Modeling the reversible decoherence of mesoscopic superpositions in dissipative environments

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A model is presented to describe the recently proposed experiment [J. Raimond, M. Brune, and S. Haroche, Phys. Rev. Lett **79**, 1964 (1997)] in which a mesoscopic superposition of radiation states is prepared in a high-*Q* cavity that is coupled to a similar resonator. The dynamical coherence loss of such a state in the absence of dissipation is reversible and can be observed in principle. We show how this picture is modified due to the presence of the environmental couplings. Analytical expressions for the experimental conditional probabilities and the linear entropy are given. We conclude that the phenomenon can still be observed provided the ratio between the damping constant and the intercavities coupling does not exceed about a few percent. This observation is favored for superpositions of states with a large overlap.

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Controlling coherence properties of quantum systems has become an increasingly important subject, given the central role they play in modern technology [1] as well as in fundamental aspects of quantum theory, such as the quantum-classical transition [2]. Recently, the impressive development of very refined experimental techniques opened up the possibility of testing all sorts of theoretical ideas and exploring quantum phenomena at a mesoscopic level. The constructing and monitoring of a superposition of radiation states was recently achieved in the context of cavity QED [3]. Shortly afterwards, it was noted that a slight modification of that experimental setup could be used to learn about a reversible decoherence mechanism of the same superposition of states: when the high-Q cavity containing the superposition state is coupled to another resonator, the mesoscopic quantum coherence should, in principle, first decay rapidly, then exibit sharp revivals with the period of energy exchange between the two cavities. This idea, presented in Ref. [4], is centered around a unitary process which introduces a new time scale related to the "tunneling" of the superposition between the two cavities. The well-known deleterious environmental effects are completely left out of the proposal. It is the purpose of the present contribution to explicitate these effects and to give quantitative limits for the observation of the phenomenon.

The experiment proposed in Ref. [4] involves a high-Q cavity C_1, which is coupled to another resonator C_2 initially empty, located between two low-Q cavities (Ramsey zones R_1 and R_2) fed with classical fields. The cavity C_1 stores a small coherent field $|\alpha(0)\rangle$ (an average number of photons n varying from 0 to 10). The transition between the two near-atomic levels, denoted as $|e\rangle$ and $|g\rangle$, is resonant with the fields in cavities R_1 and R_2, while it is slightly off-resonant with the field in cavity C_1 (this detuning δ is large enough to avoid any energy transfer between the atom and the field inside C_1). The fields in R_1 and R_2 are chosen so that their action on the atoms is given in both cases by $|e\rangle \rightarrow (1/\sqrt{2})(|e\rangle+|g\rangle)$, $|g\rangle \rightarrow (1/\sqrt{2})(-|e\rangle+|g\rangle)$. The coupling between the field in the cavity C_1 and the atom is measured by the "Rabi frequency" Ω [5]. Due to the order of magnitude of δ, the atom-field interaction leads essentially to $1/\delta$ dispersive frequency shifts. In this way, the atom-cavity coupling produces an atomic-level-dependent dephasing of the field: when in level $|e\rangle$, the atom changes the cavity field phase by an angle $\phi \equiv \Omega^2 t/\delta$, yielding a cavity state $e^{-i\phi}|\alpha(0)e^{-i\phi}\rangle$, where t is the time that the atom takes to cross C_1. An atom in level $|g\rangle$ leaves in C_1 the state $|\alpha(0)e^{i\phi}\rangle$. After the interaction of the atom with the cavity, the atomic states are mixed again in R_2. Finally, the atom is detected by the field ionization counters D_e and D_g, either in state $|e\rangle$ or in state $|g\rangle$. Since R_2 erases any information on the atomic state in C_1, the detection projects the cavity state onto the macroscopic superposition (cat state)

$$|\psi\rangle=(1/\sqrt{2})[e^{-i\phi}|\alpha(0)e^{-i\phi}\rangle \pm |\alpha(0)e^{i\phi}\rangle], \qquad (1)$$

where the $+$ signal applies for a detection in $|g\rangle$ and the $-$ signal for detection in $|e\rangle$ [9,3]. It is considered that during the preparation of the cat state the coupling between C_1 and C_2 plays no role, provided the preparation time is much shorter than the time scale of energy exchange. Now a second circular Rydberg atom is prepared to cross the same apparatus. During the passage of the second atom through this apparatus, the interaction between the cavity C_1 and C_2 is turned on. Due to this interaction, the energy is oscillating between C_1 and C_2 and the field between them becomes entangled. When the second atom is detected by one of the detectors, one can have information about the interference process, evaluating the correlation signal. The correlation signal η, defined as the difference between the conditional probabilities $P_{ee}(t)$ and $P_{eg}(t)$ to detect the second atom in state $|e\rangle$ provided the first was detected in $|e\rangle(|g\rangle)$, is proportional to the overlap of the two-cavity-field states. From this correlation signal, one can see how the quantum correlation quickly disappears and its revivals with the period of the energy exchange between the two cavities.

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Below, we introduce a model which describes the experiment proposed in Ref. [4], including the effects of the environment,

$$H=\hbar\omega a_1^\dagger a_1+\hbar\omega a_2^\dagger a_2+\hbar\frac{\gamma}{2}(a_1^\dagger a_2+a_2^\dagger a_1)+\sum_k \hbar\omega_k b_{1k}^\dagger b_{1k}+\sum_k \hbar\nu_k b_{2k}^\dagger b_{2k}+\sum_k (\hbar\beta_{1k}b_{1k}^\dagger a_1+\text{H.c.})+\sum_k (\hbar\beta_{2k}b_{2k}^\dagger a_2+\text{H.c.}). \tag{2}$$

The first two terms on the right-hand side (rhs) of Eq. (2) stand for the two resonators. Their coupling is given by the third term of the rhs of the same equation. This choice for the coupling is based on the fact that the time evolution of a superposition of coherent states remains a superposition of coherent states at later times under the dynamics given by the first three terms on the rhs of Eq. (2) [6]. The inclusion of nonresonant terms in this coupling would therefore completely destroy the simple picture proposed in Ref. [4]. The presence of an environment and its coupling to the two resonators is modeled by the standard collection of harmonic oscillators [with frequencies ω_k (ν_k) in the cavity 1 (2)] interacting separately with the two resonators. The couplings are again of the rotating-wave approximation (RWA) form, which is well justified in this context [7]. In Eq. (2), β_{1k} and β_{2k} stand for the coupling constants.

The dynamics of the full system described by Eq. (2) obeys Schrödinger's equation and the corresponding state vector is a pure state $|\psi(t)\rangle$. Since we are interested in the dynamics of systems 1 and 2 only, we deduce a master equation from Eq. (2) by means of the usual Born-Markov approximation and get for $\rho_s(t)=\text{Tr}_{\text{env}}(|\psi(t)\rangle\langle\psi(t)|)$,

$$i\frac{d\rho_s(t)}{dt}=(-i\omega-k)a_1^\dagger a_1\rho_s+(-i\omega-k)\rho_s a_1^\dagger a_1+2ka_1\rho_s a_1^\dagger+(-i\omega-k)a_2^\dagger a_2\rho_s+(-i\omega-k)\rho_s a_2^\dagger a_2+2ka_2\rho_s a_2^\dagger-i\gamma(a_1^\dagger a_2\rho_s-\rho_s a_1^\dagger a_2)-i\gamma(a_1 a_2^\dagger\rho_s-\rho_s a_1 a_2^\dagger), \tag{3}$$

where we have defined $k\equiv D_1(\omega)|\beta_1(\omega)|^2=D_2(\omega)|\beta_2(\omega)|^2$ to be the damping constants of the cavities; $D_{1,2}(\omega)$ stands for the density of states at the resonator's frequency ω, and the continumm limit has been taken with respect to the environmental frequencies. This situation corresponds precisely to the one proposed in Ref. [4], namely two resonating identical cavities. It is also possible to consider nonresonant cavities with different quality factors and obtain analytical results. This analysis is, however, more involved and beyond the scope of the present work.

Note that the reduced density which obeys Eq. (3) describes the two cavities. The solution of the analogous problem involving only one cavity can be found in several text books. The novel feature here is the terms involving operators of both cavities [e.g., the last terms on the rhs of Eq. (3)] whose physical origin is the coupling between the two resonators. It is, however, also possible in this case to find an analytical solution by noting that the set of all superoperators in the above equation form a Lie algebra [8]. For the initial condition of interest, i.e., the first cavity C_1 in the state (1) and C_2 in the vacuum (see Ref. [3]),

$$\rho_s\left(0,{g \atop e}\right)=\frac{1}{N_{g \atop e}}[e^{-i\phi}|\alpha(0)e^{-i\phi}\rangle_1\pm|\alpha(0)e^{i\phi}\rangle_1](\text{H.c.})\otimes|0\rangle_{22}\langle 0|, \tag{4}$$

we get

$$\rho_s\left(t,{g \atop e}\right)=\frac{1}{N_{g \atop e}}\Big(|\alpha_1^{(+)}(t)\rangle\langle\alpha_1^{(+)}(t)|\otimes|\alpha_2^{(+)}(t)\rangle\langle\alpha_2^{(+)}(t)|+|\alpha_1^{(-)}(t)\rangle\langle\alpha_1^{(-)}(t)|\otimes|\alpha_2^{(-)}(t)\rangle\langle\alpha_2^{(-)}(t)|\pm\{e^{\{-i\phi+1/2[|\alpha_1^{(+)}(t)|^2+|\alpha_1^{(-)}(t)|^2+|\alpha_2^{(+)}(t)|^2+|\alpha_2^{(-)}(t)|^2-2|\alpha(0)|^2]\}}\times e^{[\alpha_1^{(+)}(0)\alpha_1^{*(-)}(0)-\alpha_1^{(+)}(t)\alpha_1^{*(-)}(t)-\alpha_2^{(+)}(t)\alpha_2^{*(-)}(t)]}|\alpha_1^{(+)}(t)\rangle\langle\alpha_1^{(-)}(t)|\otimes|\alpha_2^{(+)}(t)\rangle\langle\alpha_2^{(-)}(t)|+\text{H.c.}\}\Big), \tag{5}$$

where the letters g and e are related to the two signs $\pm$ in the above equations. They correspond, in the experiment of Refs. [3,4], to measuring the first atom in the state $|g\rangle$ (or $|e\rangle$) and leaving in the high-Q cavity C_1 an odd or even "cat state," as in Eq. (1). In the above equation, index 1 (2) refers to the cavity $C_{1\,(2)}$. Also in order to obtain Eq. (5), we assume $\gamma >> k$ (this is not necessary to obtain the analytical solution but corresponds to the present physical situation) and get

$$\alpha_1^{(\pm)}(t)=\alpha(0)e^{\pm i\phi}e^{-(k+i\omega)t}\cos(\gamma t/2),$$

$$\alpha_2^{(\pm)}(t)=2i\gamma\alpha(0)e^{\pm i\phi}e^{-(k+i\omega)t}\sin(\gamma t/2), \tag{6}$$

where $\alpha(0)$ is the initial amplitude of the coherent field in cavity 1 and

$$N_{g \atop e}=2[1\pm e^{-|\alpha(0)|^2[1-\cos(2\phi)]\cos[\phi+|\alpha(0)|^2\sin(2\phi)]}]$$

is the normalization constant.

In order to describe the dynamics of cavity 1 alone, one can trace out the degrees of freedom associated with the index 2,

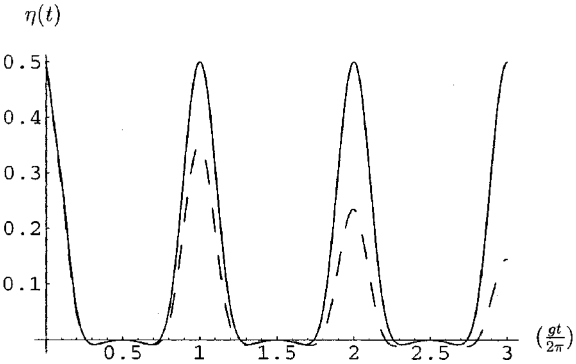

FIG. 1. Correlation signal $\eta(t)$ for $k/\gamma=0$ (full line) and $k/\gamma=0.01$ (dashed line) for $|\alpha(0)|^2=3.3$ and $\phi=0.98$ rad.

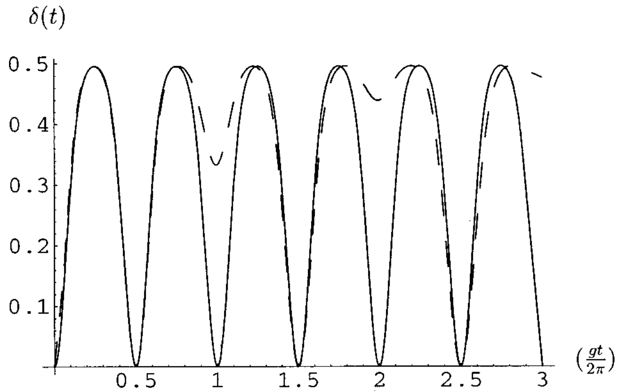

FIG. 2. Same as Fig. 1 for the linear entropy $\delta(t)$.

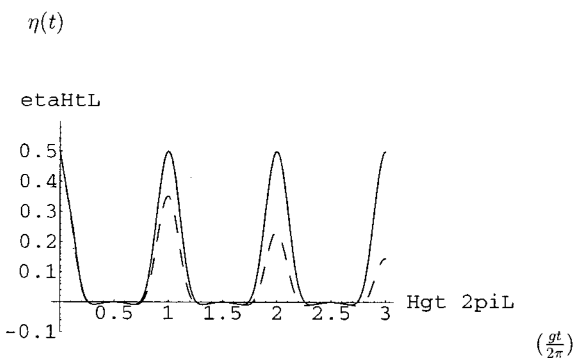

FIG. 3. Correlation signal $\eta(t)$ for $k/\gamma=0$ (full line) and $k/\gamma=0.01$ (dashed line) for $|\alpha(0)|^2=3.3$ and $\phi=0.4$ rad.

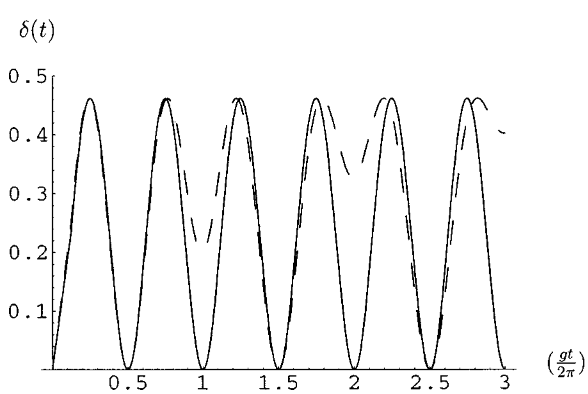

FIG. 4. Same as Fig. 3 for the linear entropy $\delta(t)$.

$$
\begin{aligned}
\rho_1\left(t,\begin{matrix}g\\e\end{matrix}\right) &= \mathrm{Tr}_2\left(\rho_s\left(t,\begin{matrix}g\\e\end{matrix}\right)\right)\\
&= (1/N_{\substack{g\\e}})\,\big(|\alpha_1^{(+)}(t)\rangle\langle\alpha_1^{(+)}(t)| + |\alpha_1^{(-)}(t)\rangle\langle\alpha_1^{(-)}(t)|\\
&\quad \pm \{e^{\{-(1/2)D^2(\alpha(0)e^{-i\phi},\alpha(0)e^{i\phi})[1-e^{-2kt}\cos^2(\gamma t/2)]\}}\\
&\quad \times e^{\{-i\phi+|\alpha(0)|^2[1-e^{-2kt}\cos^2(\gamma t/2)]\sin(2\phi)\}}\\
&\quad \times |\alpha_1^{(-)}(t)\rangle\langle\alpha_1^{(+)}(t)| + \mathrm{H.c.}\}\big), \qquad (7)
\end{aligned}
$$

where $D(\alpha(0)e^{-i\phi},\alpha(0)e^{i\phi})$ is the distance between the states in the superposition and is given by $D(\alpha,\beta)=|\alpha-\beta|$. Note here that if we take the limit $\gamma=0$, we recover the usual reduced density matrix of a superposition in a dissipative environment [9,10]. Turning on the coupling between the two cavities brings in a new time scale in the problem, i.e., the characteristic time for energy exchange between the two cavities. The interesting point is that this latter time dependence is periodic and therefore completely different from the exponential which characterizes decoherence in general. Of course, if these two time scales are sufficiently different, the proposed "reversibility" of decoherence might be observed. We now turn to the quantitative question of how different γ and k have to be in order for the phenomenon to be observable. For this purpose, we calculate the conditional probability $P_{ee}(t)$ and $P_{ge}(t)$ to detect the second atom in state e provided the first was detected in $e(g)$. Using Eq. (7), these probabilities are given by

$$
P_{ge}^{ee}(t)=\frac{1}{2}\left(1-\mathrm{Re}\left\{e^{-i\phi}\,\mathrm{Tr}_F\left[e^{-2i\phi a^\dagger a}\rho_1\left(t,\begin{matrix}g\\e\end{matrix}\right)\right]\right\}\right) \qquad (8)
$$

or, explicitly,

$$
\begin{aligned}
P_{ge}^{ee}(t) &= \frac{1}{2}-\frac{1}{2N_{\substack{g\\e}}}\big[2e^{-D^2(\alpha_1^{(-)}(t),\alpha_1^{(+)}(t))}\\
&\quad \times\cos[\phi+|\alpha(0)|^2e^{-2kt}\cos^2(\gamma t/2)\sin(2\phi)]\\
&\quad \pm e^{-(1/2)D^2(\alpha(0)e^{-i\phi},\alpha(0)e^{i\phi})[1-e^{-2kt}\cos^2(\gamma t/2)]}\\
&\quad \times(\cos\{|\alpha(0)|^2[1-e^{-2kt}\cos^2(\gamma t/2)]\sin(2\phi)\}\\
&\quad + e^{-(1/2)D^2(\alpha_1^{(-)}(t)e^{-i\phi},\alpha_1^{(+)}(t)e^{i\phi})}\\
&\quad \times\cos\{2\phi+|\alpha(0)|^2\sin(2\phi)[1-e^{-2kt}\cos^2(\gamma t/2)\\
&\quad +2e^{-2kt}\cos^2(\gamma t/2)]\})\big]. \qquad (9)
\end{aligned}
$$

The correlation signal η is defined as $\eta(t)=P_{ee}(t)-P_{ge}(t)$. Also in order to characterize decoherence, we calculate the linear entropy [12,11] $\delta(t)=1-\mathrm{Tr}(\rho_1^2(t,{}^g_e))$,

$$
\begin{aligned}
\delta(t) &= \frac{2}{N_{\substack{g\\e}}^2}\{(1+e^{[-(1/2)D(\alpha(0)e^{-i\phi},\alpha(0)e^{i\phi})]})^2\\
&\quad -(e^{[-(1/2)D(\alpha_1^{(-)}(t),\alpha_1^{(+)}(t))]}\\
&\quad + e^{[1/2D(\alpha_1^{(-)}(t),\alpha_1^{(+)}(t))-(1/2)D(\alpha(0)e^{-i\phi},\alpha(0)e^{i\phi})]})^2\}, \qquad (10)
\end{aligned}
$$

where $\alpha_1^{(\pm)}$ are defined in Eq. (6). Since the expression for the correlation signal is rather lengthy and not very illumi-

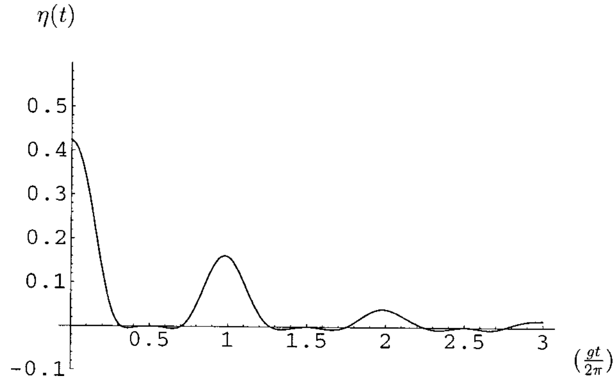

FIG. 5. Correlation signal for the case in Fig. 3, but $k/\gamma = 0.05$.

nating, we simplify it in the small overlap limit, i.e., $\langle \alpha(0)e^{-i\phi}|\alpha(0)e^{i\phi}\rangle \ll 1$. In this case, we get for the correlation signal the following simple expression:

$$\eta(t) \approx \frac{1}{2}\cos\{|\alpha(0)|^2[1-e^{-2kt}\cos^2(\gamma t/2)]\sin(2\phi)\} \times e^{\{-(1/2)D^2(\alpha(0)e^{-i\phi},\alpha(0)e^{i\phi})[1-e^{-2kt}\cos^2(\gamma t/2)]\}}. \tag{11}$$

Note that both expressions contain the "distance" factor, which is directly related to decoherence, as can be seen in Eqs. (10) and (11). The $\cos^2(\gamma t/2)$ factor contained in $\alpha_1^{(\pm)}$ in the expresssion for the linear entropy is responsible for the reversible decoherence phenomenon. The same factor is present in the correlation signal, which in addition has a term like $\cos\{|\alpha(0)|^2[1-e^{-2kt}\cos^2(\gamma t/2)]\sin(2\phi)\}$. This term, provided $k \ll \gamma$, will oscillate with a period corresponding to the energy exchange period between the two cavities.

In Figs. 1 and 2, we show the correlation signal and linear entropy for the case in which the coherent states in the superposition are distant, in the sense that the small overlap approximation is valid ($\phi = 0.98$ rad). In Fig. 1, we clearly see the reversible decoherence peaks in the case of no dissipation, as in Ref. [4]. The effect of dissipation, which can also be appreciated in this figure, is, roughly speaking, to superimpose a decaying exponential on this curve. For the value in the figure, $k/\gamma = 0.01$, the second peak is still about 70% in intensity with respect to the first one. Already for $k/\gamma = 0.05$ it has disappeared. The revivals observed in this figure are also revealed in the linear entropy. It starts at zero, indicating a pure state, and attains its maximum when the correlation signal is a minimum. At this time the state in cavity C_1 corresponds to a statistical mixture. The fact that the maximum value of $\delta(t)$ is 0.5 in this case is related to the small overlap in the component of the initial superposition. Given the form of the coupling, one sees from Eq. (5) that the state generated in the second cavity C_2 will be very similar to the one in the first and therefore we will also have a small overlap. In this case, the reduced density ρ_1 will approximately be a statistical mixture. This is clearly different in the cases of Figs. 3 and 4, where we have a larger overlap $\phi = 0.4$ rad between the two components of the superposition. Other than that, the figures are similar with similar interpretations. The significant difference resides in the fact that the effect of decoherence is slower in this case. In Fig. 5, we show that the second peak in the correlation signal can still be seen with reasonable intensity for $k/\gamma = 0.05$ (see also Fig. 6).

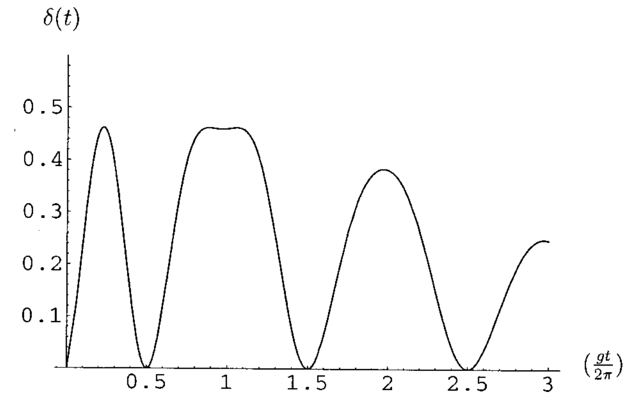

FIG. 6. Linear entropy for the case in Fig. 3, but $k/\gamma = 0.05$.

From the model we presented here, we conclude that the "reversible decoherence" proposed in Ref. [4] can in fact be observed provided (a) the coupling between the two cavities is well modeled by $\gamma(a_1^\dagger a_2 + a_1 a_2^\dagger)$, as discussed in the text and, (b) the ratio k/γ should not exceed a few percent and the distance between the components of the superpositions will definitely influence this value.

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