Randomness does not destroy interference

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(Received 24 August 2001; published 30 January 2002)

We present a simple explanation that Ericson fluctuations in nuclear, atomic, molecular, and mesoscopic systems originate from the interference between random-partial-width amplitudes in regime of strongly overlapping resonances.

DOI: 10.1103/PhysRevA.65.034701 PACS number(s): 34.80.−i, 24.60.Ky

Consider a quantum collision that proceeds through a formation and decay of an intermediate system. For sufficiently high excitation energies of the intermediate system its average level spacing is much smaller than the total decay width of the resonance levels. In this case, the collision cross section is simultaneously dominated by a large number of overlapping resonances, the amplitudes of which interfere strongly. Assuming that this interference is of a random nature, Ericson showed [1] that this gives rise to fluctuations in the cross sections. Following the paper [1], a quantitative explanation and description of Ericson fluctuations (EF) is usually given in terms of the cross-section energy-autocorrelation function.

In this paper, we present a simple alternative explanation that EF originate from the interference of randomly populated overlapping resonances. We show that if this interference is neglected, EF do not occur. Our explanation is based on the random-walk property stating that the displacement is about a characteristic length of the single random step times the root square of the number of steps. The presented explanation is relevant in a view of the recent misinterpretation [2,3] that EF occur as a result of the absence of interference between different randomly populated overlapping resonances. We point out that both the effect in the e^--H_2 scattering [2,3] and EF arise from the interference between different overlapping resonances.

The present clarification is important because of (i) universality of EF in a wide variety of fields, e.g., nuclear collisions, unimolecular reactions [4], coherent electron transport in nanostructures [5], etc., and (ii) the significant role of EF in foundation of random-matrix theory (RMT) of open quantum systems [6].

Consider the cross section, $\sigma(E)=|t(E)|^2$ with $t(E)=\Sigma_n f_n(E)$ [2,3],

$$f_n(E)=a_n/(E-\epsilon_n+i\Gamma/2), \tag{1}$$

in the regime of EF, $D/\Gamma\ll 1$, where D and Γ are average level spacing and total resonance width, accordingly. Assume that a_n are real quantities having random signs. Decompose

$$\sigma(E)=\sigma_{n=n'}(E)+\sigma_{n\neq n'}(E), \tag{2}$$

where $\sigma_{n=n'}(E)=\Sigma_n|f_n(E)|^2$ does not contain interference terms, while $\sigma_{n\neq n'}(E)=\Sigma_{n\neq n'}f_n(E)f_{n'}(E)^*$ is due to the resonance interference. Consider first $\sigma_{n=n'}(E)$. Decompose $a_n^2=\overline{a_n^2}+\delta r_n$, where overbar stands for the averaging over resonances and δr_n have random signs, $\overline{\delta r_n}=0$, with $|\delta r_n|\sim\overline{a_n^2}$. We have $\sigma_{n=n'}(E)=\tilde{\sigma}_{n=n'}(E)+\delta\sigma_{n=n'}(E)$, where

$$\tilde{\sigma}_{n=n'}(E)=\overline{a_n^2}\sum_n 1/[(E-\epsilon_n)^2+\Gamma^2/4], \tag{3}$$

and

$$\delta\sigma_{n=n'}(E)=\sum_n \delta r_n/[(E-\epsilon_n)^2+\Gamma^2/4]. \tag{4}$$

Decompose $\epsilon_n=nD+\delta\epsilon_n$, where D is a smooth function of the energy, while $\delta\epsilon_n$ have random signs, $\overline{\delta\epsilon_n}=0$, with $|\delta\epsilon_n|\sim D$.

Consider

$$\tilde{\sigma}_{n=n'}(E)-\overline{a_n^2}\sum_n 1/[(E-nD)^2+\Gamma^2/4]=R_1+R_2, \tag{5}$$

where

$$\begin{aligned}R_1&=-\overline{a_n^2}\sum_n \delta\epsilon_n^2/[(E-\epsilon_n)^2+\Gamma^2/4][(E-nD)^2+\Gamma^2/4]\\&\simeq-\overline{a_n^2}\;\overline{\delta\epsilon_n^2}\sum_n 1/[(E-nD)^2+\Gamma^2/4]^2\\&\simeq-8\overline{a_n^2}(D/\Gamma^3)\int_{-\infty}^{\infty}dx/(x^2+1)^2\\&=-4\pi\overline{a_n^2}D/\Gamma^3,\end{aligned} \tag{6}$$

and

$$\begin{aligned}R_2=2\overline{a_n^2}\sum_n (E-nD)\delta\epsilon_n/[(E-\epsilon_n)^2+\Gamma^2/4]\\\times[(E-nD)^2+\Gamma^2/4].\end{aligned} \tag{7}$$

R_2 is a sum of independent random variables having random signs (due to the random signs of $\delta\epsilon_n$). Therefore, the sum (7) can be evaluated invoking the random-walk property. The effective number of terms in the sum (7) $\sim\Gamma/D\gg 1$ and the characteristic value of these terms $\sim D/\Gamma^3$. This yields

$$R_2\sim\pm\overline{a_n^2}D^{1/2}/\Gamma^{5/2}. \tag{8}$$

1050-2947/2002/65(3)/034701(3)/$20.00

Employing the Poisson summation formula and summing up the resulting geometric series, we obtain

$$\overline{a_n^2}\sum_n 1/[(E-nD)^2+\Gamma^2/4]=\overline{a_n^2}\left\{\int_{-\infty}^{\infty} dx/[(E-Dx)^2+\Gamma^2/4] \quad +2\sum_{m=1}^{\infty}\int_{-\infty}^{\infty} dx\cos(2\pi mx)/[(E-Dx)^2+\Gamma^2/4]\right\}$$
$$=(2n\overline{a_n^2}/D\Gamma)\{1+2\exp(-\pi\Gamma/D)[\cos(2\pi E/D)$$
$$-\exp(-\pi\Gamma/D)]/|1-\exp[2\pi i(E/D)-\pi\Gamma/D]|^2\}. \tag{9}$$

Thus, from formulas (5), (6), (7), (8), and (9), we find that, for $D/\Gamma\ll 1$,

$$\tilde{\sigma}_{n=n'}(E)=(2\pi\overline{a_n^2}/D\Gamma)[1\pm O(D/\Gamma)^{3/2}]\to 2\pi\overline{a_n^2}/D\Gamma. \tag{10}$$

Notice that $\delta\sigma_{n=n'}(E)$ is the sum of $\sim(\Gamma/D)\gg 1$ terms. These terms have random signs and characteristic values $\sim\overline{a_n^2}/\Gamma^2$ yielding

$$\delta\sigma_{n=n'}(E)\sim\pm(\Gamma/D)^{1/2}\overline{a_n^2}/\Gamma^2\sim\pm(D/\Gamma)^{1/2}\tilde{\sigma}_{n=n'}(E). \tag{11}$$

Altogether we have

$$\sigma_{n=n'}(E)=(2\pi\overline{a_n^2}/D\Gamma)[1\pm O(D/\Gamma)^{1/2}]\to 2\pi\overline{a_n^2}/D\Gamma. \tag{12}$$

This shows that, for $D/\Gamma\ll 1$, $\sigma_{n=n'}(E)$ is almost energy independent. It can depend on E only through smooth energy dependencies of D and Γ. Therefore, $\sigma_{n=n'}(E)$ cannot produce EF.

Consider $\sigma_{n\neq n'}(E)$, which originates from interference between different resonance states. $\sigma_{n\neq n'}(E)$ is the double sum containing $\sim(\Gamma/D)^2$ terms. These terms have random signs and characteristic values $\sim\overline{a_n^2}/\Gamma^2$. Employing the random-walk properties, we obtain

$$\sigma_{n\neq n'}(E)\sim\pm(\Gamma/D)\overline{a_n^2}/\Gamma^2\sim\pm\sigma_{n=n'}(E). \tag{13}$$

This demonstrates that randomness of a_n does not suppress resonance interference.

Consider the energy averaged $\sigma_{n\neq n'}(E)$ over the energy interval $\mathcal{I}\gg\Gamma$ around $\overline{E}$:

$$\langle\sigma_{n\neq n'}(E)\rangle=\int_{-\infty}^{\infty} dE W(E,\overline{E})\sigma_{n\neq n'}(E)$$
$$=\sum_{n\neq n'} a_n a_{n'}/(\overline{E}-\epsilon_n+i\Gamma/2+i\mathcal{I}/2)(\overline{E}-\epsilon_{n'}-i\Gamma/2+i\mathcal{I}/2)+i\mathcal{I}$$
$$\times\sum_{n\neq n'} a_n a_{n'}/(\overline{E}-\epsilon_{n'}-i\Gamma/2-i\mathcal{I}/2)(\overline{E}-\epsilon_{n'}-i\Gamma/2+i\mathcal{I}/2)(\epsilon_{n'}-\epsilon_n+i\Gamma), \tag{14}$$

where $W(E,\overline{E})=(\mathcal{I}/2\pi)/[(E-\overline{E})^2+\mathcal{I}^2/4]$. Taking into account random signs of a_n and invoking again the random-walk property, one can easily find

$$\langle\sigma_{n\neq n'}(E)\rangle\sim\pm(\Gamma/\mathcal{I})^{1/2}\sigma_{n=n'}(E). \tag{15}$$

This demonstrates that random signs of a_n do result in $\langle\sigma_{n\neq n'}(E)\rangle=0$ indicating that $\sigma_{n\neq n'}(E)$ fluctuates around its zero energy average value. The amplitude of these fluctuations is of the order of $\sigma_{n=n'}(E)=\langle\sigma(E)\rangle$.

The above consideration is applicable for inelastic scattering, no matter whether random a_n are chosen to be real or complex. For elastic scattering, a_n are positive random quantities distributed in accordance with Porter-Thomas distribution [6]. In this case, EF originate from the interference between different $\delta a_n=a_n-\overline{a_n}$, as well as between δa_n and $\overline{a_n}$, where δa_n have random signs and $|\delta a_n|\sim\overline{a_n}$.

The above consideration explicitly shows that EF do originate from the interference between the amplitudes corresponding to different strongly overlapping resonance states. Without this interference, the cross section is almost energy independent and EF are gone. The difference between EF and the effect [2,3] is that the former assumes random signs (or random phases if a_n are complex) of a_n, which corresponds to the universal limit of RMT [6]. In contrast, the work [2,3] discusses resonance-interference effect, which is clearly beyond RMT. Other nonuniversal resonance-interference effects have recently been discussed in Refs. [7].

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