Scheme for the preparation of multiparticle entanglement in cavity QED

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(Received 31 May 2001; published 14 March 2002)

Here we present a quantum electrodynamics model involving a large-detuned single-mode cavity field and n identical two-level atoms. One of its applications for the generation of multiparticle entangled states of various kinds (Greenberger-Horne-Zeilinger states and different class of so-called W states) is analyzed. The theoretical prediction for the model of $n=2$ is made that is consistent with the experimental result by considering the possible three-atom collisions.

DOI: 10.1103/PhysRevA.65.042102 PACS number(s): 03.65.Ud, 03.67.Lx, 42.50.Dv

I. INTRODUCTION

Quantum entanglement, first noted by Einstein, Podolsky, and Rosen (EPR) [1] and Schrödinger [2], is one of the essential features of quantum mechanics. Its famous embodiment $\Phi^{\pm}=(1/\sqrt{2})(|11\rangle \pm |00\rangle), \Psi^{\pm}=(1/\sqrt{2})(|10\rangle \pm |01\rangle)$ was shown by Bell [3] to have stronger correlations than allowed by any local hidden variables theory. It is known that the analysis of entanglement and its properties for multiparticle states is much more complex than that for bipartite states. The Greenberger-Horne-Zeilinger (GHZ) state [4,5] $\Phi^{ABC}=(1/\sqrt{2})(|111\rangle + |000\rangle)$, a canonical three-particle entanglement state, exhibits the contradiction between local hidden variables theories and quantum mechanics even for nonstatistical predictions, as opposed to the statistical ones for the EPR state.

There have been many papers discussing multiparticle entanglement and its applications [6,7]. In paper [8], the authors have proved that there exists another kind of genuine tripartite entangled W states such as $W=(1/\sqrt{3})(|001\rangle + |010\rangle + |100\rangle)$, which is inequivalent to the GHZ state in the sense that they cannot be converted to each other even under stochastic local operations and classical communication (SLOCC); that is, through LOCC but without imposing that it should be achieved with certainty [9].

The GHZ state is maximally entangled in several senses [10]. For instance, it maximally violates Bell-type inequalities; the mutual information of measurement outcomes is maximal; it is maximally stable against (white) noise, and one can locally obtain an EPR state shared between any two of the three particles from a GHZ state with unit probability. On the other hand, when any one of the three qubits is traced out the remaining two qubits result in a separable state, which means the entanglement of the GHZ state is fragile in the sense of particle loss.

Conversely, the entanglement of the W state has the highest degree of endurance against qubit loss, which is argued as an important property in those situations where any one of the three particles decides not to cooperate with the others [8]. Its general form is $W_n=(1/\sqrt{n})|n-1,1\rangle$, where $|n-1,1\rangle$ denotes all the totally symmetric states including $n-1$ zeros and 1 one. For this state, the concurrence, which is related to the formation entanglement, of any reduced density operators $\rho_{k.u}$, $C_{k,u}(\rho_{k.u})$, equals $2/n$, which indicates the maximal entanglement achievable for any reduced two particles of system in any pure state [8,11]. As the states that could be converted to each other under SLOCC belong to the same class, there are at least two inequivalent classes of multiparticle entanglement states: the GHZ-state class and the W-state class [8].

Recently, it has been realized that quantum entanglement plays a key role in many quantum applications such as quantum teleportation [12], and quantum computation [13], quantum cryptography [14]. Furthermore, multiparticle states have been shown to have many advantages over the two-particle Bell states in quantum cloning [15,16], quantum teleportation [17], and superdense coding [18]. Then the preparation of the entangled states, especially multiparticle states, becomes a critical technique in quantum information processing.

Many schemes using optical systems, nuclear magnetic resonance, cavity QED, and ion trap have been proposed for the generation of entangled states. Experimentally, two-particle entangled states have been realized in both cavity QED [19] and ion traps [20]. But in most of the previous schemes for quantum-information processing in cavity QED and ion traps, the cavity and ion motion both act as memories, which store the information of an electronic system and then transfer it back to this electronic system after the conditional dynamics. Thus the decoherence of the cavity field becomes one of the main obstacles for the implementation of quantum information in the cavity field, while in the ion traps it is very difficult to achieve the joint ground state of the ion motion and the heating of the ions. In paper [21], Sørensen and Mølmer have proposed a scheme to realize quantum computation in the ion traps via virtual vibrational excitations. The same group [22] also proposed a scheme for the generation of the GHZ states in ion traps, which looses the requirement of the full control of the ion motion and has already been implemented experimentally [23]. Recently, Zheng and Guo [24] have proposed a novel cavity QED scheme for the two-atom entanglement preparation and quantum-information processing whose experimental imple-

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mentation has also been reported by Osnahgi *et al* [25].

In this paper, we present a generalized Jaynes-Cummings (JC) model involving a single-mode cavity field and n identical two-level atoms, which results in the above cavity QED scheme for the two-atom entanglement and quantum information processing in the case of $n=2$ [24]. Its application for the generation of the multiparticle entanglement states of various kinds, including both the GHZ-state class and the novel W-state class, is analyzed. As an example, we also analyze the experiment data of the Bell-state preparation [25]. This generalized JC model does not require the transfer of quantum information between the atoms and the cavity, and the cavity is only virtually excited. Then the requirement for the quality of the cavities is greatly reduced. Its efficient decoherence time is greatly prolonged. Due to the special characters of this model, it is especially handy for the preparation of W class states.

II. THE GENERALIZED JC MODEL

Consider the model of n identical two-level atoms simultaneously interacting with a single-mode cavity field, the interaction Hamiltonian in the interaction picture can be written as

$$H_i=g\sum_{j=1}^{n}\left(e^{-i\delta t}a^{\dagger}s_j^{-}+e^{i\delta t}as_j^{+}\right), \tag{1}$$

where $s_j^{+}=|1\rangle_{jj}\langle 0|$ and $s_j^{-}=|0\rangle_{jj}\langle 1|$, with $|1\rangle_j$ and $|0\rangle_j$ ($j=1,2,\ldots,n$) being the excited and ground states of the jth atom, $a^{\dagger}$ and a^{-} are, respectively, the creation and annihilation operator for the cavity model, g is the atom-cavity coupling strength, and δ is the detuning between the atomic transition frequency w_0 and cavity frequency w. In the case of $\delta\gg g$, there is no energy exchange between the atomic system and the cavity. Then the effective Hamiltonian obtained by adiabatically eliminating the atomic coherence is given by

$$H=\lambda\left[\sum_{i,j=1}^{n}\left(s_j^{+}s_i^{-}aa^{\dagger}-s_j^{-}s_i^{+}a^{\dagger}a\right)\right], \tag{2}$$

where $\lambda=g^2/\delta$. This case can be viewed as a generalized Jaynes-Cummings model Hamiltonian describing a cavity mode interacting with n atoms. When $n=1$, the Hamiltonian,

$$H=\lambda(|1\rangle\langle 1|aa^{\dagger}-|0\rangle\langle 0|a^{\dagger}a), \tag{3}$$

which represents the far-off-resonant case of the Jaynes-Cummings model [26].

When $n=2$, the Hamiltionian,

$$H=\lambda\left[\sum_{j=1,2}\left(|1\rangle_{jj}\langle 1|aa^{\dagger}-|0\rangle_{jj}\langle 0|a^{\dagger}a\right)+\left(s_1^{+}s_2^{-}+s_1^{-}s_2^{+}\right)\right], \tag{4}$$

which has been shown to be useful in the generation of two-atom maximally entangled states, the realization of quantum controlled-NOT gates, and quantum teleportation with dispersive cavity QED [24]. Since the procedure in this scheme is essentially insensitive to thermal fields and photon decay, it opens a promising perspective for complex entanglement manipulations [25].

III. GENERATION OF MULTIPARTICLE ENTANGLED STATES

Now we consider the case of multiatom. Assume that the cavity field is initially in the vacuum state, then the Hamiltonian reduces to

$$H=\lambda\left(\sum_{j=1}^{n}\left|1\right\rangle_{jj}\left\langle 1\right|+\sum_{i,j=1,i\neq j}^{n}s_j^{+}s_i^{-}\right). \tag{5}$$

It is obvious that there is no quantum information transfer between the atoms and cavity. For the case of $n=3$, the Hamiltonian can be written as

$$H=\lambda\left[\sum_{j=1,2,3}|1\rangle_{jj}\langle 1|+(s_1^{+}s_2^{-}+s_1^{-}s_2^{+}+s_1^{+}s_3^{-}+s_1^{-}s_3^{+}+s_2^{+}s_3^{-}+s_2^{-}s_3^{+})\right]. \tag{6}$$

The first term describes the Stark shifts in the vacuum cavity, and the rest terms describe the dipole coupling between any of the two atoms induced by the cavity mode.

Assume the atoms are initially in the state $|001\rangle$, then the state evolution of the system can be represented by

$$W_3(t)=\frac{e^{-i3\lambda t}+2}{3}|001\rangle+\frac{e^{-i3\lambda t}-1}{3}(|010\rangle+|100\rangle). \tag{7}$$

With the choice of $\lambda t=(2\pi/9)$, we obtain the three-partite W states [8,27]

$$W_3=\frac{1}{\sqrt{3}}\left(e^{i(2\pi/3)}|001\rangle+|010\rangle+|100\rangle\right), \tag{8}$$

where the common phase factor $e^{-i(5\pi/6)}$ has been discarded.

In the general case of n atoms which are initially in the state $|0\rangle_{1,2,\ldots,n-1}|1\rangle_n$, the evolution of the state goes as follows:

$$W_n(t)=\frac{e^{-in\lambda t}+n-1}{n}|0\rangle_{1,2,\ldots,n-1}|1\rangle_n+\frac{e^{-in\lambda t}-1}{n}|n-2,1\rangle_{1,2,\ldots,n-1}|0\rangle_n, \tag{9}$$

where $|n-2,1\rangle_{1,2,\ldots,n-1}$ denotes the symmetric $(n-1)$-particle states involving $n-2$ zeroes and 1 one. With the different choices of the evolution time, one can get various n particle states of the W-state class. This result can be understood from the properties of the Hamiltonian: the parity bit of the state is unchanged in the evolution process governed by this Hamiltonian. Then the population becomes distributed on all the states with the same parity bit which form the state of W-state class.

Obviously, only in the case of $n \leqslant 4$ can $|(e^{-in\lambda t}+n-1/n)|$ equals $|(e^{-in\lambda t}-1/n)|$, which means we can prepare the maximal entanglement W state directly in this evolution process for this case. Generally, if we measure the nth atoms at sometime t, and get $|0\rangle_n$, the other $n-1$ atoms determinately result in the state,

$$W_{n-1}=\frac{1}{\sqrt{n-1}}|n-2,1\rangle. \tag{10}$$

Then in this way, we can get the $(n-1)$-particle maximally entangled W state with the probability of $|(\sqrt{n-1}/n)(e^{-in\lambda t}-1)|^2$, which is approximately proportional to the inverse of the atom number n and gets its maximal value with $t=\pi/n$.

Furthermore, we can also prepare the states of the GHZ class conveniently using this generalized Jaynes-Cummings model. Assume there are four atoms in the cavity, which are initially in the state $|0011\rangle$. Then the evolution processing under this four-atom Hamiltonian is

$$|\phi\rangle=\frac{1}{6}(e^{-i6\lambda t}+3e^{-i2\lambda t}+2)|0011\rangle+\frac{1}{6}(e^{-i6\lambda t}-3e^{-i2\lambda t}+2)|1100\rangle+\frac{1}{6}(e^{-i6\lambda t}-1)(|1001\rangle+|0101\rangle+|1010\rangle+|0110\rangle). \tag{11}$$

Also with the choice of $\lambda t=\pi/3$, we obtain a state belonging to the GHZ-state class

$$|\phi\rangle=\frac{e^{-i(\pi/3)}}{2}(|0011\rangle+i\sqrt{3}|1100\rangle). \tag{12}$$

Noticeably, although any n-particle W state can be generated straightforwardly in the present scheme, the m-particle GHZ state where $m \geqslant 5$ cannot be prepared directly in the similar way.

It is well known that entangled states involving higher numbers of particles can be generated from entangled states involving lower numbers of particles by employing the same procedure as entanglement swapping [28]. The basic ingredients include some entangled states involving lower numbers of particles and a Bell-state measurement device. It has been proved that there are at least two classes of inequivalent multiparticle entanglement states that could not be converted to each other under SLOCC [8]. Then in this entanglement swapping procedure, the preparation of the W class states involving higher numbers of particles must employ W class states, while the preparation of the GHZ class states involving higher numbers of particles needs GHZ class states [29]. We have shown that both classes of three-particle states can be generated in the present scheme. Bell-state measurement can also be realized in this generalized Jaynes-Cummings model of the $n=2$ case [24]. Thus any multiparticle state of either the W-state class or the GHZ-state class can be prepared in this scheme of the QED cavity.

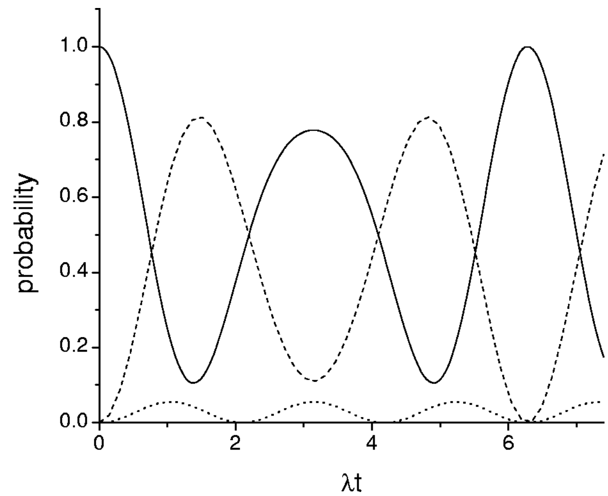

FIG. 1. The graph of the measurement probabilities $P(e_1,g_2)$, $P(g_1,e_2)$,$P(e_1,e_2)$ and $P(g_1,g_2)$ vs λt. The solid line denotes the probability $P(e_1,g_2)$, the dashed line denotes $P(g_1,e_2)$, and the dotted line denotes $P(e_1,e_2)$ and $P(g_1,g_2)$.

IV. EXPERIMENTAL IMPLEMENTATION

The discussion on the experimental matters is similar to that of paper [24]. The two atoms experiment of preparing EPR pairs using the present model of $n=2$ case has been implemented recently [25]. It is known that there are probabilities of 0.78,0.19,0.025 to have 0,1,2 atoms in one atom pulse, respectively, and events in which only one atom is detected in the two pulses are recorded. Thus in approximately 25% of these events, there are in fact two atoms in one of the pulses, one of them escaping detection. These three-atom collision events are a source of error. Then in addition to the probabilities $P(e_1,g_2)$,$P(g_1,e_2)$, there are also some spurious channels probabilities $P(e_1,e_2)$,$P(g_1,g_2)$ caused by the possible three-atom collision in the Bell-state preparation process. All these probabilities could be calculated in detail using the present multiatom model,

$$P(e_1,e_2)=P(g_1,g_2)=0.028[1-\cos(3\lambda t)],$$
$$P(e_1,g_2)=0.514+0.375\cos(2\lambda t)+0.111\cos(3\lambda t), \tag{13}$$
$$P(g_1,e_2)=0.430-0.375\cos(2\lambda t)-0.055\cos(3\lambda t),$$

where the two atom pulse are assumed initially in excited and ground-state, respectively, and the state discrimination errors are omitted. The result is shown in Fig. 1. The experimental results of paper [25] have shown the existence of $P(e_1,e_2)$ and $P(g_1,g_2)$. More elaborate experiments would reveal the oscillation of these probabilities with the interacting time.

One of the difficulties for this scheme is that this generalized Jaynes-Cummings model requires the atoms to be sent through the cavity simultaneously, otherwise there will be some error. We will show that the influence of the time difference is not as severe as expected, even assuming the third atom in the excited state enters the cavity $10\% t_0$ later than the other two ground-state atoms (the time difference between these two atoms is nonsignificant) in the generation of the W state. It is easy to see that those three atoms will be finally prepared in the state $W_3(0.90t_0)$. Again assume the third atom leaves the cavity $10\% t_0$ earlier than the other two atoms, then the final state will become

$$W_3'(0.90t_0)=\frac{e^{-i3\lambda t}+2}{3}|001\rangle+\frac{e^{-i3\lambda t}-1}{3}e^{-i0.1\lambda t_0}(|010\rangle+|100\rangle). \quad (14)$$

If we still choose $\lambda t_0 = 2\pi/9$, then

$$|\langle W_3(0.90t_0)|W_3(t_0)\rangle|^2 \simeq 0.99, \quad (15)$$

$$|\langle W_3'(0.90t_0)|W_3(t_0)\rangle|^2 \simeq 0.99.$$

Obviously the preparation operation is only slightly affected.

V. CONCLUSION

In this paper, we have presented a generalized Jaynes-Cummings model involving a single-mode cavity field and n identical two-level atoms. We consider its application for the generation of multiparticle entangled states of various kinds (GHZ states and a different class of so-called W states). We also analyzed the experiment of the $n=2$ case model [25] and explain its results by considering the possible three-atom collisions. The most distinct advantage of this model is that the cavity is initially in vacuum state and no quantum information transfer between the atoms and the cavity is required. Thus the requirement for the quality factor of the cavity is greatly reduced and then the implementation is foreseeable.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China.

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