

Dynamically turning off interactions in a two-component condensate

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(Received 23 October 2001; published 1 March 2002)

We propose a mechanism to change the interaction strengths of a two-component condensate. It is shown that the application of $\pi/2$ pulses allows us to alter the effective interspecies-interaction strength as well as the effective interaction strength between particles of the same kind. This mechanism provides a simple method to transform spatially stable condensates into unstable ones and vice versa. It also provides a means to store a squeezed spin state by turning off the interaction for the internal states and thus allows to gain control over many-body entangled states.

DOI: 10.1103/PhysRevA.65.033625

PACS number(s): 03.75.Fi, 42.50.-p, 42.50.Ct

I. INTRODUCTION

The experimental achievement of atomic Bose-Einstein condensation (BEC) [1–4] has stimulated extensive theoretical and experimental studies in this area. One of the most impressive examples of the applications of BEC is to use condensates with internal degrees of freedom to generate quantum entanglement [5–11], which is the essential ingredient for many quantum-information protocols [12]. It has been shown that the coherent collisional interactions in BECs allow to generate substantial many-particle entanglement in the spin degrees of freedom of a two-component condensate [5] during the free evolution of the condensates. The whole time evolution of the internal degrees of freedom is determined by the interaction strengths between the condensed particles. Thus it is desirable to control these interaction strengths by some external means since this opens the possibility to engineer many-particle entangled states.

One possibility is to change the atomic-interaction potential directly by applying an external magnetic field, which changes the scattering length. If one uses Feshbach resonances [13–15] this method allows for considerable changes in the interaction properties of BECs. In this paper we will propose another method to externally control the interaction strengths using $\pi/2$ pulses. This will not directly change the interatomic-interaction potential but will rather implement an *effective* Hamiltonian with an interaction strength depending on external parameters, which can be adjusted easily. We will use this effective Hamiltonian to study the influence of changing the interaction strengths on the external wave function. Furthermore, we will be able to control the time evolution of the internal degrees of freedom, which are solely determined by the effective interaction strengths. In particular, we will show how to turn off the Hamiltonian for the internal states, i.e., the internal states will evolve like those in an ideal gas.

Let us briefly explain the basic idea for changing the interaction strengths of BECs using $\pi/2$ pulses: If we fix the spatial mode function of a two-component condensate the dynamics of these two components is described by the Hamiltonian [5,8]

$$H_{\text{int}} = \chi \tilde{J}_z^2, \quad (1.1)$$

where \tilde{J}_z is the z component of an angular momentum operator $\tilde{\mathbf{J}} = \{\tilde{J}_x, \tilde{J}_y, \tilde{J}_z\}$ given by

$$\begin{aligned} \tilde{J}_x &= \frac{1}{2}(a^\dagger b + b^\dagger a), \\ \tilde{J}_y &= \frac{i}{2}(b^\dagger a - a^\dagger b), \\ \tilde{J}_z &= \frac{1}{2}(a^\dagger a - b^\dagger b), \end{aligned} \quad (1.2)$$

where a and b are bosonic destruction operators, respectively, for particles in internal states 1 and 2 with a spatial-mode function $\varphi_{1(2)}$. The parameter χ is determined by the interaction properties of the two-component condensate. We apply $\pi/2$ Raman-laser or microwave pulses to the condensate, which rotate the spin around the x or y axis by an angle $\pi/2$ depending on the phase of the pulse as we will show in Sec. II B. The idea is to use a sequence of $\pi/2$ pulses that rotates H_{int} to create contributions to the Hamiltonian given by χJ_x^2 and χJ_y^2 for a time $\zeta \delta t$ while the system evolves with χJ_z^2 for a time δt .

Time averaging leads to an effective Hamiltonian, which can be written as

$$\tilde{H}_{\text{int}}^{\text{eff}} = \frac{\chi \tilde{J}_z^2 + \zeta \chi (\tilde{J}_y^2 + \tilde{J}_x^2)}{1 + 2\zeta} \equiv \chi \frac{(1 - \zeta) \tilde{J}_z^2 + \zeta \tilde{\mathbf{J}}^2}{1 + 2\zeta}. \quad (1.3)$$

The operator $\tilde{\mathbf{J}}^2$ is a constant of motion and the Hamiltonian, therefore, is equivalent to $H_{\text{int}}^{\text{eff}} = \tilde{\chi} \tilde{J}_z^2$ up to a (time-dependent) global phase. Thus, the application of the $\pi/2$ pulses effectively leads to a change in the interaction parameters from χ to $\tilde{\chi} = \chi(1 - \zeta)/(1 + 2\zeta)$. For $\zeta = 1$ we find that the effective Hamiltonian $H_{\text{int}}^{\text{eff}}$ vanishes. The internal states then evolve like those in a noninteracting gas.

The paper is organized as follows. In Sec. II we will introduce the model. After writing down the Hamiltonian of a two-component condensate interacting with a classical laser or microwave field we will define a specific series of pulses applied to the condensate. Then we will calculate the time-averaged Hamiltonian and determine the dependence of the

effective interaction strengths on parameters of the external field. In Sec. III we will study possible applications of this effective Hamiltonian. We will investigate the influence of changing the interaction strength on the spatial wave functions of the condensate as well as on the evolution of the internal atomic degrees of freedom. Section IV is devoted to the discussion of approximations and possible imperfections in our model. We conclude in Sec. V with a discussion and summary of our results.

II. MODEL

In this section we start with the Hamiltonian of a two-component condensate interacting with an external field. We investigate the effect of $\pi/2$ pulses on the condensate and specify a specific series of pulses. We show that this specific choice of pulses effectively leads to a change in the interaction strengths of the condensate.

A. Hamiltonian

We consider a two-component BEC consisting of N atoms in different atomic hyperfine levels 1 and 2 coupled by a time- (t -) dependent classical field with Rabi frequency $\Omega(t)$ (internal Josephson effect [16–19]). The classical field can be realized by either a Raman laser or by a microwave field applied to the condensate. We assume Ω to be \mathbf{x} independent, i.e., there is negligible momentum transfer to the condensates due to the interaction with the classical field. The Hamiltonian of this system is given by $H=H_{\text{BEC}}+H_L$, where $H_{\text{BEC}}=H_1(\mathbf{u})+H_2(\mathbf{u})$ and H_L describe the two-component condensate and the interaction of external field and condensate, respectively [20–22]. These terms are given by (with $\hbar=1$)

$$H_k(\mathbf{u}) = \int d^3\mathbf{x} \psi_k^\dagger \left(-\frac{\nabla^2}{2m} + V_k + \sum_l \frac{u_{kl}}{2} \psi_l^\dagger \psi_l \right) \psi_k, \quad (2.1a)$$

$$H_L = \int d^3\mathbf{x} \left[\frac{\Omega(t)}{2} \psi_1^\dagger \psi_2 + \text{H.c.} \right]. \quad (2.1b)$$

Here, $\{k, l\} \in \{1, 2\}$ and $\psi_k \equiv \psi_k(\mathbf{x})$ is a bosonic field operator, which annihilates a particle at position \mathbf{x} in hyperfine state $|k\rangle$. The trapping potential for particles in state k is denoted by $V_k \equiv V_k(\mathbf{x})$ and the mass of the atoms is m . The interaction strengths are given by $\mathbf{u} = \{u_{11}, u_{22}, u_{12}\}$, where for simplicity we assume $u_{11} = u_{22} \equiv u$ for the interaction between atoms in the same internal state, and $u_{12} \neq u$ as is the case in Na [22]. Furthermore we assume the trapping potential for the different internal states to be equal, i.e., $V_1 = V_2 \equiv V$. We denote the first excitation energy of V by ω and the size of the single-particle ground state in the potential V by a_0 .

H_L describes the interaction of the external field with the condensate. We assume that this external field is used to apply a sequence of $\pi/2$ pulses to the condensates. While it is turned on, the Rabi frequency is constant $\Omega(t) = \Omega_0$. The duration of the $\pi/2$ pulses is thus given by $t_d = \pi/2\Omega_0$, which is assumed to be much shorter than the time scale

determined by the evolution of the condensates due to H_k , i.e., $\Omega_0 \gg \omega$ and $\Omega_0 \gg Nu/a_0^3$ [23]. Next we investigate the time evolution of the system while the external field is turned on.

1. Interaction with the external field

While H_L is turned on, i.e., $\Omega \neq 0$ it is the dominant part of the Hamiltonian H and we neglect contributions of H_k . A $\pi/2$ pulse is characterized by

$$\Omega(t) = |\Omega(t)| e^{i\alpha}, \quad \int_{-\infty}^{\infty} |\Omega(t)| dt = \pi/2 \quad (2.2)$$

with a phase α , and it implements the following time evolution for the bosonic field operators in the Heisenberg picture:

$$U_\alpha^\dagger \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} U_\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -ie^{i\alpha} \\ -ie^{-i\alpha} & 1 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \quad (2.3)$$

The inverse transformation is given by $U_\alpha^\dagger = U_\alpha^{-1} = U_{\alpha+\pi}$.

2. Spin operators

To get an intuitive picture of the effect of $\pi/2$ pulses on the condensate we define the spin operator $\mathbf{J} = \{J_x, J_y, J_z\}$ by

$$\begin{aligned} J_x &= \frac{1}{2} \int d^3\mathbf{x} (\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1), \\ J_y &= \frac{i}{2} \int d^3\mathbf{x} (\psi_2^\dagger \psi_1 - \psi_1^\dagger \psi_2), \\ J_z &= \frac{1}{2} \int d^3\mathbf{x} (\psi_1^\dagger \psi_1 - \psi_2^\dagger \psi_2). \end{aligned} \quad (2.4)$$

In the Heisenberg picture these operators are transformed by the $\pi/2$ pulses according to

$$U_\pi J_z U_0 = J_y, \quad U_{-\pi/2} J_z U_{\pi/2} = J_x. \quad (2.5)$$

Thus the application of U_0 and $U_{\pi/2}$ to the condensate corresponds to a rotation of the spin \mathbf{J} around the x and y axis by an angle $\pi/2$, respectively.

3. Series of $\pi/2$ pulses

We want to consider a specific sequence of pulses applied repeatedly to the condensates. One sequence of pulses is shown in Fig. 1(a) while Fig. 1(b) shows the whole series of pulses. As can be seen from Fig. 1(a) the condensate first evolves freely for a time δt . Then a $\pi/2$ pulse rotates the spin instantaneously around the y axis by an angle $\pi/2$ and back after a time $\zeta \delta t$. Immediately afterwards, we rotate the spin by $\pi/2$ around the x axis and then back after time $\zeta \delta t$. This sequence of $\pi/2$ pulses is repeated as shown in Fig. 1(b). Each sequence takes a time $t_c = (1 + 2\zeta) \delta t$, neglecting the time t_d needed to apply a pulse.

There are thus four time scales in this model: (i) t_d the duration of a $\pi/2$ pulse; (ii) t_c , which is the time needed for applying a sequence of $\pi/2$ pulses; (iii) $t_{\text{BEC}} = 1/\omega$, which

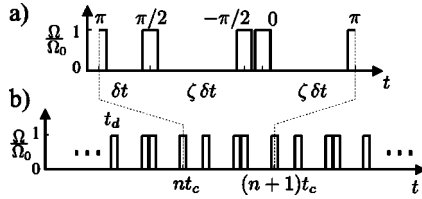


FIG. 1. Schematic plot of the sequence of laser pulses Ω/Ω_0 against time t in arbitrary units. (a) Sequence of $\pi/2$ pulses $\Omega(t)$ applied to the two-component condensate between nt_c and $(n+1)t_c$. The angle α of the $\pi/2$ pulses is shown above the corresponding pulse. Graph (b) shows the whole series of pulses, which consists of $\pi/2$ pulse sequences shown in (a).

determines the time scale of the free evolution of the condensates; and (iv) $t_{\text{int}} = a_0^3/uN$, which is the time scale set by the interactions between the particles [23]. These four time scales are assumed to satisfy the relation $t_d \ll t_c \ll t_{\text{BEC}} \approx t_{\text{int}}$, which can easily be achieved by an appropriate choice of external parameters.

4. Time-averaged Hamiltonian

We now want to study the time evolution of the condensates when the pulses specified in Sec. II A 3 are applied to the system. The time-evolution operator \mathcal{U}^M at time $t = Mt_c$, i.e., after applying M pulse sequences is given by

$$\mathcal{U}^M = \prod_{l=1}^M \mathcal{U}, \quad (2.6)$$

where

$$\begin{aligned} \mathcal{U} &= U_{\pi} e^{-iH_{\text{BEC}}\zeta\delta t} U_0 U_{-\pi/2} e^{-iH_{\text{BEC}}\zeta\delta t} U_{\pi/2} e^{-iH_{\text{BEC}}\delta t} \\ &\equiv e^{-iH_{\text{eff}}t_c}. \end{aligned} \quad (2.7)$$

To first order in t_c , we find for the effective Hamiltonian

$$\begin{aligned} H_{\text{eff}} &= \frac{H_{\text{BEC}} + \zeta(U_{\pi/2}H_{\text{BEC}}U_{-\pi/2} + U_0H_{\text{BEC}}U_{\pi})}{1 + 2\zeta} \\ &= H_1(\tilde{\mathbf{u}}) + H_2(\tilde{\mathbf{u}}), \end{aligned} \quad (2.8)$$

where $\tilde{\mathbf{u}} = \{\tilde{u}_{11}, \tilde{u}_{22}, \tilde{u}_{12}\}$ with $\tilde{u}_{11} = \tilde{u}_{22} \equiv \tilde{u}$ and

$$\begin{aligned} \tilde{u} &= \frac{u + (u + u_{12})\zeta}{1 + 2\zeta}, \\ \tilde{u}_{12} &= \frac{u_{12} + 2u\zeta}{1 + 2\zeta}. \end{aligned} \quad (2.9)$$

The effective interaction strengths \tilde{u} and \tilde{u}_{12} appearing in the time-averaged Hamiltonian H_{eff} depend on the parameter ζ . Experimentally this parameter can easily be changed by adjusting laser or microwave parameters. In Fig. 2 we show the dependence of the interaction strength on the parameter ζ . The situation $\zeta=0$ corresponds to the case of applying no pulses. We have assumed $u > u_{12}$, i.e., a spatially stable two-component condensate. By increasing ζ we find that \tilde{u} de-

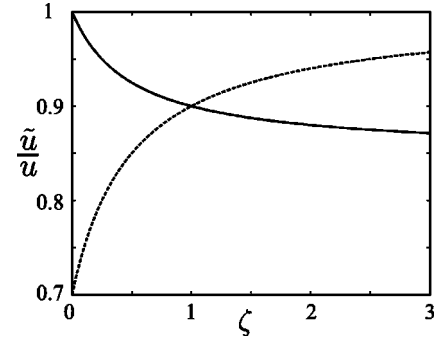


FIG. 2. Interaction strength \tilde{u} (solid curve) for particles in the same hyperfine level and \tilde{u}_{12} (dashed curve) for particles in different hyperfine levels as a function of ζ . For $\zeta=0$ we assumed that $\tilde{u}_{12} = u_{12} = 0.7u = 0.7\tilde{u}$.

creases and \tilde{u}_{12} increases. At $\zeta=1$ the two effective interaction strengths are crossing. This situation corresponds to the case where the internal Hamiltonian $H_{\text{int}}^{\text{eff}} = 0$ as discussed in Sec. I.

Qualitatively we can understand how an effective change in the interaction strengths arises by looking at the time evolution of a condensate with initially all the particles in state $|k\rangle$. If no laser pulses are applied to the condensate, the phase accumulated due to the interaction during a time $\zeta\delta t$ is proportional to $u\zeta\delta t$. However, if a $\pi/2$ pulse is applied the condensate is put into a superposition state of the internal states $|1\rangle$ and $|2\rangle$ according to Eq. (2.3). In this case the phase accumulated during a time $\zeta\delta t$ is proportional to $(u + u_{12})\zeta\delta t/2$. Then a second $\pi/2$ pulse brings the condensate back to the internal state $|k\rangle$. Effectively the two $\pi/2$ pulses thus lead to a change of the interaction strength.

B. Two-mode approximation

In this section we use the two-mode approximation for deriving the coupled Gross-Pitaevskii equations (GPE) for the condensate. Then we investigate the time evolution of the internal atomic degrees of freedom.

1. External degrees of freedom

We assume that the condensate can be described using only one spatial-mode function for each component, i.e.,

$$\psi_1(\mathbf{x}, t) = a\varphi_1(\mathbf{x}, t) \quad \text{and} \quad \psi_2(\mathbf{x}, t) = b\varphi_2(\mathbf{x}, t), \quad (2.10)$$

where $\varphi_k(\mathbf{x}, t)$ is the spatial wave function of condensate k and a (b) is the bosonic annihilation operator for particles in condensate 1 (2). We put this ansatz into the Hamiltonian (2.8); assume the state of the condensate to be

$$|\psi\rangle = \frac{(a^\dagger)^{N_1}(b^\dagger)^{N_2}}{\sqrt{N_1!N_2!}}|0\rangle, \quad (2.11)$$

with N_1 particles in condensate 1, N_2 particles in condensate 2, and $|0\rangle$ being the vacuum state. Minimizing the expression

$$\left\langle \psi \left| i \frac{\partial}{\partial t} - H_{\text{eff}} \right| \psi \right\rangle \quad (2.12)$$

with respect to the wave functions $\varphi_{1,2}(\mathbf{x})$, we find the coupled GPE equations

$$\begin{aligned} i \frac{\partial \varphi_1}{\partial t} &= \left(-\frac{\nabla^2}{2m} + V + \tilde{u} N_1 |\varphi_1|^2 + \tilde{u}_{12} N_2 |\varphi_2|^2 \right) \varphi_1, \\ i \frac{\partial \varphi_2}{\partial t} &= \left(-\frac{\nabla^2}{2m} + V + \tilde{u} N_2 |\varphi_2|^2 + \tilde{u}_{12} N_1 |\varphi_1|^2 \right) \varphi_2, \end{aligned} \quad (2.13)$$

where $\varphi_{1,2} \equiv \varphi_{1,2}(\mathbf{x}, t)$ and \tilde{u} and \tilde{u}_{12} are time dependent.

2. Internal degrees of freedom

We want to simplify the model further by assuming $N_1 - N_2$ being of $O(\sqrt{N})$ and setting $\varphi_1 = \varphi_2 \equiv \varphi$ in the ansatz for the bosonic field operators. Minimizing the terms of $O(N)$ in expression (2.12) we find the GPE

$$i \frac{\partial \varphi}{\partial t} = \left(-\frac{\nabla^2}{2m} + V + N \frac{\tilde{u} + \tilde{u}_{12}}{2} |\varphi|^2 \right) \varphi \quad (2.14)$$

for the wave function. If φ fulfills the above GPE, the terms of $O(\sqrt{N})$ vanish in Eq. (2.12) and the time evolution of the internal atomic degrees of freedom is given by the Hamiltonian (up to a global phase)

$$H_{\text{int}}^{\text{eff}} = \tilde{\chi} \tilde{\mathbf{J}}_z^2, \quad (2.15)$$

where

$$\tilde{\chi} = \chi \frac{1 - \zeta}{1 + 2\zeta} \quad \text{with} \quad \chi = (u - u_{12}) \int d^3 \mathbf{x} |\varphi|^4. \quad (2.16)$$

At $\zeta = 1$ the parameter $\tilde{\chi} = 0$. There will thus be no dynamics of the internal degrees of freedom even for $N_1 \neq N_2$. The internal spin operator $\tilde{\mathbf{J}}$ is obtained from \mathbf{J} by using ansatz (2.10) and is explicitly given in Eq. (1.2). A physical interpretation of the evolution of the internal atomic states was already given in the Introduction.

Putting the two-mode ansatz (2.10) into the time-evolution operator \mathcal{U}^M we find

$$\mathcal{U}_{\text{int}}^M = \prod_{l=1}^M [\tilde{U}_{\pi} e^{-iH_{\text{int}} \zeta \delta t} \tilde{U}_0 \tilde{U}_{-\pi/2} e^{-iH_{\text{int}} \zeta \delta t} \tilde{U}_{\pi/2} e^{-iH_{\text{int}} \delta t}], \quad (2.17)$$

where \tilde{U}_α is obtained from U_α by replacing the bosonic field operators $\psi_1, (\psi_2)$ with the corresponding annihilation operators a (b).

III. APPLICATIONS

In this section we study possible applications of changing the interaction strength between the condensate discussed

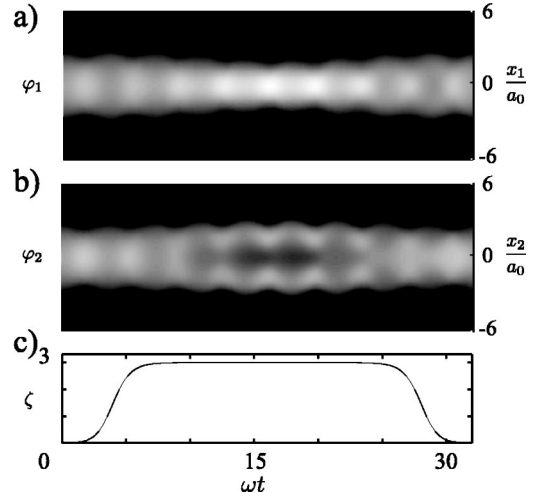


FIG. 3. Condensate wave functions φ_1 and φ_2 as a function of time and spatial coordinates x_1 and x_2 are shown in graphs (a) and (b), respectively. Light regions indicate large condensate density, dark regions do not contain condensed particles. (c) Parameter ζ as a function of time. The parameters are $N_1 = 5200$, $N_2 = 4800$, trapping frequency $\omega = 2\pi \times 40$ Hz, the interaction strength is assumed to be $2u_{12} = u = 3 \times 10^{-3} \omega a_0$.

above. First we show how stable condensates can be destabilized and vice versa. Then we investigate how a squeezed state of a two-component condensate can be preserved by turning off the interactions for the internal states.

A. (Un)stabilizing a two-component condensate

The properties of multicomponent condensates such as spin-domain formation have been studied extensively both experimentally [20,24–27] and theoretically [19,28–30] in the last few years for constant interaction strengths. Here, we study the effect of varying ζ and thus changing the interaction strengths between the condensed particles on the condensate wave functions φ_k and the spatial stability. For $\tilde{u} > \tilde{u}_{12}$ two initially overlapping condensates should remain spatially stable while for $\tilde{u} < \tilde{u}_{12}$ they separate.

We solve numerically Eq. (2.13) in one spatial dimension. The trapping potential is assumed to be harmonic, $V(x) = m\omega^2 x^2/2$, where ω is the trap frequency and $a_0 = \sqrt{\hbar/m\omega}$ is the ground-state size. Figures 3(a,b) show the condensate wave functions φ_1 and φ_2 , respectively. We change $\zeta(t)$ as shown in Fig. 3(c). Initially $\zeta(0) = 0$ and we assume that $u > u_{12}$, i.e., the two condensates are strongly overlapping. When $\zeta(t) > 1$ the repulsion between atoms in different hyperfine states separates the condensates in space since then $u_{12} > u$ (as can be seen from Fig. 2). As soon as $\zeta(t)$ becomes smaller than 1 again, the two condensates become overlapping again. Note that the time scale for the separation of the two condensates depends also on the imbalance of the condensate particle numbers $N_1 - N_2$. For $N_1 = N_2$ the two condensates do not separate for the parameters chosen in Fig. 3.

B. Spin-squeezed condensate states

Next we want to consider the behavior of the internal degrees of freedom and show that by choosing $\zeta=1$ and thus making the internal Hamiltonian $H_{\text{int}}=0$ we can store a spin-squeezed state.

1. The squeezing parameter

The entanglement properties of the atoms can be expressed in terms of the variances and expectation values of the angular momentum operators $\tilde{\mathbf{J}}$. Of particular interest is the squeezing parameter ξ^2 defined by [5]

$$\xi^2 = \min_{\mathbf{n}_{1,2,3}} \frac{N(\Delta\tilde{\mathbf{J}}_{\mathbf{n}_1})^2}{\langle\tilde{\mathbf{J}}_{\mathbf{n}_2}\rangle^2 + \langle\tilde{\mathbf{J}}_{\mathbf{n}_3}\rangle^2}, \quad (3.1)$$

where $\tilde{\mathbf{J}}_{\mathbf{n}} \equiv \mathbf{n} \cdot \tilde{\mathbf{J}}$ and the $\mathbf{n}_{1,2,3}$ are mutually orthogonal unit vectors. If $\xi^2 < 1$ the state of the atoms is nonseparable (i.e., entangled) as has been shown, e.g., in [5]. The parameter ξ^2 thus characterizes the atomic entanglement, and the states with $\xi^2 < 1$ are often referred to as ‘‘spin-squeezed states’’ [31].

2. Preserving a spin-squeezed condensate state

We assume an initial state of the form

$$|\psi\rangle = \frac{(a^\dagger + b^\dagger)^N}{\sqrt{N!}} |0\rangle \quad (3.2)$$

created by applying a $\pi/2$ pulse with $\alpha = \pi/2$ to a condensate of particles in internal state $|1\rangle$. The evolution of this initial state according to the Hamiltonian $H_{\text{int}}^{\text{eff}}$ with constant $\tilde{\chi}$ has been studied extensively in [5] and leads to one-axis squeezing as defined in [31]. Initially $\xi^2=1$ and is then rapidly reduced. After reaching a minimum value, the entanglement parameter ξ^2 increases again. Our aim is to control the interaction parameter $\tilde{\chi}$ such that after ξ^2 has reached its minimum value, further evolution of the system is suppressed. In Fig. 4 we show a comparison of the time evolution with the time-evolution operator $\mathcal{U}_{\text{int}}^M$ and the effective Hamiltonian $H_{\text{int}}^{\text{eff}}$. The squeezing parameter $\xi^2(t)$ is shown in Fig. 4(a) and the time dependence of $\zeta(t)$ is shown in Fig. 4(b). As soon as ξ^2 has reached its minimum value, ζ goes rapidly towards 1 and thus prohibits further evolution of ξ^2 . The squeezing parameter ξ^2 remains at its minimum value, which is close to the minimum value $\xi_m^2 = (3/N)^{2/3}/2$ that can be reached by one-axis squeezing [31]. Note that the minimum squeezing parameter that is reached by the evolution according to $\mathcal{U}_{\text{int}}^M$ is smaller than expected from the Hamiltonian H_{eff} as long as $t_c \ll t_{\text{BEC}}$ is fulfilled (cf. Fig. 4). We find, however, that this difference is always very small and vanishes if we further decrease t_c compared to the value used in Fig. 4.

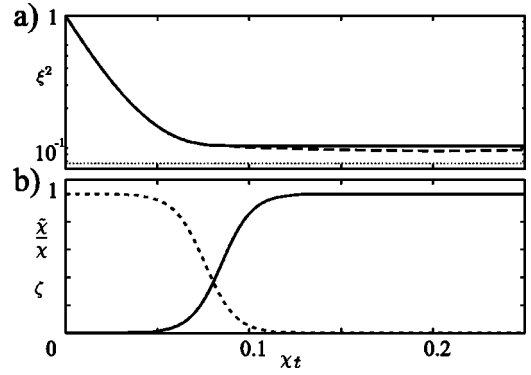


FIG. 4. (a) Squeezing parameter ξ^2 as a function of time. The dotted line shows the minimum squeezing parameter ξ_m^2 achievable by one-axis squeezing as defined in the text. The solid curve shows the squeezing parameter obtained by solving the Schrödinger equation using the effective Hamiltonian $H_{\text{int}}^{\text{eff}}$ defined in Eq. (2.15). The dashed curve shows the squeezing parameter obtained from the time-evolution operator $\mathcal{U}_{\text{int}}^M$ with $\chi t_c = 5 \times 10^{-3}$. (b) Parameter ζ (solid curve) and resulting relative interaction strength $\tilde{\chi}$ (dashed curve) as functions of time. The numerical calculation was done for $N=50$.

IV. DISCUSSION

There are two different kinds of approximations in our scheme. First, the Hamiltonian H_{eff} we use is time averaged over the duration of a sequence of pulses t_c and second, we use a two-mode description of the two-component condensate for describing the dynamics of the internal states. We will discuss these two approximations separately since they are independent of each other. Also, experimentally it is not possible to exactly realize $\pi/2$ pulses. Therefore, we will also discuss the influence of imperfections in the $\pi/2$ pulses.

A. Approximations

1. Time averaging

While H_L is turned on, we neglect the free time evolution of the condensate due to H_{BEC} completely. Typically the time evolution due to the applied $\pi/2$ pulses will take place on a time scale t_d of a few nanoseconds while the typical time scale for the free evolution of the condensate t_{BEC} is of the order of milliseconds. The neglect of the free evolution of the condensates during a pulse will lead to an error of the order of $t_d/t_{\text{BEC}} \approx 10^{-4}$ and is thus well justified. The second step in calculating the effective Hamiltonian is to average over one sequence of pulses. This will typically lead to an error of the order of t_c/t_{BEC} . In Fig. 4 we compare the time evolution according to the time-averaged Hamiltonian with the time evolution given by $\mathcal{U}_{\text{int}}^M$ for $t_c \chi = 5 \times 10^{-3}$ and find a very small deviation between the two results.

2. Two-mode approximation

The form of the effective Hamiltonian H_{eff} is equivalent to the standard form of the Hamiltonian for two-component BECs. Therefore we expect the same range of validity for

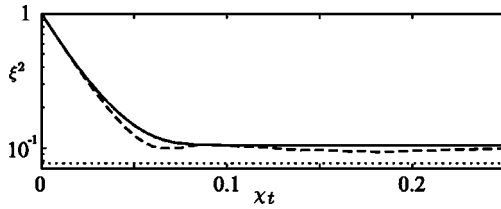


FIG. 5. Squeezing parameter ξ^2 as a function of time. The dotted line shows the minimum squeezing parameter ξ_m^2 achievable by one-axis squeezing as defined in the text. The solid curve shows the squeezing parameter obtained by solving the Schrödinger equation using the effective Hamiltonian $H_{\text{int}}^{\text{eff}}$ defined in Eq. (2.15). The dashed curve is obtained by calculating the time-evolution according to the time evolution operator $\mathcal{U}_{\text{int}}^M$ with $\chi t_c = 5 \times 10^{-3}$ and a 1% error in the intensity of the $\pi/2$ pulses. The ensemble average over $R=2000$ realizations is shown. The other parameters are equal to those chosen in Fig. 4.

H_{eff} as for the original Hamiltonian H . This also applies to the two-mode approximation introduced in Sec. II B.

B. Imperfect $\pi/2$ pulses

It is experimentally possible to adjust the phase α of the $\pi/2$ pulses very precisely, while it is much harder to exactly fulfill the integral condition in Eq. (2.2). Therefore we investigate the influence of a violation of this condition on the time evolution of our system. We assume a random Gaussian error of 1% in the value of the integral in Eq. (2.2) for each pulse applied to the condensate and calculate the resulting time evolution in the two-mode approximation. Figure 5 shows the result for the squeezing parameter averaged over $R=2000$ different realizations. As can be seen from Fig. 5, an error in the duration and intensity of the pulses does not

lead to a qualitatively different behavior of the system. For some of the realizations we obtain a smaller squeezing parameter ξ^2 than expected from one-axis squeezing. In this case the error in the $\pi/2$ pulses leads to some two-axis squeezing, which yields a smaller squeezing parameter than pure one-axis squeezing [31].

V. CONCLUSIONS

In this paper we have introduced a method to change the interaction strength of a two-component condensate by $\pi/2$ pulses. We have shown that applying a specific series of pulses to the condensate leads to an effective time-averaged Hamiltonian, which is of the form of the original two-component Hamiltonian with an interaction strength depending on parameters of the external field.

As applications of this scheme we have proposed to use this Hamiltonian for turning a stable condensate into an unstable one and vice versa. We have also shown that it is possible to store a spin-squeezed state of a condensate for, at least, in principle, an arbitrarily long time.

Finally, we want to point out that the method to change the interaction strengths of BECs discussed in this paper can experimentally be realized with current technology. It is intended to serve as a tool to gain further insight into the properties of BECs as well as to aid in engineering many-particle entangled states.

ACKNOWLEDGMENTS

We thank Lu-Ming Duan and F. Schmidt-Kaler for stimulating discussions. This work was supported by the Austrian Science Foundation (Project No. Z30-TPH, Wittgenstein-Preis and SFB “Control and measurement of Coherent Quantum Systems”).

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