Squeezing is good at low information rates

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We evaluate the performance of a squeezed-state channel, in which classical information is conveyed by squeezed states. The evaluation is carried out by calculating the expurgated lower bound for the reliability function. As a result, we find that using squeezed states improves the channel performance near the zero information rate.

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I. INTRODUCTION

Recently quantum continuous channels, in which classical information is conveyed by quantum states parametrized continuously, have been widely noticed because they have various relevant applications [1–3]. The general formulas concerning the capacity and the reliability function for such channels were proved by Holevo and coworkers [4,5]. The capacity represents the ultimate capability of information transmission. This means we can transmit information at any rate R below the capacity within an arbitrary small error probability. Here the information rate R is defined by $(\ln M)/n$ when we transmit M messages with n use of the channel, that is, a block code of length n. On the other hand, the reliability function $E(R)$ shows the speed of the exponential decay of error probability P_e at any rate R below the capacity: $P_e \approx \exp[-nE(R)]$. The reliability function gives much more detailed and practical description of the asymptotical channel performance than the capacity. The importance of the reliability function is recognized well in the classical information theory, and extensive studies have been devoted to it [6]. On the analogy from the classical case, Holevo defined the random coding bound $E_r(R)$ and the expurgated bound $E_{ex}(R)$ for the quantum channel, and proved that these give the lower bounds for the reliability function $E(R)$ truly [4]. Note that only the random coding bound for mixed input states is yet to be proved. The random coding bound $E_r(R)$ gives good evaluation of the channel performance at high information rate, and is defined such that the value of R satisfying $E_r(R)=0$ is equivalent to the channel capacity. Thus the channel coding theorem can be shown immediately from the random coding bound [4]. On the other hand the expurgated bound $E_{ex}(R)$ is good at the information rate below the so-called ‘‘cutoff rate.’’

When we intend to find a block code to send messages faithfully and efficiently, it is important to evaluate beforehand the reliability function of the channel as the target performance of coding [7–9]; the evaluation is carried out by calculating its lower bounds. The random coding and the expurgated bounds for classical Gaussian channel can be obtained easily [6]. On the contrary, it is much more difficult to get these bounds for a quantum Gaussian channel. In fact it is known that the random coding bound cannot be obtained analytically, and the way to get the expurgated bound was found only for coherent-state channel [4,5]. This paper extends the result to more general case, that is, we calculate the expurgated bound for noiseless squeezed-state channel by considering suboptimal *a priori* probability distribution. As a result we find that using squeezed states improves the channel performance at information rates below the so-called ‘‘cutoff rate.’’ This conclusion is important, because communication at low rates is typical for many practical situations, such as, for example, in cryptographic applications.

The properties of squeezed-state channel has been investigated on the basis of two information theoretical quantities, the mutual information and the capacity. As mentioned above, this paper reveals many effects of squeezed states by turning our attention to the reliability function. To clarify the historical meaning of our result, let us recall the previous works. In 1970s, Yuen and Shapiro [10–12] fully revealed effects of the squeezed states on the semiclassical communication process with photon counting schemes that involve homodyne and heterodyne measurement processes. They proved that the maximum mutual information for an optimum noiseless squeezed-state channel with the homodyne measurement is $\ln(1+2N_t)$, showing the maximum signal-to-quantum noise ratio, where N_t represents the signal photon number. On the other hand, due to the general formula of the capacity [4], we can evaluate the ultimate capability of the quantum channel, considering more general quantum measurements including so-called entangled measurements. In particular, we can give the rigorous formulation and proof for Gordon's conjecture that the capacity of noiseless-coherent-state channel under input power constraint N_t is given by $(N_t+1)\ln(N_t+1)-N_t\ln N_t$ [4], which corresponds to the Yuen-Ozawa bound [13]. This implies that using squeezed states cannot improve the capacity [14]. However, squeezed states provide a simple coding scheme for achieving the Gordon's capacity $(N_t+1)\ln(N_t+1)-N_t\ln N_t$. Our previous paper showed [14] the capacity of certain noiseless squeezed-state channel achieves the Gordon's capacity by coding based on only real number alphabet, while we cannot

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avoid using complex number alphabet in the case of the coherent-state channel.

This paper is organized as follows. In Sec. II we remind the definitions of quantum continuous channel and formulate the general expurgated bound for the error probability. Then we specialize it to the case of Gaussian channel in one mode. In Sec. III, by evaluating the expurgated bound, we find that using squeezed states improves the channel performance at low information rates.

II. THE QUANTUM EXPURGATED BOUND

A. Case of general quantum-continuous channel

We shall describe the expurgated bound for quantum-continuous channels following [5]; the bound is expected to enable us to investigate the channel performance at any information rate below the cutoff rate. For reader's convenience, we start with recalling a general formulation of the quantum-continuous channel. Take as the input alphabet $\mathcal{A}$ an arbitrary Borel subset in a finite-dimensional Euclidean space $\mathcal{E}$. The quantum-continuous channel is described by a weakly continuous mapping $x \to \rho_x$ from the input alphabet $\mathcal{A}$ to the set of density operators in $\mathcal{H}$.

Let us consider also the product channel in the Hilbert space $\mathcal{H}^{\otimes n} = \mathcal{H} \otimes \cdots \otimes \mathcal{H}$, where a density operator

$$\rho_w = \rho_{x_1} \otimes \cdots \otimes \rho_{x_n}$$

corresponds to a word of length n, $w = (x_1, \ldots, x_n) \in \mathcal{A}^n$. As in the classical case, we should impose an additive constraint on the signals of quantum continuous channel

$$f(x_1) + \cdots + f(x_n) \leqslant nE, \tag{2.1}$$

where f is a fixed continuous positive function on $\mathcal{E}$. To carry out the random coding procedure for a quantum-continuous channel with such a constraint, we consider an *a priori* probability distribution $\pi = \pi(dx)$ on $\mathcal{A}$, satisfying

$$\int_{\mathcal{A}} f(x) \pi(dx) \leqslant E. \tag{2.2}$$

We denote by $\mathcal{P}_1$ a set of *a priori* probability distributions satisfying this inequality. The random coding procedure plays an essential role in the derivation of the expurgated bound.

Now we define

$$E_{ex}(R) = \max_{1 \leqslant s} [\max_{0 \leqslant p} \max_{\pi \in \mathcal{P}_1} \tilde{\mu}(\pi, s, p) - sR], \tag{2.3}$$

where $\tilde{\mu}$ is the quantum Gallager function given by

$$\tilde{\mu}(\pi, s, p) = -s \ln \int_{\mathcal{A}} \int_{\mathcal{A}} \exp\{[f(x) + f(y) - 2E]\} \times (\mathrm{Tr}\sqrt{\rho_x}\sqrt{\rho_y})^{1/s} \pi(dx) \pi(dy). \tag{2.4}$$

In [5] we derived the following expurgated bound for the error probability of the channel using codes of size $M = e^{nR}$, with codewords of length n,

$$P_e(n, e^{nR}) \lesssim e^{-nE_{ex}(R)},$$

where R is an information rate below the channel capacity C and $P_e(n, M)$ denotes the error probability achieved with the optimal code consisting of M code words of length n and the optimal quantum detection process described by a positive operator-valued measure.

To make clear the meaning of expurgated bound, let us recall some other quantities characterizing the quantum-continuous channel. When the information rate is less than the channel capacity, $R < C$, we can estimate the logarithmic rate of convergence of the error probability $P_e(n, e^{nR})$ by the *reliability function* defined as

$$E(R) = -\liminf_{n \to \infty} \frac{1}{n} \ln P_e(n, e^{nR}). \tag{2.5}$$

The expurgated bound gives a lower bound for such defined reliability function. For higher rates there is a better bound given by the random coding without expurgation [5]. This bound is called the *random coding bound* $E_r(R)$, which is analytically much less tractable than the expurgated bound. The quantity characterizing the channel performance at low information rate is the value $E(+0)$ of the reliability function at the zero rate. Fortunately we can obtain not only lower bound but also upper bound for $E(+0)$ [5],

$$E_{ex}(0) \leqslant E(+0) \leqslant -2 \min_{\pi \in \mathcal{P}_1} \int \int \ln \mathrm{Tr} |\sqrt{\rho_x}\sqrt{\rho_y}| \pi(dx) \pi(dy). \tag{2.6}$$

In particular, in the pure state case, upper and lower bounds coincide, that is, we have

$$E(+0) = E_{ex}(0). \tag{2.7}$$

Using the quantum Gallager function $\tilde{\mu}(\pi, s, p)$, we can define another interesting quantity, the *cutoff rate*

$$\tilde{C} = \max_{\pi \in \mathcal{P}_1} \max_{0 \leqslant p} \tilde{\mu}(\pi, 1, p), \tag{2.8}$$

which gives the channel performance at an intermediate information rate. The cutoff rate is a concept widely used in practical applications of classical information theory [15].

In Sec. III we evaluate the channel performance at low information rates by means of the expurgate bound. We should remark that this is based on the assumption that the expurgated bound $E_{ex}(R)$ gives a good approximation to the reliability function $E(R)$ below the cutoff rate. Strictly speaking, the expurgated bound is a mere lower bound for the reliability function. In particular, the expurgated bound is obtained by assuming square-root measurement as the quantum detection process. Hence it seems that we cannot eliminate the possibility that the optimum quantum detection process yields different results. Fortunately Eq. (2.7) shows that the value $E(+0)$ of the reliability function at zero rate is equal to $E_{ex}(0)$. This indicates that our assumption holds at least near zero information rate.

B. Case of quantum Gaussian channel

Although our interest focuses on noiseless quantum channels with squeezed states, we should consider a more general category of states, namely, Gaussian states [5,14]; this general formulation provides us with a powerful tool of calculating various quantities characterizing the channels. In the following we recall the definition of Gaussian density operator in one mode.

We consider quantum system described by operators q and p satisfying the Heisenberg canonical commutation relation (CCR)

$$[q,p]=i\hbar I, \quad [q,q]=0, \quad [p,p]=0. \tag{2.9}$$

Let $\mathcal{H}$ be the Hilbert space of irreducible representation of CCR. Introducing the unitary operator in $\mathcal{H}$ for a vector $z=[z_q,z_p]'$,

$$V(z)=\exp i(z_q q+z_p p), \tag{2.10}$$

we define the Gaussian density operator as follows:
The density operator ρ is called *Gaussian*, if its quantum characteristic function has the form

$$\mathrm{Tr}\,\rho V(z)=\exp\left[i m^t z-\frac{1}{2}z^t\alpha z\right], \tag{2.11}$$

where m is a two-dimensional column vector $[m_q,m_p]^t$ and

$$\alpha=\begin{bmatrix}\alpha^{qq} & \alpha^{qp}\\ \alpha^{qp} & \alpha^{pp}\end{bmatrix} \tag{2.12}$$

is a real symmetric matrix.
The mean m can be an arbitrary vector; the necessary and sufficient condition on the correlation matrix α is the uncertainty relation

$$\alpha^{qq}\alpha^{pp}-(\alpha^{qp})^2\geqslant\hbar^2/4, \tag{2.13}$$

where the state ρ is pure if and only if the equality holds.

Let us consider the quantum Gaussian channel $m\to\rho_m$, where ρ_m is the quantum Gaussian density operator with the mean m and the fixed correlation matrix α. In addition we assume the energy constraint (2.1) with $f(m)=m^t m/2$. Then we can take the *a priori* Gaussian distribution with the correlation matrix Σ:

$$\pi(dm)=\frac{1}{2\pi\sqrt{\det\Sigma}}\exp\left[-\frac{1}{2}m^t\Sigma^{-1}m\right]dm, \tag{2.14}$$

and compute the expurgated bound as follows (see [5])

$$\tilde{\mu}(\pi,s,p)=2psE+\frac{s}{2}\ln\det[(I-p\Sigma)\{I-p\Sigma +[sg_{1/s}(\sqrt{\det\alpha}/\hbar)\alpha]^{-1}\Sigma\}], \tag{2.15}$$

where

$$g_s(d)=\frac{1}{2d}\,\frac{(d+1/2)^s+(d-1/2)^s}{(d+1/2)^s-(d-1/2)^s}. \tag{2.16}$$

III. EXPURGATED BOUND FOR SQUEEZED-STATE CHANNELS

In this section we mainly discuss *noiseless squeezed-state channel*, $m\to\rho_m$. Squeezed state, which is just a pure Gaussian state, in the one-mode case is conventionally represented as $S(\zeta)|0\rangle$ with $\zeta=\gamma e^{i\theta}$ and $S(\zeta)=\exp([\zeta^* a^2-\zeta(a^\dagger)^2])$, where a is the annihilation operator of the mode. This physical parametrization is related to Eq. (2.11) via formulas

$$\alpha^{qq}=\frac{\hbar}{2}[\cosh 2\gamma-\sinh 2\gamma\cos\theta]$$

$$\alpha^{pp}=\frac{\hbar}{2}[\cosh 2\gamma-\sinh 2\gamma\cos\theta]$$

$$\alpha^{qp}=\frac{\hbar}{2}\sinh 2\gamma\sin\theta. \tag{3.1}$$

In the following we put $\theta=0$ for simplicity and denote by $\alpha(\gamma)$ the correlation matrix with the elements (3.1). To see the effects of squeezing, we consider a transmitter energy constraint, given by taking

$$f(m)=\frac{1}{2}[m^t m+Sp\,\alpha(\gamma)], \tag{3.2}$$

in Eq. (2.1). Then the constraint on *a priori* probability distribution (2.2) takes the form

$$\frac{1}{2}Sp(\Sigma+\alpha(\gamma))\leqslant E, \tag{3.3}$$

and hence the Gallager function $\tilde{\mu}(\pi,s,p)$ is modified by replacing the term E in Eq. (2.15) with $E-Sp\,\alpha(\gamma)/2$.

When ρ_0 is a coherent state, that is $\gamma=0$, the expurgated bound can be computed as [4,5]

$$E_{ex}(R) =\begin{cases}2N_t(1-\sqrt{1-e^{-R}}), & R<\ln\vartheta(2N_t)\\ 2[N_t+1-\vartheta(2N_t)]+\ln\vartheta(2N_t)-R, & \text{otherwise,}\end{cases} \tag{3.4}$$

where

$$\vartheta(x) = \frac{1+\sqrt{x^2+1}}{2}.$$

Here N_t represents an average number of signal photons corresponding to energy bound E, $E=\hbar(N_t+1/2)$. On the other hand, we have not yet found the way to perform analytically maximization in Eq. (2.3) and to compute the expurgated bound when ρ_0 is a squeezed state.

In this paper we evaluate the expurgated bound for the squeezed-state channel, by considering suboptimal *a priori* probability distribution. That is, we restrict the *a priori* distributions to Gaussians with correlation matrix of the form

$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}, \tag{3.5}$$

where $\sigma_1\sigma_2=0$. Among *a priori* distributions that have the form (3.5) and satisfy the constraint (3.3), the optimal one is given by

$$\sigma_1 = 2E_n(\gamma), \quad \sigma_2=0, \quad \text{if } \gamma \geqslant 0$$

$$\sigma_1 = 0, \quad \sigma_2 = 2E_n(\gamma), \quad \text{otherwise,} \tag{3.6}$$

where

$$E_n(\gamma) = E - \frac{1}{2} Sp\,\alpha(\gamma) = \hbar(N_t - \sinh^2\gamma). \tag{3.7}$$

In the following we restrict ourselves to the case $\gamma \geqslant 0$ for simplicity and denote by $\hat{\pi}$ the *a priori* distribution with elements (3.6). Although such degenerate *a priori* distribution might not maximize $\tilde{\mu}$ in Eq. (2.3), it allows us to evaluate the expurgated bound for noiseless squeezed-state channels. Indeed, the degenerate *a priori* distribution of the form (3.5) is known to be optimal in several cases.

(i) Among semiclassical photodetections, the homodyne detection of the squeezed-state channel with squeezing parameter $\gamma = \pm \ln\sqrt{2N_t+1} \equiv \pm\gamma_0$ is optimal [12]. The homodyne detection corresponds to a channel with the degenerate *a priori* distribution.

(ii) The degenerate *a priori* distribution achieves the capacity of squeezed-state channel with $\gamma = \pm\gamma_0$ [14].

(iii) The degenerate *a priori* distribution is optimal for $E_{ex}(0)$ of squeezed-state channel [5].

The quantum Gallager function for this *a priori* distribution $\hat{\pi}$ is obtained as

$$\tilde{\mu}(\hat{\pi},s,p) = 2psE_n(\gamma) + \frac{s}{2}\ln[1-2pE_n(\gamma)]\left[1-2pE_n(\gamma) + \frac{4E_n(\gamma)}{s\hbar}e^{2\gamma}\right]. \tag{3.8}$$

Using this function, we obtain the approximation to the expurgated bound as follows:

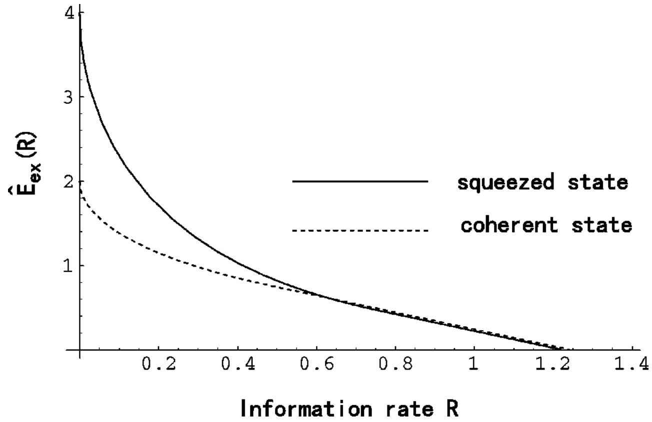

FIG. 1. $\hat{E}_{ex}(R)$ with respect to information rate R for the squeezed-state channel with $\gamma=\gamma_0$ and the coherent-state channel with $N_t=1$, where the information rate R and the expurgated bound $\hat{E}_{ex}(R)$ are measured in nats.

$$\hat{E}_{ex}(R) = \max_{1\leqslant s}[\max_{0\leqslant p}\tilde{\mu}(\hat{\pi},s,p) - sR] \quad [\leqslant E_{ex}(R)]. \tag{3.9}$$

Here $\hat{E}_{ex}(R)$ and $E_{ex}(R)$ have the same value at zero rate [5]

$$E_{ex}(0) = \hat{E}_{ex}(0). \tag{3.10}$$

Calculating Eq. (3.9) and finding the optimum squeezing parameter $\gamma=\gamma_0$ (see the Appendix), we obtain

$$\hat{E}_{ex}(R) = \begin{cases} 2N_t(N_t+1)(1-\sqrt{1-e^{-2R}}), & R \leqslant R_0 \\ \hat{C} - R, & \text{otherwise,} \end{cases} \tag{3.11}$$

where

$$R_0 = \frac{1}{2}\ln\vartheta(4N_t(N_t+1)) \tag{3.12}$$

and $\hat{C} \equiv \hat{E}_{ex}(R_0) - R_0$ is calculated as

$$\hat{C} = 2N_t(N_t+1) + 1 - \vartheta(4N_t(N_t+1)) + \frac{1}{2}\ln\vartheta(4N_t(N_t+1)). \tag{3.13}$$

Note that $\hat{C}$ gives a lower bound for the cutoff rate $\tilde{C}$ given by Eq. (2.8).

In Fig. 1 we present graphs of $\hat{E}_{ex}(R)$ for the squeezed state channel with $\gamma=\gamma_0$ and the coherent-state channel, when $N_t=1$. Note that the information rate R satisfying $\hat{E}_{ex}(R)=0$ is equal to $\hat{C}$. This figure shows that using squeezed states under the transmitter energy constraint noticeably increases the value of $\hat{E}_{ex}(R)$ at low information rates R. From this we conclude that the squeezing improves the channel performance at low information rate. Strictly speaking, we cannot evaluate channel performance precisely by the expurgated bound $\hat{E}_{ex}(R)$. In order to confirm our statement we should evaluate the reliability function $E(R)$ directly. Fortunately Eqs. (2.7) and (3.10) shows that the value $E(+0)$ of the reliability function at zero rate is equal to $\hat{E}_{ex}(0)$. Now, from Eqs. (3.4) and (3.11), we can find that the value of $\hat{E}_{ex}(0)$ [equal to $E(+0)$] for squeezed states, $2N_t(N_t+1)$, is larger than that for coherent states $2N_t$. Here the reliability function $E(R)$ is monotonously decreasing and the value of $E(+0)$ is representative of the behavior of $E(R)$ at low information rates. Thus we can confirm the statement that squeezing is good at low information rates. On the other hand, the squeezing is not good at high information rates. Indeed it has been shown in [14] that the channel capacity, which reflects the behavior of reliability function at high information rates, is not improved by squeezing.

IV. CONCLUDING REMARKS

As a serial work on the quantum capacity and coding theorem, we have calculated the expurgated bound for the squeezed-state channel. As a result, we have found that using squeezed states improves asymptotic channel performance at low information rates, while it does not help near the channel capacity. In this paper, on the analogy of classical case, we assume that the expurgated bound gives a faithful evaluation of the channel performance at information rates below the cutoff rate. Based on this assumption, we can conclude that the Fig. 1 shows the efficiency of squeezing. In addition, seeing that the expurgated bound coincides with the reliability function at zero rate, we have confirmed that our assumption holds at least near zero information rate. The problem to find good codes satisfying the expurgated bound remains; in the classical case such codes have been already known [7].

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APPENDIX

Let us prove Eq. (3.11). Optimizing Eq. (3.8) with respect to p, we have

$$\max_{0\leq p}\tilde{\mu}(\hat{\pi},s,p)-sR=\frac{2E_n(\gamma)e^{2\gamma}}{\hbar}+s-s\,\vartheta\left(\frac{4E_n(\gamma)e^{2\gamma}}{s\hbar}\right)+\frac{s}{2}\ln\vartheta\left(\frac{4E_n(\gamma)e^{2\gamma}}{s\hbar}\right)-sR. \tag{A1}$$

Taking derivative of Eq. (A1) with respect to s, we obtain the equation

$$\vartheta\left(\frac{4E_n(\gamma)e^{2\gamma}}{s\hbar}\right)=e^{2R}, \tag{A2}$$

the solution of which is

$$s=\frac{2E_n(\gamma)e^{2\gamma}}{\hbar}\frac{1}{\sqrt{e^{4R}-e^{2R}}}. \tag{A3}$$

If this is larger than 1, which is equivalent to

$$R\leq\frac{1}{2}\ln\vartheta\left(\frac{4E_n(\gamma)e^{2\gamma}}{\hbar}\right)\equiv R_0, \tag{A4}$$

then the maximum is achieved for the value of s given by Eq. (A3) and is equal to

$$\hat{E}_{ex}(R)=\frac{2E_n(\gamma)e^{2\gamma}}{\hbar}(1-\sqrt{1-e^{-2R}}). \tag{A5}$$

In the range

$$R>\frac{1}{2}\ln\vartheta\left(\frac{4E_n(\gamma)e^{2\gamma}}{\hbar}\right), \tag{A6}$$

we have

$$\hat{E}_{ex}(R)=\hat{C}-R, \tag{A7}$$

where $\hat{C}=\hat{E}_{ex}(R_0)-R_0$. Since

$$\frac{2e^{2\gamma}E_n(\gamma)}{\hbar}=-\frac{1}{2}[e^{2\gamma}-(2N_t+1)]^2+2N_t(N_t+1) \tag{A8}$$

holds, we can find $\gamma=\gamma_0=\ln\sqrt{2N_t+1}$ maximizes $\hat{E}_{ex}(R)$. Substituting $\gamma=\gamma_0$ into Eq. (A5), we have

$$\hat{E}_{ex}(R)=2N_t(N_t+1)(1-\sqrt{1-e^{-2R}}), \tag{A9}$$

in the range (4). Then $\hat{C}$ is calculated as

$$\hat{C}=2N_t(N_t+1)+1-\vartheta(4N_t(N_t+1))+\frac{1}{2}\ln\vartheta(4N_t(N_t+1)). \tag{A10}$$

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