Laser-induced Compton scattering at relativistically high radiation powers

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We reconsider nonlinear Compton scattering, induced by a linearly polarized laser field. The radiation power is assumed to be in the relativistic regime such that the ponderomotive energy of an electron in the field is of the order of magnitude of the electron's rest energy or even above it, i.e., $U_p \geqslant mc^2$. We investigate in detail for several scattering configurations the angular dependences of the induced nonlinear Compton processes of the order N of absorbed laser photons ω, assuming that in general the electron beam and laser beam can cross at an arbitrary angle.

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I. INTRODUCTION

Laser-induced Compton scattering was one of the first nonlinear processes that was intensively investigated immediately after the first lasers were brought into operation. Nice surveys of this early work and discussions of the various questions raised at that time can be found in the reviews of Eberly [1], Bunkin *et al.* [2], Mitter [3], and Neville and Rohrlich [4]. However, a long time before the laser was invented, Thomson and Compton scattering, induced by a classical electromagnetic background field, was discussed in considerable detail by Sen Gupta [5]. This early work was also reviewed by one of us [6], where we reported that the first theoretical investigations of stimulated nonlinear Thomson scattering can be found in papers that were written in the forming years of the development of quantum mechanics in the 1920s. In the early days of laser research, the attainable powers of laser radiation were so low that the nonlinear effects predicted were not accessible to observation. The experiment in which the second harmonic of nonlinear Compton scattering was observed is the one reported by Englert and Rinehart [7]. Therefore, one of the present authors reconsidered Compton scattering in an intense, linearly polarized laser field in a semirelativistic approximation and he envisaged also the process of laser-modified x-ray scattering in the relativistic regime [8]. At about the same time, classical Thomson scattering in a powerful, linearly polarized plane-wave radiation field was analyzed numerically to a considerable extent by Puntajer and Leubner [9], while this process was considered several years before in great detail for linear as well as circular laser polarization by Sarachik and Schappert [10]. With the advent of very powerful laser sources in recent years, the predicted energy and momentum shifts in a laser field became observable and were analyzed carefully by Meyerhofer and co-workers [11]. Their experimental results gave rise to renewed interest in the investigation of relativistic Thomson scattering in an extremely powerful laser field by Hartemann and Kerman [12], and in a series of papers by Salamin and Faisal [13]. The corresponding quantum-mechanical Compton process was reinvestigated by Narozhnyĭ and Fofanov [14], describing the powerful laser pulse by a circularly polarized electromagnetic plane-wave field. Finally, the old question why the quantum electrodynamic treatment of laser-induced processes, in which the laser dressing of the electron was treated by summing up Feynman diagrams, as in the work of Fried and Eberly [15], does not yield laser-induced energy and momentum shifts, while the classical description of the laser field does, has been reconsidered by Reiss and Eberly [16] and more recently by Körmendi and Farkas [17] who apparently find a reasonable explanation for this discrepancy.

It is the purpose of the present work to reanalyze laser-induced Compton scattering in a very powerful radiation field in which the ponderomotive energy U_p of the electron in the field is of the order of magnitude of the electron's rest mass $U_p \simeq mc^2$, or even larger. Since most calculations of this process, performed in the past, of which we became aware, were performed for a classical, circularly polarized electromagnetic plane-wave field and mainly analytic formulas were presented, it will be our aim to consider Compton scattering for a linearly polarized radiation field, in which case we expect a much richer and more complicated scattering spectrum. Moreover, we want to find out whether at the high-laser powers envisaged above, of about 10^{18} Wcm^{-2} and beyond, spin effects are of importance or, whether the treatment of nonlinear Compton scattering for a Klein-Gordon particle or a Dirac particle lead essentially to the same results. We shall consider various different scattering configurations in which the electrons are moving at high speed initially, which historically was not the usual assumption, and we shall permit the collision of the laser beam and the electron beam at an arbitrary angle and investigate the angular dependence of the cross sections of the scattered radiation of harmonics $N\omega$. Our calculations will lead in the present case of linear laser polarization to generalized Bessel functions of the form

$$B_N(x,y) = \sum_{\lambda=-\infty}^{+\infty} J_{N-2\lambda}(x) J_\lambda(y). \tag{1}$$

The evaluation of such functions has been analyzed some

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time ago in the complex plain by Leubner [18] who developed for that purpose generalized saddle-point methods and who wrote a program for the corresponding numerical calculations. His investigations were based on earlier work by Bleistein and Ursell [19,20] as well as by Nikishov and Ritus [21]. Similar work was performed much earlier by Reiss [22,23]. We started from the work of Leubner [18] to design a completely new $C++$ program for the evaluation of the generalized Bessel functions $B_N(x,y)$ for large values of the parameters x, y, and N and we found that in the parameter range 10^2-10^4 there is excellent agreement between the results of our computer program and the data obtained from Eq. (1). Therefore, we can expect that for parameter values larger than 10^4, the numerical results for the evaluation of B_N will also be correct, since in that case the saddle-point method should work even better.

In the next section we shall first derive the generalized Compton formula for scattering of a powerful laser beam by a Klein-Gordon particle of spin 0 and we shall then consider the same process for a Dirac particle of spin 1/2. In Sec. III we shall present and discuss our numerical results for various scattering geometries and combinations of parameter values. The final section will be devoted to a summary of our results and to some concluding remarks. We shall use units $\hbar=c=1$ throughout this work.

II. NONLINEAR COMPTON FORMULA

A. Scattering by a Klein-Gordon particle

The derivation of the scattering formula for a Klein-Gordon particle is relatively simple and straightforward. We start by considering the exact solution of the Klein-Gordon equation for a particle of mass m and charge e embedded in an electromagnetic plane wave of vector potential $\vec{A}(\tau)$ in the Coulomb gauge. Here, $\tau=t-\vec{n}\cdot\vec{r}$ and $\vec{n}$ is the direction of propagation of the plane-wave field. If the particle has initial energy E and momentum $\vec{p}$ and if adiabatic decoupling of the particle from the plane wave is assumed, the corresponding Gordon solution [24], normalized to the volume V, reads [25]

$$\psi_{\vec{p}}=\frac{1}{\sqrt{2EV}}\exp[-i(Et-\vec{p}\cdot\vec{r})]f(\tau) \quad (2)$$

with

$$f(\tau)=\exp\left\{ i\,\frac{\int_{-\infty}^{\tau}\left[e\vec{p}\cdot\vec{A}(\tau')-\frac{e^2}{2}\vec{A}^2(\tau')\right]d\tau'}{E-\vec{p}\cdot\vec{n}}\right\}. \quad (3)$$

Describing the powerful laser field by a monochromatic plane wave of amplitude A_0, linear polarization $\vec{\epsilon}$, frequency ω, and wave vector $\vec{k}=k\vec{n}$, represented by the vector potential

$$\vec{A}(\tau)=A_0\vec{\epsilon}\cos\omega\tau, \quad (4)$$

we obtain from Eqs. (2) and (3) the corresponding solution for the initial particle state

$$\psi_{\vec{p}}=\frac{1}{\sqrt{2EV}}\exp[-i(\bar{E}t-\bar{\vec{p}}\cdot\vec{r})]\exp[i(a\sin\omega\tau-b\sin 2\omega\tau)]. \quad (5)$$

Here we have introduced the abbreviations

$$\bar{E}=E+d,\quad \bar{\vec{p}}=\vec{p}+d\vec{n},\quad d=\frac{m^2\mu^2/4}{E-\vec{p}\cdot\vec{n}},$$

$$a=\frac{m(\mu/k)\vec{p}\cdot\vec{\epsilon}}{E-\vec{p}\cdot\vec{n}},\quad b=\frac{m^2\mu^2/8\omega}{E-\vec{p}\cdot\vec{n}},\quad \mu^2=\left(\frac{eA_0}{m}\right)^2. \quad (6)$$

The laser-dressed energy $\bar{E}$ and momentum $\bar{\vec{p}}$ fulfill the relation $\bar{E}^2=\bar{m}^2+\bar{\vec{p}}^2$ and correspond to an on-shell particle of effective mass $\bar{m}=m(1+\mu^2/2)^{1/2}$ [26]. The characteristic parameter, determining all the laser-induced nonlinear intensity effects, is given by $\mu^2=I/I_c$, where I is the average intensity of the laser field and $I_c=\alpha\omega^2/8\pi r_0^2$ ($\alpha=e^2$, $r_0=e^2/m$) represents the critical laser intensity at which $\mu^2=1$ and where the problem becomes relativistic [27], in which case the ponderomotive energy $U_p=m\mu^2/4$ is approaching the rest energy of the electron.

Writing down a corresponding wave function $\psi^*_{\vec{p}'}$ for the scattered particle of energy E' and momentum $\vec{p}'$ with appropriate coefficients a', b', and d' in Eqs. (5) and (6), we can evaluate in lowest order of perturbation theory the T-matrix element of nonlinear Compton scattering by a Klein-Gordon particle, viz.,

$$T_{fi}=-i\int d\vec{r}dt\,\psi^*_{\vec{p}'}H_{int}\psi_{\vec{p}}, \quad (7)$$

in which the interaction Hamiltonian H_{int} for a laser-dressed charged Boson of spin 0, interacting with a quantized electromagnetic field $\vec{A}'$ in the Coulomb gauge is given by [28]

$$H_{int}=ie[\vec{A}'(\tau')\cdot\vec{\nabla}-\overleftarrow{\nabla}\cdot\vec{A}'(\tau')]+2e^2\vec{A}'(\tau')\cdot\vec{A}(\tau), \quad (8)$$

where $\vec{A}(\tau)$ represents the laser field, defined in Eq. (4), and the effective vector potential $\vec{A}'(\tau')$ of the spontaneously emitted photon of frequency ω' and wave vector $\vec{k}'$ has to be evaluated from the quantized field operator $\vec{A}'$, by considering the matrix element

$$\langle 1_{\vec{k}'}|\vec{A}'|0_{\vec{k}'}\rangle=\sqrt{\frac{2\pi}{V\omega'}}\vec{\epsilon}'e^{i(\omega't-\vec{k}'\cdot\vec{r})}. \quad (9)$$

If we insert Eq. (9) into Eq. (8) and use the resulting expression in Eq. (7) together with the appropriate Gordon solutions Eq. (5), we obtain after Fourier decomposition of T_{fi} and integration over space and time,

$$T_{fi}=\sum_{N=-\infty}^{+\infty} T_N,$$

$$T_N=i(2\pi)^4\frac{em}{2V\sqrt{E'E}}\sqrt{\frac{2\pi}{V\omega'}}M_N\delta(\overline{E'}-\overline{E}+\omega'-N\omega)\,\delta^3(\overline{\vec{p}'}-\overline{\vec{p}}+\vec{k}'-N\vec{k}), \tag{10}$$

in which the matrix elements M_N are given by

$$M_N=\frac{(\overline{\vec{p}'}+\overline{\vec{p}})\cdot\vec{\epsilon}'}{m}B_N(x,y)-\left[\frac{\omega}{2m}(a'+a)(\vec{n}\cdot\vec{\epsilon}')+\mu(\vec{\epsilon}'\cdot\vec{\epsilon})\right][B_{N+1}(x,y)+B_{N-1}(x,y)]+\frac{\omega}{m}(b'+b)\times(\vec{n}\cdot\vec{\epsilon}')[B_{N+2}(x,y)+B_{N-2}(x,y)] \tag{11}$$

with the arguments x and y defined by

$$x=a'-a,\quad y=b-b', \tag{12}$$

where a,b and, similarly, a',b' are defined in Eq. (6).

The transition probability per unit space-time volume for nonlinear Compton scattering of a Klein-Gordon particle with the absorption of N laser photons ω, can be evaluated by standard methods. We find

$$w_N=\int\frac{|T_N|^2}{V\mathcal{T}}\frac{V}{(2\pi)^3}d\vec{k}'\frac{V}{(2\pi)^3}d\overline{\vec{p}'}=\frac{e^2m^2}{8\pi VE}\int\frac{|M_N|^2}{E'\omega'}d\Omega_{\vec{k}'}\omega'^2d\omega'\,\delta(\overline{E'}-\overline{E}+\omega'-N\omega)d\overline{\vec{p}'}\,\delta^3(\overline{\vec{p}'}-\overline{\vec{p}}+\vec{k}'-N\vec{k}). \tag{13}$$

In order to obtain the nonlinear differential cross sections $d\sigma_N/d\Omega_{\vec{k}'}$, for the general case where the electron beam and laser beam cross each other under an arbitrary angle, we have to divide the expression Eq. (13) by the relative average flux $j^{\mu}_{el}(j_{ph})_{\mu}$ of the ingoing electrons and photons. This expression is found by adapting the Lorentz invariant form of the relative particle flux of two-particle collisions, if one of the particles is massless [27,28]. The fluxes of electrons and photons we can obtain from

$$\vec{j}_{el}=\frac{\vec{p}}{EV}=\frac{m}{EV}\vec{\beta},\quad \vec{j}_{ph}=\frac{1}{\omega}\langle\vec{S}\rangle=\frac{A_0^2\omega}{8\pi}\vec{n}, \tag{14}$$

and therefore the relative flux is given by

$$j_{rel}=\frac{A_0^2\omega m}{8\pi EV}(1-\vec{n}\cdot\vec{\beta}). \tag{15}$$

Consequently, the nonlinear cross sections of the order N of laser-induced Compton scattering by a Klein-Gordon particle will read

$$\frac{d\sigma_N}{d\Omega_{\vec{k}'}}=r_0^2\left(\frac{\omega'}{\omega}\right)\left(\frac{m}{E'}\right)\frac{|M_N|^2}{\mu^2(1-\vec{n}\cdot\vec{\beta})}, \tag{16}$$

where $r_0=e^2/m$ is the classical electron radius. Next, we consider the energy and momentum conservation relations, expressed by the δ functions in Eq. (13). These can be squared and subtracted to yield

$$\omega'(\overline{E'}-\overline{\vec{p}'}\cdot\vec{n}')=N\omega(\overline{E}-\overline{\vec{p}}\cdot\vec{n}), \tag{17}$$

and by substituting on the left-hand side of Eq. (17) $\overline{E'}$ and $\overline{\vec{p}'}$ again from the energy and momentum conservation relations in Eq. (13), we find for the frequencies of scattered radiation the generalized Compton formula

$$\omega'=\frac{N\omega(E-\vec{p}\cdot\vec{n})}{E-\vec{p}\cdot\vec{n}'+(N\omega+d)(1-\vec{n}\cdot\vec{n}')}. \tag{18}$$

If the electron is initially at rest, E has to be replaced by m and $\vec{p}=0$, while d reduces to $U_p=m\mu^2/4$. In that case, the Compton formula Eq. (18) reads

$$\omega'=\frac{N\omega}{1+\left(\frac{N\omega}{m}+\frac{U_p}{m}\right)(1-\vec{n}\cdot\vec{n}')}. \tag{19}$$

At very high laser powers with $\mu\simeq 1$ and for a large number N of absorbed laser photons, the frequencies ω' of the scattered photons will strongly depend on the quantum-mechanical recoil effect, determined in the denominator of Eq. (19) by the factor $N\hbar\omega/mc^2$, as well as on the classical laser-induced drift motion of the electron, yielding in the denominator of that frequency formula the contribution U_p/mc^2.

B. Scattering by a Dirac particle

The treatment of laser-induced Compton scattering by a Dirac particle of spin 1/2 can follow similar lines, as in our discussion in the previous subsection for a particle of spin 0. We start from the Dirac equation for an electron moving in an arbitrary electromagnetic plane-wave field

$$(i\gamma^{\mu}\partial_{\mu}-e\gamma^{\mu}A_{\mu}-m)\psi(x)=0, \tag{20}$$

where the vector potential A_{μ} has the general form

$$A_{\mu}=A_{\mu}(k\cdot x),\quad A\cdot k=k\cdot k=0. \tag{21}$$

We use the Einstein summation convention and notation, namely $v^{\mu}w_{\mu}=v\cdot w$, where $\mu=0,1,2,3$. The solution of the above equation, Eq. (20), was derived by Volkov [29] and its explicit form can be found in the paper by Denisov and Fedorov [30]. Assuming adiabatic decoupling along the light cone in the past between the particle and the field we obtain

$$\psi(x)=[1+\kappa\gamma^{\mu}k_{\mu}\gamma^{\nu}A_{\nu}(k\cdot x)]\exp\left[-ip\cdot x-i\int_{-\infty}^{k\cdot x}S(\phi)\mathrm{d}\phi\right]u_{p}, \tag{22}$$

where u_p is a free particle solution of the equation $(\gamma^{\mu}p_{\mu}-m)u_p=0$ with p_{μ} being a four-vector of energy and momentum. The constant κ and the function $S(\phi)$ can be evaluated and we find

$$S(\phi)=\frac{eA(\phi)\cdot p}{p\cdot k}+\frac{e^2A^2(\phi)}{2p\cdot k},\qquad \kappa=\frac{e}{2p\cdot k}. \tag{23}$$

After normalization to the volume V, the required Volkov solution for an electron of initial four-momentum p_{μ} reads

$$\psi_p(x)=\sqrt{\frac{m}{VE_p}}\left[1-\frac{e\gamma^{\mu}A_{\mu}(k\cdot x)\gamma^{\nu}k_{\nu}}{2k\cdot p}\right]\exp\left[-ip\cdot x-i\int_{-\infty}^{k\cdot x}\left(\frac{eA(\phi)\cdot p}{p\cdot k}+\frac{e^2A^2(\phi)}{2p\cdot k}\right)\mathrm{d}\phi\right]u_p. \tag{24}$$

A similar solution, $\psi_{p'}(x)$, can be written down for the scattered electron of four-momentum p'_{μ}.

The T-matrix element of laser-induced Compton scattering by a Dirac particle can be easily evaluated, since the interaction Hamiltonian of this process is determined by [28]

$$H_{int}=e\overline{\psi}_{p'}(x)\gamma^{\lambda}\psi_p(x)A'_{\lambda}(x). \tag{25}$$

Therefore, by choosing the appropriate Volkov solutions Eq. (24) for the ingoing and the corresponding outgoing electron in Eq. (25) and by using the effective vector potential $A'_{\lambda}(x)$ for the scattered photon, given by Eq. (9) and taken in its covariant form, we obtain

$$\begin{aligned}T_{fi}=&-ie\frac{m}{V\sqrt{E_pE_{p'}}}\sqrt{\frac{2\pi}{V\omega'}}\int \mathrm{d}^4x\overline{u}_{p'}\left[1+\frac{e\gamma^{\sigma}A_{\sigma}(k\cdot x)\gamma^{\tau}k_{\tau}}{2k\cdot p'}\right]\gamma^{\lambda}\epsilon'_{\lambda}\left[1-\frac{e\gamma^{\mu}A_{\mu}(k\cdot x)\gamma^{\nu}k_{\nu}}{2k\cdot p}\right]u_p\exp[i(p'-p\\&+k')\cdot x]\exp\left[ie\left(\frac{p'}{p'\cdot k}-\frac{p}{p\cdot k}\right)\int^{k\cdot x}A(\phi)\mathrm{d}\phi\right]\exp\left[ie^2\left(\frac{1}{2p'\cdot k}-\frac{1}{2p\cdot k}\right)\int^{k\cdot x}A^2(\phi)\mathrm{d}\phi\right].\end{aligned} \tag{26}$$

By inserting the vector potential Eq. (4) of the laser field, used in its covariant version, we can easily perform the Fourier decomposition of the matrix element Eq. (26) and we obtain, as in the previous case, $T_{fi}=\Sigma_{N=-\infty}^{+\infty}T_N$ where the transition matrix elements T_N of the different nonlinear processes of the order N have exactly the same structure as for Compton scattering by a Klein-Gordon particle, shown in Eq. (10), except that the matrix elements M_N are different for the present process. The same conclusion also holds for the differential cross-section formula $d\sigma_N/d\Omega_{\vec{k}'}$ in Eq. (16) and for the frequencies ω' of the scattered radiation in Eq. (18). The above matrix elements M_N are found to be in the present case

$$\begin{aligned}M_N=&2(\mathcal{A}-\mathcal{C})B_N(x,y)+2\mathcal{B}[B_{N+1}(x,y)+B_{N-1}(x,y)]\\&-\mathcal{C}[B_{N+2}(x,y)+B_{N-2}(x,y)],\end{aligned} \tag{27}$$

where the coefficients $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ have the explicit form

$$\mathcal{A}=\overline{u}_{p'}\gamma^{\lambda}\epsilon'_{\lambda}u_p,$$

$$\mathcal{B}=\frac{eA_0}{4}\overline{u}_{p'}\left[\frac{\gamma^{\sigma}\epsilon_{\sigma}\gamma^{\tau}n_{\tau}\gamma^{\lambda}\epsilon'_{\lambda}}{p'\cdot n}-\frac{\gamma^{\lambda}\epsilon'_{\lambda}\gamma^{\mu}\epsilon_{\mu}\gamma^{\nu}n_{\nu}}{p\cdot n}\right]u_p, \tag{28}$$

$$\mathcal{C}=\frac{e^2A_0^2}{8(p'\cdot n)(p\cdot n)}\overline{u}_{p'}(\gamma^{\sigma}\epsilon_{\sigma}\gamma^{\tau}n_{\tau}\gamma^{\lambda}\epsilon'_{\lambda}\gamma^{\mu}\epsilon_{\mu}\gamma^{\nu}n_{\nu})u_p,$$

and the $B_N(x,y)$ are the generalized Bessel functions, defined in Eq. (1), with the arguments x and y given in Eq. (12). In writing down these expressions, we introduced the notation $k_{\mu}=\omega n_{\mu}$ where n_{μ} is a unit-four vector. The evaluation of the matrix elements $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ in Eq. (28) will not be done here analytically since these coefficients can be calculated by a computer. In evaluating the differential cross sections of nonlinear Compton scattering for a Dirac particle, we have to be aware of the two possible spin polarizations $s=\pm 1/2$ of the electron in its rest frame with respect to the z axis, taken before and after the scattering, leading to cross sections with and without spin flip during the nonlinear process. We shall therefore denote the cross sections of these processes by $d\sigma_N^{(s,s')}/d\Omega_{\vec{k}'}$ where in an abbreviated notation $(s=+,s'=+)$ and $(s=-,s'=-)$ will refer to the cross sections with no spin flip and, correspondingly, $(s=+,s'=-)$ and $(s=-,s'=+)$ will denote the cross sections with spin flip.

III. NUMERICAL EXAMPLES

For the evaluation of our numerical examples, we shall consider the scattering geometry depicted in Fig. 1. The laser beam and the electron beam are counterpropagating, as indicated by the arrows denoted by ω for the laser beam and by e^- for the electron beam. The scattered radiation of frequency ω', wave vector $\vec{k}'$ and linear polarization $\vec{\epsilon}'$ is emitted with an angle θ' and the linear polarizations $\vec{\epsilon}$ and $\vec{\epsilon}'$ are oriented in the scattering plane. The scattering angle θ', defined as the angle between the wave vectors $\vec{k}$ and $\vec{k}'$, can vary between 0° and 360°.

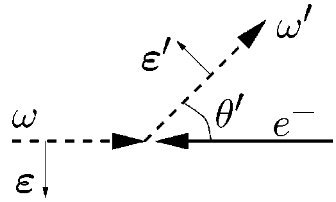

FIG. 1. Presents the scattering geometry considered. The wavevector $\vec{k}$ of the laser beam, the momentum $\vec{p}$ of the counter-propagating electrons, and the wave vector $\vec{k}'$ of the emitted photons define the scattering plane. The polarization vectors $\vec{\epsilon}$ of the laser radiation and $\vec{\epsilon}'$ of the emitted field are located in this plane.

In Fig. 2 we show the nonlinear differential cross-sections $d\sigma_N/d\Omega_{\vec{k}'}$ as a function of the scattering angle θ', evaluated from Eqs. (11) and (16) for a Klein-Gordon particle. The data turn out to be identical to those calculated for a Dirac particle, using for M_N the expression, Eqs. (27) and (28), if spin flip is not taken into account. The data are expressed in units of r_0^2 where $r_0=2.82\times10^{-11}$ m is the classical electron radius. We took for the laser frequency $\omega=1.54$ eV (as one of the frequencies of a Ti-sapphire laser) and the intensity of the laser beam is $I=10^{22}$ W cm^{-2}. The data presented in panels (a)–(f) correspond, respectively, to the initial kinetic electron energies $E-mc^2$ equal to 10, 10^3, 10^5, 10^6, 10^7, and finally 10^8 eV. The number of absorbed laser photons is for all panels $N=10^3$. The numbers along the circumferences of the circles in panels (a)–(f) indicate the scattering angles θ' in degrees. By inspecting these data, we observe that for such intense laser fields, the nonlinear Compton radiation is predominantly emitted into the forward direction with respect to the direction of propagation $\vec{n}=\vec{k}/k$ of the laser beam, as is also known to be the case for the generation of harmonics. This is even true for electron kinetic energies of the order of magnitude mc^2, as shown in panel (d). However, if the electron kinetic energy is increased still further, the scattering pattern becomes reversed so that for highly relativistic electron energies of the order of magnitude 100 MeV the Compton radiation will be emitted into the backward direction with respect to the direction of propagation $\vec{n}$ of the laser beam. Moreover, we find that with increasing electron energy, the maximum values of the differential cross sections drop down significantly as indicated by the numbers inside the circles.

In Fig. 3, the same parameter values are used in panels (a)–(f) as in Fig. 2, but instead of presenting the cross sections, we now consider the corresponding efficiency of the frequency conversion as a function of θ', measured by the ratio $\omega'/N\omega$. As we can see, for the lower electron kinetic energies this efficiency has its largest values in the forward direction $\vec{n}$ and its maximum value does not exceed 1. On the other hand, for the higher electron energies, the efficiency is large in the backward direction and its maximum values become larger than 1. This finding could suggest that in order to generate radiation of frequencies as high as possible, we should use electron beams of very high energy. But this is not true, since the power of the emitted high-frequency radiation, being proportional to the product of the conversion efficiency and the cross sections, drops down very rapidly with increasing electron energy and therefore the most favorable case turns out to be the one in which the electrons are at rest, initially. Consequently, we shall consider in our further

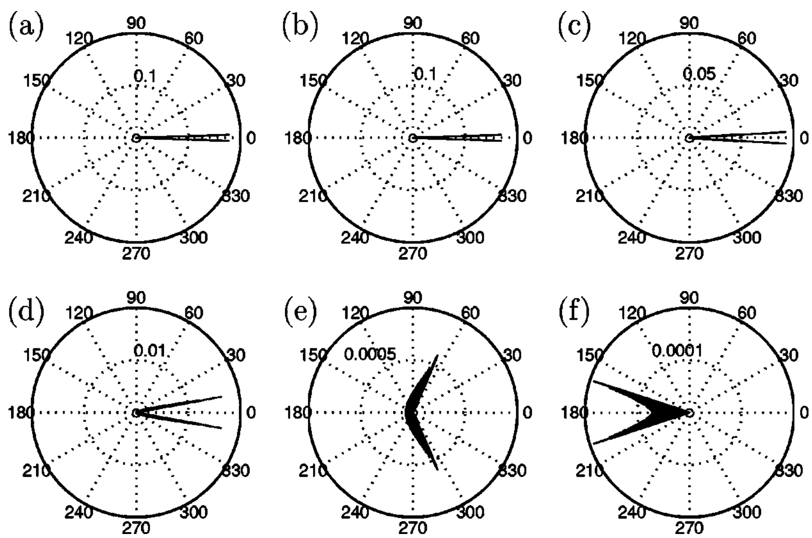

FIG. 2. Shows the differential cross sections $d\sigma_N/d\Omega_{\vec{k}'}$ evaluated for a Klein-Gordon particle and the identical cross sections $d\sigma_N^{(s,s')}/d\Omega_{\vec{k}'}$ for a Dirac particle with no spin flip, i.e., either $(s=+,s'=+)$ or $(s=-,s'=-)$. The data are measured in units of r_0^2, where $r_0=2.82\times10^{-11}$ m is the classical electron radius. The scattering angle θ', defined in Fig. 1, is measured in degrees, as indicated by the numbers along the circumferences of the circles in panels (a)–(f). The laser frequency is $\omega=1.54$ eV, the radiation intensity $I=10^{22}$ W cm^{-2}, and the number of absorbed photons $N=10^3$. In panels (a)–(f) we consider increasing electron kinetic energies, namely $E-mc^2=10$, 10^3, 10^5, 10^6, 10^7, and 10^8 eV, respectively. Evidently, for most of the electron energies considered, the nonlinear Compton radiation is predominantly emitted into the forward direction with respect to $\vec{k}$ (or $\theta'\simeq0$) and only for the highest electron energy of 100 MeV in panel (f) is the radiation emitted mainly into the backward direction in the vicinity of $\theta'=180°$.

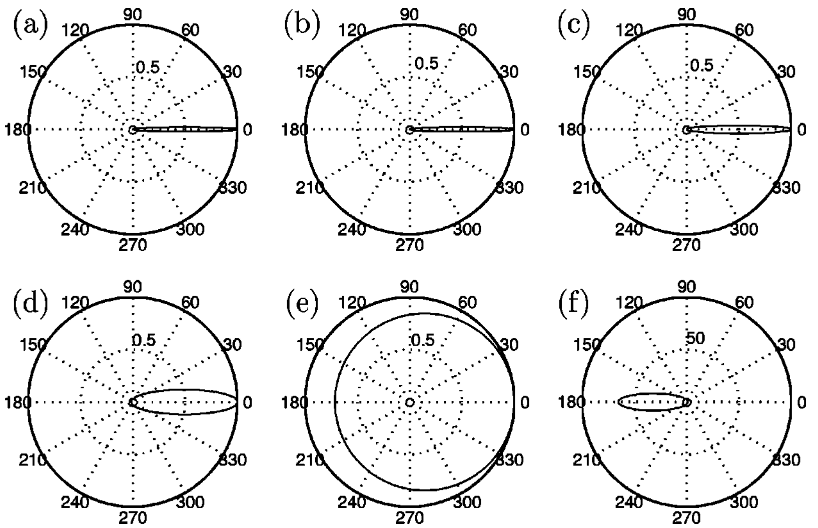

FIG. 3. Presents for the same parameter values as in Fig. 2 in panels (a)–(f) the efficiency of the frequency conversion, measured by the ratio $\omega'/N\omega$ as a function of the scattering angle θ' in degrees. This efficiency is largest in the forward direction near $\theta'=0$ for the lower electron energies in panels (a)–(d) and does not exceed the value 1, as indicated by the numbers inside the circles. For the very large electron energy in panel (f), the effect is strongest in the backward direction near $\theta'=180°$ and the maximum efficiency is becoming larger than 1.

analysis of the nonlinear Compton effect a beam of slow electrons.

In Fig. 4 are shown the differential cross sections, in the same units as before (evaluated equivalently for a Klein-Gordon or a Dirac particle without spin flip), in the upper row and the corresponding conversion efficiencies in the lower row for the parameters $N=10^3$, $\omega=1.54$ eV, and $E-mc^2=10^3$ eV. For the data in frames (a) and (b), the laser intensity is $I=10^{20}$ W cm^{-2}, for (c) and (d) we have $I=10^{21}$ W cm^{-2}, and for (e) and (f), $I=10^{22}$ W cm^{-2}. As the data show, the maximum values of the cross sections increase with increasing laser intensity but in such a way that the conversion efficiency remains roughly the same and is equal to 0.6. Let us notice, however, that during the first stage of panels [(a),(b)→(c),(d)], in which the laser intensity has increased by one order of magnitude, the cross sections have increased by almost three orders of magnitude. This is not the case in the second stage [(c),(d)→(e),(f)] where the cross sections have increased rather marginally. This indicates that in the Compton process we observe in the beginning a very rapid increase of the cross sections, which is followed by a "saturation" of the magnitude of the cross-section data, i.e., they do not increase so strongly any more with a further increase of the laser intensity as before. This suggests that there should be a most favorable choice of laser intensity below which the cross sections change significantly with increasing intensity, but above of which this change is becoming marginal. In our opinion, this optimum intensity, that has the approximate value $I\simeq 10^{21}$ W cm^{-2} for the parameter values of Fig. 3, will be the best choice for generating high-frequency radiation by means of the nonlinear Compton process (at least from the economic point of view).

In Fig. 5, we present the cross sections of Compton scattering for a Dirac particle as a function of the nonlinear order N for the large scattering angle $\theta'=178°$, the kinetic electron energy $E-mc^2=10^7$ eV, the laser frequency ω

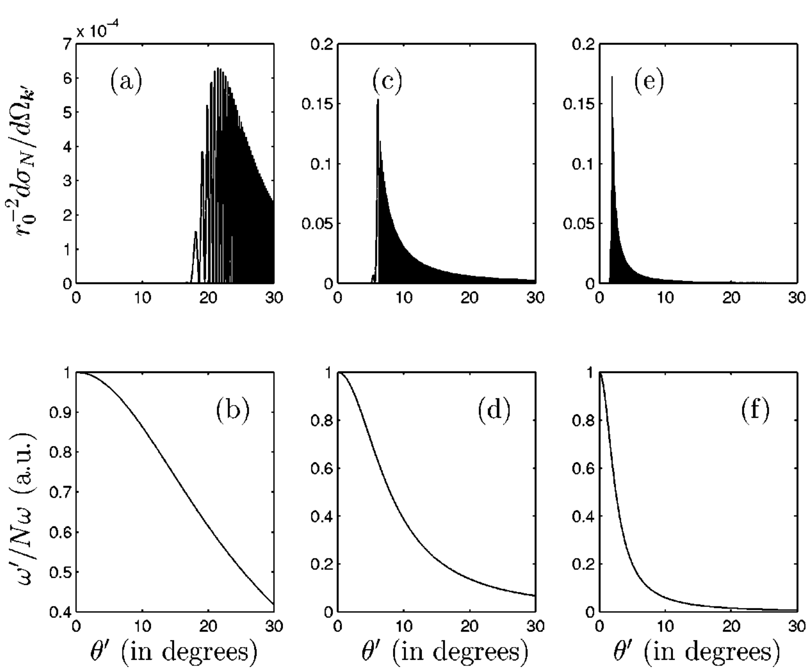

FIG. 4. Shows in the upper row the differential cross sections in units of r_0^2 as a function of the scattering angle θ' in degrees. (The data are identical for a Klein-Gordon and a Dirac particle with no spin flip.) In the lower row are presented the corresponding conversion efficiencies in atomic units (a.u.). The parameters are $\omega=1.54$ eV, $E-mc^2=10^3$ eV, and $N=10^3$. Three different laser intensities are chosen: $I=10^{20}$ W cm^{-2} in panels (a) and (b), $I=10^{21}$ W cm^{-2} in panels (c) and (d), and $I=10^{22}$ W cm^{-2} in panels (e) and (f). The maximum value of the cross sections increases with increasing intensity but apparently saturates at a laser intensity of about 10^{21} W cm^{-2}.

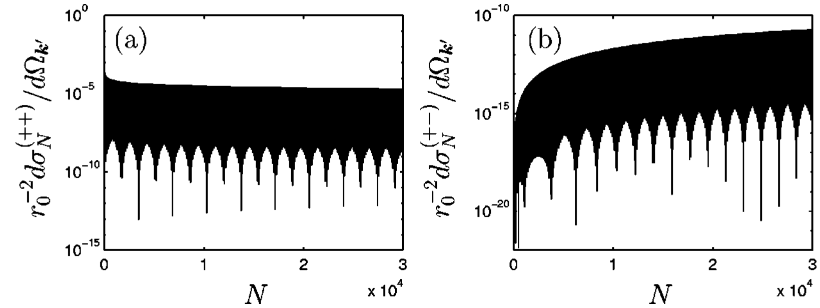

FIG. 5. Depicts the Compton cross sections in units of r_0^2 for a Dirac particle as a function of the nonlinear order N. The scattering angle is taken as $\theta' = 178°$, the electron kinetic energy is $E - mc^2 = 10^7$ eV, the laser frequency $\omega = 1.54$ eV, and the laser intensity $I = 10^{22}$ W cm^{-2}. For all data presented, we find that $d\sigma_N^{(+,+)}/d\Omega_{\vec{k}'} = d\sigma_N^{(-,-)}/d\Omega_{\vec{k}'}$ and, similarly, $d\sigma_N^{(+,-)}/d\Omega_{\vec{k}'} = d\sigma_N^{(-,+)}/d\Omega_{\vec{k}'}$. For much smaller laser intensities, the cross sections drop down to zero very rapidly, whereas for higher intensities, the spectrum of cross sections extends to very large N. Moreover, the cross-section values for no spin flip dominate by far over those for spin flip by about a factor of 10^7 and the data for no spin flip are identical to those for a Klein-Gordon particle.

$= 1.54$ eV, and the laser power $I = 10^{22}$ W cm^{-2}. All data have the property that for no spin flip, we find $d\sigma_N^{(+,+)} = d\sigma_N^{(-,-)}$, as shown in panel (a) and that for spin-flip $d\sigma_N^{(+,-)} = d\sigma_N^{(-,+)}$, depicted in panel (b). Moreover, the data for no spin flip are evidently many orders of magnitude larger than those for spin flip. For much smaller intensities than $I = 10^{22}$ W cm^{-2}, the cross-section data drop down to zero very rapidly. However, for intensities $I \gtrsim 10^{22}$ W cm^{-2}, the scattering spectrum extends to very large values of nonlinearity N and shows rapid oscillations. Nevertheless, the spin nonflip process dominates over the spin-flip effects. (We should remember that the projection of the spin is defined in the rest frame of the electron and not as the projection of the spin operator on the direction of propagation of the electron.) The above oscillations of the cross-section data are general features that we have found for all parameter values considered, stemming from the properties of the generalized Bessel functions, Eq. (1). Moreover, it appears that the cross sections for a Klein-Gordon particle are almost identical to those for a Dirac particle in the no spin-flip case. This demonstrates that even for such huge laser field intensities the spin effects are only marginally present and, therefore, from the practical point of view, the dynamics of the scattering process is very well described by the solutions of the Klein-Gordon equation. The only cases where the no spin-flip and the spin-flip processes for a Dirac-particle yield comparable cross sections occur for such scattering geometries for which the cross-section data for a Klein-Gordon particle vanish. This happens, for instance, in cases where the polarization $\vec{\epsilon}'$ of the scattered radiation is perpendicular to the scattering plane considered in Fig. 1. In this case, however, the scattering cross sections are very small, namely, by at least four orders of magnitude smaller than the cross sections evaluated for a Klein-Gordon particle, using the scattering geometry of Fig. 1.

Finally, in Fig. 6, we consider the same configuration as in panel (a) of Fig. 5 but for a much smaller range of N in which case we recognize, by using a larger scale along the abscissa, that the cross-section data are very rapidly oscillating in magnitude. This has its origin in the rapid oscillations of the values of the generalized Bessel functions in their dependence on the arguments x and y, defined in Eqs. (1) and (12), as they determine the values of the matrix elements in Eqs. (11) and (27).

IV. SUMMARY AND CONCLUSIONS

In the present work, we considered nonlinear Compton scattering for a very powerful laser field such that the ponderomotive energy U_p can be considerably larger than the rest energy mc^2 of the electron. We envisaged the case in which the electron beam and the laser beam are counterpropagating as depicted in Fig. 1. Our main conclusions were that this highly nonlinear process is most efficient for generating high-frequency x rays if the kinetic electron energy is in the keV range and if the laser power is of the order of magnitude or larger than 10^{21} W cm^{-2} for a Ti-sapphire laser of frequency $\omega = 1.54$ eV. To take even higher laser powers is not useful since the process shows the feature of saturating. Concerning the relevance of possible spin effects, we compared the data, evaluated by either treating the electron as a spinless Klein-Gordon particle or by considering a Dirac particle of spin 1/2. The general conclu-

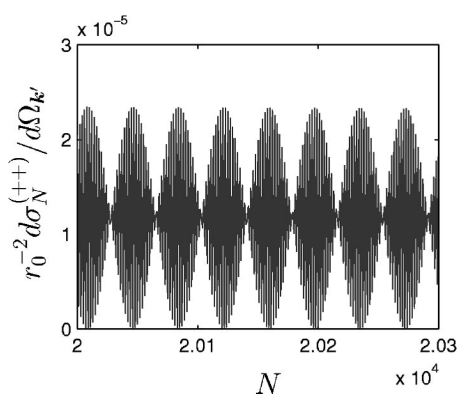

FIG. 6. Considers the same cross-section data as in Fig. 5 but only for a smaller range of N values and on an enlarged scale along the abscissa such that we can recognize the rapid oscillations of the cross sections as a function of N, having their origin in the properties of the generalized Bessel functions $B_N(x,y)$.

sion is that the cross-section data for spin flip are much smaller than those for no spin flip and that the latter data very well agree with those for a particle of spin 0. Therefore, the consideration of elementary scattering processes in very powerful laser fields requires the inclusion of spin effects only for very specific geometries. Hence, since spin effects turn out to be marginal, the question arises whether our process could not be treated entirely classical since also the laser-induced frequency shift by far exceeds the quantum-mechanical nonlinear Compton shift in Eq. (18) at the very high laser powers considered in our present work, even for very larger values N of absorbed laser photons.

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