Optimally conclusive discrimination of nonorthogonal entangled states by local operations and classical communications

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We consider one copy of a quantum system prepared with equal prior probability in one of two nonorthogonal entangled states of multipartite distributed among separated parties. We demonstrate that these two states can be optimally distinguished in the sense of conclusive discrimination by local operations and classical communications alone. This proves strictly the conjecture that Virmani *et al.* confirmed numerically and analytically. Generally the optimal protocol requires local POVM operations which are explicitly constructed. The result manifests that distinguishable information is obtained only and completely at the last operation and all prior operations give no information about that state.

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In quantum-information theory, two fascinating properties are distinguished from classical information. One is entanglement and the other is nonorthogonality. Entanglement lies at the heart of many aspects of quantum-information theory, such as quantum information [1], quantum computation [2], quantum error correction [3], and teleportation [4]. Without entanglement, many quantum tasks could not be carried out. In this sense, it is a quantum resource. It is a key point that it is impossible to discriminate perfectly between nonorthogonal quantum states if only one copy is provided. The well-known no-cloning theorem [5] demonstrates that nonorthogonal states cannot be cloned exactly. Generally, orthogonal states may be distinguished perfectly only by means of global measurements since quantum information of orthogonality may be encoded in entanglement, which may not be extracted by local operations and classical communications (LOCC) operations. Bennett *et al.* [6] showed that there exist bases of product orthogonal pure states which cannot be locally reliably distinguished despite the fact that each state in the basis contains no entanglement. Recently, Walgate *et al.* [7] demonstrated that any two orthogonal multipartite pure states can be distinguished perfectly by only LOCC operations. Virmani *et al.* [8] utilized their result [7] to show that optimal discrimination of two nonorthogonal pure states can also be achieved by LOCC in the sense of inconclusive discrimination. They also numerically and analytically confirmed that it is the case for a large set of states in conclusive discrimination. The problem of identifying two nonorthogonal states has been considered in [9] and [10] by global measurements. We have discussed the problem of discriminating two nonorthogonal product states by LOCC [11]. In this paper, we consider the issue of conclusive discrimination of two nonorthogonal entangled states and prove strictly the conjecture that the optimal discrimination by global measurements can be achieved by LOCC operations.

Suppose Alice and Bob know the precise forms of two entangled states in which one of them is shared between them. These two possible entangled states, $|\phi\rangle$ and $|\psi\rangle$, generally nonorthogonal are provided with equal prior probability. They are separated from each other and can communicate classical information only. Their aim is to identify the shared states optimally in the sense of conclusive discrimination by LOCC operations. Conclusive discrimination means that our measurement on the copy gives three outcomes which allow us to determine the prior state is $|\phi\rangle$ or $|\psi\rangle$ with certainty or "do not know." The optimization of conclusive discrimination is to obtain the maximal probability of decisive outcomes. $|\phi\rangle$ and $|\psi\rangle$ can be represented in a general form,

$$|\phi\rangle=\sum_{i=1}^{n}\sqrt{r_i}|e_i\rangle_A|\eta_i\rangle_B,$$

$$|\psi\rangle=\sum_{i=1}^{n}\sqrt{s_i}|e_i\rangle_A|\gamma_i\rangle_B, \tag{1}$$

where $\{|e_i\rangle_A\}$ form an orthonormal basis set for Alice, and the vectors $\{|\eta_i\rangle_B\}$ and $\{|\gamma_i\rangle_B\}$ are normalized and generally nonorthogonal. In [7], it was proved that the two states can be expressed as the following form in another orthonormal basis set on Alice's side:

$$|\phi\rangle=\sum_{i=1}^{n}\sqrt{r_i'}|e_i'\rangle_A|\eta_i'\rangle_B,$$

$$|\psi\rangle=\sum_{i=1}^{n}\sqrt{s_i'}|e_i'\rangle_A|\gamma_i'\rangle_B, \tag{2}$$

satisfying

$$\sqrt{r_i's_i'}\langle\eta_i'|\gamma_i'\rangle_B=\sqrt{r_j's_j'}\langle\eta_j'|\gamma_j'\rangle_B, \tag{3}$$

where $\{|e_i'\rangle_A\}$ forms another orthonormal basis set. For orthogonal states, Walgate *et al.* showed that $\langle\eta_i'|\gamma_i'\rangle_B=0$ for all $i=1,2,\ldots,n$ and proved that Alice and Bob can always distinguish between the two possible orthogonal states perfectly by LOCC operations. In the following, we suppose that the two states have been expressed as the form above and denote them still as their original form for convenience. Before our main theorem, let us introduce lemma 1.

Lemma 1. Let M be a 2×2 matrix $\binom{xy}{zt}$ whose diagonal elements are real, and U be a unitary matrix $\begin{pmatrix}\cos\theta & \sin\theta e^{i\omega}\\ \sin\theta e^{-i\omega} & -\cos\theta\end{pmatrix}$.

There exists U such that the diagonal elements of $UMU^\dagger$ are real and of which this property is independent of θ.

Proof: This lemma can be easily proved by direct computation,

$$x' = x\cos^2\theta + t\sin^2\theta + (\sin\theta\cos\theta)(ye^{-i\omega} + ze^{i\omega}),$$

$$t' = x\sin^2\theta + t\cos^2\theta - (\sin\theta\cos\theta)(ye^{-i\omega} + ze^{i\omega}). \quad (4)$$

Set $\mathrm{Im}(ye^{-i\omega}+ze^{i\omega})=0$ and there will always be an angle ω satisfying the equation which is explicitly independent of θ. This completes the proof ■. Employing Lemma 1, we can transform the two states further to the form that is expressed as Theorem 1.

Theorem 1. In a proper orthonormal basis set $\{|i\rangle\}$ on Alice's side, $|\phi\rangle$ and $|\psi\rangle$ can be expressed as the form

$$|\phi\rangle = \sum_{i=1}^{n} \sqrt{t_i}|i\rangle|\mu_i\rangle,$$

$$|\psi\rangle = \sum_{i=1}^{n} \sqrt{t_i}|i\rangle|\nu_i\rangle, \quad (5)$$

and $|\mu_i\rangle$, $|\nu_i\rangle$ satisfy the condition that the phase difference between each $\langle\mu_i|\nu_i\rangle$ and $\langle\phi|\psi\rangle$ is 0 or π.

Proof. Suppose $\langle\phi|\psi\rangle$ is real and we will show this does not lose any generality for the complex case. We also suppose that $|\phi\rangle$ and $|\psi\rangle$ have been expressed as the form of (1) and satisfy $\sqrt{r_i s_i}\langle\eta_i|\gamma_i\rangle_B = \sqrt{r_j s_j}\langle\eta_j|\gamma_j\rangle_B$. It is explicit that every $\langle\eta_i|\gamma_i\rangle$ is real. As $\Sigma_i r_i = \Sigma_i s_i = 1$, there must exist r_i, s_i and r_j, s_j satisfying $r_i \geqslant s_i, r_j \leqslant s_j$. Without no loss of generality, we set $r_1 \geqslant s_1, r_2 \leqslant s_2$. We first change the two bases $\{|e_1\rangle, |e_2\rangle\}$ into $\{|e_1'\rangle, |e_2'\rangle\}$ only. According to the result in [12], the corresponding terms on Bob's side transform as

$$\begin{pmatrix} \cos\theta & e^{-i\omega}\sin\theta \\ e^{i\omega}\sin\theta & -\cos\theta \end{pmatrix}\begin{pmatrix} \sqrt{r_1}|\eta_1\rangle \\ \sqrt{r_2}|\eta_2\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{r_1'}|\eta_1'\rangle \\ \sqrt{r_2'}|\eta_2'\rangle \end{pmatrix},$$

$$\begin{pmatrix} \cos\theta & e^{-i\omega}\sin\theta \\ e^{i\omega}\sin\theta & -\cos\theta \end{pmatrix}\begin{pmatrix} \sqrt{s_1}|\gamma_1\rangle \\ \sqrt{s_2}|\gamma_2\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{s_1'}|\gamma_1'\rangle \\ \sqrt{s_2'}|\gamma_2'\rangle \end{pmatrix}, \quad (6)$$

and

$$r_1' = r_1\cos^2\theta + r_2\sin^2\theta + \sqrt{r_1 r_2}(\cos\theta\sin\theta)(e^{-i\omega}\langle\eta_1|\eta_2\rangle + e^{i\omega}\langle\eta_2|\eta_1\rangle),$$

$$s_1' = s_1\cos^2\theta + s_2\sin^2\theta + \sqrt{s_1 s_2}(\cos\theta\sin\theta)(e^{-i\omega}\langle\gamma_1|\gamma_2\rangle + e^{i\omega}\langle\gamma_2|\gamma_1\rangle). \quad (7)$$

The matrix

$$M = \begin{pmatrix} \sqrt{r_1 s_1}\langle\eta_1|\gamma_1\rangle & \sqrt{r_1 s_2}\langle\eta_1|\gamma_2\rangle \\ \sqrt{r_2 s_1}\langle\eta_2|\gamma_1\rangle & \sqrt{r_2 s_2}\langle\eta_2|\gamma_2\rangle \end{pmatrix}$$

is transformed to $U^*MU^{\dagger *}$ [7]. In lemma 1, we see that the property that diagonal elements are real is dependent only on ω and independent of θ. So the value of ω is determined by real diagonal elements. Explicitly, its solution is given by equation

$$\mathrm{Im}(\sqrt{r_1 s_2}\langle\eta_1|\gamma_2\rangle e^{-i\omega} + \sqrt{r_2 s_1}\langle\eta_2|\gamma_1\rangle e^{i\omega}) = 0. \quad (8)$$

So $\langle\eta_1'|\gamma_1'\rangle$ and $\langle\eta_2'|\gamma_2'\rangle$ are real, positive, or negative. Then we suppose $r_1' = s_1' = t_1$ and see whether this equation always has a solution. Denote $e^{-i\omega}\langle\eta_1|\eta_2\rangle + e^{i\omega}\langle\eta_2|\eta_1\rangle = x$, $e^{-i\omega}\langle\gamma_1|\gamma_2\rangle + e^{i\omega}\langle\gamma_2|\gamma_1\rangle = y$ for short, which are real. The equation is reduced as

$$[(r_1-s_1)+(r_2-s_2)] + [(r_1-s_1)-(r_2-s_2)]\cos 2\theta + (x\sqrt{r_1 r_2} - y\sqrt{s_1 s_2})\sin 2\theta = 0. \quad (9)$$

Denote $(r_1-s_1)+(r_2-s_2)=C$, $(r_1-s_1)-(r_2-s_2)=A$, $x\sqrt{r_1 r_2} - y\sqrt{s_1 s_2} = B$. We know $|A| \geqslant |C|$ from $r_1 \geqslant s_1, r_2 \leqslant s_2$ and the equation always has a solution,

$$\theta = -\frac{1}{2}\left(\arcsin\frac{C}{\sqrt{A^2+B^2}} + \arctan\frac{A}{B}\right). \quad (10)$$

We notice the fact that $r_1 + r_2 = r_1' + r_2'$ under the unitary operation, so r_1', r_2' are also probabilities. So are s_1', s_2'. Now we find that in the new basis set $\{|e_1'\rangle, |e_2'\rangle, |e_i\rangle, i = 3, \cdots, n\}$, the two states $|\phi\rangle, |\psi\rangle$ can be expressed as

$$|\phi\rangle = \sqrt{t_1}|e_1'\rangle|\eta_1'\rangle + \sqrt{r_2'}|e_2'\rangle|\eta_2'\rangle + \sum_{i=3}^{n}\sqrt{r_i}|e_i\rangle|\eta_i\rangle,$$

$$|\psi\rangle = \sqrt{t_1}|e_1'\rangle|\gamma_1'\rangle + \sqrt{s_2'}|e_2'\rangle|\gamma_2'\rangle + \sum_{i=3}^{n}\sqrt{s_i}|e_i\rangle|\gamma_i\rangle, \quad (11)$$

where all inner products of the corresponding terms remain real. By repeating the above process for the $n-1$ terms, we could obtain the form expressed by Theorem 1. It is clear that it is also the case when $\langle\phi|\psi\rangle$ is complex. What differs in the real case is that the phase of the inner product of each corresponding term is equal to that of $\langle\phi|\psi\rangle$ or π different from it. That completes our proof.

In [9] and [10], it is proved that the optimal conclusive discrimination of two nonorthogonal states is given by $P = 1 - |\langle\phi|\psi\rangle|$ without any limitation of operations. For discriminating general states by LOCC operations, a restricted protocol is suggested in [8] that Alice performs local one-dimensional projections which would give her no information and leave Bob's particle in residual states, which could perhaps be easily distinguished from each other. In our notation, these amount to $r_i = s_i$ and $P^L = 1 - \Sigma_i r_i|\langle\eta_i|\gamma_i\rangle|$ while the optimal discrimination is $P^{\mathrm{opt}} = 1 - |\Sigma_i r_i\langle\eta_i|\gamma_i\rangle|$. If all the equations in addition to $P^L = P^{\mathrm{opt}}$ are satisfed, then the protocol is optimal. Our main idea is simlar to theirs and our conclusion demonstrates that the idea is very illuminating. However, two main obstacles are in the way. One is how to realize the equal probability of corresponding terms and the other is how to adjust the phases of all the inner products of corresponding terms to the same one. Each of them is not straightforward. To satisfy both conditions at the same time, POVM on Alice's side is required in general. In the following theorem, we try to solve the problem.

Theorem 2. Optimally conclusive discrimination between two nonorthogonal entangled states can be achieved by LOCC operations.

Proof. In Theorem 1, $|\phi\rangle,|\psi\rangle$ can be expressed as the form of Eq. (5) and satisfy the condition that the phase of each term $\langle\mu_i|\nu_i\rangle$ is the same as that of $\langle\phi|\psi\rangle$ or has π difference from that of $\langle\phi|\psi\rangle$.

If all the phases of $\langle\mu_i|\nu_i\rangle, i=1,\ldots,n$ are the same as that of $\langle\phi|\psi\rangle$, then Alice performs standard measurement on the basis set $\{|i\rangle\}$ and leaves Bob's state as $|\mu_i\rangle$ or $|\nu_i\rangle$ when $|i\rangle$ occurs. Bob performs the optimal conclusive discrimination between $|\mu_i\rangle$ and $|\nu_i\rangle$ which gives the optimal probability $P_{|i}=1-|\langle\mu_i|\nu_i\rangle|$. The overall optimal probability is averaged as

$$P^L=\sum_i t_iP_{|i}=1-\sum_i t_i|\langle\mu_i|\nu_i\rangle|$$
$$=1-\left|\sum_i t_i\langle\mu_i|\nu_i\rangle\right|=1-|\langle\phi|\psi\rangle|. \tag{12}$$

The third equality comes from the same phase of $\langle\mu_i|\nu_i\rangle, i=1,\ldots,n$, and the optimal discrimination could be realized by LOCC operations.

If there exist some terms of $\langle\eta_i|\gamma_i\rangle$ whose phases have π difference from that of $\langle\phi|\psi\rangle$, then POVM or an auxiliary system is necessarily introduced on Alice's side. Our idea is that after Alice's subsystem interacts properly with the auxiliary system S on her side, the two states including auxiliary system S can be expressed as

$$U^{AS}|s_0\rangle|\phi\rangle=\sum_{i=1}^{m}\sqrt{t_i}|s_i\rangle|\phi_i\rangle+\sum_{i=m+1}^{N}\sqrt{t_i}|s_i\rangle|i\rangle|\mu_i\rangle,$$

$$U^{AS}|s_0\rangle|\psi\rangle=\sum_{i=1}^{m}\sqrt{t_i}|s_i\rangle|\psi_i\rangle+\sum_{i=m+1}^{N}\sqrt{t_i}|s_i\rangle|i\rangle|\nu_i\rangle, \tag{13}$$

where $\langle\phi_i|\psi_i\rangle_{AB}=0$ and $\langle\mu_i|\nu_i\rangle_B$ have the same phase as that of $\langle\phi|\psi\rangle_{AB}$. Once we can express them as the form of Eq. (13), we could obtain the optimal protocol achieved by LOCC operations. If it is true, Alice can first project system S onto the orthonormal basis $\{|s_i\rangle\}$. Occurrence of $|s_i\rangle, i\leqslant m$ projects system AB onto $|\phi_i\rangle$ or $|\psi_i\rangle$, which is orthogonal to each other and can be distinguished with certainty by the protocol in [7]. Occurrence of $|s_i\rangle, i>m$ projects onto $|i\rangle|\mu_i\rangle$ or $|i\rangle|\nu_i\rangle$, which can be identified conclusively on Bob's side with optimal probability $P_{|i}=1-|\langle\mu_i|\nu_i\rangle|$. And the optimal probability overall by LOCC is

$$P^L=\sum_{i=1}^{m}t_i+\sum_{i=m+1}^{N}t_i(1-|\langle\mu_i|\nu_i\rangle|)$$
$$=1-\sum_{i=m+1}^{N}t_i|\langle\mu_i|\nu_i\rangle|$$
$$=1-\left|\sum_{i=m+1}^{N}t_i\langle\mu_i|\nu_i\rangle\right|=1-|\langle\phi|\psi\rangle|. \tag{14}$$

In the following, we will prove that we can really transform to Eq. (13). Without loss of any generality, we suppose that $\langle\phi|\psi\rangle$ is real and $\langle\phi|\psi\rangle\geqslant 0$. Moreover, set $\langle\mu_1|\nu_1\rangle>0$ and $\langle\mu_2|\nu_2\rangle<0$. First, we deal with these two terms and choose U_1^{AS} such that

$$U_1^{AS}|s_0\rangle|\phi\rangle=\sqrt{t_1}|\chi\rangle_{AS}|\mu_1\rangle+\sqrt{t_2}|\chi^{\perp}\rangle_{AS}|\mu_2\rangle+\sum_{i=3}^{n}\sqrt{t_i}|s_i\rangle|i\rangle|\mu_i\rangle,$$

$$U_1^{AS}|s_0\rangle|\psi\rangle=\sqrt{t_1}|\chi\rangle_{AS}|\nu_1\rangle+\sqrt{t_2}|\chi^{\perp}\rangle_{AS}|\nu_2\rangle+\sum_{i=3}^{n}\sqrt{t_i}|s_i\rangle|i\rangle|\nu_i\rangle, \tag{15}$$

where $\{|s_i\rangle, i=1,\ldots,n\}$ is an orthonormal basis set and $|\chi\rangle_{AS}$ and $|\chi^{\perp}\rangle_{AS}$ lie in the subspace spanned by $\{|s_i\rangle|j\rangle, i,j=1,2\}$. Our task is to find suitable forms of $|\chi\rangle_{AS}$ and $|\chi^{\perp}\rangle_{AS}$. This also means that we select proper interaction between system AS. We find that if $t_1|\langle\mu_1|\nu_1\rangle|\geqslant t_2|\langle\mu_2|\nu_2\rangle|$, then we can choose

$$|\chi\rangle=\cos\alpha|s_1\rangle|1\rangle+\sin\alpha|s_2\rangle|2\rangle,$$
$$|\chi^{\perp}\rangle=|s_1\rangle|2\rangle. \tag{16}$$

The reason to choose such forms is that we want the state of AB in the second term to be a product vector. Substituting $\{|\chi\rangle_{AS},|\chi^{\perp}\rangle_{AS}\}$ with Eq. (15), we can get

$$U_1^{AS}|s_0\rangle|\phi\rangle=|s_1\rangle(\sqrt{t_1}\cos\alpha|1\rangle|\mu_1\rangle+\sqrt{t_2}|2\rangle|\mu_2\rangle)+|s_2\rangle\sqrt{t_1}\sin\alpha|2\rangle|\mu_1\rangle+\sum_{i=3}^{n}\sqrt{t_i}|s_i\rangle|i\rangle|\mu_i\rangle,$$

$$U_1^{AS}|s_0\rangle|\psi\rangle=|s_1\rangle(\sqrt{t_1}\cos\alpha|1\rangle|\nu_1\rangle+\sqrt{t_2}|2\rangle|\nu_2\rangle)+|s_2\rangle\sqrt{t_1}\sin\alpha|2\rangle|\nu_1\rangle+\sum_{i=3}^{n}\sqrt{t_i}|s_i\rangle|i\rangle|\nu_i\rangle. \tag{17}$$

It is clear that the corresponding terms remain the same probabilities. Our aim is to make the vectors of system AB in the first corresponding terms orthogonal, which gives

$$t_1\cos^2\alpha\langle\mu_1|\nu_1\rangle+t_2\langle\mu_2|\nu_2\rangle=0. \tag{18}$$

And from the supposition that $\langle \mu_1|\nu_1\rangle > 0, \langle \mu_2|\nu_2\rangle < 0$ and $t_1|\langle \mu_1|\nu_1\rangle| \geqslant t_2|\langle \mu_2|\nu_2\rangle|$, we can see it always has a solution,

$$\alpha = \arccos\sqrt{-\frac{t_2\langle \mu_2|\nu_2\rangle}{t_1\langle \mu_1|\nu_1\rangle}}. \tag{19}$$

And the inner product of the second corresponding terms of AB has the same phase as that of $\langle\phi|\psi\rangle$, so we eliminate one negative term. If for all the negative terms we can find corresponding positive terms satisfying the above conditions, repeat the process for each pair of terms and we can resolve all the negative terms and transform to the desired form. If for the negative term we cannot find its corresponding term satisfying the conditions, we can exchange the role of negative and positive terms. In this case, $\langle \mu_1|\nu_1\rangle < 0, \langle \mu_2|\nu_2\rangle > 0$, and $t_1|\langle \mu_1|\nu_1\rangle| \geqslant t_2|\langle \mu_2|\nu_2\rangle|$. We adopt the same protocol and the only difference is that the second term is negative. However, the absolute value of negative $t_1\langle \mu_1|\nu_1\rangle$ decreases to $|t_1 \sin^2\alpha\langle \mu_1|\nu_1\rangle|$. We can continue to reduce the absolute value of the negative term until it is transformed to positive, and we can always do that since $\langle\phi|\psi\rangle > 0$ means that the sum of the positive terms is larger than that of the negative ones. So we can indeed obtain the form of Eq. (13) and achieve the optimal discrimination by LOCC alone. In our discussion, it is easy to see this is also the case for complex $\langle\phi|\psi\rangle$. That completes our proof.

We have considered only the bipartite case so far, but our protocol can be easily generalized to two multipartite entangled states. As for the case of two tripartite states, we can group system BC as one and apply the protocol between A and BC to transform as Eq. (13),

$$U^{AS}|s_0\rangle|\phi\rangle_{ABC} = \sum_{i=1}^{m} \sqrt{t_i}|s_i\rangle|\phi_i\rangle_{ABC} + \sum_{i=m+1}^{N} \sqrt{t_i}|s_i\rangle|i\rangle_A|\mu_i\rangle_{BC},$$

$$U^{AS}|s_0\rangle|\psi\rangle_{ABC} = \sum_{i=1}^{m} \sqrt{t_i}|s_i\rangle|\psi_i\rangle_{ABC} + \sum_{i=m+1}^{N} \sqrt{t_i}|s_i\rangle|i\rangle_A|\nu_i\rangle_{BC}, \tag{20}$$

where $\langle\phi_i|\psi_i\rangle_{ABC} = 0$ and $\langle\mu_i|\nu_i\rangle_{BC}$ have the same phase as that of $\langle\phi|\psi\rangle_{ABC}$. Each pair $|\phi_i\rangle_{ABC}, |\psi_i\rangle_{ABC}$, can be exactly distinguished [7], while each pair $|\mu_i\rangle_{BC}, |\nu_i\rangle_{BC}$ can be optimally discriminated by BC with $P^L_{|i} = 1 - |\langle\mu_i|\nu_i\rangle_{BC}|$. And averaging over all the possible cases gives the overall probability $P^L = 1 - |\langle\phi|\psi\rangle|$ that is optimal. It is noticeable that the optimal conclusive discrimination can be achieved by LOCC in the condition that in general the operation performed by the last one provides the distinguishable information while all operations performed beforehand give no information about $|\phi\rangle$ and $|\psi\rangle$. The operations in advance help the last one to distinguish states optimally.

In conclusion, we have found the LOCC protocol achieving the optimal conclusive discrimination between two nonorthogonal entangled states occurring with equal prior probability. Generally, local POVM operations are required. Interestingly, the protocol shows that the distinguishable information is obtained at the last operation and all the ones beforehand give no information. The result strongly implies that optimal discrimination is also achieved by LOCC for unequal prior probability. But in such situations the idea that the prior operations give no information does not work, and much more intricate transformation is required for discussing this case further.

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