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Localization and dispersivelike decoherence in vibronic states of a trapped ion

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In this paper we investigate the decoherence dynamics of nonclassical vibronic states of a trapped ion. We assume that the origin of decoherence is the coupling of the vibrational modes with classical stochastic electric fields and the finite lifetime of the electronic levels. We show that these interactions lead to a dispersivelike decoherence dynamics. The theoretical results agree with the measurements reported by Meekhof *et al.* [Phys. Rev. Lett. **76**, 1796 (1996)].

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In the last few years, the possibility of controlled generation and manipulation of quantum states has given new impulse to areas of research like quantum computing, quantum cryptography, and quantum information. One of the major problems to overcome in these areas is decoherence, which limits the fidelity of quantum manipulations. This phenomenon originates in the unavoidable interactions of the system of interest with the environment, which uncontrollably perturbs the system.

Cold trapped ions are considered one of the most promising systems to implement controlled manipulation of quantum states. In this case the experimental conditions are almost ideal. For this reason, decoherence as large as is observed in the vibronic Rabi oscillations of a trapped $^9Be^+$ ion [1,2] was not expected. The observed effect, in the fluorescence signal, presents an increasing damping of the oscillations as the motional excitation increases. The experimental results may be fitted by a sum where each Fock component is damped, while oscillating with its proper Rabi frequency. For this reason it was believed that the decoherence source could not give rise to transitions between different energy quantum numbers and models using dispersive interactions, such as laser amplitude noise [3] and other effects [2,4,5], have been presented in the literature. An important step in elucidating the sources of decoherence was given in Ref. [6], which showed that the spontaneous decay of the intermediate electronic state, in the Raman excitation, is responsible for part of the observed decoherence. Nevertheless, this effect is not large enough to explain either the magnitude of the vibronic damping or its motional dependence.

It is interesting to note that, up to now, the electrical noise environment was not considered to play an important role in this decoherence process. This attitude is very reasonable: the interaction of stochastic fields with the vibrational modes produces a motional heating that is linear in time [7–9], with a slow rate compared with the time scale of the Rabi oscillations. Furthermore, this type of interaction, at first view, would induce a decay of Rabi oscillations with many frequencies involved [4]. This would be in contradiction with the observed dispersivelike decoherence behavior. Contrary to this belief, we will show that the observed decoherence mainly originates in this interaction. In addition to this decoherence source, we will also take into account effects due to the finite lifetime of the intermediate electronic level [6].

Let us consider a three-level ion of charge e and mass M, confined to move in one direction of a Paul trap and oscillating with frequency ν. The ion is illuminated by two laser beams which drive a two-photon optical Raman transition between two ionic hyperfine levels $|\downarrow\rangle$ and $|\uparrow\rangle$, with splitting energy $\hbar\omega_0$, via an intermediate level $|I\rangle$, which has a natural radiative decay rate $\gamma_{\rm tot}$. The Rabi frequencies associated with the optical transitions are denoted by Ω_{L1} and Ω_{L2}. The frequencies ω_1 and ω_2 of the laser beams are chosen such that the first blue sideband is excited, $\omega_1-\omega_2=\omega_0+\nu$, and the detuning to the intermediated level is Δ [1]. This scheme gives rise to the effective interaction Hamiltonian [10]

$$H_{\rm ion}=-i\eta\hbar\Omega[\sigma^\dagger a^\dagger f(a^\dagger a;\eta)-\text{H.c.}], \tag{1}$$

where $\Omega=\Omega_{L1}\Omega_{L2}^*/\Delta$ is the vacuum Rabi frequency associated with the effective Raman transition (we took $\Omega_{L1}=\Omega_{L2}$) and η is the Lamb-Dicke parameter. The operator $\sigma=|\downarrow\rangle\langle\uparrow|$ is the electronic flip operator, $a(a^\dagger)$ is the annihilation (creation) operator associated with the oscillatory motion of frequency ν, and

$$f(a^\dagger a;\eta)=\sum_{n=0}^{\infty}(n+1)^{-1}e^{-\eta^2/2}L_n^{(1)}(\eta^2)|n\rangle\langle n|, \tag{2}$$

with $L_n^{(1)}(x)$ being Laguerre polynomials.

The influence of the classical stochastic electrical field $E(t)$ will be modeled by the introduction of a stochastic Hamiltonian $\tilde{H}(t)=i\hbar[u(t)\,a^\dagger-u^*(t)\,a]$, with $u(t)=ieE(t)\exp(i\nu t)/\sqrt{2M\hbar\nu}$ [8]. We will assume that $E(t)$ is a stationary Gaussian analytic signal, with correlation $\langle\langle E^*(t)E(t')\rangle\rangle=\langle|E|^2\rangle\exp(-|t-t'|/T)$. Working up to second order in the stochastic interaction strength, the evolution of the average density matrix reads [9]

$$\frac{d\rho(t)}{dt}=\frac{-i}{\hbar}[H_{\rm ion},\rho(t)]+\frac{1}{2\tau_1}\{[a,\rho(t)a^\dagger]+[a\rho(t),a^\dagger]\}+\frac{1}{2\tau_1}\{[a^\dagger,\rho(t)a]+[a^\dagger\rho(t),a]\}+\mathcal{K}_{\rm sp}[\rho], \tag{3}$$

where the Rabi frequencies were assumed to be much smaller than ν (resolved sideband limit) and a Markovian

approximation was possible as T is much shorter than the characteristic heating time τ_1 [8]. $\mathcal{K}_{\rm sp}[\rho]$ accounts for the effects of the finite lifetime of the intermediate electronic level [6] and its leading terms are given by

$$\mathcal{K}_{\rm sp}[\rho]=\frac{\Gamma}{2}(\{[\sigma,\rho(t)\sigma^\dagger]+[\sigma\rho(t),\sigma^\dagger]\}+\{[\sigma^\dagger,\rho(t)\sigma] +[\sigma^\dagger\rho(t),\sigma]\}+[\sigma_z\rho(t)\sigma_z-\rho(t)]). \quad (4)$$

The effective spontaneous decay rate Γ [11] and τ_1 are

$$\frac{1}{\tau_1}=\frac{e^2\langle|E|^2\rangle}{M\hbar\nu}\frac{T}{1+\nu^2T^2}, \qquad \Gamma=\frac{\gamma_{\rm tot}\Omega}{2\Delta}. \quad (5)$$

Note that the heating term in Eq. (3) is the same that would be obtained for a harmonic oscillator coupled to a stochastic field [9]. Thus τ_1 gives the motional heating rate when the lasers are not present [7].

In the following, we will show that the evolution given by Eq. (3) may lead to a behavior that mimics a dispersive decoherence. It will be convenient to work in the dressed basis $\{|\uparrow,0\rangle,|n,\pm\rangle=(1/\sqrt{2})(|\downarrow,n\rangle\mp|\uparrow,n+1\rangle),n=0,1,2,\ldots\}$. In the n subspace spanned by the vectors $|n,\pm\rangle$ the matrix elements corresponding to the density operator are written as

$$\langle n,s|\rho(t)|n,s'\rangle=\begin{pmatrix} P_n^{(+)}(t) & C_n(t) \\ C_n^*(t) & P_n^{(-)}(t)\end{pmatrix}, \quad (6)$$

where $(s,s')=(\pm)$. The population of the state that decouples from the Raman dynamics is denoted by

$$\tilde{P}_\uparrow(t)=\langle\uparrow,0|\rho(t)|\uparrow,0\rangle. \quad (7)$$

The evolution of the dressed coherences and populations can be obtained from Eq. (3). For the dressed coherences we have

$$\frac{dC_n(t)}{dt}=-i2\Omega_nC_n(t)+\frac{1}{\tau_1}\{\mathcal{L}_{\rm diff}[C_n]+\mathcal{L}_{\rm pop}[C_n]\} +\Gamma\mathcal{L}_{\rm sp}[C_n], \quad (8)$$

where the Rabi frequencies are given by [10]

$$\Omega_n=\eta\Omega\,(n+1)^{-1/2}L_n^{(1)}(\eta^2)e^{-\eta^2/2}, \quad (9)$$

while for the dressed populations we obtain

$$\frac{d\tilde{P}_\uparrow(t)}{dt}=\frac{1}{\tau_1^*}\{p_0^{(+)}(t)-\tilde{P}_\uparrow(t)\}, \quad (10)$$

$$\frac{dp_0^{(+)}(t)}{dt}=\frac{1}{2\tau_1^*}\{p_1^{(+)}(t)+\tilde{P}_\uparrow(t)-2p_0^{(+)}(t)\} +\frac{1}{\tau_1}\{p_1^{(+)}(t)-p_0^{(+)}(t)\}, \quad (11)$$

$$\frac{dp_n^{(+)}(t)}{dt}=\frac{1}{\tau_1^*}\mathcal{L}_{\rm diff}[p_n^{(+)}]+\frac{1}{\tau_1}\mathcal{L}_{\rm pop}[p_n^{(+)}]. \quad (12)$$

Here we have defined $p_n^{(\pm)}(t)=[P_n^{(+)}(t)\pm P_n^{(-)}(t)]/2$ and $\tau_1^*=\tau_1/(1+\tau_1\Gamma)$. For the initial conditions we will consider, we have $p_n^{(-)}(t)=0$ for all n. In the above equations we have introduced the operators $\mathcal{L}_{\rm diff}$, $\mathcal{L}_{\rm pop}$, and $\mathcal{L}_{\rm sp}$, which act on a set of functions $\{f_n(t)\}_{n=0}^\infty$ as

$$\mathcal{L}_{\rm diff}[f_n]=\frac{1}{2}[f_{n+1}+f_{n-1}-2f_n], \quad (13)$$

$$\mathcal{L}_{\rm pop}[f_n]=(n+1)f_{n+1}+nf_{n-1}-(2n+1)f_n, \quad (14)$$

$$\mathcal{L}_{\rm sp}[f_n]=-\frac{1}{4}\{f_{n+1}+f_{n-1}+6f_n\}. \quad (15)$$

The operator $\mathcal{L}_{\rm diff}$ corresponds to a diffusive process in a discrete space. The operator $\mathcal{L}_{\rm pop}$ is the generator for the time evolution of the populations of a harmonic oscillator subject to the action of a classical stochastic field [9]. Disregarding terms of order $\Gamma\eta^2$, the operator $\mathcal{L}_{\rm sp}$ accounts for the spontaneous emission and generates a diffusivelike process in the space of dressed coherences.

In deriving the rate equations for the coherences and populations we have assumed $\Omega_n\gg 1/\tau_1,\Gamma$, discarding the rapid oscillatory coupling terms between the coherences and the populations. Furthermore, the function $[\sqrt{n}+\sqrt{(n+1)}]^2/4$, for $n\geqslant 1$, was replaced by $(n+1/2)$. We have checked that these approximations are indeed very good and the numerical results obtained from Eqs. (8)–(12) are indistinguishable from those obtained using the full master equation [Eq. (3)].

Now we will analyze the solution of the previous set of equations for the experimental conditions of Ref. [1]. The observable measured in this experiment is the fluorescence signal $P_\downarrow(t)=\Sigma_{n=0}^\infty\langle\downarrow,n|\rho(t)|\downarrow,n\rangle$, which may be written in terms of the dressed coherences and the population $\tilde{P}_\uparrow(t)$ as

$$P_\downarrow(t)=\frac{1}{2}\left(1-\tilde{P}_\uparrow(t)+\sum_{n=0}^{\infty}[C_n(t)+C_n^*(t)]\right). \quad (16)$$

Let us consider first the experimental situation in which the ion starts in a Fock state with n_0 excitations: $|\downarrow,n_0\rangle$. As a numerical example we consider $n_0=5$. Figure 1(a) shows the behavior of the real part of the coherence $C_{n_0}(t)$. It decays exponentially while oscillating with its own free frequency Ω_{n_0}. This behavior may be very well fitted by

$$C_{n_0}(t)\simeq(1/2)e^{-i2\Omega_{n_0}t}e^{-[3/2\Gamma+2(n_0+1)/\tau_1]t}. \quad (17)$$

In this case, and for any initial small n_0 ($n_0<10$), we have found the same general behavior. These fits can be read from Eq. (8) considering that the total contribution of the first neighbors $C_{n_0\pm1}(t)$ is small. In the following we will see how this effect arises.

Figure 1(b) shows the behavior of the real parts of the coherences $C_{n_0\pm1}(t)$. They start at $C_{n_0\pm1}(0)=0$, and are initially being fed by $C_{n_0}(t)$. Therefore, they approximately oscillate in phase with the initial condition. As their amplitudes grow, they are affected by the contribution of their

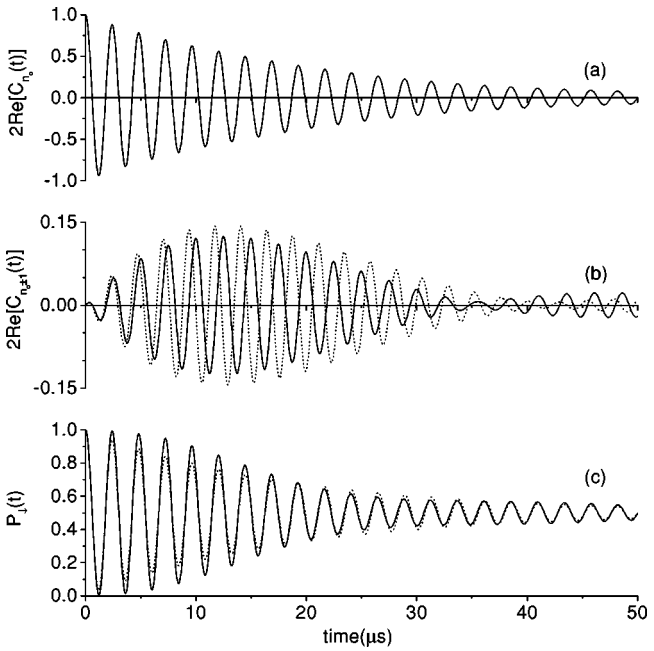

FIG. 1. Coherences and $P_{\downarrow}(t)$ for an initial Fock state $n_0=5$. (a) $2\mathrm{Re}[C_{n_0}]$; (b) full line, $2\mathrm{Re}[C_{n_0-1}]$; dotted line, $2\mathrm{Re}[C_{n_0+1}]$; (c) full line, $P_{\downarrow}(t)$; dotted line, $\{1+2\mathrm{Re}[C_{n_0}(t)]\}/2$. The parameters are the same as in the experiment of Ref. [1]: $\eta=0.202, \Omega/2\pi=475$ kHz, $\Delta/2\pi=12$ GHz, $\gamma_{\mathrm{tot}}/2\pi=19.4$ MHz. Using Eq. (18), we derived $1/\tau_1=4.14$ quanta/ms.

proper frequencies $\Omega_{n_0\pm1}$. In consequence, in a symmetrical way, their phases are continuously advancing and retarding with respect to the phase of $C_{n_0}(t)$, so that in an intermediate regime they oscillate in opposition of phase. In this manner, their contribution to the time evolution of $C_{n_0}(t)$ almost vanishes. Higher coherences $C_{n_0\pm r}$ have a similar behavior, but are less populated.

Figure 1(c) shows the experimentally observable $P_{\downarrow}(t)$. The more striking result is that this function oscillates exactly at the frequency Ω_{n_0}. This follows from the symmetrical behaviors of the $C_{n_0\pm r}$ with respect to $C_{n_0}(t)$. In this manner, the proper irreversible dynamics mimics a *dispersivelike* decoherence.

The spreading of the initial condition over the dressed coherence space is reflected in a nonexponential decay of $P_{\downarrow}(t)$. In fact, at short times, the contribution of the neighbors adds in phase with $C_{n_0}(t)$. Later on, these contributions are practically null, and $P_{\downarrow}(t)$ depends mostly on $C_{n_0}(t)$. As this spreading is governed by both the reversible and the dissipative dynamics, it does not resemble a classical case. In fact, due to the interference between the oscillations at the proper Rabi frequencies $\Omega_{n_0\pm1}$ and the oscillations at frequency Ω_{n_0} of $C_{n_0}(t)$, the maximum occupation of $C_{n_0\pm1}(t)$ is approximately of order $|(n_0+1)/\tau_1-\Gamma/4|/|\Omega_{n_0\pm1}-\Omega_{n_0}|$. Therefore, for small n_0, the combined action of the reversible and dissipative dynamics produces a *localizationlike* phenomenon. This means that $|C_{n_0\pm1}(t)|\ll|C_{n_0}(t)|$, which implies that $P_{\downarrow}(t)$ will have appreciable contributions only from $C_{n_0}(t)$. In consequence, the decay of $P_{\downarrow}(t)$ becomes exponential for small values of n_0 (cf. Eq. (17)).

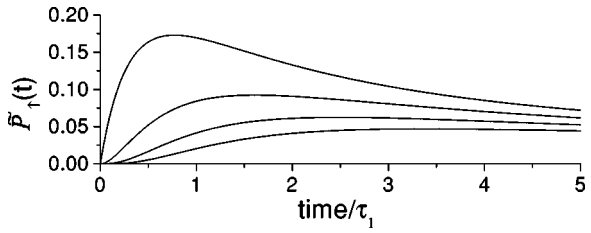

FIG. 2. $\tilde{P}_{\uparrow}(t)$ for different initial Fock states. From top to bottom $n=0,1,2,3$. The parameters are the same as in Fig. 1.

An intriguing result found in the experiment of Ref. [1] is that the fluorescence signal, for initial Fock states, may be approximately modeled by $P_{\downarrow}(t)\approx[1+\cos(2\Omega_{n_0}t)e^{-\gamma_{n_0}t}]/2$ [1]. The phenomenological decay constants were fitted as $\gamma_{n_0}\approx\gamma_0(n_0+1)^{\alpha}$, with $\alpha\approx0.7$. This exponential decay and the values of γ_0 and α can be inferred from the analysis we have presented.

For the case in which the ion starts in the ground state, the localization mechanism is extremely effective $[2|\mathrm{Re}[C_1(t)]|<0.03]$. This allows us to deduce, from Eq. (17), the following relation between the experimental decay constant γ_0, the heating rate $1/\tau_1$, and the decay rate Γ:

$$\gamma_0=\frac{3}{2}\Gamma+\frac{2}{\tau_1}. \tag{18}$$

This relation gives us the key to controlling decoherence in this kind of experiment as the origin of the decoherence rate is now clear. From the experimental values $\gamma_0\approx11.9$ kHz, $\gamma_{\mathrm{tot}}/2\pi=19.4$ Mhz, $\Omega/2\pi=475$ kHz, and $\Delta/2\pi=12$ GHz [1], we estimate a vibrational heating rate $1/\tau_1\approx4.14$ quanta/ms. This value is consistent with those found in the experiments on heating of the motional state [7].

For higher initial Fock states, the localization phenomenon becomes less effective but it does persist [see Fig. 1(c)]. In this situation the best exponential fit to $P_{\downarrow}(t)$ is obtained by discarding the contribution of neighbor coherences. This elucidates the ''mysterious'' value $\alpha\approx0.7$, which arises from choosing the best value such that $\gamma_0(n+1)^{\alpha}\approx(3/2)\Gamma+2(n+1)/\tau_1$. For n between 0 and 16 we obtain the best fitting with $\alpha\approx0.85$. Note that the specific value of α is not universal and depends strongly on the values of Γ and τ_1 [12].

It is important to remark that for initial Fock states with $n_0\gtrsim7$ the dispersivelike behavior persists, whereas the localization phenomenon becomes very weak. This breakdown of the localization gives rise to a highly nonexponential decay of $P_{\downarrow}(t)$. However, the exponential fitting found in the experiments is recovered if one assumes that, for large Fock states, small impurities are present in the initial conditions.

Until now, we have not analyzed the behavior of the asymmetrical factor $\tilde{P}_{\uparrow}(t)$ [6]. From its evolution, Eq. (10), we realize that this term can induce an appreciable asymmetry in the oscillatory behavior of $P_{\downarrow}(t)$ only if, at the beginning, the lower motional populations are occupied. Figure 2

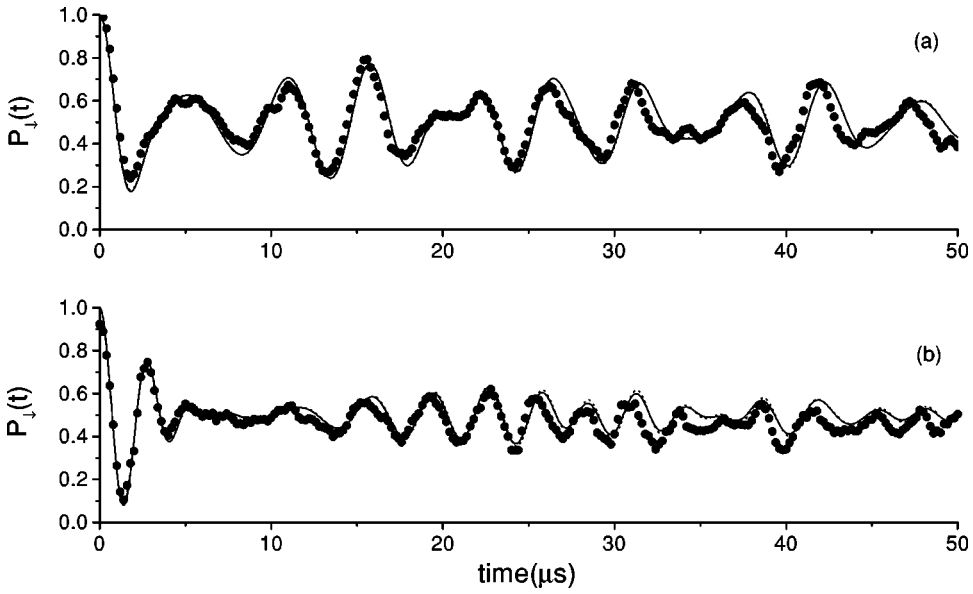

FIG. 3. $P_\downarrow(t)$ for an initial vibrational thermal state with $\bar{n}=1.3$ (a) and an initial coherent state with $\bar{n}=3.1$ (b). The parameters are the same as in Fig 1. Full line: numerical result. Dotted line: experimental fit [Eq. (19)] with $\gamma_n \approx \gamma_0(n+1)^{0.7}$. Circles: experimental data from Ref. [1].

shows the behavior of $\tilde{P}_\uparrow(t)$ for different initial Fock states. The maximum occupation occurs when the ion starts in the ground state $|\downarrow,0\rangle$. In particular, for this case, in $t=100\ \mu$s we found $\tilde{P}_\uparrow=0.16$, which agrees with the experimental data [1,6].

The localization phenomenon and the dispersivelike behavior also occur for other kinds of initial conditions. In Fig. 3 we show our results for the evolution of the fluorescence signal for an initial thermal state and an initial coherent state, with an average vibrational number $\bar{n}=1.3$ and $\bar{n}=3.1$, respectively. In these two cases, at the initial time, all the dressed coherences have approximately the same occupation as their first neighbors. For these situations one can show that each coherence evolves independently with a behavior given by Eq. (17). This result implies that, indeed, for these initial conditions, the fluorescence signal can be very well fitted by the function

$$P_\downarrow(t)=\frac{1}{2}\left(1+\sum_{n=0}^{\infty} P_n \cos(2\Omega_n t)e^{-\gamma_n t}\right), \qquad (19)$$

where P_n are the initial occupations in the Fock basis. In fact, this expression is identical to the modeling of the experimental data made in Ref. [1], where the phenomenological decay rate $\gamma_n \simeq \gamma_0(n+1)^{0.7}$ was used. In these two cases our results and the experimental fits are indistinguishable on a scale lower than the experimental errors.

In conclusion, we have shown that the interplay of the coherent Jaynes-Cummings dynamics with noise sources that feed energy into the system may give rise to a dispersivelike decay of quantum coherences. This dispersivelike behavior is independent of the origin of the irreversible dynamics, and will also be present in thermal reservoirs. This effect, together with the localization phenomenon, was used to explain the experimental results concerning decoherence of the vibronic states of trapped ions [1]. The model described agrees very well with the experimental measurements.

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[11] We are modeling the experiment with three levels only. A more detailed description involves the hyperfine manifolds ($F=2,S_{1/2}$),($F=1,S_{1/2}$), and $^2P_{1/2}$ of the $^9Be^+$ ion and the polarization and power of each beam. We have estimated that the value of Γ is not appreciably modified as long as the powers of the lasers are about the same.
[12] For the $\Omega/2\pi \approx 630$ kHz unpublished data, sent to us by D. Wineland and D. Liebfried, we found $\alpha \approx 0.7$.