Experimental investigation on excitability in a laser with a saturable absorber

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In this work, we investigate experimentally the excitable properties of a solid-state laser with an intracavity saturable absorber. The main test of excitability is the characterization of the system response to short-pulse external stimuli. We present an original method to excite the system by injecting light pulses into the saturable absorber. This technique avoids imposing frequency tuning between the input laser and the passive laser, which is a necessary condition for field injection. We show that under certain conditions the laser behaves as an excitable system. The observed excitable regime is consistent with theoretical predictions for class-*B* lasers with saturable absorber.

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I. INTRODUCTION

A system is said to be excitable whenever it has a stable stationary state, and reacts to an external perturbation in two qualitatively different ways depending on the size of the stimulus. Whenever the perturbation is smaller than a certain magnitude known as threshold, the system will evolve towards the stationary state performing a short excursion in its physical variables. On the other side, if the perturbation exceeds the threshold then a large excursion in the variables will develop before returning to the initial state. The interest in excitable systems comes from chemistry and biology, with a variety of applications such as reaction-diffusion systems, cardiac tissue, and neural modeling [1,2]. More recently, excitability was identified in certain optical systems. In particular, some specific laser systems were shown to be excitable: a CO_2 laser with saturable absorber [3] and a semiconductor laser with optical feedback [4,5]. From the experimental point of view, the excitability properties of a system can be identified in different ways. A paradigmatic test consists in perturbing the system with short impulses and measuring the response. The signature of excitability is the fact that the system begins to respond with a large pulse as long as the input amplitude remains greater than a certain value (excitability threshold). In addition, neither the amplitude nor the shape of the excitable pulse depend on the perturbation itself. If the input amplitude is below the threshold, the evolution of a measured physical variable may show a small linear response or even no response at all. Once the system is excited, it evolves and settles back to the initial state during what is called the *refractory time*.

Another test we consider is the response of the system to a periodic perturbation. This has been recently characterized for a semiconductor laser with optical feedback in the excitable regime [6]. When the period of the external forcing approaches the refractory time, the system can display different locking sequences as a parameter is changed. This kind of response to periodic forcing can be used to build confidence on the excitability of the system under study.

A third test that has been recently proposed is to relate the statistics of the interspike time distribution of a noise-driven excitable system to the geometry of the phase space, and then to compare the evolution of the histograms as the parameters are changed to those of a simple model. This analysis was performed in semiconductor lasers with optical feedback [7], which turns out to be a natural scenario for this test since even the intrinsic noise highly influences the system dynamics [8,5,9].

For a laser with saturable absorber, some theoretical investigation was made in order to describe excitability [3,10]. According to these works, the excitable regime is characterized by the presence of three fixed points: a node, a saddle, and a repellor. The excitability threshold is well determined by the presence of the saddle, and the large excursion in the phase space that gives an excitable pulse follows the unstable manifold of the saddle. Moreover, a transition to oscillations in those models (Q-switch regime) is identified by a saddle-loop homoclinic bifurcation. It is worth to remark that the Q-switch pulses for this situation appear with finite amplitude after the bifurcation, yielding coexistence between a stable state and a limit cycle. In a further bifurcation, the node and the saddle collide (transcritical bifurcation), leaving the limit cycle as the only stable solution. This dynamical evolution was already known from the early days of gas lasers with saturable absorbers [11], but only recently was interpreted, within a certain range of parameters, in terms of excitability [3,10].

In this work we report, to the best of our knowledge, the first solid-state with saturable absorber-excitable laser: a Nd:YAG (yttrium aluminum garnet) laser with intracavity Cr:YAG. We establish experimentally a range of control parameters, namely, the pumping rate and the area of the laser mode on the saturable absorber, where the conditions for excitability take place. We then perform two of the above-

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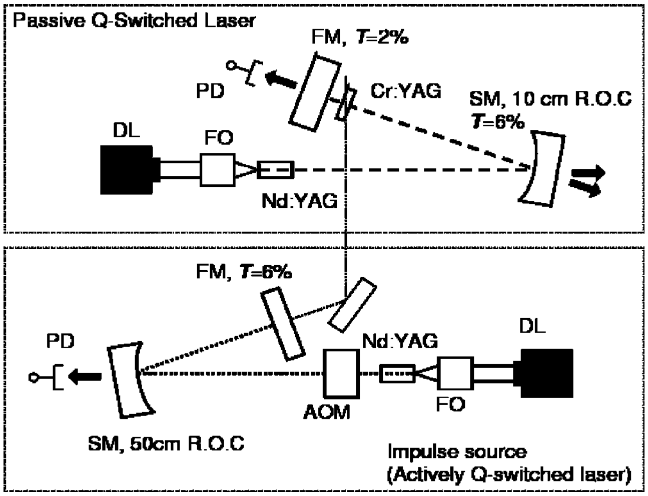

FIG. 1. Sketch of the experimental setup: DL, pumping laser diode; FO, focusing optics; SM, spherical mirror; FM, flat mirror; PD, photodiode detector.

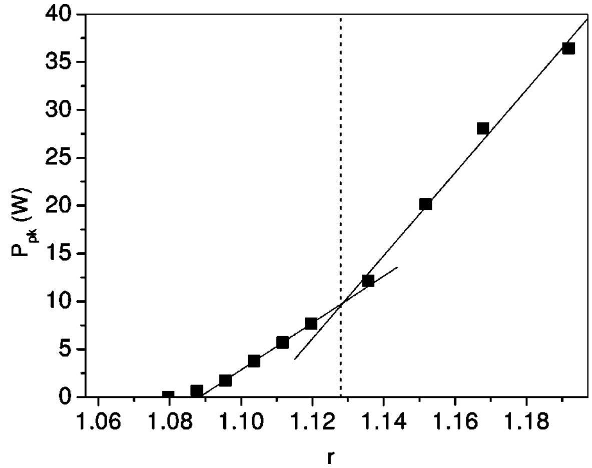

FIG. 2. Experimental determination of the second threshold. For a given laser mode size on the saturable absorber, the change in the slope of the laser peak pulse power vs pumping rate *r* is determined by the onset of the second threshold condition. The pumping rate is the pump power normalized to its threshold value.

mentioned experimental tests to evaluate excitability: perturbing the system with small external light pulses to determine the existence of a threshold, and studying locking sequences when the period of the train of stimuli approaches the natural oscillation time.

In previous experimental works on optical excitable lasers, the external perturbation is applied either to the pumping current in semiconductor lasers, or in the electromagnetic field in gas lasers with saturable absorber. In this work, we present a method to perturb the system that consists in injecting light pulses from a second (actively Q switched) laser into the saturable absorber. This has the advantage of being an all optical-perturbation method that does not require frequency tuning between the cavities. This resonance requirement becomes a strong condition for the realization of optical-field injection in general. In particular, it turns out to be a major drawback in multimode lasers for which the resonance might not be well established, since in all practical cases it is difficult to guarantee the matching of all the longitudinal active modes.

We organize this work as follows: in Sec. II we describe the experimental setup and present the results; Sec. III is devoted to the analysis and discussion of the results; finally, in Sec. IV we present our conclusions.

II. EXPERIMENT

The experimental setup is schematically shown in Fig. 1. The system under study consists of a diode-pumped Nd:YAG laser in a v-folded cavity, with an intracavity slab of 95%-low signal transmission Cr:YAG as a saturable absorber [12]. The cavity is formed by a flat end mirror coated for high reflection at 1.064 μm on one surface of the Nd:YAG rod, a concave 6% transmission folding mirror and a flat 2% transmission mirror. The total output coupling of the cavity is 14%. The pump source is a $2W$ continuous laser diode.

The Cr:YAG slab is placed between the two partially reflecting mirrors. The two control parameters of the passive Q-switched laser are the pumping power normalized to the laser threshold, namely, the pumping rate ($r \equiv R/R_{th}$), and the area of the laser mode on the Cr:YAG normalized to its value at the second threshold ($a \equiv A/A_{2th}$). The area at the second threshold was determined experimentally measuring the change of the laser power slope as the pump is increased (Fig. 2). The a value is then calculated using the relation $A_{2th} = A_0 \sqrt{R_{th}/R_0}$ [13], where R_0 is the value indicated in Fig. 2 and A_0 is the actual area for which the curve was obtained. For the present cavity configuration, the pumping threshold is $R_{th} = 1.42W$, and the laser spot diameter at the second threshold is 480μ ($1/e$ diameter of the field).

The laser mode size is $a = 0.06$ on the flat mirror, and $a = 1.83$ on the concave mirror. These values are obtained by back propagating measurements of the beam spot size outside the cavity. The area of the laser mode on the saturable absorber—which is given by its relative position between the mirrors—dramatically determines the dynamical behavior of the laser: for a mode size larger than $a = 0.58$, the laser exhibits a progressive buildup of the Q-switch regime as the pump power is increased, beginning with small amplitude modulations of the continuous-wave solution that lead to high peak power pulses as the pumping power is further increased. However, if the mode size is smaller than the above-mentioned value, an increase of the pump power over the laser threshold causes the laser to oscillate directly in the form of high peak power pulses. In this experiment, the laser mode size on the saturable absorber is $a = 0.07$. The system shows a hysteretic response to the pump power between the onset and extinction of the laser oscillation, as it is shown in Fig. 3. In this figure, the ($r = 1$) pumping rate was modulated with a 100 Hz, $\Delta r = 0.06$ peak-to-peak triangular signal. From now on, we set the pump current to the value indicated in Fig. 3 as $X(r = 0.97)$, where the off state of the system is stable.

To perform the excitability test, the system is perturbed with light pulses on the saturable absorber. The source of

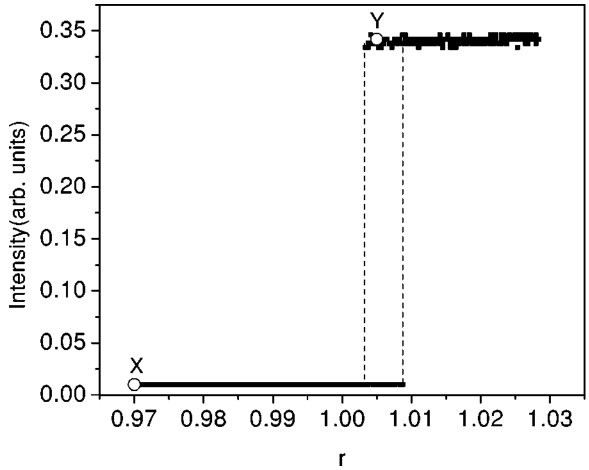

FIG. 3. Dependence of the laser output on pumping. The system shows a hysteretic behavior near the threshold. The experimental trace is the envolvent of the signal, also showing that the laser starts to oscillate with a definite pulse amplitude. The point X sets the laser condition for the excitability tests (see text).

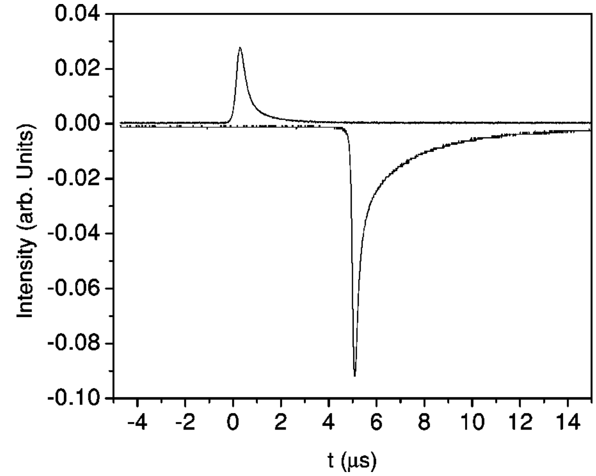

FIG. 4. Oscilloscope trace of the two lasers' outputs, showing the delayed response (inverted for clarity) of the passive cavity to the impulse injected on the saturable absorber. The pump parameter for this situation is $r=0.97$.

these pulses is another diode-pumped Nd:YAG laser, actively Q-switched by means of an intracavity acousto-optic modulator. This perturbation is injected on the saturable absorber in an angle of about 40° relative to the optical axis of the passive cavity, thus avoiding coupling between the fields (see Fig. 1). The repetition rate of the test pulses can be varied from a few hertz to 4 kHz, and the pulse width was measured to be less than 1 μs. We recall that close to the extinction of passive Q switch, at the point Y of Fig. 3, the Q-switch period is ≈ 1.3 ms. We thus choose the frequency of the impulse train to be much smaller than this natural Q-switch frequency: the time between two consecutive injection pulses is then fixed to 2.32 ms.

For low injected pulse energies, the passive system shows no response. When the injected pulse energy overcomes a certain value, the system begins to respond with a pulse, that follows the stimulus as it is shown in Fig. 4, where the output pulse has been inverted for clarity. To obtain a quantitative, systematic description of this effect, we have measured four different observables. Figure 5(a) shows the average output power as a function of the injection energy (E), measured using a Melles Griot 13PEM001 power meter. We also performed statistics on the pulse response of the system. The number of output pulses normalized to the total injection pulses is shown in Figure 5(b). In both figures, a steplike response of the system is observed, and the transition from no response to almost constant response takes place at $E_c \approx 12$ μJ. The width of the transition turns out to be $\Delta E/E_c \approx 0.02$, which means that the transition is quite sharp.

The other two measured magnitudes are the mean amplitude response and the mean time delay between the stimulus and the response, with their respective standard deviations [Figs. 6(a) and 6(b)]. It is worth to notice that there is a discontinuity in the amplitude of the response as the injection energy changes from $E<E_c$ to $E>E_c$. Besides, the time delay of the response becomes larger as the input energy gets closer to E_c, and decreases to an almost constant value of

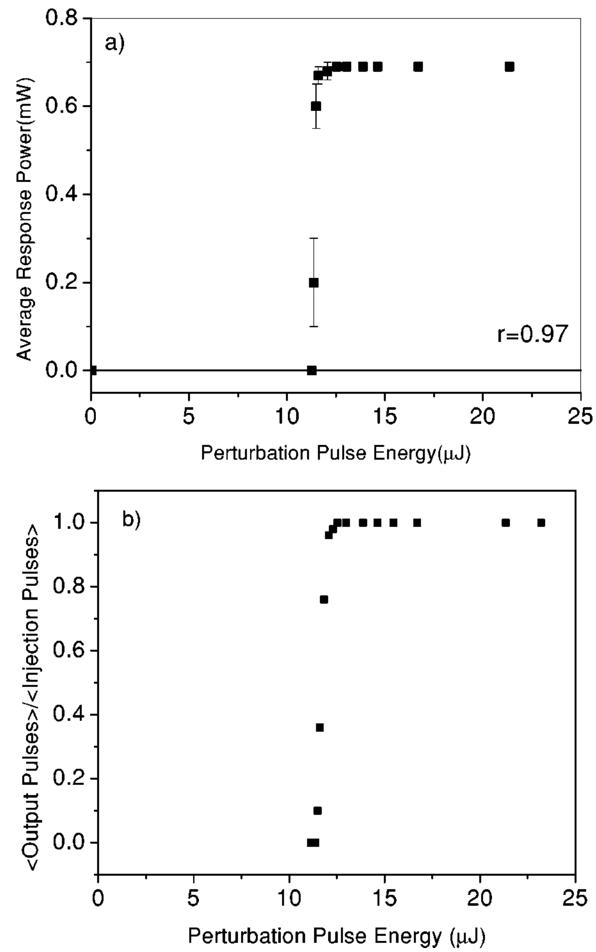

FIG. 5. Behavior of the system under the perturbation on the saturable absorber. (a) Average output power, and (b) ratio of the mean response occurrence to the impulse occurrence; vs increasing impulse energy.

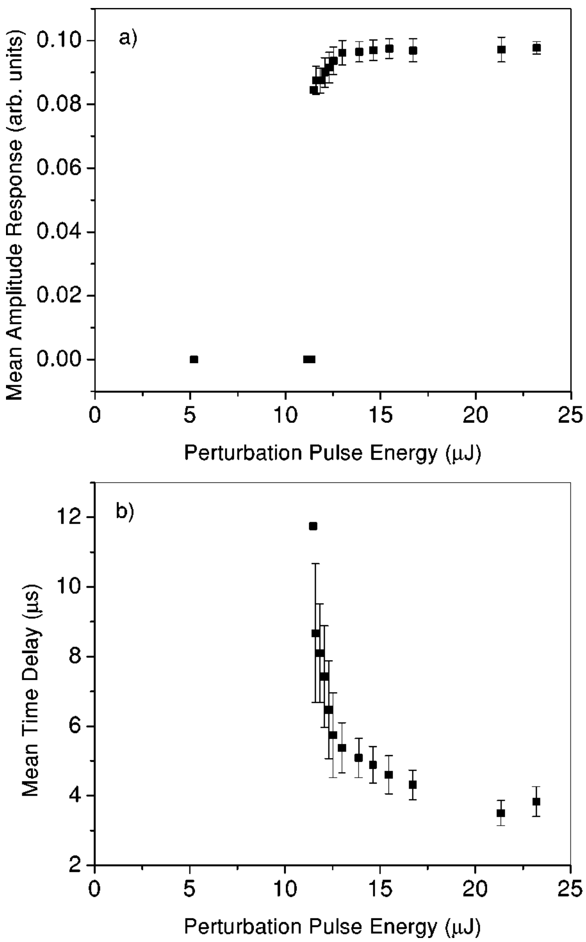

FIG. 6. Evolution of (a) the mean-amplitude response and (b) the mean delay between the impulse and the response, together with their fluctuations, near the critical value of the impulse energy E_c.

4 μs as E is increased. We remark that in all cases, the fluctuations are larger near the transition range ($E \approx E_c$).

We are now concerned on the dynamical response of the system as the period of the impulse train approaches the natural Q switch interspike time (1.3 ms). In Fig. 7, we show time segments of the passive system response to increasing impulse frequencies and constant amplitude. Several locking frequencies can be observed, from 1:1 response [Fig. 7(a)] to 1:4 [Fig. 7(f)], as well as intermittence between different locking sequences.

III. DISCUSSION

There are two main results that can be remarked on the previous excitability test. The first one is that the passive system responds with a finite amplitude signal as far as the input pulses overcome a certain magnitude E_c and exhibits no appreciable response for weaker stimuli, together with the fact that the output amplitude remains unchanged for further increasing E values. The second result is that there is a characteristic impulse frequency above which the system is not able to respond to all the stimuli; rather, the system misses some consecutive impulses and eventually locks to periods that are multiples of the input period. These strong features allow us to interpret the results from the point of view of excitability.

We first analyze the case when the time between consecutive impulses is much longer than the natural oscillation time of the passive laser (i.e., much larger than all the time scales of the system). In particular, the relaxation time of the saturable absorber and the gain are 3.4 μs and 230 μs, respectively. Therefore, we can affirm that both the population in the saturable absorber and the field have already reached their stationary values at the moment the external impulse arrives. This means that the passive laser loses memory from one perturbation to the following. Also, it is worth to notice that the impulse width is much shorter than the saturable absorber relaxation time. From these considerations it is possible to infer that the population inversion in the saturable absorber suffers an increment proportional to the energy of the impulse, what can be viewed as a finite perturbation in the system. We can then relate the impulse energy at E_c to the excitability threshold. Another generic feature of an excitable system is that the response, in terms of the excursion in the phase space before returning to the stationary state, is independent of the size of the perturbation, provided the latter exceeds the threshold value. An examination of Figs. 5 and 6(a) shows that the response in terms of the intensity variable is independent of the size of the perturbation.

The fluctuations in the excited pulses rate near E_c increases dramatically. This can be observed in the experiment and also inferred from Figs. 5(a) and 6(a): while the dispersion of pulse amplitude is quite small close to the threshold, the average response power fluctuates significantly. This means that the average power changes because of fluctuations in the excited pulse rate rather than in the pulse energy itself. At this point, it becomes clear that the system is very sensitive to noise near threshold; furthermore, this effect is not observed in the amplitude, but it is translated into the phase dynamics. Actually, we can interpret the evolution and fluctuations of the mean time delay between the injected and the response pulses (jitter) under the hypothesis that the noise plays an important role near the threshold. In order to address this issue, we note that there are two dynamical features that can be extracted from Fig. 6(b): the first one is the existence of a slow flux region in the phase space close to threshold, which could explain the fact that the time delay rises abruptly as the perturbation energy approaches E_c from higher values. The other feature is the presence of large fluctuations for larger delays, i.e., close to threshold. As the system is kicked very close to this threshold, it remains in a slow flux region of the phase space and, therefore, the trajectory will be highly influenced by noise. Once the system is triggered, the trajectory quickly collapses to a deterministic path that yields the large excursion, characteristic of excitability.

In what follows, we study the situation where the impulse repetition rate becomes shorter than the refractory time, which is the time spent for the system to relax to a neighborhood of the stable state where a second critical perturbation can promote a subsequent large excursion. The experimental procedure to determine this time is to test the system with a two short pulse perturbation with a variable delay, close to

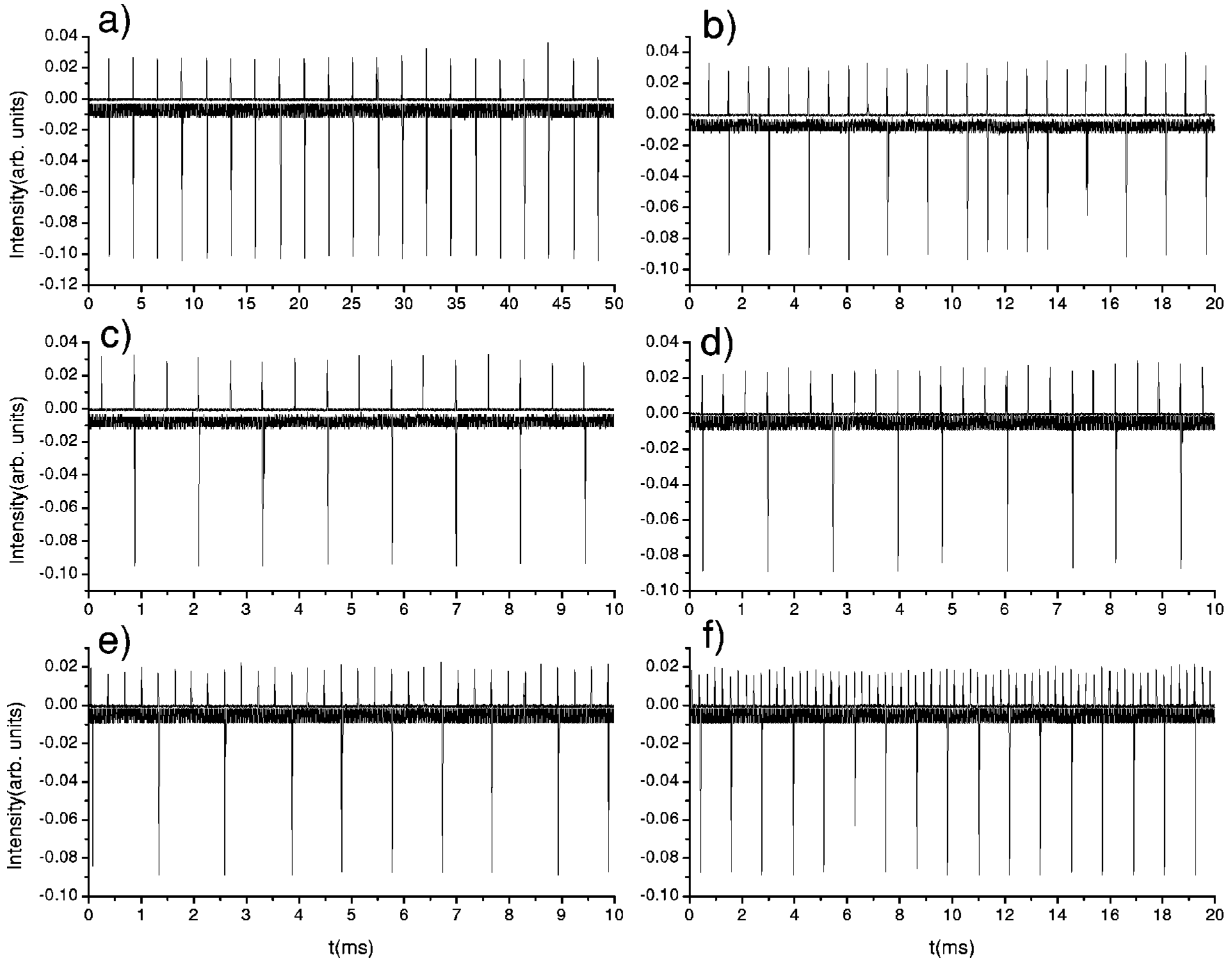

FIG. 7. Time series of the impulse (upper trace) and the response (lower trace), for different impulse-repetition rates. (a) 1:1 locking, $T=1.3$ ms; (b) 1:1–1:2 intermittence, $T=757$ μs; (c) 1:2 locking, $T=654$ μs; (d) 1:2–1:3 intermittence, $T=412$ μs; (e) 1:3–1:4 intermittence, $T=314$ μs; (f) 1:4 locking, $T=288$ μs.

E_c. The refractory time would then be given by the shorter delay for which the system responds to both pulses. We did not perform this measurement directly. However, we can estimate two bounds for the refractory time: the response pulse width and the time between pulses at the extinction of self-pulsating regime.

As we vary the frequency of the impulse train different kind of behaviors are obtained. In Fig. 7(a), the system responds synchronously for an impulse period of 1.3 ms (1:1 locking). When the impulse period is about 0.76 ms [Fig. 7(b)] the system stops responding to each of the perturbations and displays intermittence between 1:1 and a 1:2 locked states. Although this cannot be considered as a measurement of the refractory time since the active effect of the forcing on the system, it is worth noticing that the change of behavior at this repetition rate is within the above stated bounds for the refractory time. Increasing the repetition rate causes the system to follow the impulse with locking sequences at higher periods (1:2,1:3,1:4).

The response of the system to the external forcing is qualitatively different from what is observed for $a>0.58$, i.e., when the Q-switch pulses are born with small amplitude in a supercritical Hopf bifurcation [12,14]. In this situation, the output of the passive system subjected to external forcing shows a complex amplitude dynamics. As the system is periodically perturbed, the variables explore a wide region of the phase space, which is manifested as oscillations of variable amplitude. This contrasts with the above-described scenario of excitability, where the passive system responds to a stimulus either with a finite amplitude pulse or with no pulse at all. This behavior can be interpreted from the already conjectured scenario for excitability, in which the system is close to a (saddle loop) global bifurcation that creates a limit cycle. The trajectory in phase space corresponding to the excitable pulse can be highly dissipative, i.e., very contractive in the transverse direction. Therefore, significant amplitude dynamics in the excitable regime is unlikely to be observed as the system is periodically kicked.

IV. CONCLUSIONS

In this work, we study experimentally the excitable properties of a solid-state laser with saturable absorber from its response to external perturbations. We implement a method to perturb the system, which consists in injecting light pulses on the saturable absorber. Within this framework, two experimental situations are analyzed. In the first one, the time between injection pulses is much longer than the characteristic time scales of the passive system, which allows us to interpret the results as the response to isolated excitations. In the second experiment, the repetition rate of the stimuli approaches the natural time scales, thus the interpretation paradigm becomes the dynamical evolution of the system subjected to periodic forcing.

Two main features arise from the isolated perturbation test. The first one is that the dependence of the system response to the amplitude of the injection is steplike. A critical injection amplitude can be identified, and the amplitude of this response is almost constant as the injection amplitude is further increased. The second important issue is the observation of large delay fluctuations between the perturbation and the response, near the critical injection amplitude. From these experimental evidences, we can interpret the system as excitable, which turns out to be highly influenced by noise as it is kicked close to the threshold in the phase space.

These results, together with the hysteretical behavior between the onset and extinction of the laser self-oscillations are consistent with preexistent excitability models for a laser with saturable absorber. In those models, the bifurcation to oscillations is a saddle loop, which would account for the coexistence between the limit cycle and the off solution. Furthermore, we claim that the large delays observed near the critical injection together with the high sensitivity to noise can be a signature of a saddle point determining the excitable threshold. Trajectories that cross the threshold (stable manifold of the saddle) are attracted to the fixed point (slow-flux region) along its attractive direction and then escape following the unstable manifold. We propose this mechanism to explain how the system is being trapped in a slow-flux region before the final large excursion takes place. We would like to remark that even when the underlying mechanism for excitability could be essentially captured by the already mentioned models, a complete understanding of the physics of this system might require a multimode description, as it has been shown that multimode dynamics plays a significant role for certain parameter regions [15].

Finally, we have found that the system is able to lock to different sequences, as the forcing frequency is increased beyond the natural response time. The order in which the sequences appear is consistent with previous works on excitable systems under external forcing. In addition, the fact that the dynamical information lies in the phases rather than in the amplitude further evidences the highly dissipative nature of the excitable trajectory in the phase space.

We would like to encourage future works on the response of this system to added external noise. As the type of excitability is ruled by the proximity to a global bifurcation, we expect nontrivial (multipeaked) interspike time histograms as those found in semiconductor lasers with optical feedback [7].

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