Nonadiabatic transition probabilities in the presence of strong dissipation at an avoided-level crossing point

Keiji Saito
Department of Applied Physics, School of Engineering, University of Tokyo, Bunkyo-ku, Tokyo 113-8656, Japan

Yosuke Kayanuma
Department of Mathematical Science, Graduate School of Engineering, Osaka Prefecture University, Sakai 599-8531, Japan
(Received 3 September 2001; published 8 February 2002)

Dissipative effects on the nonadiabatic transition for the two- and three-level systems are studied. When the system is affected by a strong dissipation through the diabatic states, the exact transition probability is enumerated making use of the effective master equation. In the two-level system, we consider the case where the external field is swept from not only a negative large value but also from the resonant field, and the exact transition probabilities in these cases are derived. The transition probabilities are derived for the three-level system where the three diabatic states form only one avoided-level crossing point. These probabilities are compared with that in the pure quantum case obtained by Carroll and Hioe.

DOI: 10.1103/PhysRevA.65.033407 PACS number(s): 32.60.+i, 42.50.Lc

I. INTRODUCTION

Nonadiabatic transition at an avoided-level crossing point plays a crucial role in quantum dynamical changes of states, and yields a variety of phenomena in physics and chemistry. The well-known Landau and Zener (LZ) transition probability clarifies the roles of the energy gap and the sweeping velocity of the external field in the nonadiabatic transition of the two-level system [1–4]. Although the LZ transition probability is given in the two-level system, it is approximately applicable to multilevel systems where the avoided-level crossings are effectively well described by only localized two levels. Hence it is adopted in the analyses of many experiments that treat time-dependent phenomena, such as collision of particles [5,6], optics [7,8], and magnetic phenomena [9–13]. For general multilevel systems where many levels can simultaneously affect each other, different formulas of transition probabilities are derived for several models, i.e., the one where only one level interacts with a band of levels [14–16], the generalized model for this model [17,18], and the bow-tie model where many levels form only one avoided-level crossing [19,20]. We should also note Brundobler and Elser's hypothesis, which states that the survival probability of the diabatic state with maximum or minimum slope is described by the exponential form determined by only the velocity and the off-diagonal elements in the Hamiltonian [21].

On the other hand, we must also consider the effect of dissipation, since real experiments are always exposed to thermal environment. The thermal environment causes decoherence, and the inevitable deviation of transition probability from the one of pure quantum cases occurs. This modification becomes significant in real experiments such as the adiabatic rapid passage with phonon couplings [22], the nonadiabatic transitions in localized centers in solids [23], and the nonadiabatic magnetization process in molecular magnets such as Mn_{12} and Fe_8 [24–27]. Kayanuma studied such thermal-noise effect for the two-level Landau-Zener model and derived a formula for the effective transition probability in the limiting case of strong damping dissipation by the perturbative approach [28]. The effective transition probability becomes 1/2 in the adiabatic limit due to the dissipation effect, whereas it converges to the asymptotic expression of the LZ probability in the fast-sweeping limit. Ao and Rammer carried out first-principles calculations to investigate the temperature dependence of the transition probability of the two-level system with the phonon reservoir, which corresponds to Kayanuma's situation in the high-temperature limit in case of the Ohmic spectral density. Especially, they found some compensation effect that the transition probability for zero temperature takes the same value as the Landau-Zener probability [29].

In this paper, we study such thermal-noise effect in not only the two-level system but also in the three-level system. Thereby we try to investigate the effect of multilevels with thermal noise. We exploit the method to analyze strong dissipation effect using an effective master equation instead of the perturbation approach adopted in previous studies [23,28,29]. We show that the effective master-equation approach is very convenient for deriving the transition probability in the strong-damping limit. Using this approach we first reproduce Kayanuma's formula in the two-level system when the external field is reversed from a large negative value to a large positive value. We next consider the situation where the field is swept from the resonant field (zero field) to a large positive field, and derive exact transition probability. As a result the exact relations between these cases are found. The three-level model we consider is the same model as Carroll and Hioe considered [19]. In this model, three diabatic states form only one avoided-level crossing point. Therefore the transition mechanism is quite different from the LZ mechanism, which describes transitions between local two levels. Therefore we see the effect of the multilevel not only in the pure quantum case but in the dissipative case. We adopt the effective master-equation approach and derive the transition probabilities in the strong damping limit. The probabilities are compared with those of the pure quantum case obtained by Carroll and Hioe. The expression of the probabilities are always the same regardless of level structures, although in the pure quantum case the expressions of the transition probabilities show some variations.

This paper is organized in the following way. In Sec. II, we derive the master equation when strong noise couples with the system and compare it with the master equation for the system with the phonon reservoir. Section III is devoted to the problem for the two-level system, and we derive the transition probabilities for the three-level system in Sec. IV. A summary and brief discussion are given in Sec. V.

II. MASTER EQUATION

We derive the master equation for the system with dissipation. Throughout this paper, we study the transition probability by solving the master equation. The master equation we shall consider is derived for various types of dissipative environments such as the stochastic-noise field and phonon reservoir. We here choose a stochastic-noise field as a source of dissipation and rigorously derive the master equation. As shown in Appendix A, this master equation can be obtained in the case of a phonon reservoir with the Ohmic-type spectral density at very high temperatures. The correlation of the stochastic noise is assumed to be very short. This situation is described as

$$\mathcal{H}_{\rm tot}=\mathcal{H}(t)+\sum_{\ell}\xi_{\ell}(t)X_{\ell}, \tag{1}$$

$$\langle \xi_{\ell}(t)\xi_m(t')\rangle = 2\gamma_{\ell}\delta_{\ell,m}\delta(t-t'), \tag{2}$$

where $\xi_{\ell}(t)$ is a noise that affects the system through the ℓth operator X_{ℓ}. The matrix X_{ℓ} is diagonal in the diabatic bases of the Hamiltonian $\mathcal{H}(t)$ so that the computability $[X_{\ell},X_{\ell'}]=0$ is satisfied for arbitrary ℓ and ℓ'. The noise is supposed to be the white Gaussian process. We start with the von Neumann equation for the density matrix in the interaction picture ($\hbar=1$ here and hereafter),

$$\frac{\partial \rho^{(I)}(t)}{\partial t}=\sum_{\ell}\xi_{\ell}(t)\mathcal{L}_{\ell}(t)\rho^{(I)}, \tag{3}$$

$$\xi_{\ell}(t)\mathcal{L}_{\ell}(t)\rho^{(I)}=-i\xi_{\ell}(t)[\rho^{(I)}(t),X_{\ell}(t)], \tag{4}$$

$$\rho^{(I)}(t)=\exp_{\leftarrow}\left(-\int_{t_0}^{t}du\mathcal{H}(u)\right)\rho(t)\exp_{\rightarrow}\left(i\int_{t_0}^{t}du\mathcal{H}(u)\right), \tag{5}$$

$$\dot{X}_{\ell}(t)=\exp_{\leftarrow}\left(-i\int_{t_0}^{t}du\mathcal{H}(u)\right)X_{\ell}\exp_{\rightarrow}\left(i\int_{t_0}^{t}du\mathcal{H}(u)\right). \tag{6}$$

Here $\exp_{\leftarrow}$ and $\exp_{\rightarrow}$ express the time-ordered product of exponentials. In the case of white Gaussian process (2), there are several approaches to derive the master equation [30,31]. Here we use Novikov's relation, which holds for arbitrary function $g([\xi],t)$ [31],

$$\langle \xi g([\xi],t)\rangle=\int_{t_0}^{t}dt'\langle\xi(t)\xi(t')\rangle\left\langle\frac{\delta g([\xi],t)}{\delta\xi(t')}\right\rangle, \tag{7}$$

where the symbol $[\xi]$ means that $g([\xi],t)$ is a function of the process of noise ξ and $\langle\cdots\rangle$ means the average over the noise $\xi(t)$. By use of this mathematical formula, the average over noise for Eq. (3) is reduced to

$$\begin{aligned}\frac{\partial\langle\rho^{(I)}(t)\rangle}{\partial t}&=\sum_{\ell}\langle\xi_{\ell}(t)\mathcal{L}_{\ell}(t)\rho^{(I)}(t)\rangle=\sum_{\ell}\left\langle\xi_{\ell}(t)\mathcal{L}_{\ell}(t)\exp_{\leftarrow}\left(\sum_{\ell'}\int_{t_0}^{t}du\,\xi_{\ell'}(u)\mathcal{L}_{\ell'}(u)\right)\rho^{I}(t)\right\rangle\\&=\sum_{\ell}\mathcal{L}_{\ell}(t)\int_{t_0}^{t}dt'\langle\xi_{\ell}(t)\xi_{\ell}(t')\rangle\left\langle\exp_{\leftarrow}\left(\sum_{\ell'}\int_{t'}^{t}du\,\xi_{\ell'}(u)\mathcal{L}_{\ell'}(u)\right)\mathcal{L}_{\ell}(t')\rho^{(I)}(t')\right\rangle=\sum_{\ell}\gamma_{\ell}\mathcal{L}_{\ell}^{2}(t)\langle\rho^{(I)}(t)\rangle.\end{aligned} \tag{8}$$

Here we used Novikov's relation and the properties of white noise. Thus we arrived at the master equation in the Schrödinger picture of the density matrix,

$$\frac{\partial\rho(t)}{\partial t}=-i[\mathcal{H}(t),\rho(t)]-\sum_{\ell}\gamma_{\ell}[X_{\ell},[X_{\ell},\rho(t)]], \tag{9}$$

in which we denoted $\rho(t)$ for $\langle\rho(t)\rangle$, omitting the symbol of average over noise $\langle\ \rangle$. In the following sections, we focus on the noise with short correlation (2) and large amplitude of γ, namely, the strong-damping (SD) limit. This is equivalent to the phonon reservoir with high temperature. We investigate the properties of the nonadiabatic transitions based on the master equation (9).

III. DISSIPATIVE EFFECTS IN THE TWO-LEVEL SYSTEM

A. Transition probability

In this section, we consider the noise effect in the two-level system. For the two-level system, the transition probabilities in the strong-damping case have been studied by several authors using the perturbation approach [23,29]. Here we alternatively adopt a different approach, deriving the effective master equation, which simplifies derivations of transition probabilities for the case. Here two situations are studied. We first consider the familiar situation that the external field is reversed from $-\infty$ to ∞. In this case, we demonstrate that Kayanuma's transition probability $P_{\rm SD}^{-+}$ is reproduced easily using the effective master equation. We second consider the somewhat unfamiliar but realizable situation that the external field is swept from 0 to ∞ when initially a diabatic state is occupied. In the magnetic system, this situation corresponds to the case where a spin is saturated to the down (or up) state under very strong field and then the field is switched off and the field is swept linearly in time from zero field.

The model we shall consider in this section is described as

$$\mathcal{H}_{\rm tot}(t)=\mathcal{H}(t)+\xi(t)\sigma^x, \tag{10}$$

$$\mathcal{H}(t)=-vt\frac{1}{2}\sigma^z+\Gamma\frac{1}{2}\sigma^x, \tag{11}$$

where σ^α is the $\alpha\,(=x,y,z)$ component of the Pauli matrix. The diabatic states correspond to the down state $|1\rangle$ and up state $|2\rangle$, which satisfy $\sigma^z|1\rangle=-|1\rangle$ and $\sigma^z|2\rangle=|2\rangle$, respectively. Γ is the transverse field that is responsible for the tunneling between the diabatic states. Here we take only σ^z as the operator on which the noise acts in Eq. (2). Thus the master equation (9) is concretely written as

$$\frac{\partial}{\partial t}\rho(t)=-i\frac{1}{2}[vt\sigma_z+\Gamma\sigma_x,\rho(t)]-\frac{\gamma}{2}[\rho(t)-\sigma_z\rho(t)\sigma_z]. \tag{12}$$

We define the following variables:

$$c_1=\rho_{11}-\rho_{22}, \tag{13}$$

$$c_2=\rho_{12}, \tag{14}$$

$$c_3=\rho_{21}. \tag{15}$$

The time evolutions of these variables are determined by the differential equations

$$\dot{c}_1=-i\Gamma(c_3-c_2), \tag{16}$$

$$\dot{c}_2=(-ivt-\gamma)c_2+\frac{i\Gamma}{2}c_1, \tag{17}$$

$$\dot{c}_3=(ivt-\gamma)c_3-\frac{i\Gamma}{2}c_1. \tag{18}$$

Here we consider the SD limit $\gamma\to\infty$. We first consider the variable $c_2(t)$, which is formally solved from Eq. (17). We can approximate it by the following partial integral:

$$\begin{aligned}
c_2(t)&=c_2(t_0)+\frac{i\Gamma}{2}e^{-ivt^2/2-\gamma t}\int_{t_0}^{t}du\,e^{ivu^2/2+\gamma u}c_1(u)\\
&=c_2(t_0)+\frac{i\Gamma}{2}e^{-ivt^2/2+\gamma t}\left\{\left[e^{ivu^2/2-\gamma u}\frac{c_1(u)}{ivu+\gamma}\right]_{t_0}^{t}\right.\\
&\quad\left.-\frac{\Gamma}{2v}\int_{t_0}^{t}du\,e^{ivu^2/2+\gamma u}\frac{d}{du}\left(\frac{c_1(u)}{ivu+\gamma}\right)\right\}\\
&\sim c_2(t_0)+\frac{i\Gamma}{2}\left(\frac{c_1(t)}{ivt+\gamma}-\frac{\frac{d}{dt}\left(\frac{c_1(t)}{ivt+\gamma}\right)}{ivt+\gamma}+\cdots\right)\\
&\sim c_2(t_0)+\frac{i\Gamma}{2}\frac{c_1(t)}{ivt+\gamma}.
\end{aligned} \tag{19}$$

Here we used the fact that the term $e^{-\gamma(t-t_0)}$ is negligible due to large γ, and we have neglected the higher-order terms of $(ivt+\gamma)^{-1}$ [32]. When the diabatic state $|1\rangle$ is initially occupied, namely,

$$c_1(t_0)=1,\quad c_2(t_0)=c_3(t_0)=0, \tag{20}$$

$c_2(t)$ and $c_3(t)$ are approximated as

$$c_2(t)=\frac{i\Gamma}{2}\frac{c_1(t)}{ivt+\gamma}, \tag{21}$$

$$c_3(t)=\frac{-i\Gamma}{2}\frac{c_1(t)}{-ivt+\gamma}. \tag{22}$$

These relations lead us to the simplified equation for the diabatic states. By substituting the relations (21) and (22) into the Eq. (16), we arrive at the effective master equation

$$\dot{c}_1(t)=\frac{\Gamma^2}{2iv}\left\{\frac{1}{t+i\gamma/v}-\frac{1}{t-i\gamma/v}\right\}c_1(t). \tag{23}$$

Now let us consider the first problem, i.e., $t_0=-\infty$. In this case, we can readily integrate the master equation (23) to get

$$c_1(\infty)=\exp\left(-\frac{\pi\Gamma^2}{v}\right). \tag{24}$$

We now consider the tunneling probability from the state $|1\rangle$ at $t=-\infty$ to the state $|2\rangle$ at $t=\infty$. This corresponds to the value of $\rho_{22}(\infty)$. By using the conservation of probability $\mathrm{Tr}\,\rho=1$, this transition probability $P_{\rm SD}^{-+}$ $[\equiv\rho_{22}(\infty)]$ is obtained,

$$P_{\rm SD}^{-+}=\frac{1}{2}\left[1-\exp\left(-\frac{\pi\Gamma^2}{v}\right)\right]. \tag{25}$$

This is nothing but Kayanuma's transition probability [28].

For the second case where the field is swept from the resonant field, i.e., $t_0=0$, $c_1(\infty)$ is readily calculated as

$$c_1(\infty)=\exp\left(-\frac{\pi\Gamma^2}{2v}\right), \tag{26}$$

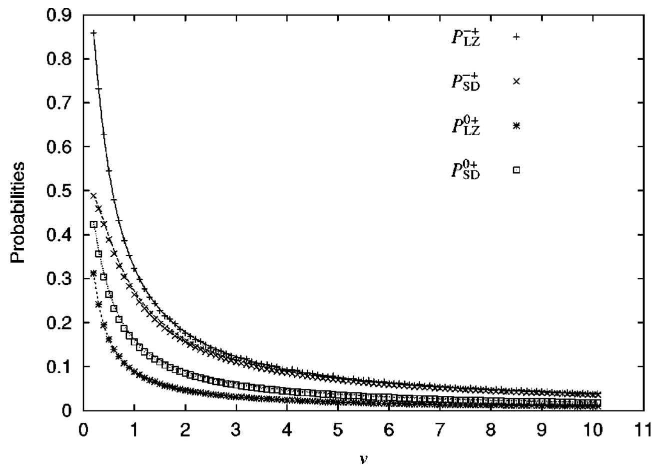

FIG. 1. Transition probabilities as a function of sweeping velocity v. Parameters are taken as $\Gamma=0.01$ and $\gamma=10.0$. Points are numerical data and lines are the analytical values. Here the subscript "LZ" means that it is a pure quantum case. P_{LZ}^{0+} is the probability in the case where the external field is swept from zero field in the pure quantum case.

which gives the transition probability P_{SD}^{0+},

$$P_{SD}^{0+}=\frac{1}{2}\left[1-\exp\left(-\frac{\pi\Gamma^2}{2v}\right)\right]. \tag{27}$$

The validity of these probabilities (25) and (27) is confirmed by numerically integrating the master equation (12). In Fig. 1, we present numerical data and the analytical results given in Eqs. (25) and (27) as a function of sweeping velocity. Here Γ is 0.01 and γ is 10.0. We show not only the dissipative case but also pure quantum cases (P_{LZ}^{-+} and P_{LZ}^{0+}). We see that formulas (25) and (27) agree with the numerical results almost perfectly.

The variable $c_1(t)$ is directly connected with the magnetization $M(t)=\mathrm{Tr}\,\sigma^z\rho(t)$. We obtain the magnetization process solving Eq. (23) as

$$M^{-+}(t)=-\exp\left[\frac{\Gamma^2}{v}\arctan\left(\frac{\gamma}{vt}\right)\right], \tag{28}$$

$$M^{0+}(t)=-\exp\left[\frac{\Gamma^2}{v}\arctan\left(\frac{\gamma}{vt}\right)-\frac{\pi}{2}\right], \tag{29}$$

where the function $y=\arctan x$ is defined in the region of $x\in[-\infty,\infty]$ and $y\in[-\pi,0]$. This shows that the magnetization process depends on the noise strength γ, whereas final magnetization does not. This is also numerically confirmed.

B. Relation between P_{SD} and P_{LZ}

The exact relation of the transition probabilities for the pure quantum case and in the strong damping case is discussed. When the field is reversed from a large negative value without dissipation; the transition probability is denoted by P_{LZ}^{-+}. Here the subscript "LZ" means that it is the pure quantum case. In the case of the field swept from the resonant point without dissipation, the transition probability is written as P_{LZ}^{0+}. As briefly shown in Appendix B, the transition probability P_{LZ}^{0+} is calculated as

$$P_{LZ}^{0+}=\frac{1}{2}\left[1-\exp\left(-\frac{\pi\Gamma^2}{4v}\right)\right]. \tag{30}$$

Thus transition probability in each case is written as follows:

$$P_{SD}^{-+}=\frac{1}{2}\left[1-\exp\left(-\frac{\pi\Gamma^2}{v}\right)\right],\quad P_{LZ}^{-+}=1-\exp\left(-\frac{\pi\Gamma^2}{2v}\right), \tag{31}$$

$$P_{SD}^{0+}=\frac{1}{2}\left[1-\exp\left(-\frac{\pi\Gamma^2}{2v}\right)\right],\quad P_{LZ}^{0+}=\frac{1}{2}\left[1-\exp\left(-\frac{\pi\Gamma^2}{4v}\right)\right].$$

These asymptotically exact probabilities satisfy the following relations (see also the Fig. 1):

$$P_{SD}^{-+}\leqslant P_{LZ}^{-+}, \tag{32}$$

$$P_{SD}^{0+}\geqslant P_{LZ}^{0+}. \tag{33}$$

The inequality (32) means that the dissipation reduces tunneling so that the state remains in the ground state. When $v\ll\Gamma^2$ in the pure quantum case, almost adiabatic tunneling from $|1\rangle$ to $|2\rangle$ occurs, whereas in the presence of dissipation, thermal excitation from ground state to the excited state represses such adiabatic evolution. Thus the inequality (32) is easily understood. On the other hand, the inequality (33) indicates the opposite property. This effect is intuitively explained as follows. The initial state $\psi(0)$ $(=|1\rangle)$ is the superposition between the ground state $|G(0)\rangle$ and the excited state $|E(0)\rangle$,

$$\psi(0)=\frac{1}{2}[|G(0)\rangle+|E(0)\rangle]. \tag{34}$$

In case of almost adiabatic evolution in the pure quantum case, $v\ll 1$, the state of the system almost follows such superposition at t,

$$\psi(t)\sim\frac{1}{2}[e^{i\phi_1(t)}|G(t)\rangle+e^{i\phi_2(t)}|E(t)\rangle], \tag{35}$$

where $e^{i\phi_1(t)}$ and $e^{i\phi_2(t)}$ are the dynamical phases. Since $|G(t)\rangle\to|2\rangle$, $|E(t)\rangle\to|1\rangle$ in the limit of $t\to\infty$, the maximum value of transition probability is $\frac{1}{2}$ in the pure quantum evolution. In the presence of the dissipation, the noise also induces such uniform distribution, because the dissipation we now consider can be regarded as a thermal effect with very high temperature. As a result, the tunneling probability is larger in the presence of the dissipation. We may say that the inequality (33) is a consequence of the special initial state, because if the initial state is not a diabatic state, e.g., $\psi(0)=|G(0)\rangle$, the inequality (33) is not realized. We expect that these characteristic relations (32) and (33) will be verified in real experiments.

IV. THREE-LEVEL SYSTEM

We here apply our approach to the multilevel systems, and investigate the dissipative effect on many levels. For the sake of definiteness, we focus our attention on the properties of the three-level system, where all diabatic levels form only one avoided-level crossing point. Because the avoided crossing structure is not formed by localized two states, the transition mechanism is very different from the LZ mechanism. Therefore this three-level system will provide much information about the effects of strong correlation of many levels. This model was first studied by Carroll and Hioe [19], and the exact transition probabilities have been obtained. The formulas for the probabilities show some varieties of expressions according to the relation between the slopes of diabatic states. The generalized arbitrary N-level system for this model is called the bow-tie model [20], and the transition probabilities and characteristic mechanisms of transitions are discussed [33]. Thus this model is quite convenient for comparison of the transition probabilities in the pure quantum case and the dissipative case. We should also note that there exist some proposals for physical realization of this model by using optical systems [19,34].

A. Analysis of transition probabilities

We here enumerate exact transition probabilities in the three-level system in the presence of dissipation. The Hamiltonian we consider is written for the diabatic bases,

$$\mathcal{H}(t)=\mathcal{H}_0(t)+\sum_k \xi_k(t)X_k, \tag{36}$$

$$\mathcal{H}_0(t)=\begin{pmatrix} 0 & \Gamma_1 & \Gamma_2 \\ \Gamma_1 & v_1 t & 0 \\ \Gamma_2 & 0 & v_2 t \end{pmatrix}, \quad X_k=\begin{pmatrix} a_1^{(k)} & 0 & 0 \\ 0 & a_2^{(k)} & 0 \\ 0 & 0 & a_3^{(k)} \end{pmatrix}. \tag{37}$$

Here the kth white noise acts on the kth operator X_k. In this case, the matrix element of the master equation (9) is written in the following form:

$$\frac{\partial \rho_{\ell,m}(t)}{\partial t}=-i[H_0(t)_{\ell,k}\rho_{k,m}(t)-\rho_{\ell,k}(t)H_0(t)_{k,m}] - \overline{\gamma}_{\ell,m}\rho_{\ell,m}(t), \tag{38}$$

where $\overline{\gamma}_{\ell,m}$ is written using the matrix elements of the operator X_k and the amplitude of the noise γ_k as

$$\overline{\gamma}_{\ell,m}=\sum_k \frac{\gamma_k}{2}(a_\ell^{(k)}-a_m^{(k)})^2. \tag{39}$$

Thus the differential equations for all matrix elements are given by

$$\dot{\rho_{00}}(t)=-i\{\Gamma_1(\rho_{10}-\rho_{01})+\Gamma_2(\rho_{20}-\rho_{02})\}, \tag{40}$$

$$\dot{\rho_{11}}(t)=-i\Gamma_1(\rho_{01}-\rho_{10}), \tag{41}$$

$$\dot{\rho_{22}}(t)=-i\Gamma_2(\rho_{02}-\rho_{10}), \tag{42}$$

$$\dot{\rho_{01}}(t)=-i\{\Gamma_1(\rho_{11}-\rho_{00})+\Gamma_2\rho_{21}-v_1 t\rho_{01}\}-\overline{\gamma}_{01}\rho_{01}, \tag{43}$$

$$\dot{\rho_{02}}(t)=-i\{\Gamma_2(\rho_{22}-\rho_{00})+\Gamma_1\rho_{12}-v_2 t\rho_{02}\}-\overline{\gamma}_{02}\rho_{02}, \tag{44}$$

$$\dot{\rho_{12}}(t)=-i\{\Gamma_1\rho_{02}-\Gamma_2\rho_{10}+(v_1-v_2)t\rho_{12}\}-\overline{\gamma}_{12}\rho_{12}. \tag{45}$$

Here we confine ourselves to the SD limit $\overline{\gamma}_{k,\ell}\to\infty$ and the initial condition $\rho_{01}(t_0)=\rho_{02}(t_0)=\rho_{12}(t_0)=0$. In the same manner as in the two-level case, $\rho_{12}(t)$ is approximated, expanding by partial integrals as follows:

$$\rho_{12}(t)=\exp\left(-i\frac{(v_1-v_2)t^2}{2}-\overline{\gamma}_{12}t\right)\int_{t_0}^t du\,\exp\left(i\frac{(v_1-v_2)u^2}{2} + \overline{\gamma}_{12}u\right)[i\Gamma_2\rho_{10}(u)-i\Gamma_1\rho_{02}(u)] \sim \frac{i\Gamma_2\rho_{10}(t)-i\Gamma_1\rho_{02}(t)}{i(v_1-v_2)t+\overline{\gamma}_{12}}. \tag{46}$$

$\rho_{02}(t)$ and $\rho_{01}(t)$ are written as

$$\rho_{02}(t)=\exp\left(i\frac{v_2t^2}{2}-\overline{\gamma}_{02}t\right)\int_{t_0}^t du\,\exp\left(-i\frac{v_2u^2}{2}+\overline{\gamma}_{02}u\right) \times\{-i\Gamma_2(\rho_{22}(u)-\rho_{00}(u))-i\Gamma_1\rho_{12}(u)\} \sim \frac{i\Gamma_2(\rho_{22}(t)-\rho_{00}(t))-i\Gamma_1\rho_{12}(t)}{-iv_2t+\overline{\gamma}_{02}}, \tag{47}$$

$$\rho_{01}(t)=\exp\left(i\frac{v_1t^2}{2}-\overline{\gamma}_{01}t\right)\int_{t_0}^t du\,\exp\left(-i\frac{v_1u^2}{2}+\overline{\gamma}_{01}u\right) \times\{-i\Gamma_1(\rho_{11}(u)-\rho_{00}(u))-i\Gamma_2\rho_{21}(u)\} \sim \frac{-i\Gamma_1(\rho_{11}(t)-\rho_{00}(t))-i\Gamma_2\rho_{21}(t)}{-iv_1t+\overline{\gamma}_{01}}. \tag{48}$$

The terms $\rho_{12}(t)$ and $\rho_{21}(t)$ in Eqs. (47) and (48) are negligible because of Eq. (46). Therefore we can approximate Eqs. (47) and (48) as

$$\rho_{02}(t)=\left(\frac{\Gamma_2}{v_2}\right)\left(\frac{1}{t+i\dfrac{\overline{\gamma}_{02}}{v_2}}\right)[\rho_{22}(t)-\rho_{00}(t)], \tag{49}$$

$$\rho_{01}(t)=\left(\frac{\Gamma_1}{v_1}\right)\left(\frac{1}{t+i\dfrac{\overline{\gamma}_{01}}{v_1}}\right)[\rho_{11}(t)-\rho_{00}(t)]. \tag{50}$$

Thus we arrive at the effective master equations defining $c_1(t)=\rho_{11}(t)-\rho_{00}(t)$ and $c_2(t)=\rho_{22}(t)-\rho_{00}(t)$,

$$\dot{c}_1 = \frac{2i\Gamma_1^2}{v_1}\left(\frac{1}{t+i\overline{\gamma}_{01}/v_1} - \frac{1}{t-i\overline{\gamma}_{01}/v_1}\right)c_1 - \frac{i\Gamma_2^2}{v_2}\left(\frac{1}{t+i\overline{\gamma}_{02}/v_2} - \frac{1}{t-i\overline{\gamma}_{02}/v_2}\right)c_2, \quad (51)$$

$$\dot{c}_2 = \frac{-i\Gamma_1^2}{v_1}\left(\frac{1}{t+i\overline{\gamma}_{01}/v_1} - \frac{1}{t-i\overline{\gamma}_{01}/v_1}\right)c_1 - \frac{2i\Gamma_2^2}{v_2}\left(\frac{1}{t+i\overline{\gamma}_{02}/v_2} - \frac{1}{t-i\overline{\gamma}_{02}/v_2}\right)c_2. \quad (52)$$

All matrix elements $\rho_{k,\ell}(t)$ are obtained if $\rho_{00}(t)$, $\rho_{11}(t)$, and $\rho_{22}(t)$ are calculated as solutions of these effective master equations.

Now let us solve these differential equations. We start with the parameters that satisfy the special relation

$$\frac{\dot{\gamma}_{01}}{|v_1|} = \frac{\overline{\gamma}_{02}}{|v_2|} = \alpha. \quad (53)$$

Here α is always positive since $\overline{\gamma}_{01} > 0$ and $\overline{\gamma}_{02} > 0$. In this special case, the equations are simplified in the form

$$\frac{d}{dt}\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = -i\left(\frac{1}{t+i\alpha} - \frac{1}{t-i\alpha}\right) M \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}, \quad (54)$$

where the matrix M is given by

$$M = \begin{pmatrix} 2a & b \\ a & 2b \end{pmatrix}, \quad a = \frac{\Gamma_1^2}{|v_1|} \text{ and } b = \frac{\Gamma_2^2}{|v_2|}. \quad (55)$$

The right-hand side of Eq. (54) has the explicit time dependence only in the prefacer. Therefore by diagonalizing the matrix M, we can obtain the scattering matrix that connects $c(-\infty)$ with $c(\infty)$. The matrix M has these eigenvalues $\lambda_\pm$,

$$\lambda_\pm = a + b \pm \sqrt{a^2+b^2-ab}. \quad (56)$$

Using a, b, and these eigenvalues $\lambda_\pm$, the final state and initial state are connected using the scattering matrix S

$$\begin{pmatrix} c_1(\infty) \\ c_2(\infty) \end{pmatrix} = S \begin{pmatrix} c_1(-\infty) \\ c_2(-\infty) \end{pmatrix}, \quad (57)$$

$$S_{1,1} = \frac{1}{2\sqrt{a^2+b^2-ab}} = [(a-b)(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}) + \sqrt{a^2+b^2-ab}(c^{-2\pi\lambda_+} + e^{-2\pi\lambda_-})], \quad (58)$$

$$S_{1,2} = \frac{b}{2\sqrt{a^2+b^2-ab}}(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}), \quad (59)$$

$$S_{2,1} = \frac{a}{2\sqrt{a^2+b^2-ab}}(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}), \quad (60)$$

$$S_{2,2} = \frac{1}{2\sqrt{a^2+b^2-ab}}[(a-b)(-e^{-2\pi\lambda_+} + e^{-2\pi\lambda_-}) + \sqrt{a^2+b^2-ab}(e^{-2\pi\lambda_+} + e^{-2\pi\lambda_-})]. \quad (61)$$

Here α does not appear, because it only gives the singular point in the Cauchy's integral to yield Eqs. (58)–(61), which is the same situation as in the two-level system. Therefore the scattering matrix does not depend on the concrete values of dissipation strength, $\overline{\gamma}_{01}$ and $\overline{\gamma}_{02}$, as far as these are large and the relation (53) is satisfied. We now obtain the probabilities for various initial states:

$$P_{\rm SD}^{-+}(0\to 0) = \frac{1}{3} + \frac{a+b}{6\sqrt{a^2+b^2-ab}}(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}) + \frac{e^{-2\pi\lambda_+} + e^{-2\pi\lambda_-}}{3}, \quad (62)$$

$$P_{\rm SD}^{-+}(0\to 1) = P_{\rm SD}^{-+}(1\to 0) = \frac{1}{3} + \frac{-2a+b}{6\sqrt{a^2+b^2-ab}}(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}) - \frac{e^{-2\pi\lambda_+} + e^{-2\pi\lambda_-}}{6}, \quad (63)$$

$$P_{\rm SD}^{-+}(0\to 2) = P_{\rm SD}^{-+}(2\to 0) = \frac{1}{3} + \frac{a-2b}{6\sqrt{a^2+b^2-ab}}(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}) - \frac{e^{-2\pi\lambda_+} + e^{-2\pi\lambda_-}}{6}, \quad (64)$$

$$P_{\rm SD}^{-+}(1\to 0) = P_{\rm SD}^{-+}(0\to 1) = \frac{1}{3} + \frac{-2a+b}{6\sqrt{a^2+b^2-ab}}(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}) - \frac{e^{-2\pi\lambda_+} + e^{-2\pi\lambda_-}}{6}, \quad (65)$$

$$P_{\rm SD}^{-+}(1\to 1) = \frac{1}{3} + \frac{a-2b}{6\sqrt{a^2+b^2-ab}}(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}) + \frac{e^{-2\pi\lambda_+} + e^{-2\pi\lambda_-}}{3}, \quad (66)$$

$$P_{\rm SD}^{-+}(1\to 2) = P_{\rm SD}^{-+}(2\to 1) = \frac{1}{3} + \frac{a+b}{6\sqrt{a^2+b^2-ab}}(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}) - \frac{e^{-2\pi\lambda_+} + e^{-2\pi\lambda_-}}{6}, \quad (67)$$

$$P_{\rm SD}^{-+}(2\to 2) = \frac{1}{3} + \frac{-2a+b}{6\sqrt{a^2+b^2-ab}}(e^{-2\pi\lambda_+} - e^{-2\pi\lambda_-}) - \frac{e^{-2\pi\lambda_+} + e^{-2\pi\lambda_-}}{3}. \quad (68)$$

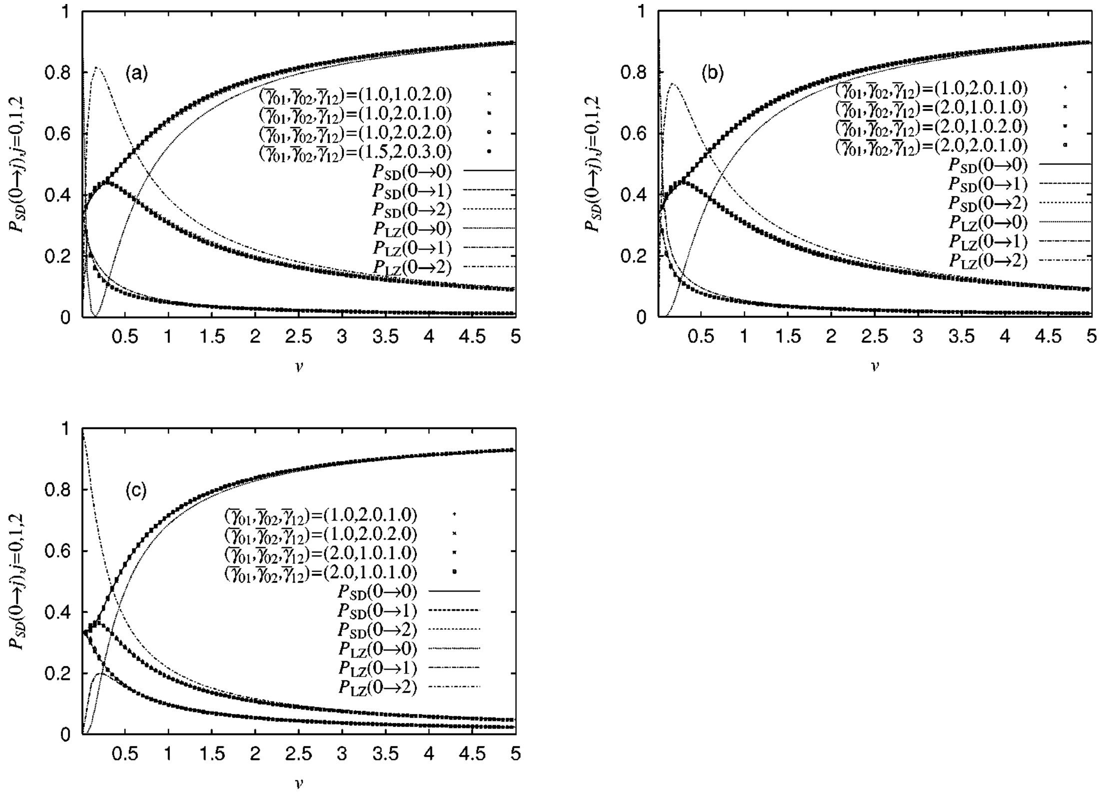

FIG. 2. Comparison of the numerical calculation with theories for various sets of $\{\overline{\gamma_{ij}}\}$. Data are plotted as a function of v. Points are numerical data, and the lines of $P_{\rm SD}^{-+}$ and $P_{\rm LZ}^{-+}$ (see Table I) are theoretical values. (a) $(\alpha_1,\alpha_2)=(1,-0.5)$, (b) (1,0.5), (c) (0.5,1).

In the adiabatic limit $v_1\to+0$ and $v_2\to+0$, all probabilities converge to 1/3 due to the strong effect of dissipation. As shown numerically in the following section, these formulas are always valid even when relation (53) is not satisfied. That is, the probabilities are little affected by the variation of the strength of dissipation $\overline{\gamma}_{01}$, $\overline{\gamma}_{012}$, and $\overline{\gamma}_{12}$.

B. Numerical investigation

We numerically integrate Eq. (38) and compare with the asymptotically exact transition probabilities obtained above for various parameter values. We write the slopes of the diabatic states v_1 and v_2 using parameter v,

$$v_1=\alpha_1 v, \tag{69}$$

$$v_2=\alpha_2 v. \tag{70}$$

Dimensionless parameters α_1 and α_2 give the ratio of v_1 to v_2 and determine the level structure. We consider three types of level structures, namely, (a) $\alpha_1\alpha_2<0$, (b) $\alpha_1>\alpha_2>0$, and (c) $\alpha_2>\alpha_1>0$. In Fig. 2, the transition probabilities $P_{\rm SD}^{-+}(0\to j)$ $(j=0,1,2)$ are shown for these cases. The probabilities are plotted as a function of the parameter v for the parameters $\Gamma_1=0.1$, $\Gamma_2=0.2$, and various sets of $(\overline{\gamma}_{01},\overline{\gamma}_{02},\gamma_{12})$. Here the sets of (α_1,α_2) are taken as (1, -0.5) for Fig. 2(a), (1,0.5) for Fig. 2(b), and (0.5,1) for Fig. 2(c), respectively. The lines are theoretical values for the SD limit $P_{\rm SD}^{-+}(0\to j)$ given by Eqs. (62)–(68) and the probabilities for pure quantum case $P_{\rm LZ}^{-+}(0\to j)$, which are already obtained by Carroll and Hioe [19]. The analytical solutions of probabilities in the pure quantum case are listed in Table I. As can be seen in these figures, we can see good agreement between the numerical data and theories, Eqs. (62)–(68). We find little dependence on the variety of $(\overline{\gamma}_{01},\overline{\gamma}_{02},\overline{\gamma}_{12})$, that is, the formulas (62)–(68) are valid even if the relation (53) is not satisfied, as long as the dissipation is very strong. This was also confirmed for $P_{\rm SD}^{-+}(1\to j)$ and $P_{\rm SD}^{-+}(2\to j)$.

As seen in Table I, the analytical expressions in the pure quantum case $P_{\rm LZ}^{-+}(0\to j)$ show some variations according to the relations of v_1 and v_2. On the other hand, the prob-

TABLE I. The transition probabilities for quantum case. Here $P=\exp(-\pi P_1^2/v_1)$ and $Q=\exp(-\pi P_2^2/v_2)$.

	$P_C^{-+}(0\to0)$	$P_C^{-+}(0\to1)$	$P_C^{-+}(0\to2)$	$P_C^{-+}(1\to1)$	$P_C^{-+}(1\to2)$	$P_C^{-+}(2\to2)$
$v_1v_2<0$	$(1-P-Q)^2$	$(1-P)(P+Q)$	$(1-Q)(P+Q)$	P^2	$(1-P)(1-Q)$	Q^2
$\lvert v_1\rvert>\lvert v_2\rvert, v_1v_2>0$	P^2Q^2	$(1-P)(1+PQ)$	$P(1-Q)(1+PQ)$	P^2	$P(1-P)(1-Q)$	$(1-P+PQ)^2$
$\lvert v_2\rvert>\lvert v_1\rvert, v_1v_2>0$	P^2Q^2	$Q(1-P)(1+PQ)$	$(1-Q)(1+PQ)$	$(1-Q+PQ)^2$	$Q(1-P)(1-Q)$	Q^2

abilities in the dissipative case (62)–(68) do not depend on such level structures. For instance, in cases of Figs. 2(a) and 2(b), where the sweeping velocities are $(v_1,v_2)=(v,-0.5v)$ and $(v_1,v_2)=(v,0.5v)$, respectively, each probability $P_{\mathrm{SD}}^{-+}(i\to j)$ for both the cases completely agrees with the other because formulas (62)–(68) have the same values for different (v_1,v_2) with the same absolute values. However in the pure quantum case, the probabilities $P_{\mathrm{LZ}}^{-+}(i\to j)$ are different between these cases as found in Table I. This is a remarkable contrast between the dissipative case and the pure quantum case.

In the slow-sweeping region $v\ll 1$, the deviation of P_{SD}^{-+} from P_{LZ}^{-+} is large. In the fast-sweeping region $v\gg 1$, P_{SD}^{-+} asymptotically converges to the behavior of P_{LZ}^{-+}. This means that the system is little affected from dissipation for fast sweeping because the time that the system stays around the avoided-level crossing point is very short. This is the same behavior as found in the two-level system in [28].

V. SUMMARY

In two- and three-level systems, we derived the effective master equations, which well describe time evolution of the system in the SD limit. Thereby we obtained analytical transition probabilities. The effective master-equation approach is quite useful because the differential equation of the system's variable becomes very simple. This approach will be applicable in the other systems whose exact transition probabilities can be analytically enumerated in the pure quantum case.

In the two-level system, we consider the two cases where the external field is swept from a large negative field and from zero field. Both situations are easily realized in real experiments. We hope that the exact relations (32) and (33) are confirmed in real experiments using the classical optical system [7] and the Cooper pair [35], and so on.

The transition time in the two-level systems has been discussed in the literature [28,36,43] when the external field is reversed from large negative field. According to Vitanov's definition [43], the transition times t^{tr} is written as

$$t^{\mathrm{tr}}\equiv\frac{\rho_{22}(\infty)}{\rho_{22}'(0)}=\frac{c_1(\infty)}{c_1'(0)} \tag{71}$$

under the initial condition of $\rho_{11}(-\infty)=1$. Vitanov derived the exact expression of transition time $t_{\mathrm{LZ}}^{\mathrm{tr}}$ in the pure quantum case as follows:

$$t_{\mathrm{LZ}}^{\mathrm{tr}}=\frac{\sqrt{1-e^{-\pi\Gamma^2/v}}}{(\Gamma/v)\cos(\chi)}, \tag{72}$$

$$\chi=\frac{\pi}{4}+\arg\Gamma\left(\frac{1}{2}-\frac{i\Gamma^2}{4v}\right)-\arg\Gamma\left(1-\frac{i\Gamma^2}{4v}\right), \tag{73}$$

where $\Gamma(x)$ is the gamma function. Equation (73) converges to

$$t_{\mathrm{LZ}}^{\mathrm{tr}}\to\begin{cases}\sqrt{\dfrac{2\pi}{v}}\cdots\dfrac{\Gamma^2}{v}\ll 1\\[2mm] 2\dfrac{\Gamma}{v}\cdots\dfrac{\Gamma^2}{v}\gg 1.\end{cases} \tag{74}$$

This asymptotic behavior agrees with that in the earlier studies [28,36]. On the other hand, the transition time of dissipative case $t_{\mathrm{SD}}^{\mathrm{tr}}$ is readily written for the definition (71),

$$t_{\mathrm{SD}}^{\mathrm{tr}}=\frac{\Gamma^2}{\gamma}\exp\left(-\frac{\pi\Gamma^2}{2v}\right). \tag{75}$$

This means that the transition time decreases as $O(1/\gamma)$ with the increase of γ. In the three-level system, the analytical expressions (62)–(68) are the solutions under the condition (53). However these solutions are valid beyond the condition (53) as shown in the numerical calculation. According to Carroll and Hioe's analytical solution (Table I), the pure quantum transition probabilities show some variations depending on the level structures. However, the probabilities in the dissipative case do not show such dependences. It is interesting to confirm experimentally this thermal effect, because the three-level system we consider can be realized experimentally [19,34].

Equation (9) is derived under the condition where the noise affects the system only through the diabatic states. In case of the off-diagonal coupling, i.e., $X_\ell=\sigma^x$, we also derived a similar master equation. In this case, we can easily show that in the strong-damping limit, all the transition probabilities become uniform regardless of the initial condition [25,38]. Thus the transition probability is affected by the coupling form with the thermal environment. Therefore it is also interesting to consider the transition probabilities for various coupling forms with finite γ.

ACKNOWLEDGMENT

The present work is partially supported by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture of Japan.

APPENDIX A

The same type of master equation as Eq. (A9) is derived in the case where the system of interest couples with the phonon bath through the diabatic states as follows [39]:

$$\mathcal{H}_{\mathrm{tot}}=\mathcal{H}(t)+\lambda\sum_{\ell}X_\ell\sum_{\omega}\gamma_\alpha(b_\omega^{(\ell)\dagger}+b_\omega^{(\ell)})+\sum_{\ell,\omega}\omega b_\omega^{(\ell)\dagger}b_\omega^{(\ell)}, \tag{A1}$$

where $\mathcal{H}(t)$ denotes the system Hamiltonian of interest and X_ℓ is the ℓth operator that interacts with the phonon system. We assume the computability between the coupling operators X_ℓ, i.e., $[X_\ell,X_\ell]=0$. The operators $b_\omega^{(\ell)\dagger}$ and $b_\omega^{(\ell)}$ are the phonon creation and annihilation operators which interact with the system through the ℓth coupling operator X_ℓ.

We use the projection-operator technique to trace out the reservoir's degree of freedom [40] and assume that the correlation between the reservoir's variables is short lived. Then we obtain the master equation for the system in the second order of coupling strength λ [41,42],

$$\frac{\partial}{\partial t}\rho(t)=\frac{1}{i\hbar}[\mathcal{H},\rho(t)]-\lambda^2\sum_\ell \Gamma_\ell\rho(t), \tag{A2}$$

where $\Gamma_\ell\rho(t)$ is given by

$$\begin{aligned}\Gamma_\ell\rho(t)=\frac{1}{\hbar^2}\int_0^\infty dt'\int_{-\infty}^{\infty}d\omega e^{i\omega t'}\Phi_\ell(\omega)\{X_\ell X_\ell(-t')\rho(t)\\ -e^{\beta\hbar\omega}X_\ell\rho(t)X_\ell(-t')+e^{\beta\hbar\omega}\rho(t)X_\ell(-t')X_\ell\\ -X_\ell(-t')\rho(t)X_\ell\}.\end{aligned} \tag{A3}$$

Here $X_\ell(-t')$ means the Heisenberg operator at time $-t'$,

$$\begin{aligned}X_\ell(-t')=\exp_{\leftarrow}\left(-\frac{i}{h}\int_{-t'}^{0}duH(u)\right)X\\ \times\exp_{\rightarrow}\left(\frac{i}{h}\int_{-t'}^{0}duH(u)\right).\end{aligned} \tag{A4}$$

In case of the phonon reservoir described in Eq. (A1), $\Phi(\omega)$ is given by

$$\phi_\ell(\omega)=\hbar\frac{I_\ell(\omega)-I_\ell(-\omega)}{e^{\beta\hbar\omega}-1}, \tag{A5}$$

where β is an inverse temperature $1/T$. $I_\ell(\omega)$ is called the spectral density.

We restrict ourselves to the case of the Ohmic spectrum

$$I_\ell(\omega)=I_\ell\omega \tag{A6}$$

and high temperature

$$T\gg 1. \tag{A7}$$

In this case, by using the fact that

$$\Phi_\ell(\omega)\to\hbar I_\ell T, \tag{A8}$$

the master equation (A2) is reduced to the simple form

$$\frac{\partial}{\partial\tau}\rho(t)=\frac{1}{i\hbar}[\mathcal{H},\rho(t)]-\lambda^2T\sum_\ell I_\ell[X_\ell,[X_\ell,\rho(t)]]. \tag{A9}$$

Although Eq. (A3) derived by the projection-operator approach is an approximation because higher-order terms of λ are neglected and the fast relaxation of the reservoir is assumed to make the equation Markovian, it can well describe the features of evolution of the system, especially in case of high temperature.

APPENDIX B

In this appendix, the transition probabilities (31) are derived. We start with the Hamiltonian

$$\mathcal{H}(t)=\frac{h}{2}(\Gamma\sigma^x-vt\sigma^z). \tag{B1}$$

We consider the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(t)=\frac{\hbar}{2}(\Gamma\sigma^x-vt\sigma^z)\Psi(t). \tag{B2}$$

Equation (B2) is concretely written defining the component similar to $\Psi(t)=(x_1(t),x_2(t))^\dagger$,

$$i\dot{x}_1(t)=\frac{vt}{2}x_1(t)+\frac{\Gamma}{2}x_2(t), \tag{B3}$$

$$i\dot{x}_2(t)=\frac{\Gamma}{2}x_1(t)-\frac{vt}{2}x_2(t). \tag{B4}$$

Now we transform the variables to the following ones:

$$\tau:=t^2, \tag{B5}$$

$$y_1(\tau):=\frac{x_1(t)}{t}, \tag{B6}$$

$$y_2(\tau):=x_2(t), \tag{B7}$$

then we obtain the equation

$$2i\tau\frac{d}{d\tau}y_1+\left(i-\frac{v\tau}{2}\right)y_1-\frac{\Gamma}{2}y_2=0, \tag{B8}$$

$$2i\frac{d}{d\tau}y_2+\frac{v}{2}y_2-\frac{\Gamma}{2}y_1=0. \tag{B9}$$

Here we used the relation $d/dt=2td/d\tau$. By using the Laplace transformation [44] defined as

$$y_k(\tau)=\int_{C_\xi}d\xi\tilde{x}_k(\xi)e^{\xi\tau}, \tag{B10}$$

we obtain the integral representation for $x_1(t)$ and $x_2(t)$ after straightforward calculation,

$$x_1(t)=At\int_{C_\xi}d\xi\left(\xi+i\frac{v}{4}\right)^{-1/2+i\Gamma^2/\delta v}\left(\xi-i\frac{v}{4}\right)^{-i\Gamma^2/8v}e^{\xi t^2}, \tag{B11}$$

$$\begin{aligned}x_2(t)=A\left(-i\frac{\Gamma}{4}\right)\int_{C_\xi}d\xi\left(\xi+i\frac{v}{4}\right)^{-1/2+i\Gamma^2/8v}\\ \times\left(\xi-i\frac{v}{4}\right)^{-1-i\Gamma^2/8v}e^{\xi t^2}.\end{aligned} \tag{B12}$$

Here the integral counter must satisfy the following condition:

$$\left[\left(2i\xi-\frac{v}{2}\right)\tilde{x}_1(\xi)e^{\xi\tau}\right]_{C_\xi}=0. \tag{B13}$$

For the variables $\xi=|\xi|e^{i\phi}$, $t=|t|e^{i\theta}$, we choose the counter with the condition

$$\phi+2\theta=3\pi \tag{B14}$$

for large $|\xi|$ noting the relation $\xi t=|\xi||t|e^{i(\phi+2\theta)}$. When the initial time is $t=(\infty)e^{i\pi}$ and the final time is $t=(\infty)e^{i0}$, the phase of ϕ varies from π to 3π from the relation (B14). We now consider the initial condition as

$$x_1(-\infty)=1. \tag{B15}$$

This condition is realized in the contour that encircles the singular point $\xi=-iv/4$ and by choosing the constant A as

$$A=e^{\pi\Gamma^2/16v}\Big/\int_\infty^{(0+)}du(-u)^{-1/2+i\Gamma^2/8v}e^{-u}. \tag{B16}$$

Thus the wave function at $t=\infty$ is calculated by using analytical continuation following the condition (B14), i.e., $\pi\to\phi\to3\pi$ [44]. Thus revival probability is calculated as

$$x_1(|t|e^{i0})=x_1(|t|e^{i\pi})\exp\left(-\frac{\pi\Gamma^2}{4v}\right)\quad(|t|\to\infty). \tag{B17}$$

This means nothing but the relation of the Landau-Zener transition.

Next we consider the case where the external field is swept from zero value [19,37], that is, $\theta=0$ and $+0\to|t|\to\infty$. In order to derive the survival probability in this case, we first note the time symmetry that the probability is of the same value as that obtained when the initial time is taken as $t=-\infty$ (the external value $=-\infty$) and the final time is $t=-0$ (the external value $=-0$). We consider the latter case $(-\infty\to t\to-0)$ because we can use the same contour as the previous case [Eq. (B17)], which satisfies the initial condition (B15). We can readily write the integral representation of the $x_1(-0)$ and obtain the transition probability P_C^{0+},

$$\begin{aligned}x_1(-0)&=A|t|\int_\infty^{(0+)}(-x)^{-1/2+i\Gamma^2/8v}\left(-x-i\frac{v}{2}\right)^{-i\Gamma^2/8v}\\&\quad\times e^{-x-iv/4|t|^2}\\&=A(1+e^{\pi\Gamma^2/4v})\int_0^\infty duu^{-1/2}e^{-u}\\&=\sqrt{\frac{1+e^{\pi\Gamma^2/4v}}{2}}.\end{aligned}$$

[1] L. Landau, Phys. Z. Sowjetunion **2**, 46 (1932).
[2] C. Zener, Proc. R. Soc. London, Ser. A **137**, 696 (1932).
[3] E. C. G. Stückelberg, Helv. Phys. Acta **5**, 369 (1932).
[4] C. Zhu and H. Nakamura, J. Chem. Phys. **101**, 10 630 (1994); **102**, 7448 (1995); **106**, 2599 (1996); Chem. Phys. Lett. **258**, 342 (1996).
[5] Y. Abe and J. Y. Park, Phys. Rev. C **28**, 2316 (1983).
[6] M. S. Child, *Molecular Collision Theory* (Academic, London, 1974).
[7] J. R. Rubbmark, M. M. Kash, M. G. Littman, and D. Kleppner, Phys. Rev. A **23**, 3107 (1981).
[8] D. Bouwmeester, N. H. Dekker, F E. v. Dorsselaer, C. A. Schrama, P. M. Visser, and J. P. Woerdman, Phys. Rev. A **51**, 646 (1995).
[9] L. Thomas, F. Lionti, R. Ballou, D. Gatteschi, R. Sessoli, and B. Barbara, Nature (London) **383**, 145 (1996); L. Thomas *et al.*, *ibid.* **383**, 145 (1996); F. Lionti, L. Thomas, R. Ballou B., Barbara, A. Sulpice, R. Sessoli, and D. Gatteschi, J. Appl. Phys. **81**, 4608 (1997).
[10] W. Wernsdorfer *et al.*, Phys. Rev. Lett. **78**, 1791 (1997); **79**, 4014 (1997).
[11] S. Miyashita, J. Phys. Soc. Jpn. **64**, 3207 (1995); **65**, 2734 (1996).
[12] H. De Raedt, S. Miyashita, K. Saito, D. García-Pablos, and N. García, Phys. Rev. B **56**, 11 761 (1997).
[13] K. Saito, S. Miyashita, and H. De Raedt, Phys. Rev. B **60**, 14 553 (1999).
[14] Y. N. Demkov, Sov. Phys. Dokl. **11**, 138 (1966).
[15] V. I. Osherov, Sov. Phys. JETP **22**, 804 (1966).
[16] Y. Kayanuma and S. Fukuchi, J. Phys. B **18**, 4089 (1985).
[17] Y. N. Demkov and V. N. Ostrovsky, J. Phys. B **28**, 403 (1995).
[18] Y. N. Demkov, P. B. Kurasov, and V. N. Ostrovsky, J. Phys. A **28**, 4361 (1995).
[19] C. E. Carroll and F. T. Hioe, J. Phys. A **19**, 1151 (1986); **19**, 2061 (1986).
[20] V. N. Ostrovsky and H. Nakamura, J. Phys. A **30**, 6939 (1997).
[21] S. Brundobler and V. Elser, J. Phys. A **26**, 1211 (1993).
[22] M. M. T. Loy, Phys. Rev. Lett. **32**, 814 (1974), J. C. Camparo and R. P. Frueholz, Phys. Rev. A **30**, 803 (1984).
[23] C. Henry and D. Lang, Phys. Rev. B **15**, 989 (1975), Y. Kayanuma, J. Phys. Soc. Jpn. **51**, 3526 (1982).
[24] W. Wernsdorfer, R. Sessoli, A. Caneschi, D. Gatteschi, and A. Cornia, Europhys. Lett. **50**, 552 (2000); J. Phys. Soc. Jpn. **69**, Suppl A, 375 (2000).
[25] M. Nishino, K. Saito, and S. Miyashita, Phys. Rev. B **65**, 014403 (2002).
[26] I. Chiorescue *et al.*, Phys. Rev. Lett. **84**, 3454 (2000); K. Saito and S. Miyashita, J. Phys. Soc. Jpn. **70**, 3385 (2001).
[27] B. Barbara, L. Thomas, F. Lionti, I. Chiorescu, and A. Sulpice, J. Magn. Magn. Mater. **200**, 167 (1999); S. Miyashita and K. Saito, J. Phys. Soc. Jpn. **70**, 3238 (2001).
[28] Y. Kayanuma, J. Phys. Soc. Jpn. **53**, 108 (1984); **53**, 118 (1984).
[29] P. Ao and J. Rammer, Phys. Rev. B **43**, 5397 (1991).
[30] R. Kubo, J. Phys. Soc. Jpn. **4**, 174 (1963).
[31] A. Novikov, Zh. Eksp. Teor. Fiz. **47**, 1919 (1964) [Sov. Phys.

JETP **20**, 1290 (1965)]; V. I. Klyatskin and V. I. Tatarskii, Usp. Fiz. Nauk. **110**, 499 (1973) [Sov. Phys. Usp. **16**, 494 (1973)].

[32] This approximation should be justified under the condition that the transition time in the pure quantum case is much larger than the dissipation time scale $1/\gamma$. See Eq. (72) for the tunneling time in the pure quantum case.

[33] Y. N. Demkov and V. N. Ostrovsky, Phys. Rev. A **61**, 032705 (2000).

[34] W. Harshwardhan and G. S. Agarwal, Phys. Rev. A **55**, 2165 (1997).

[35] Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).

[36] K. Mullen, E. Ben-Jacob, Y. Gefen, and Z. Schuss, Phys. Rev. Lett. **62**, 2543 (1989).

[37] N. V. Vitanov, Phys. Rev. A **59**, 988 (1999).

[38] Y. Kayanuma, J. Phys. Soc. Jpn. **54**, 2037 (1985). This paper also treats the case of the off-diagonal coupling and shows the nonuniform transition probability. We should note that this result is derived in a different limiting case from ours. See also Ref. [25].

[39] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983); Physica A **121**, 587 (1983).

[40] S. Nakajima, Prog. Theor. Phys. **20**, 948 (1958); R. Zwanzig, J. Chem. Phys. **33**, 1338 (1960); H. Mori, Prog. Theor. Phys. **33**, 424 (1965).

[41] W. Weidlich and F. Haake, Z. Phys. **185**, 30 (1965).

[42] K. Saito, S. Takesue, and S. Miyashita, Phys. Rev. E **61**, 2397 (2000); R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics II* (Springer-Verlag, New York, 1985).

[43] N. V. Vitanov and B. M. Garraway, Phys. Rev. A **53**, 4288 (1996).

[44] E. Majorana, Nuovo Cimento **9**, 43 (1932).