

# de Broglie wavelength reduction for a multiphoton wave packet

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An experiment is proposed that permits the observation of the reduced de Broglie wavelengths of two- and four-photon wave packets using present technology. It is suggested to use a Mach-Zehnder setup and feed both input ports with light generated by a single nondegenerate down-conversion source. The strong quantum correlations of the light in conjunction with boson enhancement at the input beam splitter allow us to detect a two- and fourfold decrease in the observed de Broglie wavelength with perfect visibility. This allows a reduction of the observed de Broglie wavelength below the wavelength of the source.

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## I. INTRODUCTION

Recently Jacobson *et al.* [1] gave a theoretical description of how to measure the effective de Broglie wavelength of a quantum system in a coherent many-particle state using a scheme employing nonlinear beam splitters. Their work concluded that many-particle systems with a sufficiently sharply defined number of constituents, in the best case prepared in a pure number state  $|N\rangle$ , should show a reduction from the single particle wavelength  $\lambda$  to the effective multiparticle wavelength  $\lambda/N$ . This is the case even if the constituents are not bound together. For bound systems such as polyatomic molecules [2,3] this fact is well known, but for unbound systems such as  $N$ -photon Fock states  $|N\rangle$  this finding is perhaps surprising.

The scheme suggested by Jacobson *et al.* uses nonlinear beam splitters in order to emulate an effective binding of the particles. In a two-mode interferometer this input beam splitter channels the unbound constituents of the system such that they simultaneously follow the “upper” interferometric channel  $u$  in superposition with all of them passing through the “lower” channel  $l$ ; this state is maximally number entangled, where inside the interferometer it reads

$$|\psi\rangle_{\text{inside}} = \frac{|N\rangle_u |0\rangle_l + |0\rangle_u |N\rangle_l}{\sqrt{2}}. \quad (1)$$

Compared to the case of a bound system this state obviously corresponds to an  $N$ -atomic molecule traversing the interferometer undivided, thus maximally shrinking the system’s de Broglie wavelength.

The output beam mixer of the interferometer (see Fig. 1) is supposed to be of the same nonlinear kind as the input beam splitter. Unfortunately, the use of nonlinear beam splitters amounts to a considerable practical problem with regards to the experimental implementation of the ideas presented in Ref. [1]: at present, neither the required large magnitudes of the nonlinearities nor their different orders for different particle number  $N$  are practically achievable.

Inspired by Jacobson’s work, but following a rather different approach, Fonseca, Monken, and Padua managed to measure the predicted halving of the wavelength for a two-photon state generated in parametric down-conversion [4].

The down-conversion source allowed them to abandon the nonlinear beam splitter in the input of their Young’s double-slit setup, and a two-particle coincidence detection scheme allowed them to get rid of the second nonlinear beam mixer. Here, I want to present a modification to their approach (borrowing ideas from [5,6] and [7]) that should allow for the observation of a fourfold reduction of the de Broglie wavelength with currently available technology [6,8].

The key idea is to use linear beam splitters in a Mach-Zehnder setup and utilize boson enhancement to generate good approximations to the ideal state Eq. (1) inside the interferometer. Combined with suitable postselection of detection events this approach, in principle, even allows for the observation of more than fourfold reductions in the measured de Broglie wavelength of unbound multiparticle systems. In all such schemes a deviation from classical behavior (single-photon or coherent states) can only be achieved by suitably tailored entangled states [4,5,9].

In parametric down-conversion, so far only variations with the wavelength of the pump beam, i.e., halving of the effective de Broglie wavelength (using one photon pair) have been observed [4,10–12]; consequently the idea presented here will allow for the observation of a reduction of the multiparticle de Broglie wavelength below that of the generating source [7,13]. Note, however, that the observation of a quartering of the joint phase of the internal degrees of freedom of four entangled ions in a trap has already been observed [14].

## II. PREPARING THE STATE

From now on we will only discuss photons passing a Mach-Zehnder interferometer (see Fig. 1). Implicitly this

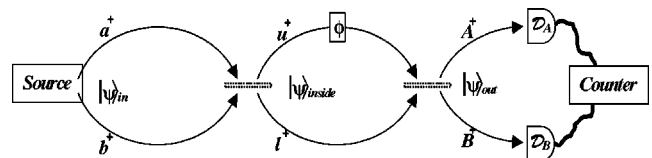


FIG. 1. Sketch of the interferometer: dotted lines outline balanced beam splitters;  $\mathcal{D}$  stands for detectors; and  $a$ ,  $b$ ,  $u$ ,  $l$ ,  $A$ , and  $B$  label the modes before, inside, and beyond the interferometer.  $\phi$  is the phase shifter in the upper channel.

discussion covers many bosonic systems and all types of two-mode interferometers.

Rather than using nonlinear beam splitters it is easier, even for two-photon states, to prepare the desired interferometric state Eq. (1) from photon pairs generated in type II (nondegenerate) parametric down-conversion. Fonseca, Monken, and Padua used a pump beam with a momentum characteristic such that both photons would always pass the same slit [4]. Hence, the state  $(|1,1\rangle_u|0,0\rangle_l + |0,0\rangle_u|1,1\rangle_l)/\sqrt{2}$  was created, where the two slots  $|\cdot, \cdot\rangle$  refer to the two orthogonal polarization modes of the photons and the indices  $u$  and  $l$  refer to either slit in their double slit setup. This state is very similar to the desired form (1) and works just as well, as long as interferometric phase changes apply to both polarization modes simultaneously. The experiment [4] confirmed some ideas presented in [1] and earlier results about joint measurements on down-converted photons [10,11].

Another recent proposal [7] suggests the use of a Mach-Zehnder layout and a projective measurement involving a third auxiliary photon (from a second down-conversion photon pair) in order to prepare state (1) for a one-photon pair inside the interferometer.

But both these approaches do not scale favorably for higher photon numbers; in the latter case [7] the use of auxiliary photons leads to extra channels and losses in the interferometer. In the former case of experiment [4] the momentum selection trick cannot be extended to higher photon numbers because multiple pump photons would be involved; this leads to a multiple product of the single-pair state mentioned above, i.e., in general to a state very different from the desired form (1). Therefore, another ingredient is needed to channel the photons through the interferometer; for this, boson enhancement can be used.

As is known from the Mandel-dip experiment [15,16,7] the preparation of state (1) using boson enhancement is straightforward for two photons: spontaneous parametric down-conversion generates single and multiple photon pairs in a two-mode squeezed vacuum [17] from the vacuum state  $|0\rangle$ ,

$$|\psi\rangle_{in} = \frac{1}{\sqrt{1-|\alpha|^2}} \sum_{n=0}^{\infty} \frac{(\alpha a^\dagger b^\dagger)^n}{n!} |0\rangle, \quad (2)$$

where  $a^\dagger$  and  $b^\dagger$  are the bosonic creation operators for the field modes of the interferometer input ports (see Fig. 1). With probability  $|\alpha|^2/(1-|\alpha|^2)$  a suitable array of lossless detectors [18], operating in coincidence, will detect two photons. This allows us to retrodict that state (2) became projected into state  $|1,1\rangle$  which entered the interferometer and became converted into the state

$$|\psi\rangle_{inside} = \frac{|2\rangle_u|0\rangle_l + |0\rangle_u|2\rangle_l}{\sqrt{2}}, \quad (3)$$

where the transformations for a balanced beam splitter,

$$a^\dagger = (u^\dagger + il^\dagger)/\sqrt{2}, \quad b^\dagger = (u^\dagger - il^\dagger)/\sqrt{2}, \quad (4)$$

have been assumed [16]. The behavior of such photon pairs has been studied intensely [4,11,12,15].

For four and more photons, our scheme only allows us to prepare states similar to the ideal state (1). Let us assume that a fourfold coincidence has been detected (see [6,8]), this occurs with probability  $|\alpha|^4/(1-|\alpha|^2)$ . It allows us to infer that the incident state was of the form  $|\psi\rangle_{in} = a^{\dagger 2} b^{\dagger 2} |0\rangle/2$ . Inside the interferometer this becomes [6] ( $\varepsilon \equiv 1$ )

$$|\psi\rangle_{inside} = \left( \frac{u^{\dagger 4}}{8} + \frac{u^{\dagger 2} l^{\dagger 2}}{4} + \frac{l^{\dagger 4}}{8} \right) |0\rangle \quad (5)$$

$$= \sqrt{\frac{3}{4}} \left( \frac{|4\rangle_u|0\rangle_l + |0\rangle_u|4\rangle_l}{\sqrt{2}} \right) + \frac{\varepsilon}{\sqrt{4}} |2\rangle_u|2\rangle_l. \quad (6)$$

Obviously the bosonic enhancement leads to the generation of the desired state of the form (1) in 75% of all cases [6], whereas the unwanted contribution  $|2\rangle_u|2\rangle_l$  only occurs in a quarter of all cases ( $\varepsilon \equiv 1$ ). Although only an approximation to our goal, this state is readily available and the desired fourfold reduction of the de Broglie wavelength can be detected using current technology [6,8].

### III. DETECTION OF THE SIGNAL

In order to determine the detectors' response we will now assume that the photons following channel  $u$  are delayed by a tunable phase  $\phi$  and are subsequently mixed with the  $l$  channel to form the detector modes  $A^\dagger$  and  $B^\dagger$ :

$$u^\dagger = e^{i\phi}(A^\dagger + iB^\dagger)/\sqrt{2}, \quad l^\dagger = (A^\dagger - iB^\dagger)/\sqrt{2} \quad (7)$$

(see Fig. 1). For the case of a single photon entering the interferometer through mode  $a^\dagger$  we thus receive the final state

$$|\psi\rangle_{out} = \frac{(i + e^{i\phi})|1\rangle_A|0\rangle_B + (1 + ie^{i\phi})|0\rangle_A|1\rangle_B}{2} \quad (8)$$

and the well known classical photodetector response probabilities

$$P_A(\phi) = \langle A^\dagger A \rangle = \frac{1}{2}(1 + \sin \phi) \quad (9)$$

and

$$P_B(\phi) = \langle B^\dagger B \rangle = \frac{1}{2}(1 - \sin \phi). \quad (10)$$

For the two-photon state (3) the corresponding expressions reflect the halving of the de Broglie wavelength

$$P_{AA}(\phi) = P_{BB}(\phi) = \frac{1}{4}(1 + \cos 2\phi) \quad (11)$$

and

$$P_{AB}(\phi) = \frac{1}{2}(1 - \cos 2\phi). \quad (12)$$

Here  $P_{AA}$ ,  $P_{BB}$ , and  $P_{AB} \equiv P_{BA}$  stand for the probabilities of detecting two photons in the channels and with the multiplicity indicated by the subscripts. Since it is difficult to detect

single photons and discriminate one from two photons arriving at the same time, the experimentally most convenient signal is  $P_{AB}$ .

The four-photon state (6) shows the expected reduction to a quarter of the de Broglie wavelength, namely,

$$P_{AAAA}(\phi) = P_{BBBB}(\phi) = \frac{9 + 12 \cos 2\phi + 3 \cos 4\phi}{64}, \quad (13)$$

$$P_{AAAB}(\phi) = P_{ABBB}(\phi) = \frac{3 - 3 \cos 4\phi}{16}, \quad (14)$$

$$P_{AABB}(\phi) = \frac{11 - 12 \cos 2\phi + 9 \cos 4\phi}{32}. \quad (15)$$

Surprisingly, despite the imperfect form ( $\varepsilon=1$ ) of the four-photon state (6),  $P_{AAAB}$  and  $P_{ABBB}$  show a pure fourfold reduction of the observed de Broglie wavelength with perfect visibility. Here  $P$  stands for the four-photon coincidence probabilities with the channels and their respective detection multiplicity indicated by the subscripts.

Since it is presently difficult to detect with single photon resolution and discriminate one from two or more photons arriving at the same time, a special detector setup might have to be used. One can employ multiport detectors as they are described in [19]; a four port in channel  $A$  suffices to see the signals  $P_{AAAA}$  and  $P_{AAAB}$  since it can split up four photons to follow four different channels (for sketches of possible realizations of four-port detectors, see, e.g., [6]).

For photon numbers  $2n$  higher than four no pure  $2n$ -fold wavelength reduction as in  $P_{AAAB}$  is attainable using our scheme. Because the problems with the generation and coincidence detection of more than four photons will also make the experiment much harder to perform, we will not write down the corresponding photodetection probabilities for six, eight, or more photons.

#### IV. SPECULATION

If one could, however, synthesize an input state of the form

$$|\psi\rangle_{in} = \frac{(a^\dagger - b^\dagger)^{2n} + (a^\dagger + b^\dagger)^{2n}}{\sqrt{2^{2n+1} \cdot 2n!}} |0\rangle, \quad (16)$$

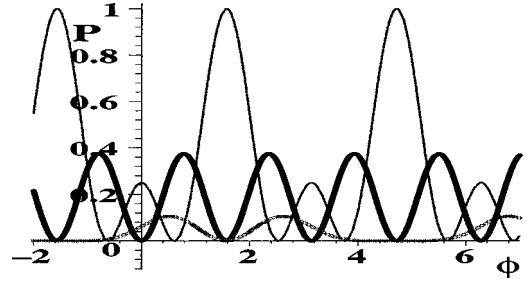


FIG. 2. The four-photon coincidence signals as a function of phase delay  $\phi$ :  $P_{AAAB}$  (thick line) and  $P_{AABB}$  (thin line) in comparison with the classical signal  $P_A P_A P_A P_B$  from Eq. (10) (dotted line).

where  $a$  and  $b$  stand for any two modes, say polarization, this state would, using the beam-splitter operation (4), transform into a ‘‘Schrödinger-kitten state’’ (1)

$$|\psi\rangle_{inside} = \frac{|2n\rangle_u |0\rangle_l + |0\rangle_u |2n\rangle_l}{\sqrt{2}}, \quad (17)$$

and would therefore also yield detector signals with perfect  $2n$ -fold wavelength reduction for any photon number  $2n$ . However, it is not known how to generate such a state with available technology. Yet, this observation stresses once more the alternatives to employing nonlinear beam splitters [1] in order to channel the particles through the interferometer.

#### V. CONCLUSION

To conclude, interference patterns of unbound particles with halved de Broglie wavelengths have been seen in parametric down-conversion. Here a scheme, using current technology is proposed, which allows us to see further de Broglie wavelength reductions for such unbound particles below the wavelength of the generating source.

At the level of four photons generated in parametric down conversion a fourfold reduction in the interference signal with perfect visibility is achievable.

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