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Optical-wave group-velocity reduction without electromagnetically induced transparency

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A Raman scheme for optical-pulse group-velocity reduction in a pure lifetime broadened system is studied. We show that this nonelectromagnetically induced transparency (NEIT) scheme has many advantages over the conventional method that critically relies on the transparency window created by an EIT process. Significant reduction of the group velocity, probe field loss, and pulse distortion are reported. In addition, rich dynamics of the propagation process are studied.

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Recent studies [1] of optical-wave propagation in a resonant medium with dramatically modified dispersion properties have opened new possibilities for wave propagation. One of the key results of these works is the significant reduction of propagation group velocity. Such a phenomenon might have profound effects on nonlinear optical processes, such as enhancement of various nonlinear interactions [2], coherent mapping of the properties of an optical wave to atomic spin waves [3], and perhaps even the possibility of novel optical devices for information technology. At the heart of all studies on such noticeable group-velocity modification is the electromagnetically induced transparency effect [4]. This key method, in the context of a three-level model, relies on a coherent mixture produced by a driving field that couples two nearly empty states. As a result, a destructive interference between optical shifted resonances cancels the linear contributions of the dispersion function of a probe pulse that is tuned to line center. Such noticeable modification of the optical properties of the medium gives rise to a steep change in the net dispersion relation, leading to a significant reduction of the group velocity, and at the same time renders an otherwise opaque medium highly transparent [5].

In reality, however, the situation is more complicated because no ideal three-level system exists. The effect of nearby hyperfine states often contributes significantly to wave propagation, thereby greatly altering the response of the system to the optical wave. Indeed, it has been shown [6] that in the case of the sodium D_1 and D_2 lines, probe field loss due to nearby hyperfine states generally dwarf that of the pure three-level system, and cause nearly an 80% reduction in the probe field intensity. In addition, it also causes as much as 30% probe-pulse broadening. This is mainly because of the small hyperfine splitting and the relatively large upper-state lifetime. The situation is better in the case of rubidium because of the large hyperfine splitting and smaller upper-state lifetime. However, the stringent requirement of having both probe and coupling laser tuned exactly on resonance, i.e., not tunable, remains. This requirement is also one of the major obstacles to the electromagnetically induced transparency EIT scheme when applied to solids where the broad upper energy band at room temperature renders ‘‘tuning to the line center’’ and ‘‘being driven transparent’’ rather meaningless [7].

In this paper, we report a four-level non-EIT (NEIT) scheme that is capable of producing a similar group-velocity reduction as that of a four-level EIT scheme. This Raman scheme, however, has many advantages over the conventional one-photon, on-resonance EIT scheme. The introduction of the one-photon detuning opens many possibilities, such as great reduction of probe field loss and enhanced gain. In particular, we will show that with properly chosen parameters this scheme can further reduce the inherent probe field attenuation that exists even in an ideal three-level EIT case. In addition, we will show rich dynamics including features such as pulse narrowing. This is to be compared to the conventional EIT process that always results in pulse broadening.

We start with a four-level system (Fig. 1) where a probe and a coupling laser are detuned from one-photon resonance, yet satisfy a two-photon resonance condition. The laser intensities should be such that the two-photon Rabi frequency times the probe pulse length is much less than unity so that depletion of the ground state can be neglected. To avoid significant group-velocity mismatch, and for mathematical simplicity, we will also assume a cw driving field. Assuming, for the ground state, $|a_0|\approx 1$, and applying the time dependent Schrödinger equation, we obtain three atomic equations of motion:

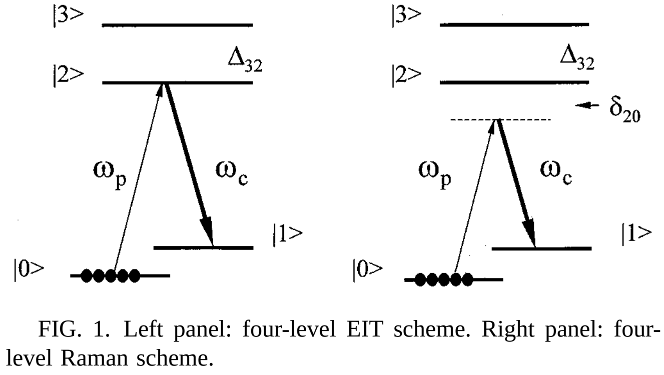

FIG. 1. Left panel: four-level EIT scheme. Right panel: four-level Raman scheme.

$$\dot{A}_1=\left(-\frac{\gamma_1}{2}\right)A_1+(i\Omega_{12}e^{-ik_ez}A_2)+(i\Omega_{13}e^{-ik_ez}A_3),$$

$$\dot{A}_2=i\left(\delta_{20}+i\frac{\gamma_2}{2}\right)A_2+(i\Omega_{21}e^{ik_ez}A_1)+(i\Omega_{20}e^{ik_rz}a_0),\quad(1)$$

$$\dot{A}_3=i\left(\delta_{20}+\Delta_{32}+i\frac{\gamma_3}{2}\right)A_3+(i\Omega_{30}e^{ik_rz}a_0)+(i\Omega_{31}e^{ik_ez}A_1).$$

As usual, $\Omega_{ij}=D_{ij}E_{p,c}/(2\hbar)$ and D_{ij} are one-half of the Rabi frequency and the relevant dipole moment for the transition $|j\rangle\rightarrow|i\rangle$, respectively. γ_1, γ_2, and γ_3 are the natural lifetimes of the states $|1\rangle$, $|2\rangle$, and $|3\rangle$, respectively. In addition, we have used $\delta_{20}=\omega_c-\omega_{21}=\omega_p-\omega_{20}$, and Δ_{32} is the spacing between states $|2\rangle$ and $|3\rangle$.

In order to correctly predict the propagation of the probe pulse, Eq. (1) must be solved together with Maxwell's equation for the probe field. For an unfocused beam within the slowly varying amplitude approximation, the wave equation for the field $E_{p0}^{(+)}$ reads

$$\frac{\partial E_{p0}^{(+)}}{\partial z}+\frac{1}{c}\frac{\partial E_{p0}^{(+)}}{\partial t}=i\frac{4\pi\omega_p}{c}P_{\omega_p0}^{(+)},\qquad(2)$$

where $P_{\omega_p0}^{(+)}=N(D_{02}A_2a_0^*+D_{03}A_3a_0^*e^{-i\Delta_{32}t})\exp(-ik_pz+i\delta_{20}t)$, and N is the concentration. With the assumption of a cw coupling field, Eqs. (1) and (2) can be solved simultaneously with the probe field expressed as a function of time t and propagation distance z,

$$E_{p0}^{(+)}(z,t)=\frac{1}{\tau\sqrt{2\pi}}\int_{-\infty}^{+\infty}d\eta\,\varepsilon_{p0}^{(+)}(0,\eta)\exp\left[-i\eta\left(\frac{t}{\tau}-\frac{z}{c\tau}\right)+i\kappa_{02}c\tau^2\left(\frac{z}{c\tau}\right)D(\eta)\right].\qquad(3)$$

Here $\varepsilon_{p0}^{(+)}(0,\eta)=(\tau E_{p0}^{(+)}(0,0)/\sqrt{2})e^{-\eta^2/4}$ is the Fourier transform of a Gaussian probe field at the entrance of the medium, $\kappa_{02}=2\pi N\omega_p|D_{02}|^2/(\hbar c)$, and $\eta=\omega\tau$ where τ is the probe pulse's e^{-1} length. We have also introduced a dimensionless dispersion function

$$D(\eta)=\frac{(\eta+i\gamma_1\tau/2)\{1+d_{00}(\Delta_2\tau/\Delta_3\tau)\}-d_{01}|\Omega_{21}\tau|^2/(\Delta_3\tau)}{|\Omega_{21}\tau|^2\{1+[d_{11}/(\Delta_3\tau)-i\gamma_1\tau/2](\Delta_2\tau)\}\left\{1-\dfrac{(\Delta_2\tau)\eta}{|\Omega_{21}\tau|^2\{1+[d_{11}/(\Delta_3\tau)-i\gamma_1\tau/2](\Delta_2\tau)\}}\right\}},\qquad(4)$$

where

$$\Delta_2=\eta+\delta_{20}\tau+i\frac{\gamma_2\tau}{2},\quad\Delta_3=\eta+\delta_{20}\tau+\Delta_{32}\tau+i\frac{\gamma_3\tau}{2},$$

$$d_{00}=\left|\frac{D_{30}}{D_{20}}\right|^2,\quad d_{11}=\left|\frac{D_{31}}{D_{21}}\right|^2,$$

and

$$d_{01}=\left|\frac{D_{30}}{D_{20}}-\frac{D_{31}}{D_{21}}\right|^2.$$

We now consider the case where $|\Omega_{21}|^2/\delta_{20}\gg\max(\gamma_1,\tau^{-1})$. In this limit $D(\eta)$ can be expanded into power series and further truncated to yield

$$D(\eta)=D_0+D_1\eta+D_2\eta^2+O(\eta^3),$$

where we have neglected higher-order terms. This is valid for a Gaussian pulse of width of τ because the major contribution to the inverse transform only occurs for $|\eta|=|\omega\tau|\leqslant5$. The main advantage of this truncated dispersion function is that its inverse Fourier transform can be analytically obtained, and phenomena that are of physical significance can be examined. We thus have, for the relative pulse intensity I_{RP},

$$I_{\mathrm{RP}}\approx\frac{\exp\{-2\kappa_{02}\!>\!J\,\mathrm{Im}[D_0]+2(\kappa_{02}zz\,\mathrm{Im}[D_1])\}/b_1}{\sqrt{b_1^2+b_2^2}}\times\exp\left[-\frac{2(t-z/V_g)^2}{\tau^2(1+b_2^2/b_1^2)b_1}\right],\qquad(5)$$

where

$$b_1=1+2\kappa_{02}\tau z\,\mathrm{Im}[D_2],\quad b_2=2\kappa_{02}\tau z\,\mathrm{Re}[D_2],\qquad(6a)$$

$$\frac{1}{V_g}=\frac{1}{c}+\kappa_{02}\tau^2\left\{\mathrm{Re}[D_1]-\mathrm{Im}[D_1]\frac{b_2}{b_1}\right\},\qquad(6b)$$

$$W_{\mathrm{RP}}=\sqrt{b_1(1+b_2^2/b_1^2},$$

where W_{RP} is the relative pulse width.

From Eqs. (5) and (6) we see that $\mathrm{Im}[D_0]<0(>0)$ represents the gain (loss) to the probe field. While $\mathrm{Im}[D_1]$ always contributes to the gain, the modified group velocity is mainly determined by $\mathrm{Re}[D_1]$. In addition, $\mathrm{Im}[D_2]<0(>0)$ gives rise to pulse narrowing (broadening), whereas $\mathrm{Re}[D_2]$ always contributes to pulse broadening. In the case of EIT (see below), $\mathrm{Im}[D_0]>0$ and $\mathrm{Im}[D_2]>0$, therefore, the probe field will always experience loss and broadening. To further examine the advantages and rich dynamics of the proposed scheme, let us assume $\delta_{20}<\Delta_{32}$ as in the case of rubidium [8]. This condition allows further expansion of $D(\eta)$ in the power of δ_{20}/Δ_{32}. After some algebra, we obtain

$$\mathrm{Im}[D_0]_{\mathrm{Raman}} \approx \mathrm{Im}[D_0]_{\mathrm{EIT}} + \left(\frac{\delta_{20}}{\Delta_{32}}\right)\frac{(\gamma_1\tau)}{2|\Omega_{21}\tau|^2}(d_{00}-d_{01}-d_{11}), \tag{7a}$$

$$\mathrm{Re}[D_1]_{\mathrm{Raman}} \approx \mathrm{Re}[D_1]_{\mathrm{EIT}} + \left(\frac{\delta_{20}}{\Delta_{32}}\right)\frac{(d_{00}-d_{01}-d_{11})}{|\Omega_{21}\tau|^2}, \tag{7b}$$

$$\mathrm{Im}[D_1]_{\mathrm{Raman}} \approx \mathrm{Im}[D_1]_{\mathrm{EIT}} + \frac{(\gamma_1\tau)(\delta_{20}\tau)}{|\Omega_{21}\tau|^4} \times \left\{1+\left(\frac{\delta_{20}}{\Delta_{32}}\right)(d_{00}-d_{01}-2d_{11})\right\}, \tag{7c}$$

$$\mathrm{Re}[D_2]_{\mathrm{Raman}} \approx \mathrm{Re}[D_2]_{\mathrm{EIT}} + \frac{(\delta_{20}\tau)}{|\Omega_{21}\tau|^4} \times \left\{1+\left(\frac{\delta_{20}}{\Delta_{32}}\right)(d_{00}-d_{01}-2d_{11})\right\}, \tag{7d}$$

$$\mathrm{Im}[D_2]_{\mathrm{Raman}} \approx \mathrm{Im}[D_2]_{\mathrm{EIT}} + \left(\frac{\delta_{20}}{\Delta_{32}}\right)\frac{(\gamma_2\tau)}{|\Omega_{21}\tau|^4}(d_{00}-d_{01}-2d_{11}), \tag{7e}$$

where

$$\mathrm{Im}[D_0]_{\mathrm{EIT}} = \frac{(\gamma_1\tau)}{2|\Omega_{21}\tau|^2} \approx \mathrm{Im}[D_0]_{\mathrm{three-Level/EIT}} > 0, \tag{8a}$$

$$\mathrm{Re}[D_1]_{\mathrm{EIT}} \approx \frac{1}{|\Omega_{21}\tau|^2},$$

$$\mathrm{Im}[D_1]_{\mathrm{EIT}} = \frac{(\gamma_2\tau)}{2(\Delta_{32}\tau)|\Omega_{21}\tau|^2}(d_{00}-d_{01}-d_{11}), \tag{8b}$$

$$\mathrm{Re}[D_2]_{\mathrm{EIT}} \approx \frac{(d_{00}-d_{01}-d_{11})}{(\Delta_{32}\tau)|\Omega_{21}\tau|^2} - \frac{(\gamma_2\tau)^2(d_{00}-d_{01}-2d_{11})}{4(\Delta_{32}\tau)|\Omega_{21}\tau|^4},$$

$$\mathrm{Im}[D_2]_{\mathrm{EIT}} \approx \frac{(\gamma_2\tau)}{2|\Omega_{21}\tau|^4} > 0. \tag{8c}$$

In deriving these results we have consistently neglected terms that are $O((\gamma_1\tau)^2)$, $O(1/(\Delta_{32}\tau)^2)$, or $O(1/|\Omega_{21}\tau|^6)$. To make the above expansion valid, however, we must have $|\Omega_{21}|^2/\delta_{20} \geqslant 5/\tau$ and $|\delta_{20}| \geqslant 5|\Omega_{21}|$ to prevent the probe pulse from overlapping with the resonance. Equations (7) and (8) are expressed in such a way that when the appropriate limit for the detuning is taken, the correct EIT results are easily recovered. The detuning dependent NEIT terms give rise to characteristic features and to the rich dynamics discussed previously. To be more specific, let us consider the D_1 line of cold rubidium atoms with both lasers circularly polarized. It can be shown [9] in this case that $d_{00}-d_{01}-d_{11}<-2$. First, we investigate the attenuation of the probe field. Since Δ_{32} and δ_{20} always have the same sign and $d_{00}-d_{01}-d_{11}<0$, Eqs. (7a) and (8a) predict that $\mathrm{Im}(D_0)_{\mathrm{Raman}} < \mathrm{Im}(D_0)_{\mathrm{EIT}}$, therefore reducing the probe field loss. In fact, the loss of our four-level Raman scheme will be even less than that of an ideal three-level EIT scheme.

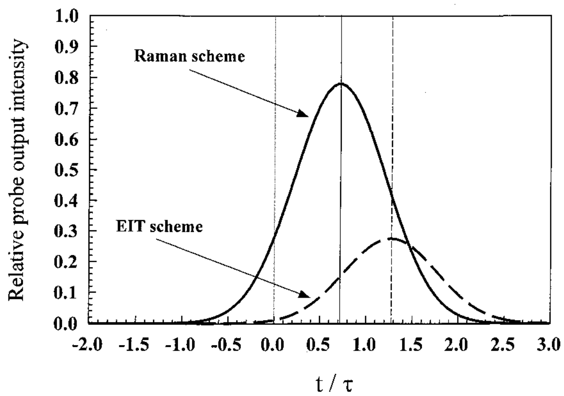

FIG. 2. Plot of probe-field relative intensities for the four-level Raman scheme and four-level EIT scheme in the case of Rb (Na has an even greater difference). Parameters: $\kappa_{02}=5\times10^{11}$ cm^{-1} s^{-1}, $\gamma_1/2\pi=30$ kHz, $\tau=10$ μs, $z=1$ mm, $\gamma_2/2\pi=\gamma_3/2\pi=6$ MHz, $\Omega_{21}/2\pi=12$ MHz, $\Delta_{32}/2\pi=820$ MHz, and $\delta_{20}/2\pi=150$ MHz. The group velocity of the Raman scheme is about 1.8 times faster than that of the EIT scheme, whereas the Raman pulse width is slightly narrower. Notice that the EIT probe intensity suffers almost three times more loss than in the case of the Raman method.

Next, we examine the probe propagation velocity. From Eqs. (5)–(8) we obtain

$$\left(\frac{1}{V_g}\right)_{\mathrm{Raman}} = \left(\frac{1}{V_g}\right)_{\mathrm{EIT}} + \kappa_{02}\tau^2\left(\frac{\delta_{20}}{\Delta_{32}}\right)\frac{(d_{00}-d_{01}-d_{11})}{|\Omega_{21}\tau|^2}. \tag{9}$$

Therefore, under the same conditions, the probe will travel slightly faster in the Raman scheme than in the EIT scheme. Although there is a slight cancellation to the gain due to $\mathrm{Im}(D_1)_{\mathrm{Raman}} < \mathrm{Im}(D_1)_{\mathrm{EIT}}$ [this can be seen from Eqs. (7c) and (8b)], the Raman gain from $\mathrm{Im}(D_0)$ dominates since $|\Delta_{32}| \ll |\Omega_{21}\tau|^2$.

In the case of pulse broadening, we inspect D_2 given by Eqs. (7d), (7e) and (8c). We first note that since $d_{00}-d_{01}-d_{11}<0$, $\mathrm{Im}(D_2)_{\mathrm{Raman}} < \mathrm{Im}(D_2)_{\mathrm{EIT}}$, regardless of the sign of the detuning [see Eqs. (7e) and (8c)]. If $\delta_{20}>0(<0)$, the leading term in $\mathrm{Re}(D_1)_{\mathrm{EIT}}<0(>0)$, whereas the leading contribution from $\mathrm{Re}(D_1)_{\mathrm{Raman}}>0(<0)$. Therefore, Raman pulse broadening is always smaller than that of the EIT scheme. In fact, it is possible to achieve $\mathrm{Im}(D_2)<0$, which yields pulse narrowing. This is to be compared with the EIT scheme where pulse broadening always occurs. In Fig. 2, we show a representative case that is based on rubidium parameters. The improvement in performance is obvious.

In conclusion, we have shown a four-level Raman scheme that is superior to conventional one-photon, on-resonance four-level EIT schemes. We have shown that with the parameters chosen, the Raman scheme will give a comparable group-velocity reduction with much less probe field loss. Correspondingly, the pulse broadening will also be smaller than that of the EIT scheme. Furthermore, we have shown rich dynamics of the Raman scheme where probe-pulse amplification and narrowing may occur, a region of propagation that has never been studied before. We remark that the one-photon resonance requirement associated with the conventional EIT scheme makes it difficult to apply in solid medium without extremely low temperatures ($\sim$5 K) to reduce the linewidth [7]. At room temperature, an upper energy band that is not strongly overlapped by the coupling laser will not be driven transparent, and hence will collectively contribute as a loss mechanism. On the other hand, in the case of the Raman scheme, the one-photon detuning can be chosen much larger than the upper energy bandwidth, thereby preserving the probe-pulse field strength while achieving a comparable reduction in group velocity. Finally, we point out that tunability is another obvious advantage of the Raman scheme that does not exist in the case of EIT. These features could make our technique very useful in optical device designs that have potential applications in telecommunications.

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