Universal continuous-variable quantum computation: Requirement of optical nonlinearity for photon counting

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Although universal continuous-variable quantum computation cannot be achieved via linear optics (including squeezing), homodyne detection, and feed-forward, inclusion of ideal photon-counting measurements overcomes this obstacle. These measurements are sometimes described by arrays of beam splitters to distribute the photons across several modes. We show that such a scheme cannot be used to implement ideal photon counting and that such measurements necessarily involve nonlinear evolution. However, this requirement of nonlinearity can be moved ''off-line,'' thereby permitting universal continuous-variable quantum computation with linear optics.

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I. INTRODUCTION

The chief attraction of quantum computation is the possibility of solving certain problems exponentially faster than any known method on a classical computer [1], and a significant effort is underway to realize a physical quantum computer [2,3]. Optical realizations of a quantum computer are particularly appealing because of the robust nature of quantum states of light against the effects of decoherence as well as the advanced techniques for state preparation, photon manipulation, and photodetection. Both discrete-variable (qubit-based) [4–6] and continuous-variable (CV) [7] schemes offer significant potential as optical quantum computers. However, the lack of a strong optical nonlinearity is a considerable hurdle for optical quantum computation [3].

A proposal by Knill, Laflamme, and Milburn (KLM) [5] describes how the measurement of photons can be employed to induce a nonlinear transformation in a qubit-based optical quantum computer and how this procedure can be done efficiently in a nondeterministic way. These remarkable results suggest that measurement in a CV system may be used to induce nonlinear evolution as well (although measurements of CV observables may not be possible in the von Neumann sense; see [8]). A scheme proposed by Gottesman, Kitaev, and Preskill (GKP) [6] uses photon-number measurement to induce a nonlinear transformation. If photon counting overcomes the obstacle of creating optical nonlinearities, then CV quantum computation may be feasible, just as the qubit-based linear optical quantum computer may be feasible [5]. Here we present three key results: (1) universal quantum computation over continuous variables can be achieved using linear optics, homodyne measurement with feed-forward, and photon counting; (2) the desired photon-counting projective measurement cannot be performed using linear optics and existing photodetectors, and necessarily involves an optical nonlinearity; and (3) the nonlinear transformations can be brought ''off-line'' to prepare quantum resources for a linear optical CV quantum computation to succeed in a deterministic way.

This paper is outlined as follows. We first review the Clifford group for continuous variables, which consists of linear optics transformations (including squeezing). The construction of a cubic phase state of GKP is outlined as a possible means to implement a nonlinear transformation, necessary for universal CV quantum computation. This gate requires photon-counting measurements, and we demonstrate that linear optics, homodyne measurement, and ideal photodetection are insufficient to implement such schemes. A nonlinear interaction is shown to be a necessary component of any photon-counting measurement. An analysis of truncated Hilbert spaces is included, and the paper concludes with a discussion on the use of photon-counting measurements (and their associated nonlinear transformations) off-line in a CV quantum computation.

II. CLIFFORD-GROUP TRANSFORMATIONS

The requirement of nonlinear transformations for universal CV quantum computation [7] can be understood by considering n harmonic oscillators, corresponding to n independent optical field modes, with annihilation operators $\{\hat{a}_i;\, i=1,\ldots,n\}$. Linear optical transformations of these modes are described by unitary phase-space displacements (by mixing with ''classical fields'' at beam splitters)

$$D_i(\alpha)=\exp(\alpha\hat{a}_i^\dagger-\alpha^*\hat{a}_i), \tag{1}$$

with $\alpha\in\mathbb{C}$; these transformations comprise the Heisenberg-Weyl group HW(n). For a classical pump field, parametric amplification invokes one-mode squeezing operations

$$S_i(\eta)=\exp[\tfrac{1}{2}(\eta\hat{a}_i^{\dagger 2}-\eta^*\hat{a}_i^2)] \tag{2}$$

and two-mode squeezing operations

$$S_{ij}(\eta)=\exp[\tfrac{1}{2}(\eta\hat{a}_i^\dagger\hat{a}_j^\dagger-\eta^*\hat{a}_i\hat{a}_j)] \tag{3}$$

for $\eta\in\mathbb{C}$ [9]. (Although squeezing utilizes an optical nonlinearity of order two or higher, the transformation is regarded as being linear because the resultant Heisenberg operator equations of motion are linear.) Squeezing operations (both one and two mode) generate the symplectic group $\mathrm{Sp}(2n,\mathbb{R})$.

The squeezing operation $S_i(\eta)$ with η real maps the canonical position as

$$S_i(\eta):\hat{q}_i=\sqrt{\hbar/2}(\hat{a}_i+\hat{a}_i^\dagger)\rightarrow\exp(-\eta)\hat{q}_i; \tag{4}$$

thus, the infinitely squeezed displaced vacuum

$$\lim_{\eta\rightarrow\infty} S_i(\eta)D_i(q/\sqrt{2\hbar})|0\rangle, \tag{5}$$

with $q\in\mathbb{R}$, is the (unnormalizable) position eigenstate $|q\rangle_i$. These position eigenstates are often employed as a computational basis for CV quantum computation, and are approximated in experiment by finite squeezing [7]. Two-mode squeezing $S_{ij}(\eta)$ acts in a similar fashion on the normal and antinormal modes $\hat{a}_i\pm\hat{a}_j$, and the infinitely squeezed two-mode vacuum

$$|\Theta\rangle_{ij}=\lim_{\eta\rightarrow\infty} S_{ij}(\eta)|0\rangle, \tag{6}$$

where $\eta\in\mathbb{R}$ is the EPR state satisfying

$${}_{ij}\langle qq'|\Theta\rangle_{ij}=\delta(q-q'). \tag{7}$$

Two-mode squeezing also allows us to implement a unitary SUM gate [6,10], defined as

$$(\mathrm{SUM})_{ij}=\exp\left(-\frac{i}{\hbar}\hat{q}_i\hat{p}_j\right)=\exp[\tfrac{1}{2}(\hat{a}_i^\dagger+\hat{a}_i)(\hat{a}_j^\dagger-\hat{a}_j)]. \tag{8}$$

This gate acts on the computational basis of position eigenstates according to

$$(\mathrm{SUM})_{ij}:\quad |q_i\rangle_i|q_j\rangle_j\rightarrow|q_i\rangle_i|q_i+q_j\rangle_j. \tag{9}$$

The ith mode is referred to as the control and the jth mode as the target.

Phase-space displacements and squeezing together give rise to a finite-dimensional group known as the Clifford group [6,10]. For n modes, the Clifford-group is the semidirect product group $[\mathrm{Sp}(2n,\mathbb{R})]\mathrm{HW}(n)$ generated by all Hamiltonians that are inhomogeneous quadratics in the canonical operators $\{\hat{a}_i,\hat{a}_i^\dagger,i=1,\ldots,n\}$. The above unitary representation of the Clifford group is a subgroup of all unitary transformations on n modes. As such, they are insufficient to generate arbitrary unitary transformations and thus cannot perform universal quantum computation. The addition of a nonlinear operation such as that provided by the $\chi^{(3)}$, or optical Kerr, nonlinearity [11] suffices, in principle, to perform universal CV quantum computation, but is not feasible in quantum optical implementations due to the lack of sufficiently strong nonlinear materials with low absorption. However, as stressed by Lloyd and Braunstein [7], *any* nonlinear coupling on a *single mode* could allow for universal CV quantum computation, as opposed to the qubit case where a nonlinear coupling between qubits is required.

III. THE CUBIC-PHASE GATE

One is led to ask whether measurements can be used to induce nonlinear evolution, following the related example for qubit-based linear-optics quantum computation [5]. First, we consider Clifford-group transformations conditioned on the results of projective-valued measurements (PVMs) in the computational basis (i.e., von Neumann measurements in the basis $\{|q\rangle,\ q\in\mathbb{R}\}$). As shown in [10], such measurements and feed-forward are efficiently simulatable on a classical computer and thus (under the assumption that universal quantum computation is *not* efficiently simulatable classically) are also insufficient for universal quantum computation. These results also include a more realistic computational basis of finitely squeezed states and realistic homodyne measurement of quadratures. Thus, in the following, we will consider Clifford-group transformations conditioned on homodyne measurements to be part of "linear optics."

Possibly, measurements in a different basis can be employed to induce a nonlinear transformation. Such a scheme has been proposed (in the context of quantum computation with finite-dimensional qudits rather than CVs) by GKP using measurements of photon number. Specifically, these measurements are described by a PVM,

$$\{\Pi_n=|n\rangle\langle n|,\quad n=0,1,2,\ldots\}, \tag{10}$$

for a single oscillator, where $|n\rangle$ is the eigenstate of the number operator $\hat{N}=\hat{a}^\dagger\hat{a}$ with eigenvalue n. In what follows, we refer to this PVM as the *photon-counting PVM*. The scheme of GKP is briefly outlined in the following, and relies on the creation of a so-called "cubic-phase state" $|\gamma\rangle$, which is the (unnormalizable) state defined as

$$|\gamma\rangle=\int dq\,\exp(i\gamma q^3)|q\rangle. \tag{11}$$

The cubic-phase state $|\gamma\rangle$ can be prepared using squeezing, phase-space displacement, and photon counting. Consider the two-mode squeezed vacuum state $S_{12}(\eta)|0\rangle$, $\eta\in\mathbb{R}$, and a large momentum displacement of the first mode, to obtain the two-mode state

$$|w,\eta\rangle=D_1(iw)S_{12}(\eta)|0\rangle, \tag{12}$$

with $w\in\mathbb{R}$. By performing a measurement of the photon-number (described by the photon-counting PVM) on the first mode, a measurement result of n photons projects the second mode of the pair into a cubic-phase state $|\gamma'\rangle$ to a good approximation if w is sufficiently large (details can be found in [6,9]), where $\gamma'\propto n^{-1/2}$. This state can be transformed into $|\gamma\rangle$ with γ of order unity using one-mode squeezing.

The cubic phase state can be used to implement a nonlinear transformation on an arbitrary state $|\psi\rangle_i$ of an optical mode as follows [6]. A $(\mathrm{SUM})_{ij}^{-1}$ gate [see Eq. (8)] is executed with $|\psi\rangle_i$ as the control and $|\gamma\rangle_j$ as the target. A position measurement is performed on the target, projecting the control into the state

$$|\psi'\rangle_i = {}_j\langle q=a|(\text{SUM})_{ij}^{-1}|\psi\rangle_i|\gamma\rangle_j = \exp[i(\hat{q}_i+a)^3]|\psi\rangle_i \tag{13}$$

for a measurement outcome a. Invoking the Clifford-group transformation

$$\begin{aligned} U(a) &= \exp[i\hat{q}_i^3 - i(\hat{q}_i+a)^3] \\ &= \exp(-ia^3/4)\exp[-3ai(\hat{q}_i+a/2)^2] \end{aligned} \tag{14}$$

(which can be implemented using linear optics) on the state $|\psi'\rangle_i$ gives a net transformation equivalent to applying

$$V_\gamma = \exp(i\gamma\hat{q}_i^3) \tag{15}$$

on $|\psi\rangle_i$. We refer to the transformation V_γ, implemented in this manner, as the *cubic-phase gate*. This nonlinear transformation could be used in combination with Clifford-group operations to perform universal quantum computation over continuous variables.

What is fundamentally important about this result is that *all* transformations involved, including (1) the preparation of the two-mode squeezed state, (2) the transformations needed to prepare $|\gamma\rangle$, (3) the $(\text{SUM})^{-1}$ gate, and (4) the transformation $U(a)$ (which is quadratic in $\hat{q}$), are implementable using Clifford-group (linear-optics) transformations. The resulting cubic-phase gate is fully deterministic. The key component that allows for the nonlinear transformation is the measurement of photon number. In the following, we argue that such a measurement possesses "hidden" nonlinear evolution (i.e., the equivalent of an optical Kerr or higher-order nonlinearity), and we can make this nonlinearity explicit.

IV. PHOTON COUNTING

Due to the classical simulatability results of [10], any CV quantum-information process that initiates with finitely or infinitely squeezed vacua and employs only Clifford-group transformations (phase-space displacements and one- and two-mode squeezing), homodyne detection, and classical feed-forward can be simulated efficiently on a classical computer. If the cubic-phase gate leads to universal quantum computation with continuous variables, then it must not satisfy the conditions of this theorem. (Otherwise, quantum computation could be simulated efficiently on a classical computer, which is believed to be impossible.) The key component of the cubic-phase gate that does not satisfy the conditions of this theorem is the photon-counting PVM; thus, we conclude that the photon-counting PVM cannot be implemented using only Clifford-group transformations and homodyne measurement.

In the following, we show that this transformation cannot be implemented even with the addition of photodetectors. We demonstrate that the photon-counting PVM necessarily requires nonlinear evolution.

Photon counting using threshold detectors

The photon-counting PVM employed by GKP consists of projections in the Fock-state basis $\{|n\rangle, n=0,1,\ldots\}$. For such a measurement to be performed, one requires photodetectors that can measure the number of photons in a mode, i.e., distinguish the state $|n\rangle$ from $|n'\rangle$, $n\neq n'$. We refer to such a photodetector as *discriminating*. However, such discriminating photodetectors do not yet exist [12]. All photodetectors in current use effectively measure whether there are no photons ($n=0$), or at least one photon ($n>0$) in a mode. (Note that a discriminating photodetector, on the other hand, must be able to count intrinsically indistinguishable photons, that is, propagating photons in the same temporal, spatial, and polarization mode [13].) We refer to existing photodetectors as *threshold detectors* and to a unit-efficiency detector as an *ideal threshold detector* (ITD). The PVM for an ITD is

$$\{\Pi_0 = |0\rangle\langle 0|, \quad \Pi_{>0} = \hat{I} - |0\rangle\langle 0|\}, \tag{16}$$

where $\hat{I}$ is the identity operator. The second projector $\Pi_{>0}$ projects onto the infinitely large subspace of states with one or more photons due to saturation of the threshold detector [14].

It is possible to use ITDs to distinguish up to a finite number k of photons by using linear optics to couple the input mode to multiple ITD modes [15]. For example, one could use an array of beam splitters to distribute the photons of the input state over N modes such that it is highly unlikely that more than one photon is in any of the N modes [5,15]. ITDs are then used at each mode, and the probability of undercounting photons is at most $k(k-1)/2N$. For small k (as in KLM), the photon-counting PVM can be approximated with high probability by using a sufficiently large N.

In a related fashion, the visible-light photon counter (VLPC) [16] has been constructed to discriminate between one and two photons with a high degree of confidence, but the measurement does not correspond to the photon-counting PVM; rather the VLPC is effective at distributing photons throughout the photosensitive region, with localized regions acting as threshold devices (as with standard photodetectors). In other words, the single-mode input field is distributed amongst many localized modes in the photodetector, each region operating as a threshold photodetector. Consequently the VLPC has much in common with the proposed detection of photons via arrays of beam splitters to split the signal field (as discussed above), with an ITD existing at each output port.

Whereas the use of multiple ITDs and linear optics approximates a discriminating photodetector if the Hilbert space can be truncated as in qubit-based linear optical quantum computation, this scheme breaks down for CV quantum computation. Without *a priori* knowledge of the maximum number of photons in a mode (for CV, this number is infinite), one would require an infinite number of auxiliary modes and ITDs. (Below, we discuss resource issues even if the Hilbert space is truncated.) Thus, the photon-counting PVM *cannot* be performed using linear optics (Clifford-group transformations), homodyne measurement, and a finite number of ITDs.

Photon counting with nonlinear evolution

In order to implement the photon-counting PVM, it is illustrative to employ a model of photodetection based on homodyne measurement and nonlinear optics (i.e., by employing a Hamiltonian that is cubic or higher in the photon creation and annihilation operators). In this model, a measurement of the phase shift in a probe field allows for a quantum nondemolition measurement of the photon number in the signal field [17]. Consider the probe field to be a coherent state with large amplitude. We interact this probe field with an arbitrary signal field $|\psi\rangle$ in a Kerr medium with interaction Hamiltonian

$$\hat{H}_{\rm int}=\hbar\chi\hat{N}_{\rm signal}\hat{N}_{\rm probe}, \tag{17}$$

where χ is proportional to the third-order nonlinear susceptibility. After an interaction time t, homodyne measurement is then used to infer the phase shift ϕ in the probe field. Although such a homodyne measurement does not project the probe field into a phase state, one can, in principle, project to a state with arbitrarily small uncertainty $\Delta\phi$ in phase. A particular value of ϕ can be used to infer the photon number $n=\phi/(\chi t)$ to the nearest integer value, with corresponding uncertainty $\Delta n=\Delta\phi/(\chi t)$. However, the photon number is only obtained modulo $N=2\pi/(\chi t)$; the periodicity of the phase does not give a true photon-number measurement [11]. This measurement projects the signal field $|\psi\rangle$ into the subspace spanned by the number states $|n_j\rangle$, where $n_j=(\phi+2\pi j)/\chi$.

To implement the photon-counting PVM without issues of periodicity, we can couple the signal field to a pointer with an unbounded domain, such as the position q or momentum p of a probe. For example, the radiation pressure on a mirror is proportional to the flux of (monochromatic) photons that strike it, and a suitable coupling Hamiltonian would be [18]

$$\hat{H}_{\rm int}=\lambda\hat{N}_{\rm signal}\hat{q}_{\rm probe}, \tag{18}$$

which is also nonlinear. For a probe field initially in the momentum eigenstate $|p=0\rangle$, after an interaction time t, a measurement of momentum p of the probe collapses the probe field into a momentum eigenstate $|p\rangle$ and thereby the signal field into a number state with $n=p/(\lambda t)$ (again to the nearest integer value).

The resulting photon-number measurement in either scheme will carry with it an error (related to measurement precision, and converting from continuous to discrete quantities). GKP require that $\Delta n \ll n^{1/3}$ for a functioning cubic phase gate; this condition places limits on the acceptable measurement errors.

Thus, measurement of photon number can be described as a nonlinear interaction plus homodyne measurement. This result gives insight into the reason why such measurements can induce a nonlinear transformation. Specifically, this model of photon-number measurement is excluded from the conditions for efficient classical simulation [10].

Truncation of photon number

Naturally, one should be suspicious of the periodicity involved in the first measurement scheme [employing the Kerr interaction of Eq. (17)] and also of the unbounded nature of the second scheme [employing Eq. (18)]. Any physical realization of a CV quantum-information process must have finite energy, and thus the Hilbert space can effectively be truncated at some highest energy with photon number $n_{\max}$. The issue of energy arises in the second scheme as well, where the momentum increases without bound and the energy is not bounded above or below. Even before infinite energy becomes an issue, a relativistic description must be applied. Coupling to momentum instead of the position does not eliminate these difficulties: the displacement itself must also be bounded by the physical boundaries of the laboratory. Truncation of the Hilbert space is an option that, however, presents challenges [19]; for example, the system is no longer described by continuous variables but rather by large-dimensional qudits (the generalization of qubits to higher dimensions).

An advantage of a truncated Hilbert space is that the linear-optics scheme involving finite-N ITD modes becomes well defined. Implementing these photon-counting-measurement schemes thus becomes a matter of resources. Consider the linear-optics measurement scheme for maximum photon number $n_{\max}$. The probability of undercounting k photons is at most $k(k-1)/2N$, for N the number of ITD modes. Thus, for a fixed probability of undercounting, the required number of ITD modes scales as $N\propto n_{\max}^2$, although detector inefficiencies somewhat complicate the issue. For the (nonlinear) photon-number-measurement scheme involving the Kerr interaction of Eq. (17), however, one does not require additional modes or detectors, so this quadratic scaling does not apply. All that is needed for the Kerr interaction scheme is an increase in phase resolution that behaves as $\Delta\phi\propto n_{\max}^{-1}$. Similar resolution arguments apply to the position-pointer scheme. Thus, even if one were to consider a truncated Hilbert space, it may be more practical to employ a nonlinear measurement scheme rather than a multimode ITD scheme; this result may be true for the KLM and GKP schemes as well. For true CV quantum computation, however, multiple ITD arrays cannot suffice, and nonlinear evolution is a *necessary* component of photon-number measurement.

V. CONCLUSIONS

Despite the need for nonlinear evolution for photon counting, the cubic-phase gate has a considerable advantage over the use of nonlinear interactions directly. Specifically, this gate can be used to remove the use of nonlinear materials from the computation and utilize them only in the preparation of cubic-phase states. In other words, the photon-number measurement can be performed ''off-line,'' and the cubic-phase states can be viewed as a quantum resource to be prepared prior to the computation. This way, the states used in the the computation need not pass through any optical Kerr nonlinearities with their high absorption, thus avoiding the loss associated with using such materials. Also, if the procedure for producing cubic-phase states possesses noise or other sources of error, imperfect cubic-phase states can be purified to produce a smaller number of states with higher fidelity [6]. Again, an advantage is that this purification can be done off-line and is not part of the computation. This concept is similar to the KLM scheme, where the ''difficult'' gates are implemented off-line on suitable ancilla states and then quantum teleported onto the encoded states when needed. In our scheme, one simply prepares a sufficient number of cubic-phase states prior to the computation, and the entire process may then occur using only linear optics and homodyne measurement. A key advantage of this scheme is that the teleportation can be performed deterministically.

In summary, we have shown that universal CV quantum computation can be obtained using linear optics (phase-space displacements and squeezing), homodyne measurement with classical feed-forward, and a realization of the photon-counting PVM. We describe the PVM for current (ideal) photodetectors, and demonstrate that such detectors cannot be used to implement the photon-counting PVM with linear optics alone. This photon-counting PVM carries with it implicit nonlinear evolution, and we discuss how it can be implemented in a CV system using a Kerr interaction (or another nonlinear Hamiltonian) and homodyne measurement. The resource requirements of this measurement scheme compared with using linear optics and current photodetectors are outlined. Finally, an advantage of this scheme is its use in the nonlinear gate of GKP, which removes the nonlinear operations from the computation and reduces them to ''off-line'' preparation of ancilla states. These results place the implementation of strong nonlinear CV quantum gates, and thus universal CV quantum computation, in the realm of experimental accessibility.

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[1] P. W. Shor, in *Proceedings of the* 35*th Annual Symposium on Foundations of Computer Science* (IEEE Press, Los Alamitos, CA, 1994).
[2] A. Steane, Rep. Prog. Phys. **61**, 117 (1998).
[3] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[4] I. L. Chuang and Y. Yamamoto, Phys. Rev. A **52**, 3489 (1995).
[5] E. Knill, R. Laflamme, and G. J. Milburn, Nature (London) **409**, 46 (2001).
[6] D. Gottesman, A. Kitaev, and J. Preskill, Phys. Rev. A **64**, 012310 (2001).
[7] S. Lloyd and S. L. Braunstein, Phys. Rev. Lett. **82**, 1784 (1999).
[8] M. Ozawa, in *Quantum Communication, Computing, and Measurement*, edited by O. Hirota, A. S. Holevo, and C. M. Caves (Plenum Press, New York, 1997), p. 233.
[9] C. M. Caves, C. Zhu, G. J. Milburn, and W. Schleich, Phys. Rev. A **43**, 3854 (1991).
[10] S. D. Bartlett, B. C. Sanders, S. L. Braunstein, and K. Nemoto, Phys. Rev. Lett. **88**, 097904 (2002).
[11] G. J. Milburn and D. F. Walls, Phys. Rev. A **28**, 2065 (1983).
[12] K. J. Resch, J. S. Lundeen, and A. M. Steinberg, Phys. Rev. A **63**, 020102(R) (2001).
[13] K. J. Blow, R. Loudon, S. J. D. Phoenix, and T. J. Shepherd, Phys. Rev. A **42**, 4102 (1990).
[14] P. L. Kelley and W. H. Kleiner, Phys. Rev. **136**, A316 (1964).
[15] P. Kok and S. L. Braunstein, Phys. Rev. A **63**, 033812 (2001).
[16] J. Kim, S. Takeuchi, Y. Yamamoto, and H. H. Hogue, Appl. Phys. Lett. **74**, 902 (1999).
[17] B. C. Sanders and G. J. Milburn, Phys. Rev. A **45**, 1919 (1992).
[18] A. F. Pace, M. J. Collett, and D. F. Walls, Phys. Rev. A **47**, 3173 (1993).
[19] V. Bužek, A. D. Wilson-Gordon, P. L. Knight, and W. K. Lai, Phys. Rev. A **45**, 8079 (1992).