}W^m$	1.79	$E2/M_5$	1.807	17	1.2×10^{-7}	$6.5\times10^{7}(b)$	4×10^{12}
$^{183}W^m$	0.544	$E1/N_1$	0.592	48	1.1×10^{-14}	$2240(b)$	8×10^{14}
$^{188}Re^m$	2.63	$E4/M_2$	2.682	52	5.8×10^{-4}	$1.1\times10^{15}(b)$	2.4×10^{14}
$^{205}Pb^m$	2.4	$E2/M_5$	2.484	84	1.2×10^{-10}	$3.2\times10^{7}(b)$	1.7×10^{15}
$^{235}U^m$	0.0735	$E3/O_5$	0.096	22.5	433	$3.7\times10^{20}(b)$	2.3×10^{14}
$^{235}U^m$	0.0735	$E3/O_4$	0.105	31.5	161	$3.7\times10^{20}(b)$	8×10^{14}

to attain a significant contribution in the rate of the laser-initiated IC process in comparison to the rate of total IC of the stripped Tc characterized by α^*_{tot}.

VII. HARD UV LASER-IGNITED INTERNAL CONVERSION PROCESSES

Now we reinvestigate the laser-induced energetically forbidden IC channels of $^{107}Ag^m$, $^{183}W^m$, and $^{235}U^m$ isomers that were studied earlier [27] and that have a small energy defect (Δ) of about 20–48 eV. These materials are completed by $^{90}Nb^m$, $^{188}Re^m$, $^{205}Pb^m$, and by an other line of $^{183}W^m$ having Δ between 17 and 84 eV.

It was found above that the bound-free transitions always dominate, and the most leading channel is the $l=L+1,n=-1$ one, i.e., the one photonic case is the dominant one (the $n<0$ values give the absorbtion branch) and α has the form $\alpha=\beta I$ in this case, where I is the laser intensity. Therefore, we have carried out the calculation with $n=-1$ for small kinetic energy of the outgoing IC electron, viz., we give the threshold values of the laser-induced IC coefficients. The obtained β values of our calculation are given in Table IV. We have, however, two intensity criteria for the applied laser that impose upper limits for the applicable laser intensities (I_{max}) that are given in W/cm^2 units in Table IV, too.

In the case of $^{107}Ag^m$ and $^{235}U^m$ the metastable state of the nucleus decays by an electric multipole decay mode of order $L=3$ (denoted as $E3$) and in case of $^{90}Nb^m$ the decay mode is a mixed $M2+E3$ transition. Its $E3$ component is discussed here. Furthermore, an $E1$ and an $E2$ transition of $^{183}W^m$, an $E2$ transition of $^{205}Pb^m$, and the $E4$ component of a mixed $M3+E4$ transition of $^{188}Re^m$ is investigated. The transition energies ($\hbar\omega_{\alpha\beta}$) [23], the symbols of the electronic shells, that take part in the IC process, the binding energies (E_B) of the electron on the shell in question [23], the photon energies ($\hbar\omega_L$) of the applied laser beam ($\hbar\omega_L=\Delta$ with $\Delta=E_B-\hbar\omega_{\alpha\beta}$), the calculated β values [in $(W/cm^2)^{-1}$ units] and the total laser-free IC coefficients [28,29] of the transitions investigated are listed in Table IV.

The intensities required to reach $\alpha\sim10^{-4}$–$10^{-2}\alpha_{tot}$ values are available nowadays using, e.g., free electron lasers (FEL) [26], synchrotron radiation sources [30], or table-top hard UV devices [31] which operate in the hard UV region.

The ignition of the IC process on the shell in question may be detected by measuring the x-ray fluorescent signal emitted by the atom when the vacancy, that has been created in the shell by laser-induced IC process, is filled up. So the induced IC may be detected measuring the x-ray signal and the inducing laser pulse in coincidence. The fluorescent signal vanishes as $\hbar\omega_L<\Delta$, thus the method is very sensitive for the applied laser photon energy, therefore it is expected that one can determine the nuclear transition energy by tuning the laser photon energy to the threshold ($\hbar\omega_L=\Delta$). These types of experiment are expected to be useful as tools for a more, accurate determimation of the nuclear transition energy.

Finally, we hope that by carrying out the suggested experiments, one may start observations of laser-induced electron-nucleus processes. Moreover, we think that carrying out such experiments would be worthy and it may help in clarifying the complex nature of laser-assisted electron-nucleus processes.

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[1] *Atoms in Intense Laser Fields*, edited by M. Gavrila (Academic Press, Boston, 1992).

[2] N. B. Delone and V. P. Krainov, *Multiphoton Processes in Atoms* (Springer-Verlag, Berlin, 1994).

[3] W. Becker, R.R. Schlicher, and M.O. Scully, Phys. Lett. A **106**, 441 (1984) .

[4] J.I. Gersten and M.H. Mittleman, Phys. Rev. Lett. **48**, 651 (1982).

[5] S. Matinyan, Phys. Rep. **298**, 199 (1998).
[6] P. Kálmán and J. Bergou, Phys. Rev. C **34**, 1024 (1986); P. Kálmán *ibid.* **37**, 2676 (1988); Phys. Rev. A **38**, 5258 (1988); Phys. Rev. C **39**, 2452 (1989); Phys. Rev. A **39**, 3200 (1989).
[7] P. Kálmán Phys. Rev. A **43**, 2603 (1991); P. Kálmán and T. Keszthelyi, *ibid.* **44**, 4761 (1991).
[8] S. S. Schweber, H. A. Bethe, and F. de Hoffmann, *Mesons and Fields*, Vol. I (Row, Peterson and Company, Evanston, IL, 1955).
[9] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Photons and Atoms* (Wiley, New York, 1989).
[10] W.C. Henneberger, Phys. Rev. Lett. **21**, 838 (1968).
[11] J. M. Eisenberg and W. Greiner, *Nuclear Theory, Vol. 2., Excitation Mechanisms of the Nucleus* (North-Holland, Amsterdam, 1970).
[12] P. Kálmán and T. Keszthelyi, Phys. Rev. C **49**, 324 (1994).
[13] T. Bükki and P. Kálmán, Phys. Rev. C **57**, 3480 (1998).
[14] P. Kálmán and T. Bükki, Phys. Rev. C **63**, 02-7601 (2001).
[15] It can be seen easily that Eqs. (24) and (25) are identical to the results obtained from the expansion of the Henneberger transformed potential $V(\vec{x}+e\vec{Z}/m)$ (Ref. [9], p. 347, exercise b).
[16] M. Gavrila, in *Atoms in Intense Laser Fields* (Ref. [1]), p. 435.
[17] I. S. Gradshteyn and I. M., Ryzhik, *Tables of Integrals, Series, and Products* (Academic Press, New York, 1994). For the expansion of associated Legendre polinoms, see formula (8.812).
[18] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (U.S. GPO, Washington, D.C., 1964). For the expansion of the Bessel function, see Eq. (9.1.10), for the expansion of the spherical Bessel function, see Eq. (10.1.2).
[19] See Eq. (9.2.5) in Ref. [18].
[20] See Eq. (10.1.8) in Ref. [18].
[21] M. E. Rose, in *Internal Conversion Processes*, edited by J. H. Hamilton (Academic Press, New York, 1966).
[22] K. Alder *et al.*, Rev. Mod. Phys. **28**, 432 (1956).
[23] R. B. Firestone and V. S. Shirly, *Tables of Isotopes*, 8th ed. (Wiley, New York, 1996).
[24] M.O. Krause, J. Phys. Chem. Ref. Data **8**, 307 (1979); M.O. Krause and J.H. Oliver, *ibid.* **8**, 329 (1979).
[25] B. Zon and F.F. Karpeshin, Phys. Lett. B **383**, 367 (1996).
[26] Max Cornacchia *et al.*, in Performance and Design Concepts of a Free Electron Laser Operating in the X-ray Region, Photonic West '97 Conference, San Jose, California, 1997; Sandra G. Biedron *et al.*, in Proceedings of the 19th International Committee for Future Accelerators, Advanced Beam Dynamics Workshop on Future Light Sources, Arcidosso, Italy, 2000.
[27] P. Kálmán and J. Bergou, Phys. Rev. C **34**, 1024 (1986); P. Kálmán *ibid.* **37**, 2676 (1988); **39**, 2452 (1989); T. Bükki and P. Kálmán, *ibid.* **57**, 3480 (1998).
[28] F. Rösel *et al.*, At. Data Nucl. Data Tables **21**, 91 (1978).
[29] I.M. Band and M.B. Trzhaskovskaya, At. Data Nucl. Data Tables **55**, 43 (1993).
[30] *Handbook on Synchrotron Radiation*, edited by Ernst-Eckhard Koch *et al.* (North-Holland, Amsterdam, 1987), Vol. 2.
[31] Ch. Spielmann *et al.*, Science **278**, 661 (1997); A. Rundquist *et al.*, *ibid.* **280**, 1412 (1998); T. Ditmire *et al.*, Phys. Rev. A **51**, R902 (1995); G. Sommerer, H. Rottke, and W. Sandner, Laser Phys. **9**, 1 (1998); M. Schnürer *et al.*, Phys. Rev. Lett. **83**, 722 (1999); M. Schnürer *et al.*, *ibid.* **85**, 3392 (2000).