Qubit rotation by stimulated Raman adiabatic passage

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We introduce a procedure for qubit rotation, alternative to the commonly used method of Rabi oscillations of controlled pulse area. It is based on the technique of stimulated Raman adiabatic passage and, therefore, it is robust against fluctuations of experimental parameters. Furthermore, our work shows that it is, in principle, possible to perform quantum logic operations via stimulated Raman adiabatic passage. This opens up the search for a completely new class of schemes to implement logic gates.

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The growing interest in quantum computation stimulates the search for schemes to prepare and manipulate quantum states [1]. The basic operation on a single qubit is the rotation. This is usually achieved via coherent Rabi oscillations. For a two-level optical transition Rabi oscillations are simply produced by direct coupling using a laser field. If the qubit is formed by two ground states instead, Rabi flopping is then implemented by Raman transitions via a virtual excited state, and a fundamental logic gate has been demonstrated using this configuration [2]. Fundamental quantum logic operations based on (conditional) Rabi flopping have also been implemented in cavity QED experiments [3]. To perform qubit rotation utilizing Rabi flopping, a complete control of the pulse area is required. An imperfect control leads obviously to errors in the computation procedure.

In this work, we introduce an alternative procedure for qubit rotation. It is based on the technique of stimulated Raman adiabatic passage (STIRAP [4]) and, therefore, it is robust against fluctuations of several experimental parameters such as the pulse area. Furthermore, our proposal shows that STIRAP is not only a powerful tool to transfer population between quantum states [5], including here the creation of coherent superpositions [6–10] and entangled states [11], but can also play a role in the implementation of a quantum logic operation.

Consider the generic linear superposition of two long-living atomic states $|1\rangle$ and $|2\rangle$,

$$|i\rangle = \alpha|1\rangle + \beta|2\rangle. \tag{1}$$

The state rotated about a unit vector $\boldsymbol{n}$ through an angle ζ reads

$$|f\rangle = \hat{R}_{\boldsymbol{n}}(\zeta)|i\rangle, \tag{2}$$

with $\hat{R}_{\boldsymbol{n}}(\zeta)$ being an element of the SU_2 group defined as

$$\hat{R}_{\boldsymbol{n}}(\zeta) = \exp\left(-i\frac{\zeta}{2}\boldsymbol{n}\cdot\hat{\boldsymbol{\sigma}}\right) = \cos\frac{\zeta}{2} - i\boldsymbol{n}\cdot\hat{\boldsymbol{\sigma}}\sin\frac{\zeta}{2}, \tag{3}$$

where $\boldsymbol{n} = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$, and $\hat{\boldsymbol{\sigma}} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli's spin operators $\sigma_x = |1\rangle\langle 2| + |2\rangle\langle 1|$, $\sigma_y = i(|2\rangle\langle 1| - |1\rangle\langle 2|)$, $\sigma_z = |1\rangle\langle 1| - |2\rangle\langle 2|$.

Mapping of the state (1) into the state (2) can be achieved easily using a single STIRAP sequence, as described for example in Ref. [9]. However, in that scheme the Stokes and pump pulses that link the different states must satisfy well defined relationships with the coefficients of the initial and final superpositions. Therefore, that procedure cannot be considered as a general rotation, because for a given pulse sequence, not every state will be transformed in the same way. As it will be shown here, the use of multiple STIRAP sequences allows indeed true rotations, with the axis and angle of rotation being uniquely defined by the parameters of the laser pulses.

In our scheme, shown in Fig. 1, three ground states $|1\rangle$, $|2\rangle$, and $|3\rangle$ are coupled via a single excited state $|4\rangle$ by different laser fields. We assume that due to their polarizations and frequencies, each laser field drives only one transition. The ground states $|1\rangle$ and $|2\rangle$ define our qubit, while the state $|3\rangle$ is an auxiliary state that will be occupied only in the intermediate phase of the rotation procedure.

The detunings of the three laser fields are the same, i.e.

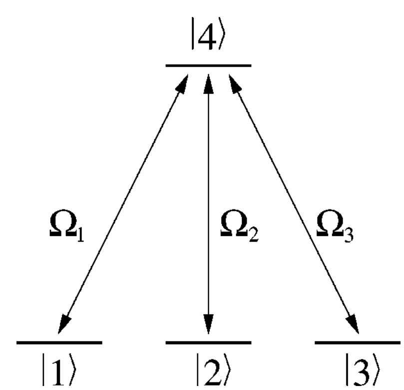

FIG. 1. Interaction scheme for the rotation of a qubit by STIRAP. The three ground states $|1\rangle$, $|2\rangle$, and $|3\rangle$ are coupled to the excited state $|4\rangle$ by three different laser fields. The qubit is defined by the ground states $|1\rangle$ and $|2\rangle$. State $|3\rangle$ is an auxiliary state occupied only in the intermediate phase of the rotation procedure.

the system is at multiphoton resonance that is a necessary condition for STIRAP. The Schrödinger equation of this system reads

$$\frac{d}{dt}|\psi(t)\rangle = -\frac{i}{\hbar}\hat{H}(t)|\psi(t)\rangle, \tag{4}$$

where $|\psi(t)\rangle$ denotes the state vector of the four-level system and the Hamiltonian $\hat{H}(t)$ is given in the interaction picture and in the rotating-wave approximation (RWA) as

$$\hat{H}(t) = \hbar\Delta|4\rangle\langle 4| + \frac{\hbar}{2}\sum_{i=1}^{3}(\Omega_i(t)|i\rangle\langle 4| + \text{H.c.}). \tag{5}$$

Here Δ is the laser detuning. The pulsed Rabi frequencies $\Omega_1(t)$ and $\Omega_2(t)$ are taken to have essentially the same envelopes: $\Omega_1(t) = \Omega(t)\cos\chi$, $\Omega_2 = \Omega(t)\exp(i\eta)\sin\chi$, where χ and η are fixed angles. The pulses 1,2 (i.e., the fields that yield the Rabi frequencies $\Omega_1(t)$ and $\Omega_2(t)$) and 3 are delayed with respect to each other, however, for an efficient STIRAP process they must have a significant overlap. In the following $\Omega(t)$ will be taken as real.

Our procedure consists of two STIRAP processes. In the first one, the fields 1 and 2 are the pump fields, whereas the field 3 plays the role of the Stokes field. In this STIRAP process the field 3 has the same phase as that of fields 1 and 2. The pulses are applied in the counterintuitive order, i.e., the Stokes pulse arrives before the pump ones. The transfer process can be described as follows: The pump fields 1 and 2 define a dark (or noncoupled) state $|S_{NC}\rangle$

$$|S_{NC}\rangle = -\sin\chi|1\rangle + e^{i\eta}\cos\chi|2\rangle, \tag{6}$$

in the subspace spanned by the states $\{|1\rangle,|2\rangle\}$ [12]. The orthogonal state (the coupled or bright state) $|S_C\rangle$ is

$$|S_C\rangle = \cos\chi|1\rangle + e^{i\eta}\sin\chi|2\rangle, \tag{7}$$

from which the population can be transferred to state $|3\rangle$ if all the three fields are on. By decomposing the initial superposition $|i\rangle$ onto $|S_C\rangle$ and $|S_{NC}\rangle$,

$$|i\rangle = \langle S_{NC}|i\rangle|S_{NC}\rangle + \langle S_C|i\rangle|S_C\rangle \tag{8}$$

with

$$\langle S_{NC}|i\rangle = -\alpha\sin\chi + \beta e^{-i\eta}\cos\chi \tag{9a}$$

$$\langle S_C|i\rangle = \alpha\cos\chi + \beta e^{-i\eta}\sin\chi \tag{9b}$$

we find that only the component along $|S_C\rangle$ will be affected by the STIRAP process and eventually mapped to the target state $|3\rangle$. Indeed, the Hamiltonian in Eq. (5) can be rewritten in the form

$$\hat{H}(t) = \hbar\Delta|4\rangle\langle 4| + \frac{\hbar}{2}(\Omega(t)|S_C\rangle\langle 4| + \Omega_3(t)|3\rangle\langle 4| + \text{H.c.}). \tag{10}$$

Inserting Eq. (10) into Eq. (4) one obtains a Schrödinger equation that describes an ordinary STIRAP process in a three-level system. The dark state $|\psi_D(t)\rangle$ of the Hamiltonian in Eq. (10) is given by

$$|\psi_D(t)\rangle = \frac{1}{\sqrt{\Omega^2(t) + \Omega_3^2(t)}}(\Omega_3(t)|S_C\rangle - \Omega(t)|3\rangle). \tag{11}$$

Using the same argument as in the case of STIRAP, it can be easily shown that in the adiabatic limit the system is left, after the first pulse sequence, in the superposition of the three ground states

$$|\psi\rangle = \langle S_{NC}|i\rangle|S_{NC}\rangle - \langle S_C|i\rangle|3\rangle, \tag{12}$$

without populating the excited state during the evolution. We recognize in Eq. (12) that the component of $|i\rangle$ along the noncoupled state $|S_{NC}\rangle$ is untouched, whereas the orthogonal bright component is transferred to the target state $|3\rangle$.

The second step of the rotation procedure is a reverse STIRAP process that maps the state $|3\rangle$ back to the qubit subspace. In this STIRAP process the phase of field 3 is in general different from that of fields 1 and 2. The pulses are applied in the reverse order with respect to the first step of the rotation procedure: the pulses 1 and 2 (which play now the role of the Stokes pulses) arrive before the pulse 3 (the pump). The fields 1 and 2 have the same Rabi frequencies as in the first step of the procedure. In this way the state $|\psi\rangle$ prepared by the first STIRAP process, Eq. (12), is initially a dark state for the three laser pulses because: (a) the state $|3\rangle$ is initially not coupled (counterintuitive pulse order); (b) the state $|\psi\rangle$ has no component along $|C\rangle$, and, therefore, it is decoupled from the fields 1 and 2. The darkness of $|\psi\rangle$ allows the implementation of the second STIRAP process although all the ground states are initially populated. In this process the state $|3\rangle$ is transferred back to the qubit subspace. More precisely, it will be mapped on the coupled state $|S_C\rangle$ because the state $|S_{NC}\rangle$ is a decoupled state also in this second STIRAP process. The component of the initial qubit $|i\rangle$ along the coupled state $|S_C\rangle$ and the new component along the same state obtained by mapping back the state $|3\rangle$ will differ by a phase. This phase is determined by the phase difference δ between the relative phase of the field 3 with respect to the fields 1 and 2 in the two STIRAP processes. Clearly, for identical phases the system certainly goes back to the initial state $|i\rangle$. In the general case a similar calculation that yielded Eq. (12) shows that in the adiabatic limit the component of $|\psi\rangle$ along $|3\rangle$ is mapped back onto the subspace $\{|1\rangle,|2\rangle\}$ according to

$$\langle 3|\psi\rangle|3\rangle \rightarrow e^{-i\delta}\langle 3|\psi\rangle|S_C\rangle, \tag{13}$$

so that the final state is

$$|\psi_f\rangle = \langle S_{NC}|i\rangle|S_{NC}\rangle + e^{-i\delta}\langle S_C|i\rangle|S_C\rangle. \tag{14}$$

By substituting the expressions (9) for the coefficients and taking into account Eqs. (2) and (3), we obtain

$$|\psi_f\rangle = e^{-i\delta/2}\hat{R}_{\boldsymbol{n}}(\delta)|i\rangle, \tag{15}$$

where $\boldsymbol{n}=(\sin 2\chi \cos \eta, \sin 2\chi \sin \eta, \cos 2\chi)$. Apart from a global phase $-\delta/2$ the states $|i\rangle$ and $|\psi_f\rangle$ are connected through the rotation $\hat{R}_{\boldsymbol{n}}(\delta)$. If the qubit is isolated, then this global phase is unimportant. If the qubit is part of a larger system, e.g., there are several qubits that form a quantum computer, then the global phase is clearly relevant, however, it may be incorporated into the algorithm being implemented on the quantum computer.

The geometric interpretation of the rotation procedure described above is straightforward: the axis of rotation $\boldsymbol{n}$ is defined by the polar angle 2χ and the azimuthal angle η, which characterize the relative amplitude and phase of the laser pulses acting on the states $|1\rangle$ and $|2\rangle$, respectively. In the Hilbert space the state vector $|S_{NC}\rangle$, Eq. (6), is parallel with this axis, therefore, it is unaffected by the rotation. The state vector $|S_C\rangle$, Eq. (7), is perpendicular to the rotation axis, and it is rotated through an angle δ about this axis.

Our rotation procedure is sensitive to the relative phase and relative amplitude of the pulses 1 and 2. Moreover, the method is also sensitive to the phase δ, the phase difference of the pulse 3 in the first and second STIRAP processes. However, it is robust against the fluctuation of the pulse shapes and pulse areas.

Our analysis is supported by numerical calculations. In these calculations, the envelope of the different pulsed Rabi frequencies has been taken as Gaussian, although any sufficiently smooth pulse shape is suitable. The width of the pulses and their overlaps have been chosen to satisfy well the usual STIRAP conditions [4]. For different choices of the initial state of the qubit (1) we have solved numerically the Schrödinger equation (4) and determined the time-dependent state of the system during the rotation procedure composed of two STIRAP processes. An example of our calculations is shown in Fig. 2 where the populations of the three ground states are shown together with the pulse sequence that effects the qubit rotation. The time evolution in the case of identical phases for the two STIRAP processes is also reported for comparison. In order to analyze quantitatively the quality of our rotation procedure, we have calculated the fidelity $F = |\langle\psi_f|\psi_\infty\rangle|^2$, where $|\psi_\infty\rangle$ is the numerically calculated state after the rotation procedure and $|\psi_f\rangle$ the state of the rotated qubit. We have verified that the fidelity is unity within numerical error. We have also verified that under usual STIRAP conditions the excited state occupation is negligible throughout the rotation procedure.

In conclusion we have shown that it is possible to perform qubit rotations by stimulated Raman adiabatic passage, and correspondingly, proposed a rotation procedure. The procedure is composed of two STIRAP processes. The first step consists in projecting the original qubit on two perpendicular states: the dark and the bright states, as defined by the pump pulses. The component of the qubit along the bright state is then transferred to an auxiliary state. In the second STIRAP process, the auxiliary state is mapped back onto the bright state, with a phase shift δ with respect to the initial component of the qubit along this state. The resulting state corresponds indeed to a rotation of the qubit, with the axis and angle of rotation determined uniquely by the parameters of the laser fields.

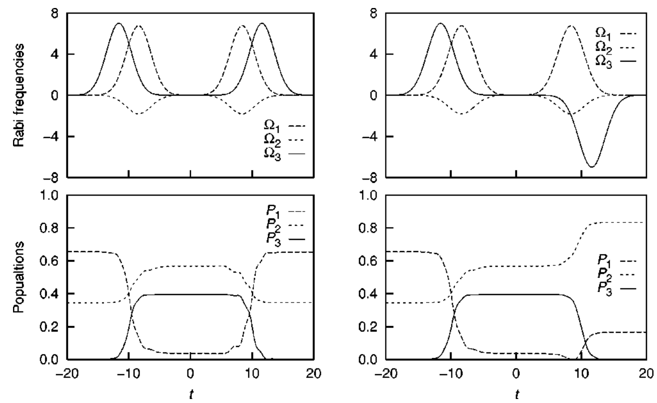

FIG. 2. Time evolution of the ground-state populations and pulse sequences for two different STIRAP procedures. In the procedure shown in the left column, the second STIRAP process is precisely the reverse of the first one (top left) and the system returns back to its original state (bottom left). In the procedure shown in the right column, in the second STIRAP process the phase of the pulse 3 is shifted by $\delta=\pi$ with respect to the the pulses 1 and 2 (top right). In this case the system does not return back to its original state, and the qubit is rotated through an angle π about a unit vector characterized by the polar angle 2χ and the azimuthal angle η. Parameters of the calculation are $\alpha=\cos \pi/5$, $\beta=\sin \pi/5$, $\Delta=0$, $\chi=-\pi/12$, and $\eta=0$. The pulses have Gaussian shape $\exp[-(t\pm T/2\pm t_0)^2/2\tau^2]$, with $\tau=2$, $t_0=1.6$, $T=20$. The time and the Rabi frequencies are measured in arbitrary units.

Furthermore, our paper shows that it is, in principle, possible to perform quantum logic operations via stimulated Raman adiabatic passage. This opens up the search for new schemes for logic gates based on STIRAP.

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