Strong-field approximation to the relativistic channeling of electrons in the presence of electromagnetic waves

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We present a study of the interaction of a relativistically planar channeled electron with an intense electromagnetic field. Using a *S*-matrix approach in the strong-field approximation, it is shown that the crystal periodicity affects drastically the excitation process, suppressing the possibility of multiphoton absorption except for some particular cases. This selective excitation opens the possibility to control the dynamics of the channeling process by means of an external field. Explicit expressions for the *S*-matrix *N*-photon excitation rates together with the corresponding conservation laws are obtained from the relativistic quantum-mechanical Dirac equation.

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I. INTRODUCTION

Channeling in crystal lattices occurs when an accelerated charged particle is introduced into a crystalline target at sufficiently large energy. Depending on the crystal orientation, the particle's trajectory may be aligned with a crystal axis (axial channeling) or with a direction parallel to a crystal plane (planar channeling), and the penetration may reach anomalous depths. Although the possibility of this effect was already pointed out very early by Stark [1], it was demonstrated experimentally 50 years later by Rol *et al.* [2], when the result of the ion sputtering was found to depend strongly on the orientation of the target crystal. After the discovery, the theoretical and experimental work increased rapidly and extended to the case of channeling of electrons and positrons [3].

In this paper we will investigate the excitation dynamics of a planar channeled electron under the influence of an external electromagnetic (e.m.) field. As a main result, we demonstrate that the crystal periodicity introduces a momentum-conservation condition that affects the efficiency of the different *N*-photon channels of excitation, leading to the strong suppression of photon absorption in a broad range of situations. The multiphoton excitation of channeled particles has been already addressed by Avetissian and co-workers [4,5] by assuming an electromagnetic wave copropagating with the electron, and with a frequency resonant to the (Doppler-shifted) lower-energy-level transitions. In the present case, however, we are interested in a complementary situation where the channeled electron is excited to a final state lying in the crystal quasicontinuum. Since the transition is produced by the interaction of the electron with an external intense optical field, the strong field approximation (SFA) constitutes a more appropriated procedure in comparison to the discrete level approach in Ref. [4].

SFA theories have been developed in the context of ionization of atoms in strong laser fields. Among them, the so-called Keldish-Faisal-Reiss theory [6–8] is based on the *S*-matrix approach, where the final state is approximated by a Volkov state, which describes the evolution of a free electron driven by the electromagnetic wave. SFA theories describe most of the relevant aspects of the atomic ionization, including multiphoton absorption and multiphoton excitation above the ionization threshold.

Although employed mainly in the atomic and molecular context, *S*-matrix SFA approaches can be used in any general situation in which the field interaction energy is comparable with the energies of the matter system. In fact, for the higher energy bound states, the intensity of the field required to promote an electron to the continuum does not have to be very high, and yet SFA can be used. On the other hand, SFA requires the matter potential to be approximately constant over the complete interaction time. In our case it suffices with a moderate intensity field, (10^{12}–10^{13} W/cm^2), while the crystal stability can be ensured by a sufficiently short pulse (about 100 fs), which still enclose enough cycles to ensure the adiabatic limit involved in the theoretical approach. It should be mentioned that, in the case of atom ionization by strong field, the adiabatic assumption is correct even in the case of a few cycles pulse length, and there is no reason to think that in this channeling case the thing should be different.

II. GEOMETRY OF THE SYSTEM AND DESCRIPTION OF THE UNPERTURBED CHANNELED ELECTRON

Let us consider the interacting geometry depicted in Fig. 1. A relativistic electron introduced in a crystal with a small tilt angle θ respect to the *x axis*, interacts with a copropagating or a counterpropagating electromagnetic wave. For particular crystal orientations, the electron is confined transversely to trajectories close to the initial injection axis. In the case of planar channeling, the electron is injected almost parallel to a crystal plane [3,9], avoiding the coincidence with any crystal symmetry axis to inhibit axial channeling. We will assume an electromagnetic plane-wave field, which propagates in the *x axis* and linearly polarized in the *y axis*, i.e., orthogonal to the crystal plane.

Since the channeled particle is injected with a relativistic velocity, the crystal potential may be well approximated by a spatial average over the crystal plane coordinates, as it is done in the so-called continuum model [3],

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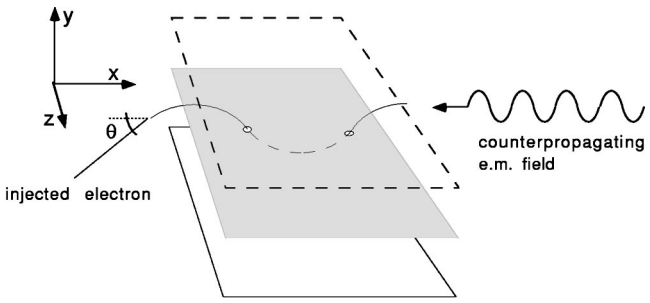

FIG. 1. The system to be studied consists of a relativistic electron channeled along a crystal plane and interacting either with a copropagating or with a counterpropagating electromagnetic field (only the counterpropagating case has been plotted in the figure). As a consequence of the continuum approximation, the planes are assumed to be uniformly charged.

$$V(y)=\frac{1}{L_xL_z}\int_{-L_x/2}^{L_x/2}\int_{-L_z/2}^{L_z/2}U(x,y,z)dxdz, \tag{1}$$

where L_x and L_y are the crystal plane dimensions, and $U(x,y,z)$ is the crystal potential. With this effective potential the electron motion in the crystal channel, driven by the external e.m. field, will be confined to the polarization plane xy.

In the most general case, the quantum description of the electron's dynamics is described by the Dirac equation

$$\{c\boldsymbol{\alpha}\cdot(\hat{\mathbf{p}}-e\mathbf{A}/c)+\beta mc^2+V(y)\}\Phi(\mathbf{x})=E_B\Phi(\mathbf{x}). \tag{2}$$

Let us first consider the unperturbed channeling situation, $\mathbf{A}=\mathbf{0}$. Since the averaged potential depends only on the y coordinate, a general positive-energy solution of the channeled electron can be written as

$$\Phi(\mathbf{x})=\int dp_y\sqrt{\frac{mc^2}{E_{p_x,p_y}}}u^1_{p_x,p_y}\exp[i(p_xx+p_yy)/\hbar]\xi_{p_y}, \tag{3}$$

where $u^1_{p_x,p_y}$ is the positive-energy solution of a free Dirac electron [10,11], $E_{p_x,p_y}=\sqrt{c^2p_x^2+c^2p_y^2+m^2c^4}$,

$$u^1_{p_x,p_y}=\sqrt{\frac{E_{p_x,p_y}+mc^2}{2mc^2}}\begin{pmatrix}1\\0\\0\\\dfrac{c(p_x+ip_y)}{E_{p_x,p_y}+mc^2}\end{pmatrix}. \tag{4}$$

Introducing Eq. (3) into Eq. (2), with $\mathbf{A}=\mathbf{0}$, one can obtain the following form for the Dirac equation in momentum space:

$$E_Bu^1_{p_x,p_y}\xi_{p_y}=\{c\alpha_xp_x+c\alpha_yp_y+\beta mc^2\}u^1_{p_x,p_y}\xi_{p_y}+\int dp_y'\sqrt{\frac{E_{p_x,p_y}}{E_{p_x,p_y'}}}V_{p_y-p_y'}u^1_{p_x,p_y'}\xi_{p_y'}, \tag{5}$$

V_p being the Fourier transform of the interplanar potential at the spatial frequency $p/\hbar$. Due to the nature of the averaged potential, the channeling along a crystal plane is only possible if the energy of the electron's transversal dynamics is moderate, i.e., nonrelativistic. In this case, it is justified to approximate Eq. (5) to second order of cp_y/E_{p_x}.

$$E_{p_x,p_y}=\sqrt{c^2p_x^2+c^2p_y^2+m^2c^4}\approx E_{p_x}\left(1+\frac{c^2p_y^2}{2E_{p_x}^2}\right), \tag{6}$$

$$\sqrt{\frac{E_{p_x,p_y}+mc^2}{2E_{p_x,p_y}}}\approx\sqrt{\frac{E_{p_x}+mc^2}{2E_{p_x}}}\left[1-\frac{c^2p_y^2}{4E_{p_x}^2}\left(\frac{mc^2}{E_{p_x}+mc^2}\right)\right], \tag{7}$$

$$\sqrt{\frac{2E_{p_x,p_y}}{E_{p_x,p_y}+mc^2}}\approx\sqrt{\frac{2E_{p_x}}{E_{p_x}+mc^2}}\left[1+\frac{c^2p_y^2}{4E_{p_x}^2}\left(\frac{mc^2}{E_{p_x}+mc^2}\right)\right]. \tag{8}$$

Note that, for the field intensities considered here, this approximation will remain equally valid when considering the interaction with the external electromagnetic wave.

By introducing Eqs. (6)–(8) into Eq. (5), and since the scalar potential $V_{p_y-p_y}$ is a first-order term in cp_y/E_x, the Dirac equation is reduced to the same identity for every nonzero component of the spinor $u^1_{p_x,p_y}$

$$E_B\xi_{p_y}=\left\{\frac{c^2p_y^2}{2E_{p_x}}+E_{p_x}\right\}\xi_{p_y}+\int dp_y'V_{p_y-p_y'}\xi_{p_y'}. \tag{9}$$

Computing the inverse Fourier transform in the y coordinate, we finally end with a Schrödinger-like equation [12,13]

$$\epsilon_B\xi(y)=\left\{\frac{p_y^2}{2\gamma_xm}+V(y)\right\}\xi(y), \tag{10}$$

where $E_{p_x}=\gamma_xmc^2$, and $\epsilon_B=E_B-\gamma_xmc^2$ corresponds to the nonrelativistic eigenstate energy.

Several explicit forms of the averaged crystal potential may be found in the literature [3]. Among them, those derived from the Thomas-Fermi screened two-body potentials have been widely used [3,9], and have been fine adjusted by standard Hartree-Fock many-body calculations [14]. From the theoretical point of view, model potentials are more convenient, since they allow for further analytical work while keeping the essential features of the interaction. For instance, $V(y)=-4V_0y^2/d^2$ was proposed by Avetissian *et al.* [4] in the context of computation of the multiphoton transitions between bound states of the continuum potential, for a pos-

itron interacting with a strong electromagnetic wave. The form $V(y)=-V_0\cosh^{-2}(y/b)$ has also been used to study the possible amplification of x-ray channeling radiation [5]. This later form has a better resemblance with the averaged Thomas-Fermi potentials while still allowing for an analytical diagonalization and, therefore, we shall use this potential for our calculations. The transverse energy spectrum for this case can be cast in the following form [5]:

$$\epsilon_{Bn}=-\frac{\hbar^2}{2b^2m\gamma_x}(s-n)^2 \tag{11}$$

where n can be $0,1,\ldots,[s]$, being $s=-\frac{1}{2}+\sqrt{\frac{1}{4}+2b^2m\gamma_xV_0/\hbar^2}$. Note that, as a result of the spatial averaging, the scattering with the continuum potential affects only to the transverse dynamics of the channeled particle, while the very large longitudinal momentum remains unaffected.

III. *S*-MATRIX DESCRIPTION OF A CHANNELED PARTICLE INTERACTING WITH AN ELECTROMAGNETIC WAVE

Let us now add the electromagnetic excitation to the problem. As it is well known, Eq. (2) does not accept analytical solutions for a space-time dependent vector potential. In such situations, the *S*-matrix approach offers a standard procedure to find approximated solutions [15], used specially in quantum field theory [10,16,17] and scattering [18].

A. The general relativistic case

The relativistic SFA *S*-matrix theory for the Dirac electron in an atom can be found in Refs. [19–21]. The general expression for the transition amplitude using time-reversed *S*-matrix theory has the following form:

$$S_{fi}=\lim_{t\to\infty}\langle\Psi_f^{(-)}|\Phi_i\rangle. \tag{12}$$

Although mainly used in the strong-field ionization of atoms and molecules, this approach is quite general and can be exported to any other system, provided its eigenstates can be found analytically. To our knowledge, however, this is the first time that it is applied to the relativistic channeled electron in interaction with an electromagnetic wave. In the present case, Φ_i corresponds to the unperturbed channeled electron state discussed in Sec. II, and $\Psi_f^{(-)}$ is an arbitrary final state, solution of the complete Eq. (2). Since this exact solution is not available, the success of the *S*-matrix approach consist in finding a suitable approximation. In the strong-field approximation, the interaction with electromagnetic field is assumed to be the relevant for the final state, therefore, $\Psi_f^{(-)}$ is approximated in terms of the Volkov states, $\Psi_V(\mathbf{x},t)$ [22,23]. These wave functions are solutions of Eq. (2) for $V(y)=0$ and $\mathbf{A}(\mathbf{x},t)\neq\mathbf{0}$, and describe a free electron in the presence of an electromagnetic field. In the Lorentz gauge, the form of these states for a laser field pulse of arbitrary form is [23]

$$\Psi_V^{(-)}(x)=\sqrt{\frac{mc^2}{(2\pi)^3E}}\left(1+\epsilon^r\frac{e\not{s}\not{A}(\varphi)}{2c(s\cdot p)}\right)u_{\mathbf{p}}^re^{iS} \tag{13}$$

$\epsilon^{1,2}=1$, $\epsilon^{3,4}=-1$, $\varphi=s\cdot x$ being the phase of the laser field where $s^\mu\equiv(1,\mathbf{s})$ and $\mathbf{s}$ is a unitary vector that indicates the direction of propagation of the electromagnetic field, and

$$S=-\epsilon^r\frac{p\cdot x}{\hbar}+\int_{s\cdot x}^{\infty}\left[\frac{e[p\cdot A(\varphi')]}{\hbar c(s\cdot p)}-\epsilon^r\frac{e^2A^2(\varphi')}{2\hbar c^2(s\cdot p)}\right]d\varphi'. \tag{14}$$

The *S*-matrix approach takes as a starting point the following exact relation:

$$\Psi_f(x)=\Psi_V(x)+\int d^4x'G_V(x,x')\gamma^0V(x')\Psi_f(x'), \tag{15}$$

where $G_V(x,x')$ is the Volkov Green's function [24], and keeps the lowest-order term in powers of $V(x)$. The transition amplitude obtained is [20,21]

$$\begin{aligned}(S-1)_{fi}^{SFA}&=-\frac{i}{\hbar c}\int d^4x\overline{\Psi}_V^{(-)}eA_\mu\gamma^\mu\Phi_i\\&=-\frac{i}{\hbar c}\sqrt{\frac{mc^2}{(2\pi)^3E}}\int d^4xe^{-iS}\overline{u_{\mathbf{p}}^r}\\&\quad\times\left(e\not{A}-\epsilon^r\frac{e^2(A\cdot A)\not{s}}{2c(s\cdot p)}\right)\Phi_i,\end{aligned} \tag{16}$$

where we have used Eq. (13) and we have assumed the transverse character of the e.m. field, $s\cdot A=-\mathbf{s}\cdot\mathbf{A}=0$. Separating the time and the space integrals in Eq. (16), the transition amplitude takes the form

$$\begin{aligned}(S-1)_{fi}^{SFA}&=-i\sqrt{\frac{mc^2}{(2\pi)^3E}}e\overline{u_{\mathbf{p}}^r}\int dt\,\mathcal{I}_1(t)\\&\quad+i\sqrt{\frac{mc^2}{(2\pi)^3E}}\frac{e^2\epsilon^r}{2(s\cdot p)}\overline{u_{\mathbf{p}}^r}\not{s}\int dt\,\mathcal{I}_2(t)\end{aligned} \tag{17}$$

$\mathcal{I}_1(t)$ and $\mathcal{I}_2(t)$ being:

$$\mathcal{I}_1(t)=\gamma^\mu\mathcal{I}_{1,\mu}(t)=\frac{\gamma^\mu}{\hbar}\int d\mathbf{x}e^{-iS}A_\mu e^{-i(E_Bt/\hbar)}\Phi_i, \tag{18}$$

$$\begin{aligned}\mathcal{I}_2(t)&=\frac{1}{\hbar c}\int d\mathbf{x}e^{-iS}(A\cdot A)e^{-i(E_Bt/\hbar)}\Phi_i\\&=-\frac{1}{\hbar c}\int d\mathbf{x}e^{-iS}|\mathbf{A}|^2e^{-i(E_Bt/\hbar)}\Phi_i.\end{aligned} \tag{19}$$

Once the transition amplitude $(S-1)_{fi}^{SFA}$ is known, the total transition rate can be computed as

$$W=\int \frac{Vd\mathbf{p}}{(2\pi)^3}w, \tag{20}$$

where V is the normalization volume and w is the transition probability per unit of time, which is defined as

$$w=\lim_{t\to\infty}\frac{1}{t}|(S-1)_{fi}^{SFA}|^2. \tag{21}$$

Another important magnitude is the transition rate per unit of solid angle, which has the form

$$\frac{dW}{d\Omega}=\frac{V}{(2\pi)^3}\int wp^2dp. \tag{22}$$

It should be mentioned that SFA S-matrix theory cannot be considered as a perturbation series in powers of $V(x)$ since the initial state ϕ_i is an eigenstate of the potential itself including all the properties of the crystal. The presence of this wave function produces a new behavior in the scattering section, introducing all the differences with the atom ionization or the simple Compton scattering.

B. Application to the case of a monochromatic and linearly polarized laser field

Let us focus our attention to the geometry depicted in Fig. 1, where the electromagnetic field can be a copropagating or counterpropagating linear polarized plane wave of frequency ω, $\mathbf{A}(\varphi)=A_j(\varphi)\mathbf{e}_j=A_0\cos(\omega t-\mathbf{k}\cdot\mathbf{x})\mathbf{e}_j$, where $\mathbf{e}_j$ is the field's polarization vector.

The phase factor of the Volkov function (14) now reads as

$$S=-\epsilon^r\left(\frac{p^\mu}{\hbar}+\frac{e^2A_0^2k^\mu}{4\hbar c^2(k\cdot p)}\right)x_\mu+\frac{ep_jA_0}{\hbar c(k\cdot p)}\sin(k\cdot x)$$
$$-\epsilon^r\frac{e^2A_0^2}{8\hbar c^2(k\hbar p)}\sin[2(k\cdot x)]. \tag{23}$$

The resulting exponential factor can be expanded as a series of Bessel functions

$$e^{-iS}=\exp\left\{i\epsilon^r\left[\frac{p^\mu}{\hbar}+\frac{e^2A_0^2k^\mu}{4\hbar c^2(k\cdot p)}\right]x_\mu\right\}$$
$$\times\sum_{N,n=-\infty}^{+\infty}J_{N+2n}(\eta)J_n(\xi)e^{-iN(k\cdot x)}, \tag{24}$$

where η and ξ are factors which only depend on the momentum of the Volkov function, and the frequency and amplitude of the laser field

$$\eta=\frac{ep_jA_0}{\hbar c(k\cdot p)},\quad \xi=\frac{\epsilon^re^2A_0^2}{8\hbar c^2(k\cdot p)}. \tag{25}$$

Substituting Eq. (24) in Eqs. (18) and (19),

$$\mathcal{I}_1(t)=-\gamma^j\mathcal{I}_{1,j}(t)$$
$$=-\frac{\gamma^j}{\hbar}\int d\mathbf{x}e^{-iS}A_j(\varphi)\Phi_i(\mathbf{x})$$
$$=-\frac{\gamma^jA_0}{\hbar}\sum_{N,n=-\infty}^{+\infty}\frac{N+2n}{\eta}J_{N+2n}(\eta)J_n(\xi)$$
$$\times\exp\left[i\frac{(\hbar\omega_N-E_B)t}{\hbar}\right]\tilde{\Phi}_i(\mathbf{q}_N) \tag{26}$$

and

$$\mathcal{I}_2(t)=-\frac{1}{\hbar c}\int d\mathbf{x}e^{-iS}|\mathbf{A}|^2e^{-iE_Bt/\hbar}\Phi_i$$
$$=-\frac{A_0^2}{\hbar c}\sum_{N,n=-\infty}^{+\infty}\left(\frac{(N+2n)^2}{\eta^2}J_{N+2n}(\eta)\right.$$
$$\left.+\frac{1}{2\eta}[J_{N+2n+1}(\eta)-J_{N+2n-1}(\eta)]\right)$$
$$\times J_n(\xi)e^{i(\hbar\omega_N-E_B)t/\hbar}\tilde{\Phi}_i(\mathbf{q}_N), \tag{27}$$

where $\mathbf{q}_N$, ω_N, and $\tilde{\Phi}_i(\mathbf{q}_N)$ are defined as

$$\mathbf{q}_N=\frac{\epsilon^r\mathbf{p}}{\hbar}+\left[\frac{\epsilon^re^2A_0^2}{4\hbar c^2(k\cdot p)}-N\right]\mathbf{k}, \tag{28}$$

$$\omega_N=\frac{\epsilon^rE}{\hbar}+\left[\frac{\epsilon^re^2A_0^2}{4\hbar c^2(k\cdot p)}-N\right]\omega, \tag{29}$$

$$\tilde{\Phi}_i(\mathbf{q}_N)=\int e^{-i\mathbf{q}_N\cdot\mathbf{x}}\Phi_i(\mathbf{x})d\mathbf{x}. \tag{30}$$

The time integrals appearing in Eq. (17) can now be calculated as

$$\gamma^j\int_{t_0}^t\mathcal{I}_{1,j}(\tau)d\tau=\frac{\gamma^jA_0}{\hbar}\sum_{N,n=-\infty}^{+\infty}\frac{N+2n}{\eta}J_{N+2n}(\eta)J_n(\xi)\tilde{\Phi}_i(\mathbf{q}_N)$$
$$\times\exp[i(\hbar\omega_N-E_B)(t+t_0)/2\hbar]\frac{\sin(\hbar\omega_N-E_B)T/2\hbar}{(\hbar\omega_N-E_B)/2\hbar} \tag{31}$$

and

$$\int_{t_0}^{t}\mathcal{I}_2(\tau)d\tau=-\frac{A_0^2}{\hbar c}\sum_{N,n=-\infty}^{+\infty}\left(\frac{(N+2n)^2}{\eta^2}J_{N+2n}(\eta)+\frac{1}{2\eta}[J_{N+2n+1}(\eta)-J_{N+2n-1}(\eta)]\right)J_n(\xi)\tilde{\Phi}_i(\mathbf{q}_N)$$
$$\times\exp[i(\hbar\omega_N-E_B)(t+t_0)/2\hbar]\frac{\sin(\hbar\omega_N-E_B)T/2\hbar}{(\hbar\omega_N-E_B)/2\hbar} \tag{32}$$

being $T=t-t_0$.

To compute the rate of excitation, we should use this two equations together with Eq. (17) to calculate

$$|(S-1)_{fi}^{SFA}|^2=T_1+T_2+T_3+T_4 \tag{33}$$

with

$$T_1=\frac{mc^2}{(2\pi)^3E}e^2\left[\int dt\,\mathcal{I}_{1,j}(t)\right]^+\gamma^{j+}\gamma^0u_{\mathbf{p}}^ru_{\mathbf{p}}^{r+}\gamma^0\gamma^j$$
$$\times\int dt\,\mathcal{I}_{1,j}(t), \tag{34}$$

$$T_2=\frac{mc^2}{(2\pi)^3E}\frac{\epsilon^re^3}{2(k\cdot p)}\left[\int dt\,\mathcal{I}_{1,j}(t)\right]^+\gamma^{j+}\gamma^0u_{\mathbf{p}}^ru_{\mathbf{p}}^{r+}\gamma^0\not{k}$$
$$\times\int dt\,\mathcal{I}_2(t), \tag{35}$$

$$T_3=T_2^*, \tag{36}$$

$$T_4=\frac{mc^2}{(2\pi)^3E}\left(\frac{\epsilon^re^2}{2(k\cdot p)}\right)^2\left[\int dt\,\mathcal{I}_2(t)\right]^+\gamma^{\mu+}k^\mu\gamma^0u_{\mathbf{p}}^ru_{\mathbf{p}}^{r+}\gamma^0\not{k}$$
$$\times\int dt\,\mathcal{I}_2(t). \tag{37}$$

Substituting the time integrals (31) and (32), in each term, we may calculate the transition probability per unit of time as

$$w=\lim_{t\to\infty}\frac{1}{t}|(S-1)_{fi}^{SFA}|^2=\lim_{t\to\infty}\frac{T_1+T_2+T_3+T_4}{t}$$
$$=t_1+t_2+t_3+t_4, \tag{38}$$

where

$$t_1=\frac{mc^2}{8\pi^2E}\frac{e^2A_0^2}{\hbar^2}\sum_{N=-\infty}^{+\infty}\left[\sum_{n=-\infty}^{+\infty}\frac{(N+2n)}{\eta}J_{N+2n}(\eta)J_n(\xi)\right]^2\delta(\hbar\omega_N-E_B)\tilde{\Phi}_i^+(\mathbf{q}_N)\gamma^{j+}\gamma^0u_{\mathbf{p}}^ru_{\mathbf{p}}^{r+}\gamma^0\gamma^j\tilde{\Phi}_i(\mathbf{q}_N), \tag{39}$$

$$t_2=-\frac{mc^2}{8\pi^2E}\frac{\epsilon^re^3A_0^3}{2\hbar^2c(k\cdot p)}\sum_{N=-\infty}^{+\infty}\sum_{n,n'=-\infty}^{+\infty}\left(\frac{(N+2n')^2}{\eta^2}J_{N+2n'}(\eta)+\frac{1}{2\eta}[J_{N+2n'+1}(\eta)-J_{N+2n'-1}(\eta)]\right)$$
$$\times J_{n'}(\xi)\frac{(N+2n)}{\eta}J_{N+2n}(\eta)J_n(\xi)\delta(\hbar\omega_N-E_B)\tilde{\Phi}_i^+(\mathbf{q}_N)\gamma^{j+}\gamma^0u_{\mathbf{p}}^ru_{\mathbf{p}}^{r+}\gamma^0\not{k}$$
$$\times\tilde{\Phi}_i(\mathbf{q}_N), \tag{40}$$

$$t_3=t_2^*, \tag{41}$$

$$t_4=\frac{mc^2}{8\pi^2E}\left(\frac{\epsilon^re^2A_0^2}{2\hbar c(k\cdot p)}\right)^2\sum_{N=-\infty}^{+\infty}\left[\sum_{n=-\infty}^{+\infty}\left(\frac{(N+2n)^2}{\eta^2}J_{N+2n}(\eta)+\frac{1}{2\eta}[J_{N+2n+1}(\eta)-J_{N+2n-1}(\eta)]\right)\right]^2$$
$$\times\delta(\hbar\omega_N-E_B)\tilde{\Phi}_i^+(\mathbf{q}_N)\not{k}^+\gamma^0u_{\mathbf{p}}^ru_{\mathbf{p}}^{r+}\gamma^0\not{k}\tilde{\Phi}_i(\mathbf{q}_N). \tag{42}$$

Finally, the excitation rate is

$$w=\frac{mc^2}{8\pi^2 E}\left(\frac{eA_0}{\hbar}\right)^2\sum_{N=-\infty}^{+\infty}\left[S_1^2\Delta_1-\frac{\epsilon^r eA_0}{c(k\cdot p)}S_2\,\mathrm{Re}(\Delta_2)+\left(\frac{eA_0}{2c(k\cdot p)}\right)^2 S_4^2\Delta_4\right]\delta(\hbar\omega_N-E_B), \tag{43}$$

where we have defined

$$S_1=\sum_{n=-\infty}^{+\infty}\frac{(N+2n)}{\eta}J_{N+2n}(\eta)J_n(\xi), \tag{44}$$

$$S_2=\sum_{n,n'=-\infty}^{+\infty}\left(\frac{(N+2n')^2}{\eta^2}J_{N+2n'}(\eta)+\frac{1}{2\eta}[J_{N+2n'+1}(\eta)-J_{N+2n'-1}(\eta)]\right)J_{n'}(\xi)\frac{(N+2n)}{\eta}J_{N+2n}(\eta)J_n(\xi), \tag{45}$$

$$S_4=\sum_{n=-\infty}^{+\infty}\left(\frac{(N+2n)^2}{\eta^2}J_{N+2n}(\eta)+\frac{1}{2\eta}[J_{N+2n+1}(\eta)-J_{N+2n-1}(\eta)]\right)J_n(\xi), \tag{46}$$

$$\Delta_1=\tilde{\Phi}_i^+(\mathbf{q}_N)\gamma^{j+}\gamma^0 u_{\mathbf{p}}^r u_{\mathbf{p}}^{r+}\gamma^0\gamma^j\tilde{\Phi}_i(\mathbf{q}_N), \tag{47}$$

$$\Delta_2=\tilde{\Phi}_i^+(\mathbf{q}_N)\gamma^{j+}\gamma^0 u_{\mathbf{p}}^r u_{\mathbf{p}}^{r+}\gamma^0\not{k}\tilde{\Phi}_i(\mathbf{q}_N), \tag{48}$$

$$\Delta_4=\tilde{\Phi}_i^+(\mathbf{q}_N)\not{k}^+\gamma^0 u_{\mathbf{p}}^r u_{\mathbf{p}}^{r+}\gamma^0\not{k}\tilde{\Phi}_i(\mathbf{q}_N). \tag{49}$$

IV. CONSERVATION LAWS AND CLOSING OF EXCITATION CHANNELS

As expressed in Eq. (43), the transition probability is a function of the initial momentum-space probability amplitude, $\tilde{\Phi}_i(\mathbf{q}_N)$. Let us now assume injected electron of positive energy with a wave function of the form (3), therefore, $\epsilon^r=+1$. From the δ function in Eq. (43), we obtain the following energy conservation relation:

$$E+\left[\frac{e^2A_0^2}{4\hbar c^2(k\cdot p)}-N\right]\hbar\omega=E_B. \tag{50}$$

On the other hand, an additional conservation law relates the momentum of the final and initial states, $\mathbf{p}$ and $\mathbf{p}_i$, respectively, in Eq. (28). For a positive-energy electron, this reads as

$$\mathbf{p}_i=\frac{\mathbf{p}}{\hbar}+\left[\frac{e^2A_0^2}{4\hbar c^2(k\cdot p)}-N\right]\mathbf{k}. \tag{51}$$

Since the electromagnetic field propagates along the x axis, this condition may be split into two parts

$$(p_x)_i=p_x\pm\left[\frac{e^2A_0^2}{4\hbar c^2(k\cdot p)}-N\right]\hbar k, \tag{52}$$

$$(p_y)_i=p_y \tag{53}$$

with $(k\cdot p)=k(E/c\mp p_x)$, and where $k=|\mathbf{k}|$, the top sign refers to a field copropagating with the electron, and the bottom to the counterpropagating case. Equations (50), (52), and (53) describe the energy and momentum changes due to the stimulated absorption or emission of N photons. Combining these with the energy expression for the final state, $E=\sqrt{c^2p_x^2+c^2p_y^2+m^2c^4}$, we obtain a closed formula for the energy conservation of the multiphoton process, in terms of the initial momentum and the field parameters

$$N\hbar\omega[1\mp(\beta_x)_i]=\frac{c^2(p_x)_i^2+c^2(p_y)_i^2+m^2c^4-E_B^2+\frac{A_0^2e^2}{2}}{2\gamma_B mc^2}, \tag{54}$$

where we have defined the initial energy of the electron as $E_B=\gamma_B mc^2$, and the initial relativistic velocity factor, $(\beta_x)_i=(p_x)_i/\gamma_B mc$. Since $E_B\approx\epsilon_B+(\gamma_x)_i mc^2$, with $(\gamma_x)_i=\sqrt{1/[1-(\beta_x)_i^2]}$ we have

$$N\hbar\omega\gamma_B[1\mp(\beta_x)_i]\approx\frac{(p_y)_i^2}{2m}-(\gamma_x)_i\epsilon_B+\frac{A_0^2e^2}{4mc^2}. \tag{55}$$

The interpretation of this energy conservation relation is straightforward if we take as reference system a frame propagating with the electron, with its initial velocity $(p_x)_i/(\gamma_x)_i m$. The frequency $\omega'=\omega\gamma_B(1\mp(\beta_x)_i)$ corresponds to the Doppler-shifted electromagnetic wave, and $(\gamma_x)_i\epsilon_B$ is the result of the Lorentz transform of the bound-state energy (11) from the laboratory to the moving frame. Equation (55) states the resonance condition for a (Doppler-shifted) N-photon transition from a bound state to a state lying in the crystal quasicontinuum of momentum given by Eqs. (52) and (53). This is, in essence, the crystal equivalent to the ionization of atoms by intense fields. Note, however, that in the atom case an ionization channel for any photon number N is always possible, since the initial state is distributed continuously over the momentum space and, therefore, a nonzero transition probability exists for any $(p_y)_i$ which fulfills the condition (55). This is not the case for the channeled electron, since the crystal plane periodicity forces a discretization of the electron states in the transverse coordinate of the momentum space $p_y=\tilde{n}2\pi h/d_p=\tilde{n}\Delta p_y$, being $\tilde{n}$ an integer and d_p the interplanar distance. As a consequence, in the general case the N-photon channel of excitation should be strongly suppressed, except in those particular cases in which

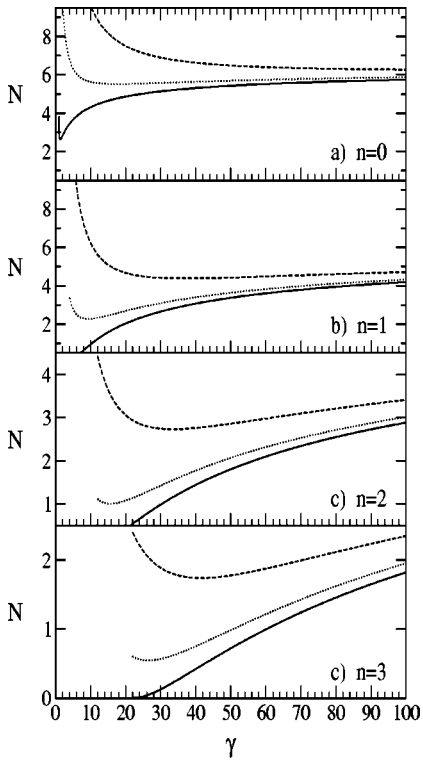

FIG. 2. This figure shows the values of $(\gamma_x)_i$ needed to open a specific N-photon excitation channel for the lowest orders of transverse momentum transferred $\tilde{n}$. All the pictures correspond to an electron channeled along the (110) plane of silicon, driven by a counterpropagating linear polarized Ti:sapphire laser, $\lambda = 800$ nm, of 3.51×10^{12} W/cm^2. The different pictures represent distinct initial bound states of the interplanar potential. The top one corresponds to the ground state case ($n=0$), the second picture to the first excited state ($n=1$), and so on. The continuous, dotted, and dashed lines represent the excitation process with zero, one, and two quanta of transverse momentum transferred ($\tilde{n}=0,\pm1,\pm2$), respectively.

$$N+\frac{(\gamma_x)_i\epsilon_B}{\hbar\omega'}\approx\frac{1}{\hbar\omega'}\left(\frac{\tilde{n}^2\Delta p_y^2}{2m}+\frac{A_0^2e^2}{4mc^2}\right) \tag{56}$$

holds for N and $\tilde{n}$ as integer numbers. As a consequence, this opens the possibility of selective excitation of channeled electrons in terms of their initial velocity, or permits its control through the variation of the electromagnetic field parameters. Figures 2(a)–2(d) show the possible N-photon channel excitations as a function of the initial electron's energy and for the lowest orders of transverse momentum transferred $\tilde{n}$. Each plot shows the result for a different initial channeling bound state. We assume planar channeling along the (110) plane of Si by selecting the potential parameters $V_0 = 20.4$ eV and $b=0.03$ nm reproducing [25], and a counterpropagating TiSa laser of 3.51×10^{12} W/cm^2 ($\lambda\simeq800$ nm). Note that the number of photons N should be an integer quantity, therefore, the figure shows clearly that, except for very particular choices of the electron's initial energy, the excitation channels are closed.

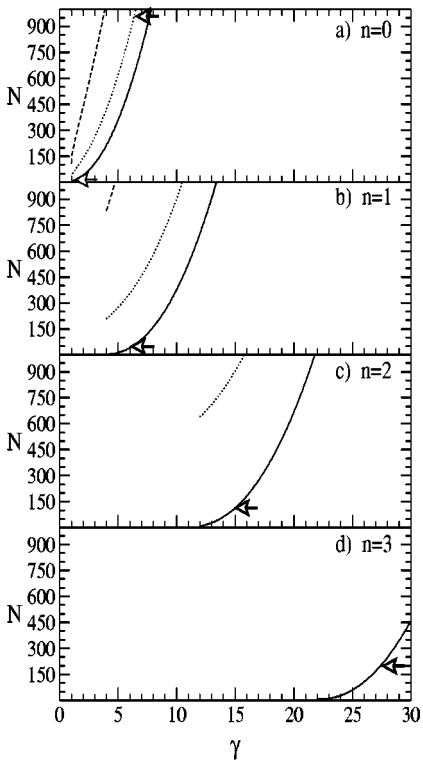

FIG. 3. The same situation as in Fig. 2 but with the external electromagnetic field copropagating with the channeled electron. The arrows represent the N_{Limit} for each transition line. The thinner arrow in (a) must only be considered as a qualitative estimation (see text). The transition lines without arrow mean that the N_{Limit} occurs for parameters beyond the plotted region.

The same figure can be done for the case of a copropagating electromagnetic field. Figures 3(a)–3(d) show again the possible N-photon channel excitations as a function of the initial electron's energy, for the lowest orders of transverse momentum transferred assuming the same crystal and laser parameters as in Fig. 2. An important increase of the number of photons needed to excite the electron, attributable to the Doppler redshift, can be observed. Under this circumstance, even when the energy and momentum constraints are fulfilled, the process may involve a very small transition probability due to the high number of photons needed. To give an idea of the order of this probability one can make use of the asymptotic expansion of the Bessel functions for large orders, $J_n(x)\approx(1/\sqrt{2\pi n})(n_e x/2n)^n$ [26], being $n_e = \lim_{n\to\infty}(1+1/n)^n$, to calculate a limit of the number of photons above which the transition probability will be negligible. The criterion to be used here will be to consider negligible the Bessel function when $n_e|x|/2n_{Limit}\leqslant0.1$. Applying it to our case one obtain the limit of the number of photons for each transverse momentum transferred as a function of the final energy of the electron

$$N_{Limit}=\frac{5n_e|e|A_0}{mc\hbar\omega'}\left(|\tilde{n}|\Delta p_y+\frac{|e|A_0}{2c}\right), \tag{57}$$

where $\omega'=\omega\gamma(1-\beta_x)$. Assuming that the electron finishes in a state of the crystal quasicontinuum and that the energy along the axial direction do not change significantly during

the evolution, one can approximate $\gamma \approx (\gamma_x)_i$ and $\beta_x \approx (\beta_x)_i$. Consequently ω', and therefore, N_{Limit}, can be expressed as a function of the initial electron's energy. Figures 3(a)–3(d) show with arrows the point when the number of photons required for the excitation process surpass the N_{Limit}. For energies above this point, the transition probability reduces drastically and we can consider that no excitation takes place, even though the energy and momentum conservation relations may be fulfilled. It should be pointed out that Eq. (57) is only valid for the case of large orders in the Bessel functions. This means that it should be taken qualitatively in all cases in which N_{Limit} is a small quantity, as, for instance, in Fig. 3(a) for the case $\tilde{n}=0$. Note also that those cases in which the arrow is not shown correspond to N_{Limit} outside the plotting region, i.e., the N-photon excitation is possible along the complete plotted line.

Finally, let us remark the fact that the photon excitation number is greater than in the counterpropagating case increases the sensitivity of the channel process to the selective excitation in terms of the laser parameters.

V. CONCLUSIONS

We have computed the explicit forms of the S-matrix transition probabilities for the N-photon absorption of a relativistic electron channeled along a crystal plane. In contrast to previous works, we consider the interaction with an intense electromagnetic wave, generated externally, which may excite the electron to high-energy states lying in the crystal quasicontinuum. Due to the crystal periodicity, we show that the energy and momentum conservation equations constraint strongly this excitation process, suppressing the multiphoton absorption except for some particular cases. Under these circumstances, the selection of a single multiphoton channel of excitation is feasible by an adequate choice of the external laser parameters, opening a broad range of possibilities for the coherent control of the channel electron's dynamics. The case of an electromagnetic field copropagating with the injected electron is also studied showing an important increase of the number of photons needed to excite the electron due to the Doppler redshift. For this case, we give an estimation of the maximum photon number for which the excitation process is not negligible. The selective excitation in the copropagating case is found to be more sensitive to the electron's energy and the transverse momentum transferred in the transition than in the counterpropagating one.

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