Spontaneous electric multipole emission in a condensed medium and toroidal moments

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The contribution of toroidal moments to the spontaneous electric emission in a continuous homogeneous nonabsorbing condensed medium is calculated. The probability of toroidal emission of multipolarity L depends on dielectric and magnetic constants as $f_L^2(\epsilon)\epsilon^{L+3/2}\mu^{L+5/2}$, where $f_L(\epsilon)$ is the local-field factor. The linear refractive-index dependence of the spontaneous emission probability for electric-dipole transitions in dielectrics in the real cavity model transforms to an n^5 dependence in the range $n \gtrsim 1/\omega a_B$.

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The dependence of the electric (E) and magnetic (M) dipole spontaneous emission probability on dielectric (ϵ) and magnetic (μ) constants of a condensed medium was first predicted by Nienhuis and Alkemade [1]. According to that work the $E1$ and $M1$ emission probabilities must increase by factors of $\epsilon^{1/2}\mu^{3/2}$ and $\epsilon^{3/2}\mu^{5/2}$, respectively, in contrast with the emission probabilities in vacuum. These results were confirmed experimentally for dielectrics in Ref. [2]. The formulas, which connect the spontaneous emission probability of arbitrary multipolarity transitions (L) in a nonabsorbing medium (W_m) and in a vacuum ($W_{\rm vac}$) were obtained in Ref. [3],

$$W_m^{\rm EL} = f_L^2(\epsilon)\epsilon^{L-1/2}\mu^{L+1/2}W_{\rm vac}^{\rm EL}, \tag{1}$$

$$W_m^{\rm ML} = f_L^2(\mu)\epsilon^{L+1/2}\mu^{L+3/2}W_{\rm vac}^{\rm ML}. \tag{2}$$

The functions f_L in Eqs. (1) and (2) correct for a factor in the local field, i.e., the difference between the local fields $\mathbf{E}_{\rm loc}$ or $\mathbf{B}_{\rm loc}$ interacting with the radiating atom, and the electromagnetic fields in a medium $\mathbf{E}_m$ and $\mathbf{B}_m$ (see details in Refs. [3, 4]).

The result of Refs. [1,3] for electric transitions, expressed by Eq. (1), is true if the following three conditions are satisfied simultaneously: (a) the long-wave approximation can be used in describing the emission process; (b) the medium has relatively small values for ϵ and μ; (c) the corresponding charge and magnetic transition moments of the radiating system are nonzero.

Now we consider electric multipole emission in a medium, which arises from toroidal transitions, and obtain a law that is analogous to Eqs. (1) and (2). Then a formula will be deduced for electric multipole emission in a medium with large values of ϵ and μ.

The interaction Hamiltonian for the electric emission of arbitrary multipolarity L (EL emission) is derived from the general formula for the Hamiltonian of the interaction of the current of the emitting system $j_{\rm fi}^{\nu}(t,\mathbf{r})$ with the vector potential of the radiation field $A_{\nu}(x)$

$$H_{\rm int}(t) = -e\int j_{\rm fi}^{\nu}(t,\mathbf{r})A_{\nu}(t,\mathbf{r})d^3r, \tag{3}$$

where e is the electron charge, $j_{\rm fi}^{\nu}(t,\mathbf{r}) = (\rho_{\rm fi}(t,\mathbf{r}), \mathbf{j}_{\rm fi}(t,\mathbf{r}))$ is the current [i.e., the matrix element of the current operator for an electron transition between initial (i) and final (f) states], and $A_{\nu}(t,\mathbf{r})$ is the vector potential of the radiation field. The vector-potential operator and the current operator are evident in the interaction picture in Eq. (3). It is known that these operators satisfy the same equations of motion and the same permutation relations as the operators of free fields in a medium in the Heisenberg picture [5]. Therefore the field operators depend on the electronic properties of the medium, where fields propagate. This enables one to take into account the medium's properties within the frame work of perturbation theory for QED. Adopted system of units is $\hbar = c = 1$.

The vector part of $A_{\nu}(t,\mathbf{r})$ in the Coulomb gauge for the case of photon emission can be presented as $\mathbf{A}(t,\mathbf{r}) = \langle 1|\hat{\mathbf{A}}(t,\mathbf{r})|0\rangle$, where

$$\hat{\mathbf{A}}(t,\mathbf{r}) = \sum_{\mathbf{k}}\sum_{\lambda=1,2}\hat{a}_{\mathbf{k},\lambda}^{+}\mathbf{A}_{\mathbf{k},\lambda}^{*}(\mathbf{r})e^{i\omega t},$$

$\hat{a}_{\mathbf{k},\lambda}^{+}$ is the photon creation operator, λ and ω are the photon polarization and energy, respectively, $\mathbf{A}_{\mathbf{k},\lambda}(\mathbf{r})$ are plane waves

$$\mathbf{A}_{\mathbf{k},\lambda}(\mathbf{r}) = \mathbf{e}_{\mathbf{k},\lambda}\left(\frac{2\pi}{\epsilon\omega}\right)^{1/2}e^{i\mathbf{kr}}, \tag{4}$$

and $\mathbf{e}_{\mathbf{k},\lambda}$ is the unit polarization vector. The factor $\sqrt{2\pi/\epsilon\omega}$ in Eq. (4) follows from the formula for the energy of a free electromagnetic field in a medium $1/8\pi\int(\mathbf{E_m D_m} + \mathbf{H_m B_m})d^3r$, and ensures the standard form for the field energy and momentum operators [3,6].

The transition current $j_{\rm fi}^{\nu}$ in Eq. (3) occurs between bound states. Therefore, angular momentum is conserved in the transition, and one can use the standard expansion of the wave function (4) in the transverse electric $\mathbf{A}_{\rm Lm}^{E}(k,\mathbf{r})$ and magnetic $\mathbf{A}_{\rm Lm}^{M}(k,\mathbf{r})$ multipole fields (see Refs. [3,7]). These fields together with the longitudinal multipole field $\mathbf{A}_{\rm Lm}^{Y}(k,\mathbf{r})$ are the vector solutions of the Helmholtz equation $\Delta\mathbf{A}(\mathbf{r})$

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$+k^2\mathbf{A}(\mathbf{r})=0$, and constitute a basis set. Here, the parameter $k=\sqrt{\epsilon\mu}\omega$ is the absolute value of the photon momentum $\mathbf{k}$ in the medium.

The Hamiltonian for radiation in the EL mode follows from Eq. (3), if one substitutes in this equation the multipole expansion for the plane wave (4) [3],

$$H_{\rm int}^{\rm EL}(t)=\exp[i(\omega-\omega_{\rm fi})t]e\left(\frac{2\pi}{\epsilon\omega}\right)^{1/2}[2\pi(2L+1)]^{1/2}\times(-i)^{L+1}\sum_m D^{L*}_{m\lambda}(\varphi_{\mathbf{k}},\vartheta_{\mathbf{k}},0)\times\int d^3r\,\mathbf{A}^{E*}_{\rm Lm}(k,\mathbf{r})\mathbf{j}_{\rm fi}(\mathbf{r}). \quad (5)$$

Here $\omega_{\rm fi}$ is the transition energy, $D^L_{m\lambda}(\varphi_{\mathbf{k}},\vartheta_{\mathbf{k}},0)$ are Wigner D functions, and the explicit form for the electric multipole field is

$$\mathbf{A}^E_{\rm Lm}(k,\mathbf{r})=\left(\frac{L+1}{2L+1}\right)^{1/2}j_{L-1}({\rm kr})\mathbf{Y}^{L-1}_{Lm}(\mathbf{n_r})-\left(\frac{L}{2L+1}\right)^{1/2}j_{L+1}({\rm kr})\mathbf{Y}^{L+1}_{\rm Lm}(\mathbf{n_r}), \quad (6)$$

where $j_L({\rm kr})$ are spherical Bessel functions [8], and $\mathbf{Y}^L_{\rm Jm}(\mathbf{n_r})$ are spherical vector functions [7].

To evaluate the integral in Eq. (5) let us introduce the electric multipole form factor

$$\langle J_fm_f|\hat{Q}^E_{\rm Lm}(k)|J_im_i\rangle=\frac{(2L+1)!!}{k^{L-1}}\left(\frac{L}{L+1}\right)^{1/2}\left(\frac{4\pi}{2L+1}\right)^{1/2}\times\int d^3r\,\mathbf{j}_{\rm fi}(\mathbf{r})\mathbf{A}^{E*}_{\rm Lm}(k,\mathbf{r}). \quad (7)$$

With this definition the form factor goes over into the charge multipole moment as k tends to zero,

$$\lim_{k\to0}\langle J_fm_f|\hat{Q}^E_{\rm Lm}(k)|J_im_i\rangle=i\omega_{\rm fi}\langle J_fm_f|\hat{Q}^C_{\rm Lm}|J_im_i\rangle, \quad (8)$$

where the standard formula is used for the charge multipole moments [9]

$$\langle J_fm_f|\hat{Q}^C_{\rm Lm}|J_im_i\rangle=\left(\frac{4\pi}{2L+1}\right)^{1/2}\int d^3r\,\rho_{\rm fi}(\mathbf{r})r^LY^*_{\rm LM}(\mathbf{n_r}). \quad (9)$$

Equations (8) and (9) are obtained easily from Eq. (7), if one calculates the integral using the known relation in the low-energy limit $\mathbf{A}^E_{\rm Lm}(k,\mathbf{r})\approx\sqrt{(L+1)/L}\mathbf{A}^Y_{\rm Lm}(k,\mathbf{r})=\sqrt{(L+1)/L}1/k\boldsymbol{\nabla}[j_L({\rm kr})Y_{\rm Lm}(\mathbf{n_r})]$, and by the continuity equation for the current $\boldsymbol{\nabla}\mathbf{j}_{\rm fi}(\mathbf{r})=i\omega_{\rm fi}\rho_{\rm fi}(\mathbf{r})$ (see details in the Ref. [3]). In nuclear physics this reduction of the electric form factor to the charge moment is known as Siegert's theorem [7]. In 1974 Dubovik and Cheshkov [10] proved that such an approximation is not correct, because the toroidal moments are lost. One obtains a stricter representation for the electric multipole form factor $\langle J_fm_f|\hat{Q}^E_{\rm Lm}(k)|J_im_i\rangle$, if one takes into account higher-order terms in the Bessel function's expansion into a power series in Eqs. (6)–(7).

Toroidal moments are a familiar part of atomic-nuclear physics now. Static toroidal moments cause parity nonconservation in atoms (see Refs. [11,12], and references therein) arising from the P-odd part of the weak interactions. Wood *et al.* measured a dipole toroidal moment of the ^{133}Cs nucleus in the atomic $6S\to7S$ transition in 1997 [11]. The dipole toroidal moment coincides with the known anapole moment of Zel'dovich [13]. In the framework of classical physics a dipole toroidal moment describes a current $\mathbf{j}(\mathbf{r})$ in a conventional solenoid folded into a small torus. Such a toroidal solenoid has a ring-shaped magnetic field situated inside the torus and a dipole moment $\mathbf{Q}^T$ directed along the axis of symmetry of the torus [10,14]: $\mathbf{Q}^T=\int d^3r\mathbf{q}^T$, where $\mathbf{q}^T=\frac{1}{10}[\mathbf{r}(\mathbf{r}\cdot\mathbf{j})-2r^2\mathbf{j}]$ is a density of the toroidal moment. A toroidal solenoid does not have a charge distribution $\rho(\mathbf{r})$ or a charge dipole moment $Q^C_{1m}\sim\int d^3r\rho(\mathbf{r})rY^*_{1m}(\mathbf{n_r})$. Magnetic dipole moments of the ring currents $\mathbf{q}^M_i=\frac{1}{2}[\mathbf{r_i}\times\mathbf{j_i}]$ from a closed circle. Therefore, a sum of this moments $\mathbf{q}^M$ can be expressed as a curl of some vector (it is easy to prove that $\mathbf{q}^M={\rm curl}\mathbf{q}^T$), and as a consequence the torus magnetic dipole moment $Q^M_{1m}\sim\int d^3r\,{\rm div}\,\mathbf{q}^MrY^*_{1m}(\mathbf{n_r})$ [15] is equal to zero too. So, contracted to a point a toroidal solenoid does not interact with static homogeneous fields. However a current penetrating into a solenoid (i.e., running through the point-like toroidal solenoid) interacts with its magnetic field. This interaction enables one to find the ^{133}Cs anapole moment in the experiment [11]. Really, nS electronic wave functions have large amplitudes at the origin, and, as a consequence, electrons in the $6S$ and $7S$ states effectively penetrate the nucleus. The electromagnetic interaction between the electronic current and the nuclear dipole toroidal moment mixes states of opposite parity (i.e., S and P electronic states) and leads to nonzero $E1$ transition amplitudes between the $6S$ and $7S$ states. It is significant that this interaction depends on the nuclear spin (unlike the usual P-odd part of the weak interaction between electron and nucleus), and its contribution to transitions between different hyperfine structure components vary [12]. Thus if the static nuclear anopole moment exists as a result of the parity nonconservation part of nuclear forces, it can be found in hyperfine spectra of atomic transitions in the optical range, as was observed in the experiment [11].

Toroidal moments of transitions could exist even if a system does not have a static toroidal moments. For a very limited number of nuclei, the toroidal moments of transitions can be extracted from data on anomalous internal conversion [16] (this is a quantum electrodynamical second-order process for the decay of a nuclear level accompanied by ejection of an inner-shell electron). This paper will demonstrate how to find a toroidal moment contribution to the probability of ordinary atomic emission, which is a first-order QED process. The medium will play a leading role in this demon-

stration. Such an overall view of toroidal moments is important because, as we now understand, the multipole parametrization of an arbitrary system of charges and currents is not complete without the toroidal moments [10].

Toroidal multipole form factors are introduced according to the relation [10,14]

$$\langle J_f m_f|\hat{Q}^E_{\mathrm{Lm}}(k)|J_i m_i\rangle = i\omega_{\mathrm{fi}}\langle J_f m_f|\hat{Q}^C_{\mathrm{Lm}}|J_i m_i\rangle + ik^2\langle J_f m_f|\hat{Q}^T_{\mathrm{Lm}}(k)|J_i m_i\rangle. \quad (10)$$

The explicit form for $\langle J_f m_f|\hat{Q}^T_{\mathrm{Lm}}(k)|J_i m_i\rangle$ is easily obtained. The auxiliary relation follows from Eqs. (7) and (8), if one expands the Bessel function $j_{L-1}(\mathrm{kr})$ in the range $kr \ll 1$ and neglects the function $j_{L+1}(\mathrm{kr})$ in Eq. (6),

$$i\omega_{\mathrm{fi}}\langle J_f m_f|\hat{Q}^C_{\mathrm{Lm}}|J_i m_i\rangle = \sqrt{4\pi L}\int d^3r\,\mathbf{j}_{\mathrm{fi}}(\mathbf{r})r^{L-1}\mathbf{Y}^{L-1*}_{\mathrm{Lm}}(\mathbf{n_r}).$$

Extracting this result from Eq. (7) one derives an exact equation for the toroidal form factors

$$\begin{aligned}\langle J_f m_f|\hat{Q}^T_{\mathrm{Lm}}(k)|J_i m_i\rangle &= -i\frac{(2L+1)!!}{k^{L-1}}\left(\frac{L}{L+1}\right)^{1/2}\left(\frac{4\pi}{2L+1}\right)\\ &\times\int d^3r\,\mathbf{j}_{\mathrm{fi}}(\mathbf{r})\left[\left(\left(\frac{L+1}{2L+1}\right)^{1/2}j_{L-1}(\mathrm{kr})\right.\right.\\ &-(\mathrm{kr})^{L-1}\frac{\sqrt{L+1}\sqrt{2L+1}}{(2L+1)!!}\Big)\mathbf{Y}^{L-1*}_{\mathrm{Lm}}(\mathbf{n_r})\\ &-\left.\left(\frac{L}{2L+1}\right)^{1/2}j_{L+1}(\mathrm{kr})\mathbf{Y}^{L+1*}_{\mathrm{Lm}}(\mathbf{n_r})\right]. \quad (11)\end{aligned}$$

The toroidal multipole moments $\langle J_f m_f|\hat{Q}^T_{\mathrm{Lm}}|J_i m_i\rangle$ result from Eq. (11) in the limit of $k\to 0$. Expanding the Bessel functions $j_{L-1}(\mathrm{kr})\approx(\mathrm{kr})^{L-1}/(2L-1)!!(1-(\mathrm{kr})^2/[2(2L+1)])$ and $j_{L+1}(\mathrm{kr})\approx(\mathrm{kr})^{L+1}/(2L+3)!!$ [8] one obtains

$$\begin{aligned}\langle J_f m_f|\hat{Q}^T_{\mathrm{Lm}}|J_i m_i\rangle &= \frac{i}{2}\left(\frac{4\pi}{2L+1}\right)^{1/2}\left(\frac{L}{L+1}\right)^{1/2}\int d^3r\,\mathbf{j}_{\mathrm{fi}}(\mathbf{r})r^{L+1}\left(\mathbf{Y}^{L-1*}_{\mathrm{Lm}}(\mathbf{n_r})\right.\\ &\left.+\frac{2}{2L+3}\left(\frac{L}{L+1}\right)^{1/2}\mathbf{Y}^{L+1*}_{\mathrm{Lm}}(\mathbf{n_r})\right). \quad (12)\end{aligned}$$

This result enables one to make a more accurate parametrization in the Hamiltonian for radiation of the EL mode in Eq. (5),

$$\begin{aligned}\int d^3r\,\mathbf{A}^{E*}_{\mathrm{Lm}}(k,\mathbf{r})\mathbf{j}_{\mathrm{fi}}(\mathbf{r}) &= \frac{ik^L}{(2L+1)!!}\left(\frac{L+1}{L}\right)^{1/2}\left(\frac{2L+1}{4\pi}\right)^{1/2}\\ &\times\left(\frac{\omega}{k}\langle J_f m_f|\hat{Q}^C_{\mathrm{Lm}}|J_i m_i\rangle\right.\\ &\left.+k\langle J_f m_f|\hat{Q}^T_{\mathrm{Lm}}|J_i m_i\rangle\right) \quad (13)\end{aligned}$$

in the range $kr\ll 1$.

As was pointed out in Refs. [14,10], (previously it had been established by Blatt and Weisskopf [15]), electric multipole emission is possible even if $\rho_{\mathrm{fi}}=0$, i.e., if all charge multipole moments are equal to zero. Now it is evident from Eq. (13).

The first-order element of the S matrix can be written through the interaction Hamiltonian Eq. (5) as $S^{(1)}_{\mathrm{fi}} = i\int dt H^{\mathrm{EL}}_{\mathrm{int}}(t)$ [9]. The probability of spontaneous emission, or a radiative width, is calculated in the framework of QED from the formula [9]

$$W = \frac{1}{2J_i+1}\sum_{m_1,m_f,\lambda}\int\frac{|S^{(1)}_{\mathrm{fi}}|^2}{t}\frac{d^3k}{(2\pi)^3},$$

where t is a large, but finite time interval (t is canceled out with an analogous interval, which arises in the numerator together with the $2\pi\delta(\omega-\omega_{\mathrm{fi}})$ from the $|S^{(1)}_{\mathrm{fi}}|^2$ as result of integration over the time [9]).

The final formula for the EL emission in a nonabsorbing medium is

$$W^{\mathrm{EL}}_m = f^2_L(\epsilon)\epsilon^{L-1/2}\mu^{L+1/2}W^{\mathrm{EL}(C)}_{\mathrm{vac}}\left(1+2\omega\epsilon\mu\frac{\mathrm{Re}\langle J_f\|\hat{Q}^T_L\|J_i\rangle\mathrm{Re}\langle J_f\|\hat{Q}^C_L\|J_i\rangle+\mathrm{Im}\langle J_f\|\hat{Q}^T_L\|J_i\rangle\mathrm{Im}\langle J_f\|\hat{Q}^C_L\|J_i\rangle}{|\langle J_f\|\hat{Q}^C_L\|J_i\rangle|^2}\right.$$
$$\left.+\omega^2\epsilon^2\mu^2\frac{|\langle J_f\|\hat{Q}^T_L\|J_i\rangle|^2}{|\langle J_f\|\hat{Q}^C_L\|J_i\rangle|^2}\right), \quad (14)$$

where $\langle J_f\|\hat{Q}^{T,C}_L\|J_i\rangle$ are the reduced matrix elements for multipole moments (9) and (12), Re and Im are the real and imaginary parts, and $W^{\mathrm{EL}(C)}_{\mathrm{vac}}$ is the regular emission probability in vacuum connected with the charge transition [9]

$$W_{\rm vac}^{{\rm EL}(C)}=e^2\frac{2(L+1)(2L+1)}{L[(2L+1)!!]^2}\omega^{2L+1}\frac{|\langle J_f\|\hat{Q}_L^C\|J_i\rangle|^2}{2J_i+1}.$$

Suppose we have an emitting system with toroidal transition moments only. (One can imagine, for example, a quantum system with a dipole toroidal moment such as a molecular wheel, where spins are added vectorially to the ring by analogy with magnetic moments of ring currents $\mathbf{q}_i^M$ of the type described above.) In this case the spontaneous emission probability in a nonabsorbing medium is

$$W_m^{{\rm EL}(T)}=f_L^2(\epsilon)\epsilon^{L+3/2}\mu^{L+5/2}e^2\frac{2(L+1)(2L+1)}{L[(2L+1)!!]^2}\omega^{2L+3}\times\frac{|\langle J_f\|\hat{Q}_L^T\|J_i\rangle|^2}{2J_i+1}. \quad (15)$$

From Eq. (15) it follows that a medium changes the spontaneous emission probability for toroidal transitions by a factor $f_L^2(\epsilon)\epsilon^{L+3/2}\mu^{L+5/2}$. This is even stronger than for magnetic transitions [compare with Eq. (2)]. This is a result of the fact that toroidal moments arise in the next higher order of $({\rm kr})^2$ than do the charge transition moments in the parametrization process of the integral $\int d^3r\mathbf{A}_{\rm Lm}^{E*}(k,\mathbf{r})\mathbf{j}_{\rm fi}(\mathbf{r})$ in Eq. (7).

The function $f_L(\epsilon)$ for toroidal transitions coincides with the function for electric transitions calculated in Ref. [3]. For the real (R) [4] and virtual (V) [17] cavity models these functions are

$$f_L^R(\epsilon)=\frac{\epsilon(2L+1)}{\epsilon(L+1)+L},\quad f_L^V(\epsilon)=\frac{\epsilon L+L+1}{2L+1}.$$

Effects of local fields on spontaneous emission in dielectric media were investigated in detail experimentally [for small values of the refractive index n) [2,18] and theoretically [4,19–22] in recent years. In nonabsorbing dielectrics the functions $(f_1^R)^2=[3n^2/(2n^2+1)]^2$ and $(f_1^V)^2=[(n^2+2)/3]^2$ modify the decay rate additionally by a factors 1.8 and 4, respectively, even if $n=2$. At large values of n the asymptotic behavior of the local-field correction factors are $(f_L^R)^2\sim[(2L+1)/(L+1)]^2$ and $(f_L^V)^2\sim n^4[L/(2L+1)]^2$. That is, the virtual cavity modifies the spontaneous emission rate appreciably at large n unlike the behavior for a real cavity.

As for absorption dielectrics the local-field correction factors are crucially important too. This was first pointed out by Burnett, Huttner, and Loudon [19]. According to their results, one should make the substitutions $f_1(\epsilon)^2\to|f_1(\epsilon)|^2$ and $\epsilon^{1/2}\to n$ in Eq. (1) for an $E1$ transition in a nonmagnetic ($\mu=1$) absorbing dielectric, where n is the real part of the refractive index. Burnett and co-workers considered a virtual cavity and demonstrated a key role of the factor $|f_1(\epsilon)|^2$ is modifying the spontaneous emission rate by considering the example of a simple model dielectric constant $\epsilon(\omega)$ based on a single medium resonance at the frequency ω_0: $\epsilon(\omega)=1+\omega_P^2/(\omega_0^2-\omega^2-i\gamma\omega)$, where ω_P is the plasma frequency of the dielectric, and γ is the loss coefficient. It was found that the factor $n|f_1(\epsilon)|^2$ can take on large values near the resonance for some values of the parameters ω_P and γ. More particularly these problems were investigated later in the Refs. [20, 21] both for real and virtual cavity models.

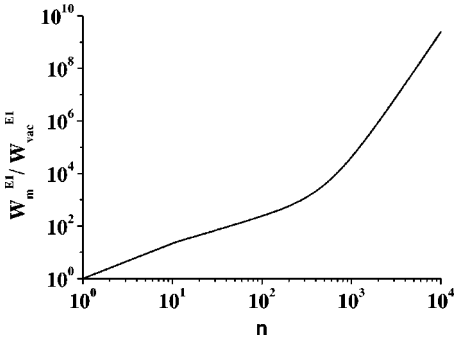

FIG. 1. Plot of $W_m^{E1}/W_{\rm vac}^{E1}$ as a function of the refractive index n for a typical atomic transition in the optical range. The plot corresponds to a real cavity. The plot for a virtual cavity can be obtained by multiplying by a factor $\sim n^4$.

The strong dependence of the $W_m^{{\rm EL}(T)}$ on ϵ and μ [see Eq. (15)] enables one to try an experiment for observation of the toroidal transition contribution to the electric-dipole spontaneous emission. The toroidal moments in Eq. (12) have the dimensions r^{L+1}, whereas the charge moments in Eq. (9) have the dimensions r^L. So, the toroidal term in the Eq. (14) becomes equal approximately to the charge term in the range $\epsilon\mu\sim 1/\omega a$, where the a is a characteristic size of the emitting system. If an atom emits an optical range photon, then $a\simeq a_B$, where a_B is the Bohr radius. Inside a dielectric medium, for example, in the real cavity model the known linear refractive-index dependence $W_m^{\rm EL}/W_{\rm vac}^{\rm EL}\simeq n$ becomes $W_m^{\rm EL}/W_{\rm vac}^{\rm EL}\simeq n^5$ in the range $n\gtrsim 1/\sqrt{\omega a_B}$. Respectively in the virtual cavity model the dependence $W_m^{\rm EL}/W_{\rm vac}^{\rm EL}\simeq n^5$ becomes $W_m^{\rm EL}/W_{\rm vac}^{\rm EL}\simeq n^9$.

The $1/2^-\leftrightarrow 1/2^+$ and $1^\pm\leftrightarrow 0^\mp$ atomic transitions are suitable for experimental investigation of the dipole toroidal moment contribution to the $E1$ emission, because there is no $M2$ component in such transitions. Using the current [9]

$$\mathbf{j}_{\rm fi}(\mathbf{r})=\frac{1}{2m_e i}\{\psi_f^*(\mathbf{r})\nabla\psi_i(\mathbf{r})-[\nabla\psi_f^*(\mathbf{r})]\psi_i(\mathbf{r})\}+\frac{\mu_e}{es}\nabla\times[\psi_f^*(\mathbf{r})\hat{\mathbf{s}}\psi_i(\mathbf{r})],$$

where m_e, μ_e, and $\mathbf{s}$ are the electron mass, magnetic moment, and spin, respectively, one can calculate the spontaneous emission probability. A typical plot of $W_m^{E1}/W_{\rm vac}^{E1}$ as a function of the refractive index n for the real cavity model is shown in Fig. 1. The plot corresponds to an atomic transition with an energy in the range 2–3 eV. The curvature of the line lies in the range $n\simeq 500-1000$.

In conclusion, the formulas obtained describe the spontaneous emission in a medium due to toroidal moments, the third fundamental family of multipole moments that belong in a class with the charge and magnetic ones. The strong dependence of the toroidal transition probability on the me-

dium's electronic properties provides an opportunity for detecting a corresponding electromagnetic emission in atomic $E1$ transitions, especially if one takes into account the recent rapid progress in the engineering of media with controlled properties. On the other hand, the development of ''spin-ring radiators'' will enable us to find ''toroidal'' $E1$ emission and the $\epsilon^{5/2}\mu^{7/2}$ effect in materials already in existence.

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