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Radiative emission due to atomic self-dressing in QED

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We study the radiative emission due to the self-dressing of a two-level atom, initially in its bare ground state, interacting with the zero-point electromagnetic field. Evolution in time leads to the formation of a dressed ground state of lower energy. This energy difference between bare and dressed ground state is taken into account by the emission of real photons. In order to describe this aspect of the self-dressing process we study the transition probability amplitude from the initial bare state to an asymptotic state consisting of the atom in its dressed ground state plus some real photons. Adopting nonperturbative techniques based on the resolvent method we find that the bare-dressed transition occurs at the lowest order with the emission of two photons. The spectral distribution obtained shows that there is a probability for emission of photons at frequency of the order of the atom's transition frequency.

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I. INTRODUCTION

Description of interaction between atoms and electromagnetic field is to some extent an open problem. In particular there are difficulties associated with the dynamical description of excited states when one wants to take into account also the virtual cloud of the field [1–4]. In fact the photons of this cloud are due to the same interaction that is responsible for the decay and they may not be clearly distinguished from the real photons emitted during the decay. The description of the interaction is instead well defined if one considers the total system, atom plus field, in its ground state. This equilibrium state, named dressed, is represented by an extended structure formed by the bare atom surrounded by a stationary cloud of virtual photons that it emits and reabsorbs [5–7]. The presence of this virtual cloud is responsible for phenomena such as the Casimir effect [8], Lamb shift [9] and mass renormalization [10] in QED, the shift of energy levels [11–13], and decaylike behavior [14] of an exciton interacting with the vibration modes of a crystal lattice and modification of the polaron mass [15] in solid state physics.

It must however be observed that such a state may be difficult to obtain in practice because any external perturbation must induce a readjustment of the virtual cloud around the atom and this must occur in a finite time. The other extreme is that the atom completely deprived of its virtual cloud is named bare. Of course it is doubtful if such a state may be obtainable in principle since most of the interactions between the atom and the external world are mediated by the exchange of virtual quanta present in the cloud.

Between these two extremes of completely bare or dressed source it is presumedly more realistic to envisage intermediate situations where the virtual cloud of the source is not the same as in equilibrium because it has been perturbed in some way. Then one must expect that the dressed source should regain its equilibrium configuration and that the physical properties of such a source should change.

One is led then to consider the evolution from bare to dressed state that is of self-dressing. This evolution has been studied using perturbation theory for times of the order of l/c (l linear dimension of virtual cloud and c light speed) showing traces of the process of virtual cloud formation around the source [16]. Self-dressing has been studied for a model consisting of a two-level atom interacting with the electromagnetic field by using techniques based on the theory of subdynamics [17] and for the Friedrichs model [18]. It has been predicted that in the evolution from the bare to the dressed system a real photon is emitted. This photon is essentially resonant with the atom's transition and thus has an energy much larger than the difference between the bare and dressed energy levels. Moreover the analysis of the evolution of the photon density generated during the self-dressing has been shown to be relevant in the causality problem [19]. The prediction of real photons' appearance during the self-dressing of the ground state has led to study the process by using standard quantum-mechanics nonperturbative techniques based on van Hove's resolvent theory [5,20]. This has been applied to the analysis of the time evolution of the permanence amplitude $A_a(t)$ in the initial bare state. It has been shown that the evolution can be decomposed in two time scales: the first, of the order of the inverse transition frequency ω_0^{-1}, corresponds to the formation of the virtual cloud around the source [21]; the second τ_a^* corresponds to a decaylike process from the higher bare energy level to the lower dressed one and is much larger than the time γ^{-1} of real photon emission from the excited state [22]. These two time scales correspond to the physical distinction that, while the virtual cloud's photons remain localized around the atom, real photons, emitted in the self-dressing process, are formed by wave packets with a frequency spread of the order of $\Gamma_a^*/\hbar = \tau_a^{*-1}$ and are localized in a region of dimension $l \sim c\hbar/\Gamma_a^*$. Because these photons propagate at velocity c, for times $t > \hbar/\Gamma_a^*$, they are localized far from the atom. This characteristic permits to use the approximation useful to describe the dynamics of self-dressing. In fact states formed by dressed ground states plus real photons are used to represent the asymptotic states in scattering theory [20].

Here we apply the van Hove theory to study the form under which the energy excess is eliminated and obtain the energy spectrum emitted during the self-dressing of the ground state of a two-level atom interacting with the electromagnetic field. In particular we shall study the transition amplitude from the initial bare ground state to a final state constituted by the dressed atom [23–25,16] plus a certain number of real photons.

II. VAN HOVE THEORY OF RESOLVENT: REAL PHOTONS SPECTRUM

The atom-field system is taken as a two-level atom interacting with the electromagnetic field within the electric dipole approximation. The Hamiltonian is given by [23]

$$H=H_0+V, \tag{1}$$

where

$$H_0=H_A+H_F=\hbar\omega_0 S_z+\hbar\sum_{\mathbf{k}j}\ \omega_k a^\dagger_{\mathbf{k}j}kja_{\mathbf{k}j} \tag{2}$$

represents the unperturbed Hamiltonian and

$$V=\sum_{\mathbf{k}j}\ (\epsilon_{\mathbf{k}j}a_{\mathbf{k}j}S_+ + \epsilon^*_{\mathbf{k}j}a^\dagger_{\mathbf{k}j}kjS_- + \epsilon_{\mathbf{k}j}a^\dagger_{\mathbf{k}j}S_+ + \epsilon^*_{\mathbf{k}j}a_{\mathbf{k}j}S_-) \tag{3}$$

the atom-field interaction. The atom is treated by the pseudospin formalism, ω_0 is the atom's transition frequency, ω_k is the frequency of the field plane mode of wave vector $\mathbf{k}$ and polarization j $(j=1,2)$, $a_{\mathbf{k}j}(a^\dagger_{\mathbf{k}j})$ are photon annihilation (creation) operators in the mode $\mathbf{k}j$; $\epsilon_{\mathbf{k}j}=-i(2\pi\omega_0^2)/(L^3\omega_k)^{1/2}\vec{\mu}_{21}\cdot\hat{\mathbf{e}}_{\mathbf{k}j}$ is the atom-field coupling constant [21] with L^3 the quantization volume, $\vec{\mu}_{21}$ the matrix element of the electric dipole operator between the bare atomic states, and $\hat{\mathbf{e}}_{\mathbf{k}j}$ the polarization vector of $\mathbf{k}j$ mode. The atomic states are $|s\rangle$, with s = a (lower), b (higher), and the field states $|\{n_{\mathbf{k}j}\}\rangle$, where $\{n_{\mathbf{k}j}\}$ represents a photon distribution with $n_{\mathbf{k}j}$ photons in the $\mathbf{k}j$ mode. The bare eigenstates of the Hamiltonian H_0 are $|s,\{n_{\mathbf{k}j}\}\rangle=|s\rangle\otimes|\{n_{\mathbf{k}j}\}\rangle$ where $\otimes$ indicates the tensor product. The energy eigenstates of H_0 with eigenvalues E_a and E_b are $|\psi_a^0\rangle=|a,\{0_{\mathbf{k}j}\}\rangle$ and $|\psi_b^0\rangle=|b,\{0_{\mathbf{k}j}\}\rangle$ representing, respectively, the atom in its lower and higher energy state with the field in its vacuum state. Because, as explained previously, one expects that the transition from the initial atomic bare state with energy E_a to the final dressed atomic state with energy E_a^* may occur with the emission of real photons, we shall study the transition amplitude $\langle\psi_a;\{n_{\mathbf{k}j}\}|U(t,0)|\psi_a^0\rangle$, where $U(t,0)$ is the unitary time evolution operator. It must be observed that the states $|\psi_a;\{n_{\mathbf{k}j}\}\rangle$ are not exact eigenstates of H, however it is possible to show that they can be used as asymptotic states to study the photon emission process [20]. In fact they can be used to describe wave packets where the photons are localized far from the atom [19]. As said in the introduction this is what one expects in our case for times longer than the decay time τ_a^* [22]. Moreover the state $|\{n_{\mathbf{k}j}\};\psi_a\rangle$ has, with enough accuracy, the energy $E_a^*+\Sigma_k n_k\hbar\omega_k$. The final atomic stationary dressed state, representing the atom-field system at the equilibrium and no photons, may be expressed by using the Brillouin-Wigner expansion [26] as

$$\begin{aligned}|\psi_a\rangle&=\sqrt{Z}|\psi_a^0\rangle+Q_a|\psi_a\rangle=\sqrt{Z}|\psi_a^0\rangle+|\psi_a^\perp\rangle\\&=\sqrt{Z}\Bigg[|\psi_a^0\rangle+\Bigg(\frac{Q_a}{E_a^*+i\eta-Q_aH_0Q_a}VP_a\\&\quad+\frac{Q_a}{E_a^*+i\eta-Q_aH_0Q_a}V\frac{Q_a}{E_a^*+i\eta-Q_aH_0Q_a}VP_a\\&\quad+\cdots\Bigg)|\psi_a^0\rangle\Bigg]\end{aligned} \tag{4}$$

with $P_a=|\psi_a^0\rangle\langle\psi_a^0|$ projector on the subspace associated to the nondegenerate bare eigenvalue E_a, $Q_a=1-P_a$ projector on the orthogonal subspace and $|\psi_a^\perp\rangle=Q_a|\psi_a\rangle$ component of $|\psi_a\rangle$ orthogonal to $|\psi_a^0\rangle$, Z is the normalization constant.

By using the resolvent operator we may express the transition amplitude from the initial bare state to a final state with the atom in its dressed ground state and $\{n_k\}$ photons as

$$\begin{aligned}&\langle\psi_a;\{n_{\mathbf{k}j}\}|U(t,0)|\psi_a^0\rangle\\&\quad=\frac{1}{2\pi i}\int_C dz\langle\psi_a;\{n_{\mathbf{k}j}\}|G(z)|\psi_a^0\rangle e^{-i(z/\hbar)t},\end{aligned} \tag{5}$$

where $G(z)=1/(z-H)$ is the resolvent operator [20] and $C=C_++C_-$ is the integration contour in the complex z plane made by lines C_+ immediately above and C_- below the real axis and followed, respectively, from right to left and from left to right. At first we consider the probability amplitude that the dressing occurs with the emission of only one real photon; this amounts to put $|\{n_{\mathbf{k}j}\}\rangle=|\mathbf{k}j\rangle$ in Eq. (5), which leads to calculate the amplitude $\langle\psi_a;\mathbf{k}j|G(z)|\psi_a^0\rangle\equiv\langle\psi_a;|a_{\mathbf{k}j}G(z)|\psi_a^0\rangle$. Because of the form of the Hamiltonian it is possible to show that

$$a_{\mathbf{k}j}G(z)=G(z-\hbar\omega)a_{\mathbf{k}j}+G(z-\hbar\omega)V^\dagger_{\mathbf{k}j}G(z) \tag{6}$$

with

$$V^\dagger_{\mathbf{k}j}=[a_{\mathbf{k}j},V]=\epsilon_{\mathbf{k}j}{}^*S_-+\epsilon_{\mathbf{k}j}S_+. \tag{7}$$

Thus

$$\begin{aligned}\langle\psi_a;\mathbf{k}j|G(z)|\psi_a^0\rangle&=\langle\psi_a|\{G(z-\hbar\omega)a_{\mathbf{k}j}\\&\quad+G(z-\hbar\omega)V^\dagger_{\mathbf{k}j}G(z)\}|\psi_a^0\rangle\\&=\langle\psi_a|G(z-\hbar\omega)V^\dagger_{\mathbf{k}j}G(z)|\psi_a^0\rangle.\end{aligned} \tag{8}$$

Due to the form of interaction (3) the right-hand side of Eq. (8) vanishes and from Eq. (5) the corresponding probability amplitude becomes

$$\langle\psi_a;\mathbf{k}j|U(t,0)|\psi_a^0\rangle=0. \tag{9}$$

Thus the transition from the bare ground state to the dressed state may not occur with the emission of only one real photon. We then consider the amplitude of transition when two photons are emitted, that is $|\{n_{\mathbf{k}j}\}\rangle=|\mathbf{k}j,\mathbf{k}'j'\rangle$. We must calculate $\langle\psi_a;\mathbf{k}'j',\mathbf{k}j|G(z)|\psi_a^0\rangle\equiv\langle\psi_a;|a_{\mathbf{k}'j'}a_{\mathbf{k}j}G(z)|\psi_a^0\rangle$. Applying two times Eq. (6) we get

$$\begin{aligned}\langle\psi_a|a_{\mathbf{k}'j'}a_{\mathbf{k}j}G(z)|\psi_a^0\rangle&\\ =\langle\psi_a|\{G(z-\hbar\omega-\hbar\omega')a_{\mathbf{k}'j'}&\\ +G(z-\hbar\omega-\hbar\omega')V^\dagger_{\mathbf{k}'j'}G(z-\hbar\omega)\}a_{\mathbf{k}j}|\psi_a^0\rangle&\\ +\langle\psi_a|\{G(z-\hbar\omega-\hbar\omega')a_{\mathbf{k}'j'}&\\ +G(z-\hbar\omega-\hbar\omega')V^\dagger_{\mathbf{k}'j'}G(z-\hbar\omega)\}V^\dagger_{\mathbf{k}j}G(z)|\psi_a^0\rangle&\end{aligned}\tag{10}$$

and taking into account that $a_{\mathbf{k}j}|\psi_a^0\rangle=0$, the matrix element of resolvent corresponding to the transition from the bare ground state with no photons to the dressed ground state with two real photons becomes

$$\begin{aligned}\langle\psi_a|a_{\mathbf{k}'j'}a_{\mathbf{k}j}G(z)|\psi_a^0\rangle&\\ =\langle\psi_a|G(z-\hbar\omega-\hbar\omega')V^\dagger_{\mathbf{k}j}G(z-\hbar\omega')V^\dagger_{\mathbf{k}'j'}G(z)|\psi_a^0\rangle&\\ +\langle\psi_a|G(z-\hbar\omega-\hbar\omega')V^\dagger_{\mathbf{k}'j'}G(z-\hbar\omega)V^\dagger_{\mathbf{k}j}G(z)|\psi_a^0\rangle.&\end{aligned}\tag{11}$$

The two terms appearing in the right-hand side of (11) may be obtained each from the other by exchanging ω with ω' and $\mathbf{k}j$ with $\mathbf{k}'j'$. We shall then limit the calculations to only one of the terms; in particular we shall fix our attention to the first one.

Since $|\psi_a\rangle$ is an exact eigenstate of total Hamiltonian H, we get

$$\begin{aligned}\langle\psi_a|G(z-\hbar\omega-\hbar\omega')V^\dagger_{\mathbf{k}j}G(z-\hbar\omega')V^\dagger_{\mathbf{k}'j'}G(z)|\psi_a^0\rangle&\\ =\frac{1}{z-E_a^*-\hbar\omega-\hbar\omega'}\langle\psi_a|V^\dagger_{\mathbf{k}j}G(z-\hbar\omega')V^\dagger_{\mathbf{k}'j'}G(z)|\psi_a^0\rangle.&\end{aligned}\tag{12}$$

Using the definition (7) for $V^\dagger_{\mathbf{k}j}$, the matrix element in the last term of Eq. (12) becomes

$$\begin{aligned}\langle\psi_a|V^\dagger_{\mathbf{k}j}G(z-\hbar\omega')V^\dagger_{\mathbf{k}'j'}G(z)|\psi_a^0\rangle&\\ =\epsilon^*_{\mathbf{k}j}\epsilon_{\mathbf{k}'j'}\langle\psi_a|\psi_a^0\rangle\langle\psi_b^0|G(z-\hbar\omega')|\psi_b^0\rangle\langle\psi_a^0|G(z)|\psi_a^0\rangle&\\ =\sqrt{Z}\epsilon^*_{\mathbf{k}j}\epsilon_{\mathbf{k}'j'}\langle\psi_b^0|G(z-\hbar\omega')|\psi_b^0\rangle\langle\psi_a^0|G(z)|\psi_a^0\rangle,&\end{aligned}\tag{13}$$

where $\langle\psi_a|\psi_a^0\rangle=\sqrt{Z}$ and $S_+=|\psi_b^0\rangle\langle\psi_a^0|$, $S_-=|\psi_a^0\rangle\langle\psi_b^0|$ have been used. Substituting Eq. (13) in Eq. (12) we get

$$\begin{aligned}\langle\psi_a|G(z-\hbar\omega-\hbar\omega')V^\dagger_{\mathbf{k}j}G(z-\hbar\omega')V^\dagger_{\mathbf{k}'j'}G(z)|\psi_a^0\rangle&\\ =\sqrt{Z}\epsilon^*_{\mathbf{k}j}\epsilon_{\mathbf{k}'j'}\frac{1}{z-E_a^*-\hbar\omega-\hbar\omega'}\langle\psi_b^0|G(z-\hbar\omega')|\psi_b^0\rangle&\\ \times\langle\psi_a^0|G(z)|\psi_a^0\rangle.&\end{aligned}\tag{14}$$

Thus we are left to calculate the quantities $G_a(z)=\langle\psi_a^0|G(z)|\psi_a^0\rangle$ and $G_b(z-\hbar\omega')=\langle\psi_b^0|G(z-\hbar\omega')|\psi_b^0\rangle$. $G_a(z)$ is connected to the permanence amplitude of remaining in the bare state $|\psi_a^0\rangle$. The nonperturbative calculation of $G_a(z)$, performed by application of the Van Hove technique, introduces both the effect of decay and that of dressing, and it is given for $z=E+i\eta$ (with η infinitesimal positive) by [22,27]

$$\begin{aligned}G_a(z)&=\frac{1}{z-E_a-R_a(z)}\\ &=\frac{1}{z-(E_a+\Delta_a^*-i(\Gamma_a^*/2))}=\frac{1}{z-E_a^*+i(\Gamma_a^*/2)},\end{aligned}\tag{15}$$

where $R_a(z)$ is the matrix element

$$\langle\psi_a^0|R(z)|\psi_a^0\rangle=\Delta_a^*-i\frac{\Gamma_a^*}{2}\tag{16}$$

of the level-shift operator

$$\begin{aligned}R(z)=V+V\frac{1}{z-H_0}V+V\frac{1}{z-H_0}V\frac{1}{z-H_0}V&\\ +V\frac{1}{z-H_0}V\frac{1}{z-H_0}V\frac{1}{z-H_0}V+\cdots&\end{aligned}\tag{17}$$

with

$$\begin{aligned}\Delta_a^*=\frac{\hbar\gamma}{2\pi\omega_0}\Bigg\{-\omega_M+\omega_0\log\left(\frac{\omega_0+\omega_M}{\omega_0}\right)&\\ +\frac{1}{2}\gamma\left[\arctan\left(2\frac{\omega_0+\omega_M}{\gamma}\right)-\arctan\left(2\frac{\omega_0}{\gamma}\right)\right]\Bigg\},&\end{aligned}\tag{18}$$

$$\Gamma_a^*=\frac{\hbar}{2\pi}\frac{\gamma}{\omega_0}\gamma\log\frac{\omega_M}{\omega_0},\tag{19}$$

and the cutoff ω_M is the frequency upper limit typical of the nonrelativistic QED's scheme.

Analogously the matrix element of the resolvent connected to the permanence amplitude of remaining in the bare state $|\psi_b^0\rangle$ is (for $z=E+i\eta$)

$$G_b(z-\hbar\omega')=\frac{1}{z-(E_b+\Delta_b^{(2)}-i\gamma)-\omega'}=\frac{1}{z-E_b^{(2)}-\omega'+i\gamma}, \tag{20}$$

where

$$\Delta_b^{(2)}=\frac{\hbar\gamma}{2\pi\omega_0}\left[-\omega_M-\omega_0\log\left(\frac{\omega_0+\omega_M}{\omega_0}\right)\right] \tag{21}$$

is the energy shift, $\hbar\gamma$ the linewidth corresponding to the ordinary decay rate from the excited state, and $E_b^{(2)}=E_b+\Delta_b^{(2)}$ is the dressed energy value of the excited state.

The approximations used to obtain the form (15) of $G_a(z)$ are the same of those used previously [22,28]. The same kind of approximations have been used to calculate $G_b(z-\hbar\omega')$. Using $G_a(z)$ and $G_b(z-\hbar\omega')$, as given by Eqs. (15) and (20), in the matrix element of $G(z)$, given by Eq. (11), between the bare and the dressed ground state with two photons, we obtain

$$\begin{aligned}\langle\psi_a|a_{\mathbf{k}'j'}a_{\mathbf{k}j}G(z)|\psi_a^0\rangle &=\sqrt{Z}\epsilon_{\mathbf{k}j}^*\epsilon_{\mathbf{k}'j'}\frac{1}{z-E_a^*-\hbar\omega-\hbar\omega'}\\ &\times\frac{1}{z-E_b^{(2)}-\omega'+i\gamma}\frac{1}{z-E_a^*+i\Gamma_a^*}+\sqrt{Z}\epsilon_{\mathbf{k}j}\epsilon_{\mathbf{k}'j'}^*\\ &\times\frac{1}{z-E_a^*-\hbar\omega-\hbar\omega}\frac{1}{z-E_b^{(2)}-\omega+i\gamma}\\ &\times\frac{1}{z-E_a^*+i\Gamma_a^*}.\end{aligned} \tag{22}$$

The probability amplitude $\langle\psi_a;\mathbf{k}j\mathbf{k}'j'|U(t,0)|\psi_a^0\rangle$ may be now obtained substituting Eq. (22) in Eq. (5) and calculating the integral over z by the residue theorem by closing the line integral over C_+ in the lower complex half-plane. It is then easily seen that for $t\gg\gamma^{-1},(\Gamma_a^*/\hbar)^{-1}$ only the pole at $E_a^*+\omega+\omega'$ contributes to the integral. By introducing the approximation $E_b^{(2)}-E_a^*\approx\hbar\omega_0$, we get

$$\begin{aligned}\langle\psi_a;\mathbf{k}j\mathbf{k}'j'|U(t,0)|\psi_a^0\rangle &\approx\sqrt{Z}\exp\left[-i\left(\frac{E_a}{\hbar}+\omega+\omega'\right)t\right]\\ &\times\left\{\frac{\epsilon_{\mathbf{k}'j'}^*\epsilon_{\mathbf{k}j}}{\left(\hbar\omega-\hbar\omega_0+\frac{i}{2}\hbar\gamma\right)\left(\hbar\omega+\hbar\omega'+\frac{i}{2}\Gamma_a^*\right)}\right.\\ &\left.+\frac{\epsilon_{\mathbf{k}'j'}^*\epsilon_{\mathbf{k}j}}{\left(\hbar\omega'-\hbar\omega_0+\frac{i}{2}\hbar\gamma\right)\left(\hbar\omega+\hbar\omega'+\frac{i}{2}\Gamma_a^*\right)}\right\}.\end{aligned} \tag{23}$$

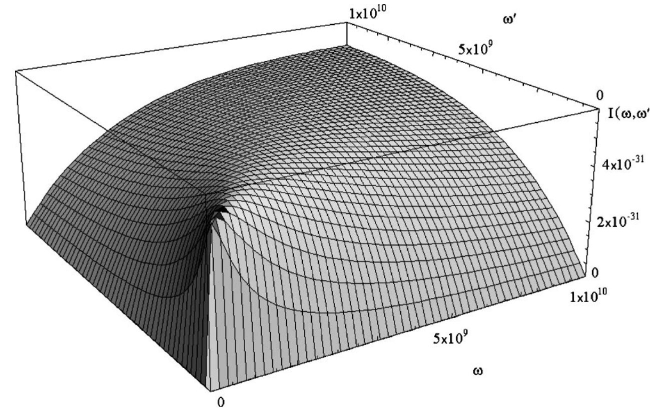

FIG. 1. The energy spectrum $I(\omega,\omega')$ is shown for $0<\omega<10^3\ \mathrm{rad\,s^{-1}}$, $0<\omega'<10^3\ \mathrm{rad\,s^{-1}}$ using arbitrary unit. In this range the value of the intensity $I(\omega,\omega')$ is negligible . For $\omega=\omega'=10^3\ \mathrm{rad\,s^{-1}}$ the value of the energy spectrum increases rapidly up to $I(\omega,\omega')\sim5\times10^{-31}$ arb. units. This value is substantially conserved up to $\omega\approx\omega'\approx10^{10}\ \mathrm{rad\,s^{-1}}$.

The probability amplitude (23) contains the contributions due to a different order of emission of the real photons. The frequency distribution of the real photons is obtained taking the modulus square of Eq. (23). Yet, under the condition $|\omega_0-\Delta_a^*/\hbar|\gg\gamma,\Gamma_a^*/\hbar$, the two amplitudes may never be large simultaneously, thus the effects due to the interference between probability amplitudes may be neglected and the probability density becomes a sum of only two terms. Finally multiplying by the density of two photon states

$$\frac{L^3 4\pi\omega^2 d\omega}{(2\pi)^3}\frac{L^3 4\pi\omega'^2 d\omega'}{(2\pi)^3} \tag{24}$$

and using the definition of $\epsilon_{\mathbf{k}j}$, the spectral distribution is obtained as

$$\begin{aligned}I(\omega,\omega')&\approx Z\left[\frac{\omega_0^2}{\pi^2}|\vec{\mu}_{21}|^2\right]^2\omega\omega'\\ &\times\left\{\frac{1}{\left[(\omega_0-\omega)^2+\frac{\gamma^2}{4}\right]\left[(\omega+\omega')^2+\frac{\Gamma_a^{*2}}{4\hbar^2}\right]}\right.\\ &\left.+\frac{1}{\left[(\omega_0-\omega')^2+\frac{\gamma^2}{4}\right]\left[(\omega+\omega')^2+\frac{\Gamma_a^{*2}}{4\hbar^2}\right]}\right\}.\end{aligned} \tag{25}$$

Its graphical representation in the regions around $\omega\approx\omega'\approx\omega_0$ and $\omega\approx\omega'\ll\omega_0$ is reported in Figs. 1 and 2. These figures indicate that the emission probability results are negligibly small when both the photons have frequencies ω, ω' different from ω_0. The frequency distribution appears to be symmetric under exchange of ω and ω' and for $\omega(\omega')\sim\omega_0$, it increases with $\omega'(\omega)$ increasing up to ω_0. For this value $\omega=\omega'=\omega_0$ the frequency distribution presents a peak that suggests that the real photons preferably are emitted both at the transition frequency.

It appears to be of interest to evaluate the probability of creation of two photons with respect to the probability that the self-dressing process occurs without real photon emission. This may be obtained by studying $|\psi_a^0(\infty)\rangle$, which represents the state that is obtained from the time evolution of the bare ground state for $t\to\infty$. Within the approximation that the atomic self-dressing occurs at the most with the emission of two real photons, we may write

$$
\begin{aligned}
|\psi_a^0(\infty)\rangle &= (e^{-i(H/\hbar)t}|\psi_a^0\rangle)_{t\to\infty} \\
&= c_0|\psi_a;\{0_{\mathbf{k}j}\}\rangle + c_1|\psi_a;\mathbf{k}j\rangle + c_2|\psi_a;\mathbf{k}j,\mathbf{k}'j'\rangle \\
&= c_0|\psi_a;\{0_{\mathbf{k}j}\}\rangle + c_2|\psi_a;\mathbf{k}j,\mathbf{k}'j'\rangle, \qquad (26)
\end{aligned}
$$

where $|c_2|^2$ coincides with the normalization factor Z present in Eq. (25) and, due to Eq. (9) it is $c_1=0$. The coefficients c_0 and c_2 are the probability amplitude that a measurement on the state $|\psi_a^0(\infty)\rangle$ reveals, respectively, no photon and two photons. From the normalization of the state (26) we obtain

$$|c_2|^2 = 1-|c_0|^2. \qquad (27)$$

The coefficient c_0 may be obtained taking into account the projection of the stationary dressed state with no real photons onto $|\psi_a^0(\infty)\rangle$

$$
\begin{aligned}
c_0 &= \langle\psi_a;\{0_{\mathbf{k}j}\}|\psi_a^0(\infty)\rangle \\
&= \langle\psi_a;\{0_{\mathbf{k}j}\}|(e^{-i(H/\hbar)t}\rangle|\psi_a^0\rangle)_{t\to\infty} \\
&= -\frac{1}{2\pi i}\left(\int_{-\infty}^{+\infty}\langle\psi_a;\{0_{\mathbf{k}j}\}|\frac{1}{z-H}|\psi_a^0\rangle e^{-i(z/\hbar)t}\right)_{t\to\infty} \\
&= (e^{-i(E_a^*/\hbar)t})_{t\to\infty}\langle\psi_a;\{0_{\mathbf{k}j}\}|\psi_a^0\rangle, \qquad (28)
\end{aligned}
$$

where the expression for $\langle\psi_a;\{0_{\mathbf{k}j}\}|\psi_a^0\rangle$, accurate up to order ϵ^2, results to be

$$\langle\psi_a;\{0_{\mathbf{k}j}\}|\psi_a^0\rangle = 1-\frac{\gamma}{4\pi\omega_0}\int_0^{\omega_M}\frac{\omega}{(\omega_0+\omega)^2}d\omega. \qquad (29)$$

Using Eq. (29) and $\gamma\sim10^9\ \mathrm{s}^{-1}$, $\omega_0\sim10^{15}\ \mathrm{rad\,s}^{-1}$, $\omega_M\sim10^{18}\ \mathrm{rad\,s}^{-1}$, (28) becomes

$$|c_0|^2 = 1-\frac{\gamma}{2\pi\omega_0}\int_0^{\omega_M}\frac{\omega}{(\omega_0+\omega)^2}d\omega \approx 1-\frac{\gamma}{\omega_0}\approx 1-10^{-6}. \qquad (30)$$

Finally from Eq. (27) we get

$$|c_2|^2 \approx \frac{\gamma}{\omega_0}\sim 10^{-6}. \qquad (31)$$

Equations (30) and (31) show that the probability of emission of two real photons during the atomic self-dressing is 10^6 times smaller than the probability that the self-dressing occurs without real photon creation [17]. On the other side the frequency distribution (25) indicates that the real photons preferably are emitted with frequencies $\sim\omega_0$.

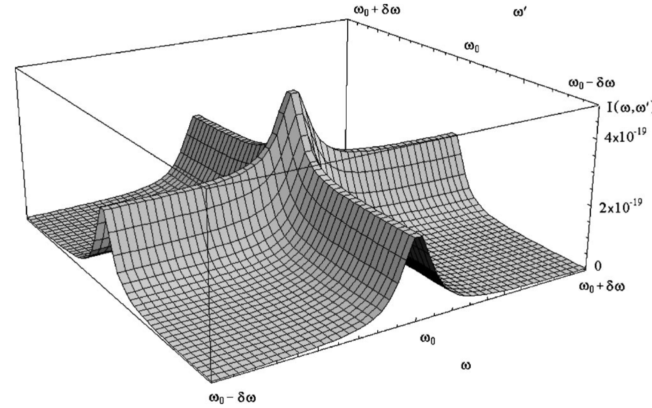

FIG. 2. The energy spectrum $I(\omega,\omega')$ is shown for $\omega\approx\omega'\approx\omega_0$ with $\omega_0=10^{15}\ \mathrm{rad\,s}^{-1}$ and $\delta\omega=10^{10}\ \mathrm{rad\,s}^{-1}$ using arbitrary units as in Fig. 1. A peak appears in $\omega=\omega'=\omega_0$ with $I(\omega,\omega')\sim5\times10^{-19}$ arb. units. For ω, ω' at the transition frequency ω_0, the photon emission probability is the largest. The comparison of Fig. 2 with Fig. 1 shows that the photon emission probability at $\omega=\omega'=\omega_0$ is much larger than that at $\omega=\omega'=10^3\ \mathrm{rad\,s}^{-1}$; the real photons are more likely emitted at the transition frequency.

If we extend the description of the atomic dressing to an ensemble of atoms, we may conclude in agreement with the results obtained that the complete dressing process requires among 10^6 atoms one to emit two real photons with frequencies $\omega=\omega'=\omega_0$ (Fig. 2). This implies that during the self-dressing of the two-level atom's ground state it is possible, in principle, to detect real photons. These for typical transition frequency $(\omega_0\sim10^{15}\ \mathrm{rad\,s}^{-1})$ carry the energy $E\sim10^{-6}\hbar\omega_0\sim10^{-25}J$, this value having the same magnitude order of the Lamb-Shift of the hydrogen atom's ground state [17,18].

III. CONCLUSIONS

In this paper we have studied the spectrum of energy emitted under the form of real photons due to the dressing process of a two-level atom initially in its bare ground state and interacting with electromagnetic field in its vacuum state. The dressing process for a such system occurs in two steps: the first consists of the formation of the virtual cloud surrounding the atomic dressed state [21]; the second is responsible for the appearance of the dissipative behavior [22] and for the energy decrease that leads the atom from the bare to the stationary dressed state.

The presence of dissipative behavior during the atomic dressing was previously obtained by the theory of subdynamics where it was predicted the appearance of radiation constituted by real photons with frequency ω_0 resonant to the atom's transition frequency [17,18].

Recently it has been studied how the self-dressing produces a density of real photons which by expanding carries the energy excess [19].

Here we obtain the spectrum of the real photons using standard quantum-mechanics nonperturbative techniques [5,20]. We find that at the lowest order the atomic self-dressing causes the emission of two real photons with frequencies ω and ω'. For ω (ω') fixed the frequency distribution results to be the largest when $\omega'(\omega)\sim\omega_0$ and it presents a peak in $\omega=\omega'=\omega_0$. The emission of this radiation is characterized by a time scale $\tau_a^*=\hbar/\Gamma_a^*$ much longer both than that associated to the virtual cloud's formation and than that characterizing the ordinary (Wigner-Weisskopf) decay from the excited state [22].

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