Polarization of photons emitted in radiative electron capture by bare high-Z ions

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The linear and circular polarization of photons emitted in radiative electron capture (or, equivalently, radiative recombination in the electron's rest frame) by bare relativistic high-Z projectiles is calculated. In this paper we illustrate for specific examples and a limited systematics the range of phenomena and the physical insight that can be gained by measuring the polarization of photons emitted in radiative electron capture into bare ions. In particular, we investigate the dependence of the polarization in the reaction plane on the charge and the energy of the projectile. In specific examples, we also consider the contribution of spin-flip processes and the polarization off the reaction plane.

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I. INTRODUCTION

In an energetic collision between a highly charged high-Z ion and a low-Z target atom, an electron may be captured by the projectile, while a simultaneously emitted photon carries away the excess energy and momentum. Since loosely bound target electrons can be considered as quasifree, the process is to a very good approximation equivalent to radiative recombination (RR), that is, the inverse of the photoelectric effect in the electron's rest frame. Experimental total and angle-differential cross sections for high-Z one-electron systems (see e.g., [1]) are well understood theoretically [2–4]. More detailed information on the reaction mechanism is provided by the strong alignment observed in radiative electron capture (REC) into an excited state of bare uranium [5], which subsequently decays into the $1s_{1/2}$ ground state by emitting a deexcitation photon. A detailed theory within a completely relativistic description of this process including all multipole orders of the photon wave function was worked out in [6]. In order to express the alignment, it was essential to take the electron direction as the quantization axis instead of the photon direction, although, in this case, the multipole expansion of the photon wave becomes more complicated.

In the present work, we discuss another way to gain more insight into the dynamics of REC or of RR, namely, by measuring the linear or circular polarization of the emitted photon. Experiments of this type are being prepared [7], and our aim is to provide theoretical results useful for designing these experiments. A discussion of general electron-photon polarization correlations can be found in the literature; see, e.g., [8–10]. While alignment measurements are sensitive to substate populations of the intermediate level, measurements of linear polarization are sensitive to interferences between right-hand and left-hand circular polarization of the photon. The basic formalism, however, is very similar. We adopt atomic units $e=m_e=\hbar=1$ unless these constants are displayed explicitly.

II. CROSS SECTION FOR PHOTOIONIZATION

For convenience, we start with the photoelectric effect [9]. Let us assume we have a single electron in the state $|\kappa_n\mu_n\rangle$ with the Dirac quantum number κ_n combining the angular momentum j_n with parity and the angular momentum projection μ_n. If the electron absorbs a photon with wave number $\boldsymbol{k}$ and helicity $\lambda=\pm 1$, it may be emitted into a continuum state with asymptotic momentum $\boldsymbol{p}$ and spin projection (on its own direction of propagation) $m_s=\pm\frac{1}{2}$. Following Ref. [6], we here adopt the direction of emission of the electron as the quantization axis. The angle-dependent cross section for photoionization by circularly polarized photons is given by

$$\sigma_\lambda^{\mathrm{ph}}(\theta)=\mathcal{N}\sum_{\mu_n}\sum_{m_s=\pm 1/2}|\langle \boldsymbol{p}m_s|\boldsymbol{\alpha}\cdot\hat{\boldsymbol{u}}_\lambda e^{i\boldsymbol{k}\cdot\boldsymbol{r}}|\kappa_n\mu_n\rangle|^2,$$

$$\mathcal{N}=\frac{\alpha}{4k}\frac{1}{2j_n+1}, \tag{2.1}$$

where $\boldsymbol{\alpha}$ denotes the set of Dirac matrices, and $\hat{\boldsymbol{u}}_\lambda$ is the unit vector for the polarization. We are now interested in the linear photon polarization in the reaction plane spanned by the vector $\boldsymbol{p}$ defining the z axis and the vector $\boldsymbol{k}$. Let the photon polarization be referred to an x-y plane perpendicular to $\hat{\boldsymbol{k}}$. Then, in general, the unit vector of linear polarization in a direction forming an angle χ with the x axis in the x-y plane,

$$\hat{\boldsymbol{u}}(\chi)=\frac{1}{\sqrt{2}}(e^{-i\chi}\hat{\boldsymbol{u}}_+ + e^{i\chi}\hat{\boldsymbol{u}}_-), \tag{2.2}$$

can be expressed by the circular polarizations $\hat{\boldsymbol{u}}_\pm$ for $\lambda=\pm 1$. Specifically, for polarization in the x and y directions, we have $\chi=0$ and $\chi=\pi/2$, respectively. If we choose the

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direction $\hat{\boldsymbol{p}}$ as the z axis, we have to perform an Euler rotation $\hat{\boldsymbol{k}}\to\hat{\boldsymbol{z}}$ between the coordinate systems with the angle θ between $\hat{\boldsymbol{k}}$ and $\hat{\boldsymbol{p}}$ and the azimuth chosen to be $\varphi=0$, so that the x axis lies in the reaction plane. The direction of a general vector $\hat{\boldsymbol{u}}$ of linear polarization in any coordinate system is hence specified by the intersection of the plane perpendicular to $\hat{\boldsymbol{k}}$ and the plane forming an angle χ with the reaction plane.

The plane photon wave rotated to the new coordinate frame

$$\hat{\boldsymbol{u}}_\lambda e^{i\boldsymbol{k}\cdot\boldsymbol{r}}=\sqrt{2\pi}\sum_{L=1}^{\infty}\sum_{M=-L}^{M=L} i^L\sqrt{2L+1}\,\boldsymbol{\mathcal{A}}_{LM}^{(\lambda)}D_{M\lambda}^{L}(\hat{\boldsymbol{k}}\to\hat{\boldsymbol{z}}) \tag{2.3}$$

is decomposed [11] into electric and magnetic multipole fields $\boldsymbol{\mathcal{A}}_{LM}^{(\lambda)}$ [6] with the phase factor of the electric part depending on λ [see Eq. (2.5)]. The Wigner D matrix rotates the fields from the $\hat{\boldsymbol{k}}$ into the $\hat{\boldsymbol{z}}$ direction. Since we are considering pure photon states, there is no need to introduce Stokes parameters explicitly.

By expanding the Coulomb-Dirac continuum wave function for the electron into partial waves $|\kappa m_s\rangle$, we can write the transition matrix elements

$$\begin{aligned}\langle \boldsymbol{p}m_s|\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{A}}_{LM}^{(\lambda)}|\kappa_n\mu_n\rangle &= \sum_{\kappa} i^{-l}e^{i\Delta_\kappa}\sqrt{4\pi(2l+1)}\\ &\quad\times\left(\begin{array}{cc|c} l & \frac{1}{2} & j\\ 0 & m_s & m_s\end{array}\right)\\ &\quad\times\langle\kappa m_s|\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{A}}_{LM}^{(\lambda)}|\kappa_n\mu_n\rangle,\end{aligned} \tag{2.4}$$

where Δ_κ is the Coulomb phase shift and $(\cdot\cdot|\cdot)$ a Clebsch-Gordan coefficient. In the general multipole matrix element

$$\begin{aligned}\langle\kappa m_s|\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{A}}_{LM}^{(\lambda)}|\kappa_n\mu_n\rangle &= \mathcal{T}_{LL}(\kappa m_s,\kappa_n\mu_n)\\ &\quad+i\lambda\left[\sqrt{\frac{L+1}{2L+1}}\mathcal{T}_{L,L-1}(\kappa m_s,\kappa_n\mu_n)\right.\\ &\quad\left.-\sqrt{\frac{L}{2L+1}}\mathcal{T}_{L,L+1}(\kappa m_s,\kappa_n\mu_n)\right],\end{aligned} \tag{2.5}$$

only the first (magnetic) *or* the second (electric) part contributes between specified electronic states owing to parity selection rules. The general Dirac matrix elements $\mathcal{T}_{L\Lambda}(\kappa m_s,\kappa_n\mu_n)$ with $\Lambda=L,L\pm1$ can be expressed by geometrical coefficients and radial integrals. They are explicitly given in Eq. (15) of Ref. [6]. By inserting the expansion (2.3) into Eq. (2.1), we obtain the cross section for circularly polarized radiation as [6]

$$\begin{aligned}\sigma_{\pm}(\theta) &= 2\pi\mathcal{N}\sum_{\mu_n,m_s}\sum_{L\bar{L}}(-1)^{M+1}i^{L-\bar{L}}\\ &\quad\times\sqrt{(2L+1)(2\bar{L}+1)}\sum_{\nu}\left(\begin{array}{cc|c} L & \bar{L} & \nu\\ M & -M & 0\end{array}\right)\\ &\quad\times\left(\begin{array}{cc|c} L & \bar{L} & \nu\\ \lambda & -\lambda & 0\end{array}\right)\\ &\quad\times\langle \boldsymbol{p}m_s|\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{A}}_{LM}^{(\lambda)}|\kappa_n\mu_n\rangle\langle \boldsymbol{p}m_s|\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{A}}_{\bar{L}M}^{(\lambda)}|\kappa_n\mu_n\rangle^{*}\\ &\quad\times P_\nu(\cos\theta).\end{aligned} \tag{2.6}$$

In all cases, $M=m_s-\mu_n$. Because the cross section depends only on the relative signs of λ, μ_n, and m_s and the latter two are summed over, it is independent of the circular polarization. Hence, the cross section σ_0 averaged over photon polarizations is

$$\sigma_0=\sigma_+=\sigma_-, \tag{2.7}$$

assuming that the electron polarization is not detected. If, however, the spin projection $m_s=\pm\frac{1}{2}$ of the electron is specified, i.e., the summation in Eq. (2.6) is discarded, cross sections depend on the relative sign of λ and m_s, so that σ_{λ,m_s} differs from $\sigma_{-\lambda,m_s}$.

When calculating cross sections σ_χ for linear photon polarization in the direction of $\hat{\boldsymbol{u}}(\chi)$, we have to use linear superpositions according to Eq. (2.2), which lead to interference terms $\sigma_\chi^{\rm int}$ between circular polarizations $\lambda=1$ and $\lambda=-1$. As a result, we may write

$$\sigma_\chi(\theta)=\sigma_0(\theta)+\sigma_\chi^{\rm int}(\theta), \tag{2.8}$$

where

$$\begin{aligned}\sigma_\chi^{\rm int}(\theta) &= \pi\mathcal{N}\sum_{\mu_n,m_s}\sum_{\lambda=\pm1}\sum_{L\bar{L}}(-1)^{M+1}i^{L-\bar{L}}e^{-i2\lambda\chi}\\ &\quad\times\sqrt{(2L+1)(2\bar{L}+1)}\sum_{\nu}\left(\begin{array}{cc|c} L & \bar{L} & \nu\\ M & -M & 0\end{array}\right)\\ &\quad\times\left(\begin{array}{cc|c} L & \bar{L} & \nu\\ \lambda & \lambda & 2\lambda\end{array}\right)\langle \boldsymbol{p}m_s|\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{A}}_{LM}^{(\lambda)}|\kappa_n\mu_n\rangle\\ &\quad\times\langle \boldsymbol{p}m_s|\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{A}}_{\bar{L}M}^{(-\lambda)}|\kappa_n\mu_n\rangle^{*}\sqrt{\frac{(\nu-2)!}{(\nu+2)!}}P_\nu^2(\cos\theta)\end{aligned} \tag{2.9}$$

and P_ν^2 is an associated Legendre polynomial [12]. Specifically, for linear polarization in the reaction plane, we have $\sigma_\parallel=\sigma_{\chi=0}$ and for the polarization perpendicular to it, we have $\sigma_\perp=\sigma_{\chi=\pi/2}$.

III. DEGREE OF POLARIZATION

For an experiment measuring radiative electron capture, equivalent to RR in the electron's rest frame, one has to transform the cross section (2.8) from the projectile system into the laboratory frame by the usual Lorentz transformation [2,4,6,13]. With a suitable detector, sensitive to linear polarization, one may measure, e.g., the angle-differential cross section $\sigma_{\parallel}(\theta)$ of the radiation polarized in the reaction plane or, alternatively, $\sigma_{\perp}(\theta)$.

An alternative measurement may consist in determining the degree of linear polarization in the direction $\hat{\boldsymbol{u}}(\chi)$ at one or several angles. This quantity is defined as

$$P_{\chi}^{\mathrm{lin}}(\theta)=\frac{\sigma_{\chi}-\sigma_{\chi+\pi/2}}{\sigma_{\chi}+\sigma_{\chi+\pi/2}}=\frac{\sigma_{\chi}^{\mathrm{int}}}{\sigma_0}. \tag{3.1}$$

The degree of circular polarization for a given spin projection m_s of the electron is defined as

$$P^{\mathrm{circ},m_s}(\theta)=\frac{\sigma_{+,m_s}-\sigma_{-,m_s}}{\sigma_{+,m_s}+\sigma_{-,m_s}}. \tag{3.2}$$

IV. RESULTS FOR RADIATIVE RECOMBINATION

It is the purpose of this section to illustrate the full range of phenomena that can be obtained by the measurement of photon polarization produced by radiative electron capture into the K shell of bare high-Z ions. For the high projectile energies considered, this is equivalent to radiative recombination (K-RR) of an electron at rest with the moving projectile. The calculations have been performed to high precision, the critical part being the Coulomb-Dirac continuum wave functions, which in all cases are supplied by two independent calculations (see Ref. [13]) allowing one to assess the accuracy. Three-digit accuracy requires one to take into account multipole orders up to $L=17$ and Legendre polynomials up to $\nu=34$ for 300 MeV/u and about twice that much for 1500 MeV/u.

In Fig. 1 we show the degree of linear photon polarization in the reaction plane as a function of the emission angle θ for various projectile charge numbers Z. One obtains a very high degree of polarization over most of the angular range. The flatness of the polarization correlation (except for the spikes in the forward and backward directions) and the weak Z dependence will be more pronounced as the collision energy decreases. Eventually, in the nonrelativistic limit, the linear polarization in the reaction plane $P_{\parallel}^{\mathrm{lin}}=1$, independent of Z [8]. In a relativistic description, $Z=92$, the softening of the forward shoulder can be partly assigned to the contribution of spin-flip transitions, which have a negative sign.

On the other hand, for increasing projectile energy, with $Z=92$ kept constant (see Fig. 2), one obtains a "crossover" at about 500 MeV/u, beyond which the linear polarization becomes increasingly negative at forward angles. This corresponds to the "crossover" observed in the photoeffect [8], taking into account the replacement $\theta\rightarrow\pi-\theta$ and the Lorentz transformation to the atomic rest frame, which compresses the angular distribution at forward angles. In fact, we very closely reproduce the results of Ref. [8], where applicable.

In analogy to Fig. 1, the degree of circular polarization is displayed in Fig. 3, assuming that the incoming electron has the spin projection $m_s=\frac{1}{2}$. At forward angles, a transition can occur only if the photon carries away the angular momentum $\lambda=1$ (with respect to the electron direction) leaving the electron with the spin projection $-\frac{1}{2}$. At backward angles, the transition is achieved by $\lambda=-1$. Corresponding results have been obtained for the photoelectric effect [8]. However, for RR we replace $\theta\rightarrow\pi-\theta$, so that the signs for the circular polarization are reversed; see Eq. (3.2) and Fig. 3. We have illustrated the effect of spin-flip transitions in more detail by

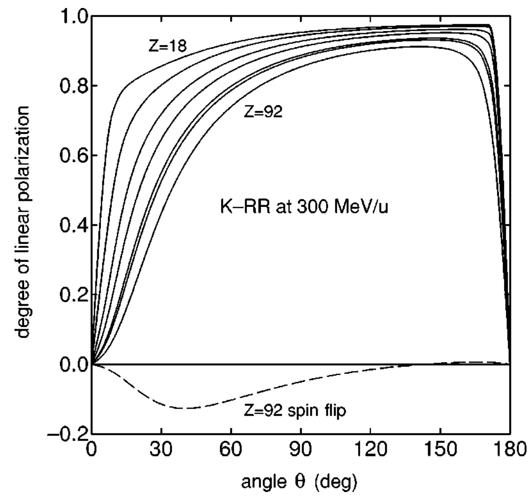

FIG. 1. Charge dependence of the linear photon polarization in the reaction plane as a function of the emission angle θ for K-RR at a projectile energy of 300 MeV/u, corresponding to a relative electron kinetic energy of 164.6 keV. The results are given successively for the charges $Z=18$, 36, 54, 66, 79, 82, and 92. As an illustration, the spin-flip contribution to the polarization for the case of $Z=92$ is also shown.

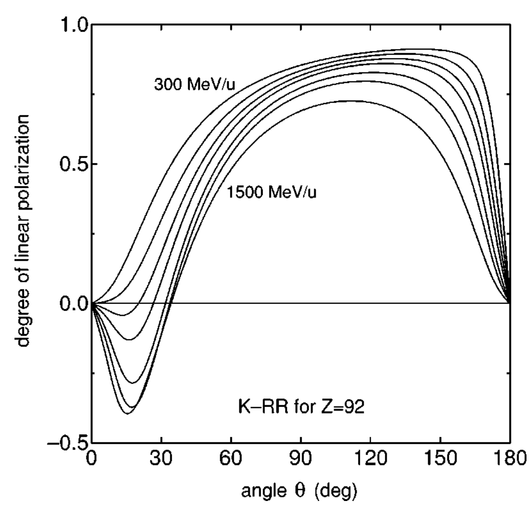

FIG. 2. Projectile energy dependence of the linear photon polarization in the reaction plane as a function of the emission angle θ for K-RR with $Z=92$. The results are given successively for the projectile energies of 300, 400, 500, 600, 800, 1000, and 1500 MeV/u, corresponding, respectively, to relative electron energies of 165, 219, 247, 329, 439, 549, and 823 keV. To obtain the photon energy, one has to add 132 keV binding energy.

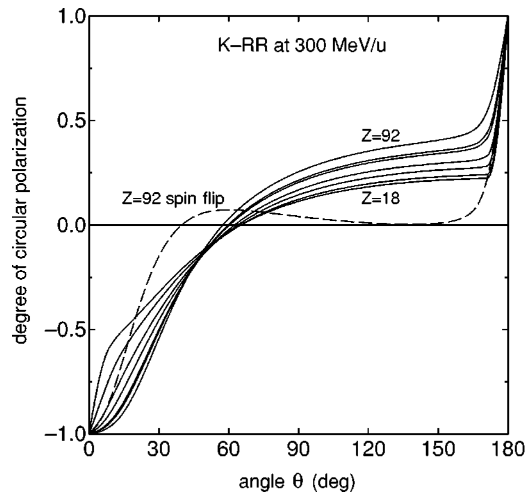

FIG. 3. Charge dependence of the circular photon polarization (assuming $m_s = \frac{1}{2}$) as a function of the emission angle θ of the photon for K-RR at a projectile energy of 300 MeV/u (relative electron energy 164.6 keV). The results are given successively for the charges $Z=18$, 36, 54, 66, 79, 82, and 92. As an illustration, the spin-flip contribution to the polarization for the case of $Z=92$ is also shown.

separately showing the spin-flip contribution for $Z=92$. Indeed, in the whole range of forward angles and at extreme backward angles, the transition is completely dominated by the spin-flip contributions. Obviously, the magnetic spin-flip effect is more pronounced here than already observed in the angle-differential cross sections [1].

While in Figs. 1 and 3 we have confined ourselves to the single projectile energy of 300 MeV/u, we present in Fig. 4 the energy dependence of linear and circular polarization between 1 MeV/u and 2 GeV/u for the fixed photon angle $\theta = 90°$ and the projectile charge $Z=92$. It is observed that

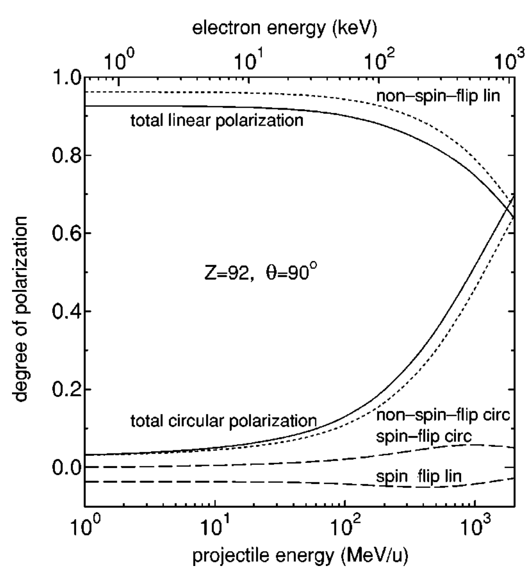

FIG. 4. Projectile energy dependence (bottom scale) and electron energy dependence (top scale) of the linear and circular polarization at the fixed photon emission angle $\theta = 90°$ for $Z=92$.

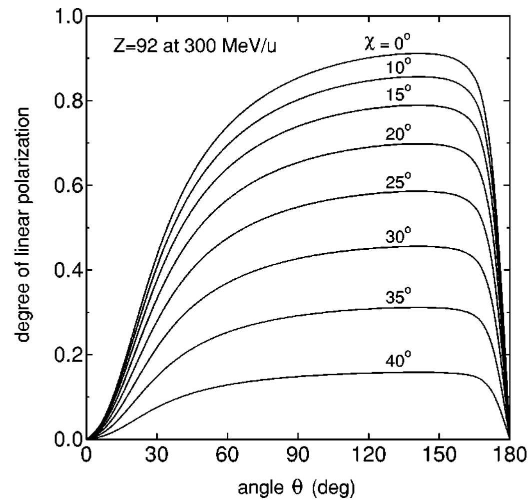

FIG. 5. Linear photon polarization in a plane forming an angle χ with the reaction plane as a function of the emission angle θ. The energy of the projectile with $Z=92$ has been taken as 300 MeV/u (electron energy 164.6 keV, photon energy 296.7 keV).

linear polarization decreases at high energies while the circular polarization increases. In both cases, spin-flip contributions play a minor role at this angle.

So far, we have considered polarization in the reaction plane. Figure 5 shows the linear polarization in a plane tilted by the angle χ with respect to the reaction plane, again as a function of the photon angle θ and for $Z=92$ and energy of 300 MeV/u. The χ dependence is uniquely determined by the angular range from 0° to 45° because it follows from Eq. (2.9) that

$$P^{\mathrm{lin}}_{\chi} = P^{\mathrm{lin}}_{\pi-\chi} = -P^{\mathrm{lin}}_{\pi/2-\chi} = P^{\mathrm{lin}}_{\chi=0} \cos 2\chi. \tag{4.1}$$

In particular, the linear polarization vanishes for $\chi = 45°$. Clearly, the linear polarization is maximal in the reaction plane.

The differential cross section for a particular polarization is obtained by multiplying the overall differential cross section, tabulated, e.g., in [13], by the degree of polarization presented here.

V. SUMMARY

Using an exact relativistic description, we have calculated the linear and circular polarization of photons emitted in radiative recombination with bare ions. This is essentially equivalent to radiative electron capture by high-Z projectiles from light target atoms, for which experiments are now under way [7], and which, in turn, have determined the choice of some of the parameters considered in this paper. We have investigated the charge, energy, and angular dependence of linear polarization as well as the charge dependence of circular polarization, both in the reaction plane, the angular dependence of linear polarization off the reaction plane, and, finally, the energy dependences of linear and circular polarizations at a fixed angle. The contribution of separately calculated spin-flip processes reveals details of the reaction mechanism. In particular, the circular polarization in the range of forward and backward angles is completely dominated by spin-flip processes.

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[1] Th. Stöhlker, X. Ma, T. Ludziejewski, H.F. Beyer, F. Bosch, O. Brinzanescu, R.W. Dunford, J. Eichler, S. Hagmann, A. Ichihara, C. Kozhuarov, A. Krämer, D. Liesen, P.H. Mokler, Z. Stachura, P. Swiat, and A. Warczak, Phys. Rev. Lett. **86**, 983 (2001).
[2] A. Ichihara, T. Shirai, and J. Eichler, Phys. Rev. A **49**, 1875 (1994).
[3] J. Eichler, A. Ichihara, and T. Shirai, Phys. Rev. A **51**, 3027 (1995).
[4] For a summary of relativistic atomic collisions, see, e.g., J. Eichler and W. E. Meyerhof, *Relativistic Atomic Collisions* (Academic Press, San Diego, 1995).
[5] Th. Stöhlker, F. Bosch, A. Gallus, C. Kozhuharov, G. Menzel, P.H. Mokler, H.T. Prinz, J. Eichler, A. Ichihara, T. Shirai, R.W. Dunford, T. Ludziejewski, P. Rymuza, Z. Stachura, P. Swiat, and A. Warczak, Phys. Rev. Lett. **79**, 3270 (1997).
[6] J. Eichler, A. Ichihara, and T. Shirai, Phys. Rev. A **58**, 2128 (1998).
[7] Th. Stöhlker (private communication).
[8] R.H. Pratt, R.D. Levee, R.L. Pexton, and W. Aron, Phys. Rev. **134**, A916 (1964).
[9] R.H. Pratt, A. Ron, and H.K. Tseng, Rev. Mod. Phys. **45**, 273 (1973).
[10] J.H. Scofield, Phys. Rev. A **40**, 3054 (1989).
[11] M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957).
[12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, New York, 1980).
[13] A. Ichihara and J. Eichler, At. Data Nucl. Data Tables **74**, 1 (2000); **79**, 1 (2001).