

Semiconductor-cavity QED in high- Q regimes: Detuning effectYu-xi Liu,¹ N. Imoto,^{1,2,3} Ş. K. Özdemir,^{1,2} Guang-ri Jin,⁴ and C. P. Sun⁴¹*The Graduate University for Advanced Studies (SOKEN), Hayama, Kanagawa 240-0193, Japan*²*CREST Research Team for Interacting Carrier Electronics,
Hayama, Kanagawa 240-0193, Japan*³*NTT Basic Research Laboratories, 3-1 Morinosato Wakamiya, Atsugi, Kanagawa 243-0198, Japan*⁴*Institute of Theoretical Physics, The Chinese Academy of Sciences, P.O. Box 2735, Beijing 100080, China*

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The nonresonant interaction between the high-density excitons in a quantum well and a single-mode cavity field is investigated. An analytical expression for the physical spectrum of the excitons is obtained. The spectral properties of the excitons, which are initially prepared in the number states or the superposed states of the two different number states by the resonant femtosecond pulse pumping experiment, are studied. A numerical study of the physical spectrum is carried out and a discussion of the detuning effect is presented.

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I. INTRODUCTION

The optical properties of the semiconductor structures have been the subject of intensive experimental and theoretical investigations during recent years. Technical progress in the semiconductor crystal growth has made it possible to control semiconductor structures in all three spatial dimensions. Experimentally, physicists can fabricate the quantum wells, in which a thin semiconductor film is sandwiched between different materials via heterojunctions so that the motion of carriers is confined in the two-dimensional thin-film plane. This confinement of carriers to two dimensions gives rise to new quantum effect not observed in bulk materials, for example, an electric-field-induced energy shift of the resonance, which is called the quantum-confined Stark effect [1]. Multidimensional quantum-confined structures, such as quantum wires and quantum dots, are expected to further improve the quantum effect of the optical devices. Experiments have shown that the confinement of carriers in these low-dimensional semiconductor structures (LDSS's) can result in novel optical-electronic effects, which may lead to fabrication of new optical components. Some new fascinating achievements in the areas, such as semiconductor microcavity (SMC) quantum electrodynamics (QED), quantum-dot microlaser and turnstile device [2], quantum computer with quantum dot [3,4], and semiconductor random laser [5] etc., encourage the study of optical-electronic properties of these microstructures.

The present trend toward smaller-scale nanostructures and the continuous development of new and improved materials have started a steady progress towards fabrication of more ideal optical microcavities. If an LDSS is placed in an SMC, the optical mode structure of the SMC will change around the LDSS. Using this effect, many interesting phenomena, such as tailoring the spontaneous-radiation pattern and rate [8–10], the coupled exciton-photon mode splitting in a semiconductor quantum microcavity [11], have been demonstrated. Therefore, the investigation of optical properties of an LDSS placed in a semiconductor cavity is very important and necessary for theoretical and experimental physicists.

It is known that the interaction between the light and these microsystems occurs via the exciton [6,7] which is an electron-hole pair bound by the Coulomb attraction. The radiation of the exciton exhibits the super-radiant character. The initial theoretical [12] and experimental [13] studies were focused on the super-radiance of the Wannier excitons in semiconductor microcrystallites. Then, the super-radiance of the Frenkel excitons was observed in J aggregates at low temperature [14,15]. In 1995, the super-radiance of high-density Frenkel excitons in a R-Phycocerythrin (R-PE) single crystal was observed at room temperature for the first time [16]. We have studied the spontaneous radiation of the Frenkel excitons in a crystal slab under the condition of low excitation with an exactly solvable model and show its superradiance nature [17]. We have also discussed the quantum-statistical properties of the output field and the semiconductor QED for the high-density excitons in a semiconductor microcavity [18,19]. In the former works, our main interest was the resonant interaction between the excitons and the cavity field. But we know that the detuning between the cavity field and the exciton always affects the radiation properties of the exciton in the quantum well [20]. So in this paper we will discuss a general model of the interaction between the high-density excitons in a quantum well and a single-mode cavity field.

In Sec. II, we give a general theoretical model of the nonresonant interaction between a single-mode cavity field and the excitons. By virtue of Schwinger's representation of the angular momentum using two boson modes, we approximately obtain the analytical solution of the system. In Sec. III, the stationary physical spectrum of the excitons, which are initially in the number state or the superposed state of two different number states, is presented. A comparison of these results with those of the resonant cases will be also given. Finally, a brief summary and conclusion are presented in Sec. IV.

II. THEORETICAL MODEL AND ANALYTICAL SOLUTION

In this section, we present a theoretical model to study the interaction between a quantum well and a single-mode cavity

field. We assume that the cavity and the quantum well are ideal, and they are in an extremely low-temperature circumstance. The quantum well interacts with the cavity field via the exciton, which is an electron-hole pair bound by the Coulomb attraction. At extremely low temperatures, the thermal momentum of the excitons is so small that the thermalized excitons can be neglected. Then there are only excitons with zero in-plane momentum. It is well known that when the density of the excitons becomes high, the ideal bosonic model of the excitons is no longer adequate (in the case of a GaAs quantum well, the ideal bosonic model becomes inadequate when the density of the excitons exceeds $1.3 \times 10^9 \text{ cm}^{-2}$ [21]) for a theoretical study. However, we can describe exciton operators as hypothetical bosonic operators. In order to deal with the deviation of the exciton operators from the ideal bosonic operators, we introduce an effective nonlinear interaction between these hypothetical ideal bosons to correct the high excitons density effect [22,23]. These considerations lead to the following Hamiltonian after the rotating-wave approximation is made [19,22]:

$$H = H_0 + H' \quad (1)$$

with

$$H_0 = \hbar \omega_1 a^\dagger a + \hbar \omega_2 b^\dagger b + \hbar g (a^\dagger b + b^\dagger a), \quad (2a)$$

$$H' = \hbar A b^\dagger b^\dagger b b - \hbar \nu (a^\dagger b^\dagger b b + b^\dagger b^\dagger a b), \quad (2b)$$

where $b^\dagger(b)$ are the creation (annihilation) operators of the excitons with frequency ω_2 , and $a^\dagger(a)$ are the creation (annihilation) operators of the cavity field with frequency ω_1 . We assume that both of them obey the bosonic commutation relation $[b, b^\dagger] = [a, a^\dagger] = 1$. g stands for the interaction strength between the cavity field and the excitons. A represents an effective interaction constant between the excitons. A is assumed as a positive-real number, which means that the biexcitons are not stable in this system of the quantum well. So, it is reasonable to consider that only excitons are present in the quantum well. ν is a positive real number that describes the phase-space filling factor. The ratio of the phase-space filling factor to the interaction constant of the excitons is about $\nu/A = 0.3$ [24]. For the sake of simplicity, we assume that the parameters A and ν are of the same order, or $A^2 < \nu$. In that case, it is convenient to give the solution of Eq. (2a) using Schwinger's representation of the angular momentum for two-mode bosonic operators [25]. Angular-momentum operators can be constructed as

$$J_x = \frac{1}{2}(a^\dagger b + b^\dagger a), J_y = \frac{1}{2i}(a^\dagger b - b^\dagger a), \quad (3a)$$

$$J_z = \frac{1}{2}(a^\dagger a - b^\dagger b), \quad (3b)$$

and total number operator as

$$\hat{N} = a^\dagger a + b^\dagger b, \quad (3b)$$

using the ladder operators a^\dagger, a of the cavity field and the exciton operators b^\dagger, b . It is obvious that \hat{N} is the constant of motion with respect to the Hamiltonian (1). Then the total angular-momentum operator can be expressed as

$$J^2 = J_x^2 + J_y^2 + J_z^2 = \frac{\hat{N}}{2} \left(\frac{\hat{N}}{2} + 1 \right). \quad (4)$$

For any fixed total particle number \mathcal{N} , the common eigenstates of J^2 and J_z are

$$|jm\rangle = \frac{(a^\dagger)^{j+m}(b^\dagger)^{j-m}}{\sqrt{(j+m)!(j-m)!}} |0\rangle, \quad (5)$$

with the eigenvalues $j = \mathcal{N}/2$, and $m = \mathcal{N}/2, \dots, -\mathcal{N}/2$, where $|jm\rangle$ are the Fock states with $j+m$ photons in the cavity and $j-m$ excitons in the quantum well, respectively. Although $j \pm m$ must be integers, j and m can both be integers or both be half-odd integers. If we define

$$\Omega = \frac{1}{2}(\omega_1 + \omega_2), \quad \Delta = \frac{1}{2}(\omega_1 - \omega_2), \quad (6)$$

then Eq. (2a) can be simplified to

$$H_0 = \hbar \Omega \hat{N} + \hbar G (\sin \theta J_x + \cos \theta J_z) = \hbar \hat{N} + \hbar G e^{-i\theta J_y} J_z e^{i\theta J_y} \quad (7)$$

in terms of an SO(3) rotation $e^{i\theta J_y}$ of $\hbar \Omega \hat{N} + \hbar G (\sin \theta J_x + \cos \theta J_z)$ with $G = \sqrt{\Delta^2 + 4g^2}$ and $\tan \theta = 2g/\Delta$. The eigenfunctions and eigenvalues of Hamiltonian (7) are, respectively,

$$|\psi_{jm}^{(0)}\rangle = e^{-i\theta J_y} |jm\rangle, \quad E_{jm}^{(0)} = \hbar(\mathcal{N}\Omega + mG). \quad (8)$$

It is clear that eigenfunction $|\psi_{jm}^{(0)}\rangle$ represents a dressed exciton state or a polariton state. Based on Eq. (8), using perturbation theory and keeping A and ν parameters up to their first order, we obtain the eigenvalues of the Hamiltonian (1) as

$$E_{jm} = E_{jm}^{(0)} + \langle jm | e^{i\theta J_y} H' e^{-i\theta J_y} | jm \rangle, \quad (9a)$$

and

$$|\psi_{jk}\rangle = |\psi_{jk}^{(0)}\rangle + \sum_{n \neq k} \frac{\langle \psi_{jn}^{(0)} | H' | \psi_{jk}^{(0)} \rangle}{E_{jk}^{(0)} - E_{jn}^{(0)}} |\psi_{jn}^{(0)}\rangle. \quad (9b)$$

In order to obtain E_{jm} and $|\psi_{jk}\rangle$, we first calculate the matrix elements of the perturbation term as follows:

$$\begin{aligned}
\langle \psi_{jn}^{(0)} | H' | \psi_{jm}^{(0)} \rangle = & h \left[A \left(\cos \frac{\theta}{2} \right)^4 + \nu \sin \theta \left(\cos \frac{\theta}{2} \right)^2 \right] (j-m-1)(j-m) \delta_{n,m} + \hbar [A(\sin \theta)^2 - \nu \sin 2\theta] (j^2 - m^2) \delta_{n,m} \\
& + h \left[A \left(\cos \frac{\theta}{2} \right)^2 \sin \theta - \nu \cos \theta \left(\cos \frac{\theta}{2} \right)^2 + \frac{\nu}{2} (\sin \theta)^2 \right] (j-m) \sqrt{(j+m)(j-m+1)} \delta_{n,m-1} \\
& + \hbar \left[\frac{A}{4} (\sin \theta)^2 - \frac{\nu}{4} \sin 2\theta \right] \sqrt{(j+m)(j+m-1)} \sqrt{(j-m+1)(j-m+2)} \delta_{n,m-2} \\
& + \hbar \left[A \sin \theta \left(\cos \frac{\theta}{2} \right)^2 - \nu \cos \theta \left(\cos \frac{\theta}{2} \right)^2 + \frac{\nu}{2} (\sin \theta)^2 \right] \sqrt{(j-m)(j+m+1)} (j-m-1) \delta_{n,m+1} \\
& + \hbar \left[A \sin \theta \left(\sin \frac{\theta}{2} \right)^2 - \nu \cos \theta \left(\sin \frac{\theta}{2} \right)^2 - \frac{\nu}{2} (\sin \theta)^2 \right] \sqrt{(j+m)(j-m+1)} (j+m-1) \delta_{n,m-1} \\
& + \hbar \left[A \sin \theta \left(\sin \frac{\theta}{2} \right)^2 - \nu \cos \theta \left(\sin \frac{\theta}{2} \right)^2 - \frac{\nu}{2} (\sin \theta)^2 \right] \sqrt{(j-m)(j+m+1)} (j+m) \delta_{n,m+1} \\
& + \hbar \left[\frac{A}{4} (\sin \theta)^2 - \frac{\nu}{4} \sin 2\theta \right] \sqrt{(j+m+1)(j+m+2)} \sqrt{(j-m)(j-m-1)} \delta_{n,m+2} \\
& + \hbar \left[A \left(\sin \frac{\theta}{2} \right)^4 - \nu \sin \theta \left(\sin \frac{\theta}{2} \right)^2 \right] (j+m)(j+m-1) \delta_{n,m}, \tag{10}
\end{aligned}$$

and then write the eigenvalues of the Hamiltonian (1) as

$$\begin{aligned}
E_{jm} = & \hbar \Omega N + \hbar G m + h [A(\sin \theta)^2 - \nu \sin 2\theta] (j^2 - m^2) \\
& + \hbar \left[A \left(\cos \frac{\theta}{2} \right)^4 + \nu \sin \theta \left(\cos \frac{\theta}{2} \right)^2 \right] (j-m-1)(j-m) \\
& + \hbar \left[A \left(\sin \frac{\theta}{2} \right)^4 - \nu \sin \theta \left(\sin \frac{\theta}{2} \right)^2 \right] (j+m)(j+m-1). \tag{11}
\end{aligned}$$

All the eigenfunctions of the Hamiltonian (1) can be easily obtained using Eq. (9b) and Eqs. (8) and (10). Then the time evolution operators of the system can be written as

$$U(t) = e^{-(1/\hbar)Ht} = \sum_{j=0}^{\infty} \sum_{m=-j}^j e^{-it(E_{jm}/\hbar)} |\psi_{jm}\rangle \langle \psi_{jm}|. \tag{12}$$

Consequently, for any initial state $|\psi(0)\rangle$, the time-dependent wave function can be obtained as $|\psi(t)\rangle = U(t)|\psi(0)\rangle$.

III. RADIATION SPECTRUM OF EXCITONS

Under the condition of ideal cavity, ideal quantum well, and extremely low temperature, both the excitons and the cavity field have zero linewidth. The assumption of ideal quantum well is important to eliminate the linewidth of excitons that may be caused by the fluctuations of the quantum well. So the only broadening mechanism comes from the detecting spectrometer for which the physical spectrum can be defined as [26,27]

$$\begin{aligned}
S(\omega) = & 2\gamma \int_0^t dt_1 \int_0^t dt_2 \exp[-(\gamma - i\omega)(t - t_2)] \\
& \times \exp[-(\gamma + i\omega)(t - t_1)] G(t_1, t_2), \tag{13}
\end{aligned}$$

where γ is the half-bandwidth of the spectrometer, and t is the time length of the excitation in the cavity. $G(t_1, t_2)$ represents the dipole correlation function of the excitons and is defined as

$$G(t_1, t_2) = \langle \psi(0) | B^\dagger(t_2) B(t_1) | \psi(0) \rangle \tag{14}$$

with the initial state $|\psi(0)\rangle$ of the system, and $B^\dagger = b^\dagger - \nu b^\dagger b^\dagger b$ ($B = b - \nu b^\dagger b b$), where the second term comes from the correction of the phase-space filling effect [28]. Taking into account the fact that $b(t) = U^\dagger(t) b U(t)$, we can obtain the correlation $G(t_1, t_2)$ as

$$\begin{aligned}
G(t_1, t_2) = & \langle \psi(0) | U^\dagger(t_2) B^\dagger U(t_2) U^\dagger(t_1) B U(t_1) | \psi(0) \rangle \\
= & \sum_{j,k,l,m,n} \langle \psi_{jl} | B^{\dagger\dagger} | \psi_{km} \rangle \langle \psi_{km} | B | \psi_{jn} \rangle \langle \psi(0) | \psi_{jl} \rangle \\
& \times \langle \psi_{jn} | \psi(0) \rangle e^{i\omega_{jl,km} t_2} e^{-i\omega_{jn,km} t_1} \tag{15}
\end{aligned}$$

with $\omega_{jl,km} = (E_{jl} - E_{km})/\hbar$ and $\omega_{jn,km} = (E_{jn} - E_{km})/\hbar$. It is evident that j is determined only by the initial state $|\psi(0)\rangle$. Then the stationary physical spectrum can be written as

$$\begin{aligned}
S(\omega) = & \sum_{j,l,k,m} \frac{2\gamma}{\gamma^2 + (\omega - \omega_{jl,km})^2} \\
& \times |\langle \psi(0) | \psi_{jl} \rangle|^2 |\langle \psi_{jl} | B^\dagger | \psi_{km} \rangle|^2. \tag{16}
\end{aligned}$$

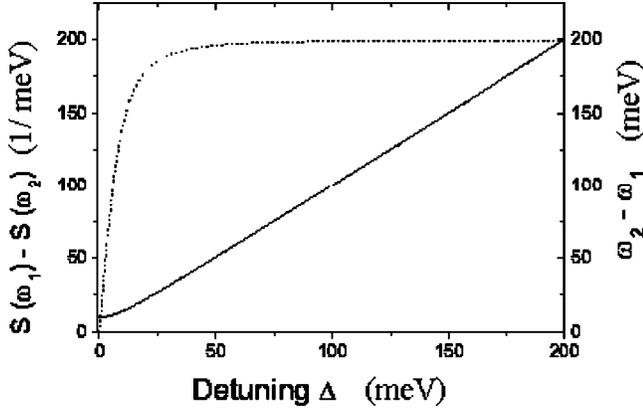


FIG. 1. 1000 times the difference between the heights of two peaks (dotted line), and the difference between positions (solid line) of two peaks are plotted as a function of the detuning Δ for $\gamma = 0.01$, $g = 6$ meV. $\omega_1 = \omega - \Omega = -G/2$ and $\omega_2 = \omega - \Omega = G/2$.

In order to fulfill the condition $\langle m' j' | B^\dagger | j m \rangle \neq 0$, both $j' = j + \frac{1}{2}$ and $m' = m - \frac{1}{2}$ must be satisfied. According to the above selection rule, Eq. (16) can be rewritten as

$$S(\omega) = \sum_{j,l,m} \frac{2\gamma}{\gamma^2 + (\omega - \omega_{jl, j-(1/2)m})^2} \times |\langle \psi(0) | \psi_{jl} \rangle|^2 |\langle \psi_{jl} | B^\dagger | \psi_{j-(1/2)m} \rangle|^2. \quad (17)$$

The eigenvalues determine the position of the spectral component and $|\langle \psi(0) | \psi_{jl} \rangle|^2 |\langle \psi_{jl} | B^\dagger | \psi_{j-(1/2)m} \rangle|^2$ determines the intensity of the spectral lines. The above spectral formula is similar to that of Ref. [19].

In the following, we will consider that the system is initially prepared in the bare-exciton state using the resonant femtosecond pulse pumping method [29]. For simplicity, we assume that $\hbar = 1$, so that all frequency quantities have a unit of energy. Moreover, we impose that initially there are no photons in the cavity. Then for a system, which is initially prepared in the single-exciton state, the physical spectrum from $\mathcal{N}=1$ to $\mathcal{N}=0$ can be simplified to

$$S(\omega) = \frac{2\gamma \left(\sin \frac{\theta}{2}\right)^4}{\gamma^2 + \left(\omega - \Omega - \frac{G}{2}\right)^2} + \frac{2\gamma \left(\cos \frac{\theta}{2}\right)^4}{\gamma^2 + \left(\omega - \Omega + \frac{G}{2}\right)^2}, \quad (18)$$

which has double peaks located at $\Omega + (G/2)$ and $\Omega - (G/2)$. It is clear that the phase-space filling factor and the interaction between the excitons, which are the results of the multiexciton presence, do not affect the physical spectrum of this system.

Figure 1 depicts the effect of the detuning Δ on the relative heights and the positions of the two peaks in the spectrum of the system described by Eq. (18). The difference between the heights of the two peaks in the physical spectrum is zero when $\Delta = 0$ (resonance case). This profile line is observed for two-level atomic systems, too. In the nonresonant case, peaks have different heights and the frequency

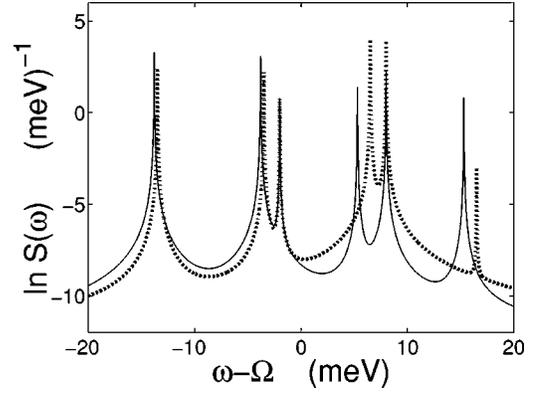


FIG. 2. $S(\omega)$ are plotted as a function of the frequency $\omega - \Omega$ for $g = 5$ meV, $A/g = 0.6$, $\gamma = 0.01$ meV when (a) $\nu = 0$ (dashed line), and (b) $\nu/A = 0.3$ (solid line).

difference between them become larger due to the detuning effect. With the increase of Δ , the height of the peak locating at $\Omega + (G/2)$ reduces gradually, whereas the height of the peak at $\Omega - (G/2)$ increases. From Eq. (18) and the condition $\tan \theta = 2g/\Delta$, it can be derived that if Δ and the coupling constant g satisfy the condition $\Delta \gg 2g$, then $\theta \approx 0$, which means the physical spectrum has approximately one peak. It is seen in Fig. 1 that when $\Delta \geq 10g$, the difference between the heights of the peaks becomes constant. A further analysis will reveal that at those values of Δ , the height of the peak located at $\Omega + (G/2)$ becomes very close to zero, therefore the spectrum can be interpreted as having a single peak.

With the increase of the pumping power, the emission spectrum of the system becomes complex. The simplest non-trivial example, for which the nonlinear terms play a role in the physical spectrum, is obtained when the quantum well has initially two excitons and there is no photon in the cavity. In that case, the physical spectrum from $\mathcal{N}=2$ to $\mathcal{N}=1$ becomes

$$S(\omega) = \sum_{l,m} \frac{2\gamma}{\gamma^2 + (\omega - \omega_{l,m})^2} |\langle \psi(0) | \psi_{1l} \rangle|^2 |\langle \psi_{1l} | B^\dagger | \psi_{1/2m} \rangle|^2 \quad (19)$$

with $\omega_{lm} = \omega_{1l, 1/2m}$. We know that there are three and two dressed exciton states for $\mathcal{N}=2$ and $\mathcal{N}=1$, respectively. According to the selection rule, there should be six peaks in the physical spectrum (19).

Figure 2, which is drawn using Eq. (19), depicts the radiation spectrum $S(\omega)$ of the excitons as a function of the frequency difference $\omega - \Omega$ under the resonant condition with or without considering the phase-space filling factor. In order to give a clear picture, the natural logarithm of the physical spectrum is plotted in Fig. 2 (in Figs. 3 and 6, as well). It is understood that the phase-filling factor does not affect the line profile of the spectrum, but changes the heights and the positions of the peaks. So in the following numerical results, we have neglected the phase-space filling effect.

Figure 3 clarifies that the detuning between the cavity field and the excitons changes not only the positions of the

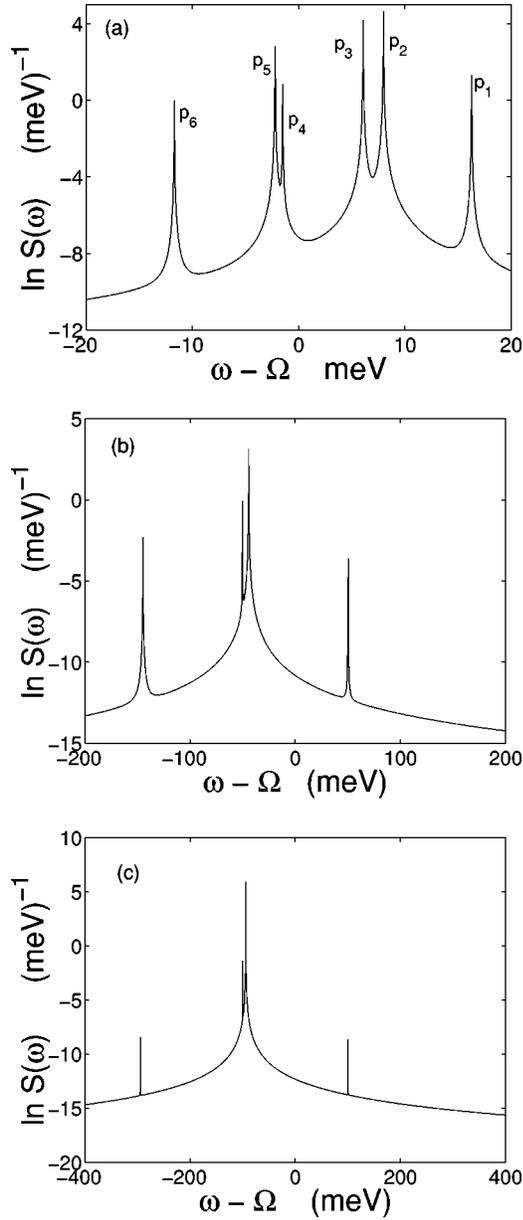


FIG. 3. $S(\omega)$ is plotted as a function of the frequency $\omega - \Omega$ for a set of parameters $g = 5$ meV, $A/g = 0.6$, $\gamma = 0.01$ meV (a), $\Delta = 2$ (b), $\Delta = 100$ (c), $\Delta = 200$ meV.

peaks, but also reduces their heights. The larger the detuning is, the stronger its effect on the physical spectrum is. As it is seen in Fig. 3, $\Delta = 2$ meV only slightly changes the position and heights of the peaks keeping their number constant. However, with increasing detuning up to $\Delta = 200$ meV, the number of the peaks in the spectrum gradually decreases because (i) the heights of some peaks become so small when compared to the main peak that they can be neglected, and (ii) some peaks are shifted so close to each other that they cannot be resolved.

The variation of the positions of the six peaks [labeled as p_1, p_2, \dots, p_6 in the spectrum of Fig. 3(a)] of Eq. (19) is plotted as a function of the detuning Δ in Fig. 4. It is seen that with the increase in the detuning, the difference between p_2 and p_3 becomes almost zero so that they cannot be re-

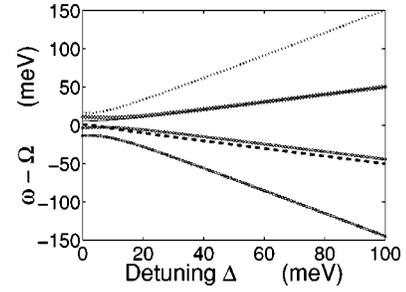


FIG. 4. Positions of peaks are plotted as a function of the detuning Δ for $\gamma = 0.01$ meV, $g = 5$ meV, $A/g = 0.6$. Curves, from top to bottom, correspond, respectively, to the positions of the peaks labeled $p_1, p_2, p_3, p_4, p_5,$ and p_6 in Fig. 3(a).

solved. Peaks p_1 and p_6 , respectively, shift to more positive and negative sides of the spectrum. The positions of p_4 and p_5 are interchanged with increasing Δ , and after this interchange their relative positions are kept the same.

It is also observed that when the half-bandwidth of the spectrometer γ and the coupling constant between the cavity field and the excitons are of the same order, the number of peaks in the physical spectrum (19) becomes two. This is because the spectrometer cannot resolve the position of some of the peaks. Figure 5 shows that under the conditions of the weak interaction between the excitons, the heights of the peaks in spectrum are equal if the cavity field resonates with the excitons. However, for the nonresonant case, the heights of the two peaks are not equal due to the detuning between the frequencies of the cavity field and that of the excitons even if there is weak interaction between the excitons.

Superposition of two different exciton states is yet another interesting case to investigate. If we consider the excitons are initially in the state $(1/\sqrt{2})(|1\rangle + |2\rangle)$ and the cavity field is in the vacuum state $|0\rangle$, then the physical spectrum can be written as

$$S(\omega) = \sum_{j=(1/2)}^1 \sum_{l,m} \frac{2\gamma}{\gamma^2 + (\omega - \omega_{jl, j-(1/2)m})^2} \times |\langle \psi(0) | \psi_{jl} \rangle|^2 |\langle \psi_{jl} | b^\dagger | \psi_{j-(1/2)m} \rangle|^2. \quad (20)$$

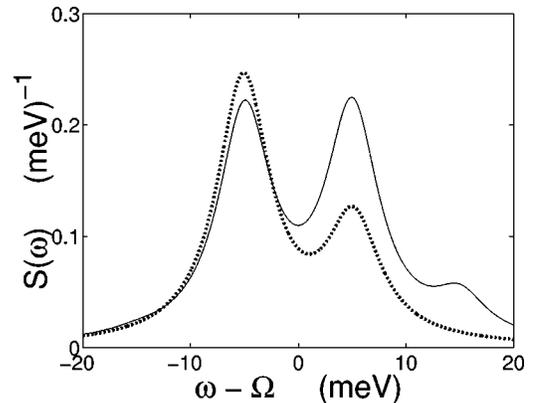


FIG. 5. $S(\omega)$ is plotted as a function of the frequency $\omega - \Omega$ for $g = 5$ meV, $A/g = 0.001$, $\gamma = 3$ meV when (a) $\Delta = 0$ (solid line), and (b) $\Delta = 2$ meV (dashed line).

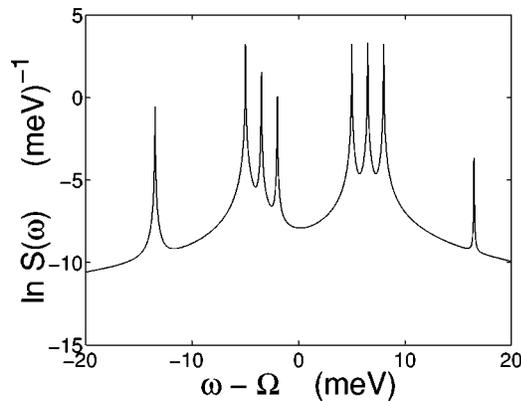


FIG. 6. $S(\omega)$ is plotted as a function of the frequency $\omega - \Omega$ for $g = 5$ meV, $A/g = 0.6$, and $\gamma = 0.01$ meV.

from which it can be found that there are eight peaks in the emission spectrum of the excitons. Figure 6 depicts the line profile of the physical spectrum $S(\omega)$ as a function of the frequency difference $\omega - \Omega$ under the resonant condition.

It is seen from Fig. 6 that the physical spectrum is a simple sum of one- and two-exciton spectra. At first sight, it seems that it is a normal and expected result. However, if the half-bandwidth of the spectrometer increases gradually the peaks of the physical spectrum given by Eq. (20) reduces from eight to one.

IV. CONCLUSIONS

In this study, the interaction between the high-density excitons in a quantum well and the cavity field is investigated under both resonant and nonresonant conditions. The model and the discussions presented in this paper are valid only when the excitons and cavity field have zero linewidth.

An analytical expression of the physical spectrum of the excitons is obtained. A discussion of the physical spectrum of the excitons, which are initially prepared in the number

state or the superposed state of the two different number states, is presented. It is observed that for a system having a single exciton state initially, the resonant interaction between the cavity field and the excitons gives the results similar to those of the two-level atomic system for which the two peaks in the physical spectrum have equal heights. However, non-resonant interaction results in the detuning which will change both the amplitudes of peaks and the frequency difference between them.

It is understood that the phase-space filling effect does not affect the line profile of the physical spectrum, but it adjusts the heights and positions of the physical spectrum. It is also shown that the number of the peaks in the physical spectrum is reduced with increase in either the detuning quantity or the half-bandwidth of the spectrometer. Under the conditions of low-exciton density and the resonant interaction between the cavity field and the excitons, if the half-bandwidth of the spectrometer has the same order as the coupling constant between the cavity field and excitons, then spectrum has two peaks of equal height. But in the case of higher exciton density or the nonresonant interaction, even if the half-bandwidth of the spectrometer has the same order as the coupling constant between the cavity field and excitons, the two peaks have different heights due to the interaction between the excitons or the detuning effect. When the system is initially in the superposed state of a single-exciton state and two-exciton state, the resultant physical spectrum is a simple sum of the spectra of the components of the superposition states.

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