PHYSICAL REVIEW A, VOLUME 65, 031402(R)

Area evolution of a few-cycle pulse laser in a two-level-atom medium

Jian Xiao,* Zhongyang Wang, and Zhizhan Xu
Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, P.O. Box 800-211, Shanghai 201800, People's Republic of China
(Received 27 August 2001; published 5 February 2002)

By solving the Maxwell-Bloch equations, we study the area evolution of a few-cycle pulse laser propagating in a resonant two-level-atom medium. We find that in short propagation distance, the pulse envelope, obtained within the slowly varying envelope approximation and the rotating-wave approximation, agrees nicely with the carrier field. In this case, the area theorem can still predict the profile of the area evolution of a few-cycle optical pulse. However, contrary to the long-pulse case, the variation of the few-cycle pulse area is caused by the pulse splitting but not by the pulse broadening or the pulse compression. Furthermore, the negative area occurs when the pulse area decreases. As a result, a pulse with area less than π is not absorbed rapidly according to the usual Beer's law of absorption but evolves to nonvanishing zero π pulse.

DOI: 10.1103/PhysRevA.65.031402 PACS number(s): 42.50.Md, 42.65.Re, 42.50.Gy

The pulse area is the integral of the Rabi frequency over time and expressed as $A(z)=(d/\hbar)\int_{-\infty}^{\infty}E_0(z,t)dt$ with $E_0(z,t)$ the electric-field envelope of the pulse and d the dipole moment. The area theorem $[dA(z)/dz=-(\alpha/2)\sin A(z)$ with α the absorption coefficient] [1] is an important theorem to describe pulse propagation, which can predict and explain the pulse broadening, pulse compression, and the self-induced transparency (SIT) that a pulse with area of $2n\pi$ (n is an integral) can propagate without loss with lower speed through a resonant absorber medium. And specially, the envelope shape of a 2π pulse remains invariable during propagation. To obtain the area theorem, the standing slowly varying envelope approximation (SVEA) [2] and the rotating-wave approximation (RWA) [3] are employed.

Recent advance in ultrafast laser technology has made it possible to generate extremely short, intense pulses with only two optical periods of less than 5 fs [4,5]. Some theoretical studies have shown the limitations of the SVEA and the RWA for ultrashort pulses. Simulation of 100 fs pulse propagation in a two-level-atom (TLA) medium [6] has demonstrated that the time-derivative behavior of the oscillatory electric field plays an essential role in the nonlinear evolution of the system, which sustains the nonlinear process through null-field points. For larger pulse area of 18 fs duration [7], the areas under individual carriers may themselves cause Rabi flopping (RF), i.e., the carrier-wave RF occurs, which leads to carrier-wave reshaping and significantly higher spectral components. All these interesting phenomena cannot be explained by the area theorem or RWA.

However, in this paper, numerical simulation shows that the area theorem can still predict the profile of the area evolution of a few-cycle pulse in a resonant TLA absorber medium. Contrary to the long pulse case, the variation of the few-cycle pulse area is not caused by the pulse broadening or the pulse compression, but by a kind of pulse splitting.

We consider the propagation of a few-cycle pulse laser along the z axis in vacuum to an input interface of a resonant TLA medium at $z=0$, then the pulse enters the medium. The Maxwell equations take the form

$$\partial_t H_y = -\frac{1}{\mu_0}\partial_z E_x,$$

$$\partial_t E_x = -\frac{1}{\varepsilon_0}\partial_z H_y - \frac{1}{\varepsilon_0}\partial_t P_x. \tag{1}$$

Here E_x, H_y are the electric and magnetic fields, respectively. The macroscopic nonlinear polarization $P_x=Ndu$ is connected with the off-diagonal density-matrix element $\rho_{12}=(u+iv)/2$ and the population difference $w=\rho_{22}-\rho_{11}$ between the upper and lower states, N is the density of the medium, d is the dipole moment. The refractive index is determined by the real part of ρ_{12} and the gain coefficient is proportional to the imaginary part of ρ_{12}. u, v, and w are determined by the Bloch equations [1],

$$\partial_t u = -\gamma_2 u - \omega_0 v,$$

$$\partial_t v = -\gamma_2 v + \omega_0 u + 2\Omega w,$$

$$\partial_t w = -\gamma_1(w-c) - 2\Omega v, \tag{2}$$

where ω_0 is the resonant frequency, γ_1 and γ_2 are the population and polarization relaxation constants, respectively, $\Omega=dE_x/\hbar$ the Rabi frequency, and w_0 is the initial population difference.

To obtain the pulse envelope, we rewrite the Maxwell-Bloch (MB) equations by substituting

$$E_+(z,t)=\frac{1}{2}E_0(z,t)\exp\{i[\omega_p t - kz - \phi(z,t)]\},$$

$$P_+(z,t)=Nd\rho'_{12}(z,t)\exp\{i[\omega_p t - kz - \phi(z,t)]\}, \tag{3}$$

where ω_p is the carrier frequency, k the wave vector, $\phi(z,t)$ the phase function, and $\rho'_{12}=(u'+iv')/2$ the envelope of ρ_{12}, into the wave equation [10]

*Email address: iamxiao@21cn.com

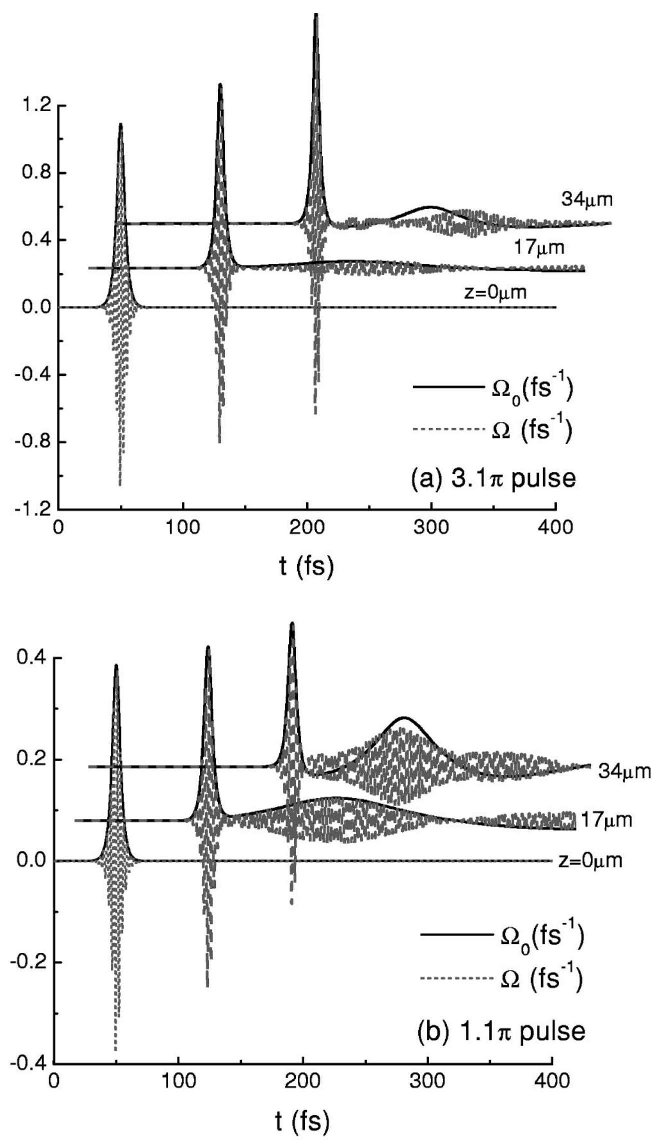

FIG. 1. The pulse envelope Ω_0 (solid line) and the pulse carrier Ω (dashed line) of the 3.1π (a) and 1.1π (b) pulses at the respective propagation distances of $z=0$ μm, 17 μm, and 34 μm.

$$\partial_z^2 E_+ - \frac{1}{c^2}\partial_t^2 E_+ = \mu_0 \partial_t^2 P_+ ,$$

and the Bloch equations (2). Then we get the envelope forms of the MB equations in SVEA and RWA,

$$\partial_z E_0 + \frac{1}{c}\partial_t E_0 = \frac{N d \mu_0 c \omega_p v'}{2}, \tag{4a}$$

$$E_0\left(\partial_z \phi + \frac{1}{c}\partial_t \phi\right) = \frac{N d \mu_0 c \omega_p u'}{2}, \tag{4b}$$

$$\partial_t u' = -\gamma_2 u' - \triangle \omega v', \tag{5a}$$

$$\partial_t v' = -\gamma_2 v' + \triangle \omega u' + \Omega_0 w, \tag{5b}$$

$$\partial_t w = -\gamma_1 (w - w_0) - \Omega_0 v'. \tag{5c}$$

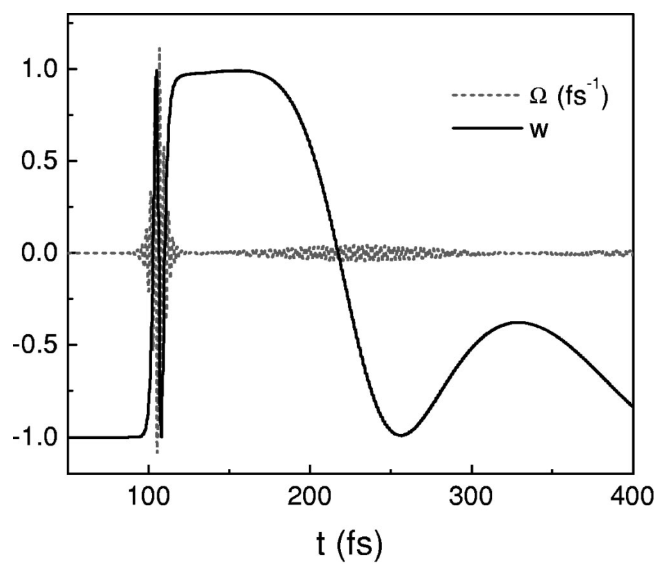

FIG. 2. The population difference w (solid line) and the pulse carrier Ω (dashed line) of the 3.1π pulse at the propagation distance of $z=17$ μm.

Here $\Omega_0 = dE_0/\hbar$ and $\Delta\omega = \omega_0 - \omega_p$. The area theorem can be derived from Eqs. (4) and (5) [1].

Equations (1), (2), (4), and (5) can be solved by using the finite-difference time-domain method for the field equations and the predictor-corrector method for the material variables [6–9]. The initial conditions are $u=v=0$, $w_0=-1$ for the absorber medium and $\Omega(t=0,z)=\Omega_m \cos[\omega_p(z-z_0)/c]\mathrm{sech}[1.76(z-z_0)/c\tau_p]$ for the input soliton pulse with Ω_m the maximum Rabi frequency and τ_p the full width at half maximum of the pulse-intensity envelope. The choice of z_0 ensures that the pulse penetrates negligibly into the medium at $t=0$. Considering the conditions of SIT, we choose the relaxation times much longer than the input pulse duration and adopt the following pulse and material parameters: $\tau_p=5$ fs, $\omega_0=\omega_p=2.3$ fs^{-1} ($\lambda=820$ nm), $z_0=15$ μm, $d=2\times10^{-29}$ A s m, $N=2\times10^{24}$ m^{-3}, $\gamma_1^{-1}=1$ ns, $\gamma_2^{-1}=0.5$ ns. The corresponding pulse area is $A=\Omega_m\tau_p\pi/1.76$, and $\Omega_m=1$ fs^{-1} corresponds to the electric field of $E_x=5\times10^9$ V/m or an intensity of $I=6.6\times10^{12}$ W/cm^2. The time increment $\triangle t$ and the space increment $\triangle z$ are chosen to ensure $c\triangle t \leqslant \triangle z$ [9].

Ideal SIT of pulses with a width of 100 fs [6] and 18 fs [7] has been simulated in resonant TLA mediums, and it is shown that the time-derivative behavior of the input field may cause local carrier modification but the pulse envelope is essentially unchanged [7]. Here we first model the area evolutions of the $3.1\pi(\Omega_m=1.0912$ fs$^{-1})$ and $1.1\pi(\Omega_m=0.3872$ fs$^{-1})$ pulses. Figure 1 shows the corresponding envelopes (solid line) and carriers (dashed line) of the two pulses at the propagation distances of $z=0$ μm, 17 μm, and 34 μm. We can see that both pulses experience similar evolutions. Despite SVEA and RWA, the envelopes coincide with the carrier fields nicely at $z=17$ μm, positive areas appear following the main pulses and they increase at $z=34$ μm. Obviously, the whole area of each pulse increases, the area evolutions are in agreement with what the

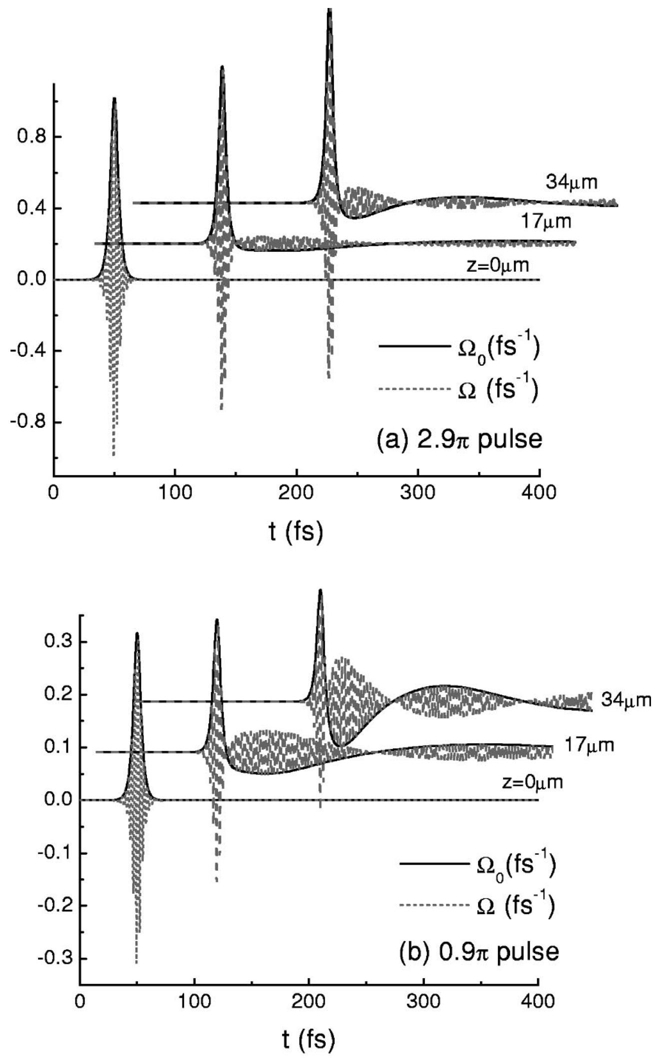

FIG. 3. The pulse envelope Ω_0 (solid line) and the pulse carrier Ω (dashed line) of the 2.9π (a) and 0.9π (b) pulses at the respective propagation distances of $z=0$ μm, 17 μm, and 34 μm.

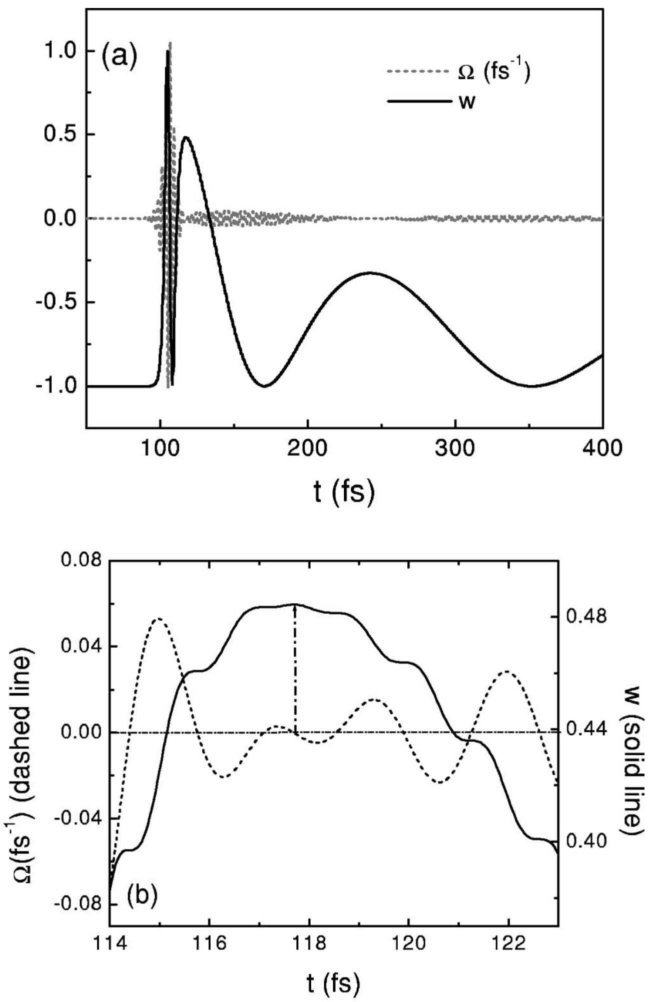

FIG. 4. (a) The population difference w (solid line) and the pulse carrier Ω (dashed line) of the 2.9π pulse at the propagation distance of $z=17$ μm. (b) The enlargement of (a) from 114 fs to 123 fs.

area theorem predicts, i.e., the areas of the two pulses will increase to 4π and 2π, respectively. However, the increases of the few-cycle pulse areas is not caused by the pulse broadening as in the case of long pulses [1], but by the pulse splitting, a different kind of splitting. To interpret the physical mechanism, we present the population difference (solid line) and the carrier field (dashed line) of the 3.1π pulse at $z=17$ μm in Fig. 2. As known, a π area can make a complete population inversion, and a 2π area may also reverse this inversion, i.e., RF occurs. Hence for a pulse area between 3π and 4π, the population is excited to the upper level completely after one RF occurs, while the remaining energy (minus 3π) in the trailing edge of the pulse may cause the stimulated radiation. For a long pulse, the trailing edge is amplified and the pulse is broadened, but for a few-cycle pulse, the pulse has passed away when the stimulated radiation occurs, the energy is retained behind the main pulse and the pulse splits.

On the other hand, we should note that the pulse envelopes coincide well with the envelopes of the carrier fields, but not completely, and the difference at $z=34$ μm is evident. This disagreement comes from the SVEA and the RWA. Because for few-cycle pulse laser, the time-derivative behavior of the electric field that was ignored within SVEA may cause oscillation features of the population [6] [see also in Fig. 4(b)], and the fast oscillation terms that were ignored within RWA may lead to carrier-wave reshaping [7]. Of course, these nonlinear behaviors may exhibit a more evident feature with further propagation or within a higher-density medium. However, Fig. 1 shows that despite these approximations, the profile of the area evolution of few-cycle pulse still follows the area theorem.

Further consideration is given to the $2.9\pi(\Omega_m=1.0208\ \mathrm{fs}^{-1})$ and $0.9\pi(\Omega_m=0.3168\ \mathrm{fs}^{-1})$ pulses with the corresponding profiles shown in Fig. 3. We should stress that the pulse area is the algebraic sum of the whole area under the envelope including the negative area that has a π difference in the carrier phase with the positive envelope [1]. Obviously, in Fig. 3, negative areas appear behind the main pulses, so the areas of the 2.9π and 0.9π pulses will decrease to 2π and 0π as what the area theorem predicts, respectively. However, the decreases are also caused by the pulse splitting, but not by the pulse compression as in the case of long pulses [1]. Figure 4 shows the population difference (solid line) and the carrier field (dashed line) of the 2.9π pulse at $z=17$ μm. From Fig. 4(a), we can see that the 2.9π pulse cannot make a complete population inversion after one RF occurs, only part of the population on the ground level is excited to the upper level and returns immediately. While just at this moment, a minor pulse with π difference in the carrier phase with the main pulse appears. Figure 4(b) is the enlargement of Fig. 4(a) from 114 fs to 123 fs, it shows the phase variation in detail. In contrast to Fig. 2, it can be inferred that the phase change of the minor pulse comes from the incomplete population inversion. For a longer pulse, the phase variation may decrease the trailing edge of the pulse and the pulse will be compressed, but for a few-cycle pulse, it leads to pulse splitting because the few-cycle pulse propagates very fast. In addition, we also calculate the propagation of the $0.1\pi(\Omega_m=0.0352\ \mathrm{fs}^{-1})$ pulse, which has similar area evolution as that of the 0.9π pulse, it does not vanish but splits into several pulses, the whole pulse area is zero. For a longer pulse, a pulse with area less than π will be broadened and absorbed in the beginning propagation distance, then it disappears rapidly according to the usual Beer's law of absorption [1]. However, for a few-cycle pulse, negative area occurs and reduces the pulse area. As a result, the pulse evolves to nonvanishing zero π pulse.

In conclusion, by solving the MB equations without and with SVEA and RWA, we get the carrier field and pulse envelope of a few-cycle pulse propagating in a resonant TLA absorber medium. It is found that the area theorem can still predict the profile of the area evolution of a few-cycle optical pulse. The variation of the few-cycle pulse area is caused by the pulse splitting but not by the pulse broadening or the pulse compression. Furthermore, negative area occurs when the pulse area decreases. In this case, a pulse with area less than π will evolve to nonvanishing zero π pulse.

This work is supported by the National Key Basic Research Special Foundation (NKBRSF) (No. G1999075200) and the Shanghai Natural Science Foundation (No. 00QA14028).

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