Measurements of intensity fluctuations in a laser with a saturable absorber

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Fluctuations of light intensity in a laser with an intracavity nonlinear absorber are studied experimentally. Measurements are made close to the tricritical point by varying the operating parameters of the laser. Experimental results are in agreement with the theoretical predictions based on the nonlinear oscillator model of the laser in which both the third- and fifth-order nonlinearities in the field amplitude are retained.

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I. INTRODUCTION

It is well known that the light from a single-mode laser operating below the threshold of oscillation where the nonlinearity of light-matter interaction plays a negligible role has the character of a narrowband thermal light. As the laser approaches and passes the threshold of oscillation, the nonlinearity of light-matter interaction becomes important. As a result of this nonlinearity, the field amplitude stabilizes at a nonzero value, relative intensity fluctuations tend toward zero, and the laser light approaches a coherent state with increasing excitation [1]. This transformation of laser light from a thermal state to a coherent state is likened to a second-order phase transition and can be described in terms of a third-order (in field amplitude) nonlinearity [2].

The physical mechanism responsible for the third-order nonlinearity in the laser is gain saturation. When a saturable absorber is added to the laser cavity, the nature of nonlinearity changes because the gain and absorption saturate, in general, at different rates. This leads to the appearance of new nonlinear phenomena such as bistability, hysteresis, and novel phase-transition analogies [3–5]. The type of fluctuation phenomena that the laser with a saturable absorber (LSA) can exhibit is extremely rich when both the atomic and field degrees of freedom are involved. Here we restrict ourselves only to class-*A* lasers where the atomic polarization and population can follow the field amplitude adiabatically. Under these conditions, the atomic variables can be eliminated from the equations of motion leaving the field amplitude as the only dynamical variable. These conditions are satisfied, for example, for a low-power He:Ne laser.

Intensity fluctuations for class-*A* LSA have been studied theoretically by a number of workers [4–9]. Experimental results for some special cases have also been reported [10]. In this paper, we present a systematic experimental investigation of the intensity fluctuations in the LSA. In our experiment, we are able to control the nature of nonlinearity and study laser light fluctuations as excitation is increased from below to above threshold. In Sec. II, we review the theoretical model that describes light-intensity fluctuations in class-*A* LSA. Section III describes the experimental procedure. Finally, Sec. IV compares experimental results with theoretical predictions.

II. THEORY

We consider a single-mode laser containing an intracavity saturable absorber. We assume both the gain and absorber media to be a collection of two-level atoms in gas phase with strong Doppler broadening of atomic lines. In the gain medium, the active atoms are "pumped" to the excited state and in the absorber cell the active atoms are pumped to the lower level. Then for small pump levels, the equation of motion for the slowly varying dimensionless complex field amplitude $\mathcal{E}(t)$ of the LSA is [7,8,10]

$$\dot{E}(t)=E[a+b|E|^2-|E|^4]+q(t), \tag{1}$$

where the pump and saturation parameters a and b are given by

$$a=(1-\alpha-\mathcal{C}/\mathcal{A})\left[\frac{8\mathcal{A}^2}{3\mathcal{B}^2(\alpha s^2-1)}\right]^{1/3}, \tag{2}$$

$$b=(\alpha s-1)\left[\frac{8\mathcal{A}}{3\mathcal{B}(\alpha s^2-1)^2}\right]^{1/3}, \tag{3}$$

and $q(t)$ is a complex Gaussian white-noise process with zero mean and variance $\langle q^*(t_1)q(t_2)\rangle=4\delta(t_1-t_2)$. Here $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are the gain, self-saturation, and loss parameters of Scully and Lamb theory [11,12]. Parameters with a bar, $\overline{\mathcal{A}}$ and $\overline{\mathcal{B}}$, refer to the absorption and self-saturation coefficients for the absorber. $\alpha=\overline{\mathcal{A}}/\mathcal{A}$ is the ratio of gain and absorption parameters and $s=(\overline{\mathcal{B}}/\overline{\mathcal{A}})/(\mathcal{B}/\mathcal{A})$ is the ratio of the saturation intensities for the gain and absorber atoms. Equation (1) was derived by using a perturbative approach to calculate the atomic polarization. For the LSA, terms up to fifth order in the field are required to ensure the establishment of a steady state for $s>1$ [5]. Equations (2) and (3) indicate that for fixed $\mathcal{A}$ and s the pump parameter a can be varied by changing the loss $\mathcal{C}$ and that, for fixed s and $\mathcal{A}$, the parameter b can be varied by changing the discharge current in the absorption cell. This changes $\overline{\mathcal{A}}$ and b via the relation $\alpha=\overline{\mathcal{A}}/\mathcal{A}$. In this way, we can explore the intensity fluctuations for the LSA in the entire threshold region.

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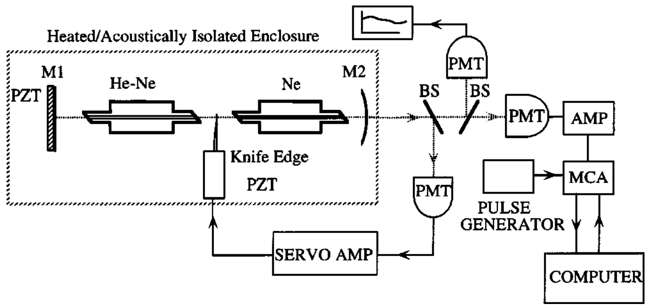

FIG. 1. An outline of the experimental apparatus. $M1$ and $M2$ are two high reflectivity mirrors, BS is a beam spliter, PMT is a photomultiplier tube, AMP is an amplifier, MCA is a multichannel analyzer, and PZT is a piezoelectric transducer.

We can convert the nonlinear Eq. (1) into a Fokker-Planck equation linear in the probability density for the electric-field amplitude. The steady-state solution of this equation describing the fluctuations of the light intensity $I=|E|^2$ is

$$P_s(I)=\mathcal{N}\exp(\tfrac{1}{2}aI+\tfrac{1}{4}bI^2-\tfrac{1}{6}I^3), \tag{4}$$

where $\mathcal{N}$ is a normalization constant. This equation predicts bistability for $b>0$ in the pump parameter range $-b^2/4<a<0$ with a first-order-like phase transition at $a=-3b^2/16$ [4]. For $b<0$, only monostable behavior is predicted for all values of a with a second-order-like phase transition at $a=0$. For $b=0$, the leading nonlinearity is of fifth order in the field amplitude. This corresponds to the distribution for a pure sextic oscillator. The point $a=0=b$ is a special point at which the LSA exhibits the so-called tricritical behavior [10].

With the help of Eq. (4), the moments of the light intensity can be computed numerically as

$$\langle I^m\rangle=\int_0^\infty dI\, P_s(I)I^m. \tag{5}$$

From this equation we can calculate the mean $\langle I\rangle$, normalized variance $\kappa_2=\langle(\Delta I)^2\rangle/\langle I\rangle^2$, and skewness $\kappa_3=\langle(\Delta I)^3\rangle/\langle I\rangle^3$ of light intensity. The variation of the mean light intensity as a function of the pump parameter a shows that as the saturation parameter b increases, the change in the light intensity as a function of the pump parameter in the threshold region becomes more abrupt. In the limit of no fluctuations, the mean intensity changes discontinuously and approaches the behavior of the most probable intensity. Similarly, from the behavior of κ_2 and κ_3 as functions of pump parameter a for different positive values of b, we find that the intensity fluctuations are enhanced in the region of threshold before they decrease as the pump parameter increases. These predictions were tested in the experiments described in the next section.

III. EXPERIMENT

The experiments were conducted on a He:Ne laser operating at 633 nm. The apparatus shown in Fig. 1 is similar to that used in earlier experiments [10]. The laser contains a Ne absorber cell in addition to a He:Ne gain cell. The ratio of the pressures in the gain and absorption cells was 10:1 giving a value of approximately $s=9$ for the ratio of the saturation intensities for the gain and absorber atoms [10]. The laser assembly was enclosed in a temperature-stabilized housing with acoustic isolation. The laser frequency drift was found to be less than 5 MHz over a period of several minutes. A small fraction of the light coming out from the laser was directed to a photomultiplier tube (PMT), which was part of a feedback loop that could hold the operating point of the laser (mean intensity) constant to about 1%. The operating point of the laser was varied by changing the position of the knife edge. This allowed us to vary the loss and therefore the pump parameter a in a controlled manner.

The main beam was detected by a fast photomultiplier tube whose output after suitable amplification was fed to a multichannel analyzer (MCA). The MCA sampled the input voltage at regular intervals of about 50 μs. The value sampled was digitized and the resulting number was used as an address in a memory. If a number n was recorded, then the contents of memory location n were incremented by 1. In this way, a histogram was built up. Since the voltage input to the MCA is proportional to the light intensity incident on the PMT, the mean, variance, and skewness of n are related to those of the light intensity by

$$\langle n\rangle=\sum nN_n\Big/\sum N_n=K\langle I\rangle, \tag{6}$$

$$\langle(\Delta n)^2\rangle=\sum(n-\langle n\rangle)^2N_n\Big/\sum N_n=K^2\langle(\Delta I)^2\rangle, \tag{7}$$

$$\langle(\Delta n)^3\rangle=\sum(n-\langle n\rangle)^3N_n\Big/\sum N_n=K^3\langle(\Delta I)^3\rangle, \tag{8}$$

where N_n is the number of counts stored in channel n and K is a scale factor that relates channel number to light intensity. From Eqs. (6)–(8) we find that the relative variance $\kappa_2\equiv\langle(\Delta I)^2\rangle/\langle I\rangle^2=\langle(\Delta n)^2\rangle/\langle n\rangle^2$ and skewness $\kappa_3\equiv\langle(\Delta I)^3\rangle/\langle I\rangle^3=\langle(\Delta n)^3\rangle/\langle n\rangle^3$ are independent of the scale factor K.

To compare the measured moments as functions of the mean light intensity (or the pump parameter a) with the theoretical predictions, we need to know the scale factor K that relates channel number to the dimensionless intensity used in the theoretical calculations. This was done by plotting the measured κ_2 against $\log_{10}\langle n\rangle$ and comparing it with the theoretical plots of κ_2 versus $\log_{10}\langle I\rangle$ for several different values of b. The scale factor K then corresponds to a shift along the $\log_{10}\langle 1\rangle$ axis. This procedure also determines b. Once K and b are known, mean intensities can be converted into pump parameter values if desired. We chose to characterize the operating point of the LSA by its mean intensity. Measurements were carried out for several different values of the discharge current in the absorber cell which correspond to different values of b. The results of this procedure are shown in Figs. 2 and 3. Experimental uncertainties are indicated by the bars attached to each data point. The uncertainties were dominated by electronic noise at low intensities and by statistical errors at higher intensities. This limited our ability to

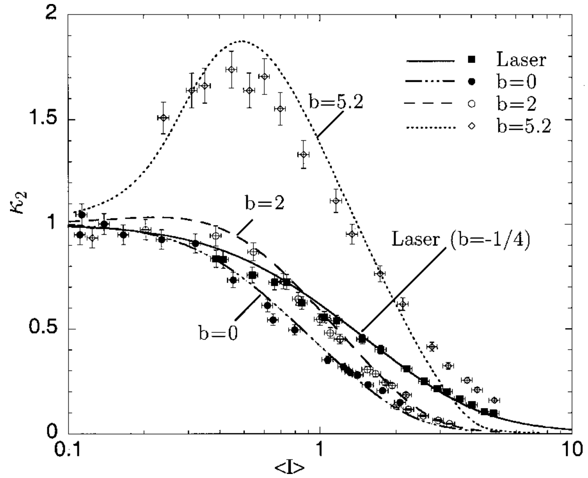

FIG. 2. Comparison of the experimentally measured and theoretically predicted normalized variance of light intensity for the LSA with $b=0$, 2, and 5.2 and the ordinary laser, formally to $b=-\frac{1}{4}$.

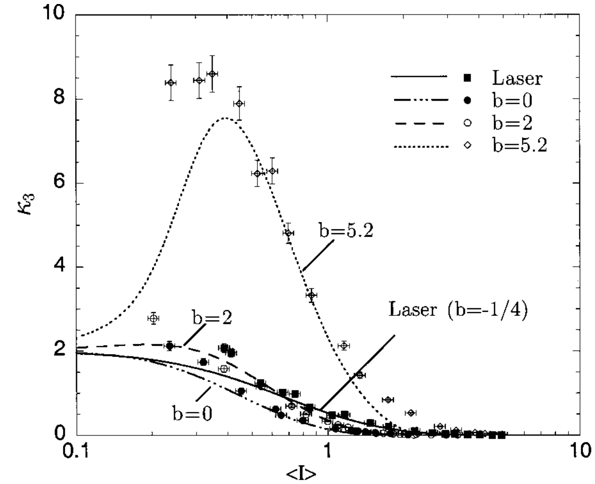

FIG. 3. Comparison of the experimentally measured and theoretically predicted normalized skewness of light intensity for the LSA with $b=0$, 2, and 5.2 and the ordinary laser, formally to $b=-\frac{1}{4}$.

explore operating points too far below threshold. The continuous curves have been computed using Eqs. (4) and (5). There is a reasonable agreement between the predictions of the theory and the experimental measurements.

The enhancement of fluctuations for $b>0$ in the region of threshold is due to rapid switching of the laser between the two steady states. For small values of b the steady states are not well developed and the switching is more frequent. When switching is more frequent, we can still use the feedback loop with a long time constant to hold the average operating point of the laser constant as long as the laser is intrinsically stable. For large values of b, the switching between the steady state is less frequent and causes large perturbations rendering the feedback loop ineffective. This, in part, is reflected in the relatively large deviations of the measured moments from the theoretical predictions for $b=5.2$.

As a check of our procedure for comparing the experiment and theory, we repeated the experiment by turning off the discharge in the absorber cell. The system then reduces to an ordinary laser. The measured values of κ_2 and κ_3 along with the theoretical predictions for the ordinary laser are also shown in Figs. 2 and 3. Once again, we find a good agreement between the experiment and theory. The ordinary laser is formally equivalent to the case $b=-\frac{1}{4}$, in which case the fifth-order nonlinearity in Eq. (1) can be ignored. Figures 2 and 3 show that for positive values of b, relative intensity fluctuations can become superthermal before decreasing to zero, whereas for negative values of b they stay subthermal and monotonically decrease to zero with increasing excitation.

IV. SUMMARY

We have investigated the transformation of fluctuations in a laser with a saturable absorber (LSA) in the region close to the tricritical point where a line of first-order phase transitions meets a line of second-order phase transitions [10]. Our measurements show that as the nature of stabilizing nonlinearity changes from the third order (ordinary laser) to fifth order (LSA) in the field amplitude, the transformation of laser-intensity fluctuations in passing the threshold undergoes a qualitative change. For the third-order nonlinearity, the relative intensity fluctuations decrease monotonically from thermal values to zero with increasing excitation, whereas for the fifth-order nonlinearity the relative intensity fluctuations are enhanced in the threshold region, becoming superthermal, before decreasing to zero as excitation increases. A nonlinear oscillator model that includes nonlinear terms up to fifth order in field amplitude adequately describes the transformation of fluctuations for the LSA in the entire threshold region near the tricritical point.

ACKNOWLEDGMENTS

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