Quantum mechanics gives stability to a Nash equilibrium

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We consider a slightly modified version of the rock-scissors-paper (RSP) game from the point of view of evolutionary stability. In its classical version the game has a mixed Nash equilibrium (NE) not stable against mutants appearing in small numbers. We find a quantized version of the RSP game for which the classical mixed NE becomes stable.

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I. INTRODUCTION

Long played as a children's pastime, or as an odd-man-out selection process, the rock-scissors-paper (RSP) game is a game for two players typically played using the players' hands. The two players opposing each other, tap their fist, in their open palms three times (saying rock, scissors, paper) and then show one of three possible gestures. The rock wins against the scissors (crushes it) but loses against the paper (is wrapped into it). The scissors wins against the paper (cuts it) but loses against the rock (is crushed by it). The paper wins against the rock (wraps it) but loses against the scissors (is cut by it).

In a slightly modified version of the RSP game both players get a small premium ϵ for a draw. This game can be represented by the following payoff matrix:

$$\begin{pmatrix} & R & S & P \\ R & -\epsilon & 1 & -1 \\ S & -1 & -\epsilon & 1 \\ P & 1 & -1 & -\epsilon \end{pmatrix}, \tag{1}$$

where $-1<\epsilon<0$. The matrix of the usual game is obtained when ϵ is zero in the matrix (1).

One cannot win if one's opponent knew which strategy was going to be picked. For example, picking rock consistently, all the opponent needs to do is pick paper and he would win. Players find soon that in case predicting opponent's strategy is not possible, the best strategy is to pick rock, scissors, or paper at random. In other words, the player selects rock, scissors, or paper with a probability of $\frac{1}{3}$. In case opponent's strategy is predictable, picking a strategy at random with a probability of $\frac{1}{3}$ is not the best thing to do unless the opponent is doing the same [1].

We explore evolutionarily stable strategies (ESSs) in a quantized RSP game in its modified form. Originally defined by Smith and Price [2] as a behavioral phenotype, an ESS cannot be invaded by a mutant strategy when a population is playing it. A mutant strategy does things in different ways than most of a population does. Smith and Price considered a symmetric game where the players are anonymous. Let $P[u,v]$ be the payoff to a player playing u against the player playing v. Strategy u is an ESS if for any alternative strategy v, the following two requirements are satisfied:

$$P[u,u] \geqslant P[v,u], \tag{2}$$

and in the case $P[u,u]=P[v,u]$,

$$P[u,v] > P[v,v]. \tag{3}$$

Requirement (2) is in fact the Nash condition and says that no single individual can gain by unilaterally changing his/her strategy from u to v. An ESS is in fact a stable Nash equilibrium (NE) in a symmetric game and its stability is against a small group of mutants [2,3].

A straight analysis of the modified RSP game of matrix (1) shows that playing each of the three different pure strategies with a fixed equilibrial probability $\frac{1}{3}$ constitutes a mixed NE. However it is not an ESS because ϵ is negative [3].

In an earlier paper [4] we showed that in the quantized version of certain asymmetric games between two players, it is possible to make appear or disappear an ESS that is a pure strategy NE by controlling the initial state used to play the game. Because a classical game is embedded in its quantized form, therefore, it is possible that a pure strategy NE remains intact in both classical and certain quantized forms of the same game but is an ESS in only one form. Later we presented an example [5] of a symmetric game between two players for which a pure-strategy NE is an ESS in the classical version of the game but not so in a quantized form even when it remains NE in both versions. This is more relevant because the idea of an ESS was originally defined for symmetric contests. We also showed [6] that mixed strategy ESSs can be related to entanglement and can be affected by quantization for three-player games. However this is not the case for two-player games when the quantum state is in a simpler form proposed by Marinatto and Weber (MW) [7] in their scheme to quantize a two-player game in the normal form.

MW [7] expanded on the scheme proposed by Eisert, Wilkens, and Lewenstein [8] for the game of prisoner's dilemma. They showed that the dilemma does not exist in a quantum version of the game. The motivation of MW was to remove the need of an unentangling gate in the scheme of Eisert *et al.* [9,10]. In our effort to extend the ideas of evolutionary game theory toward quantum games we found MW's scheme more suitable for the following reasons.

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(a) In the usual setup of a symmetric bimatrix evolutionary game, two pure strategies are assumed such that players can play a mixed strategy by their probabilistic combination. In a similar way, players in MW's scheme can play a mixed strategy by applying the two unitary operators in their possession with classical probabilities that are both non-zero.

(b) The usual definition of "fitness" of a mixed strategy in evolutionary games [1] can be given a straight-forward extension in MW's scheme [4]. It is done when in the quantum game, playing a pure strategy means that a player uses only one unitary operator out of the two.

(c) The theory of ESSs in evolutionary game theory is developed mostly for situations when players are anonymous and possess a discrete number of pure strategies. We find that the ESS idea can be extended towards quantum settings more easily in MW's scheme than in the scheme of Eisert *et al.* involving a continuum of the pure strategies that players have as an option to play. The idea of an ESS as a stable equilibrium is confronted with problems when players have an access to a continuum of pure strategies [11].

In this paper we want to extend our previous results regarding effects of quantization on evolutionary stability for a modified version of the RSP game. This game is different because now classically each player possesses three pure strategies instead of two. A classical mixed NE exists that is not an ESS. Our motivation is to explore the possibility that the classical mixed NE becomes an ESS for some initial quantum state. We show that such a quantum state not only exists but is also easy to find.

II. QUANTIZED RSP GAME

Using simpler notation, $R\sim 1, S\sim 2, P\sim 3$, we quantize this game via MW's scheme [7]. We allow the two players to be in possession of three unitary operators I, C, and D defined as

$$I|1\rangle=|1\rangle, \quad C|1\rangle=|3\rangle, \quad D|1\rangle=|2\rangle,$$

$$I|2\rangle=|2\rangle, \quad C|2\rangle=|2\rangle, \quad D|2\rangle=|1\rangle,$$

$$I|3\rangle=|3\rangle, \quad C|3\rangle=|1\rangle, \quad D|3\rangle=|3\rangle,$$

where $C^{\dagger}=C=C^{-1}$, $D^{\dagger}=D=D^{-1}$, and I is identity operator. We also start with a general payoff matrix for two players, Alice and Bob, each having three strategies,

$$\begin{bmatrix} & 1 & 2 & 3 \\ 1 & (\alpha_{11},\beta_{11}) & (\alpha_{12},\beta_{12}) & (\alpha_{13},\beta_{13}) \\ 2 & (\alpha_{21},\beta_{21}) & (\alpha_{22},\beta_{22}) & (\alpha_{23},\beta_{23}) \\ 3 & (\alpha_{31},\beta_{31}) & (\alpha_{32},\beta_{32}) & (\alpha_{33},\beta_{33}) \end{bmatrix}, \tag{4}$$

where α_{ij}, β_{ij} are payoffs to Alice and Bob, respectively, where Alice pays i and Bob plays j and $1\leqslant i,j\leqslant 3$. Suppose Alice and Bob apply operators C, D, and I with probabilities p, p_1, $(1-p-p_1)$ q, q_1, and $(1-q-q_1)$, respectively. Let us represent the initial state of the game by ρ_{in}. After Alice plays her strategy, the state changes to

$$\rho_{\text{in}}^{A}=(1-p-p_1)I_A\rho_{\text{in}}I_A^{\dagger}+pC_A\rho_{\text{in}}C_A^{\dagger}+p_1D_A\rho_{\text{in}}D_A^{\dagger}. \tag{5}$$

The final density matrix after Bob too has played his strategy is

$$\overset{A,B}{\rho_f}=(I-q-q_1)I_B\rho_{\text{in}}^{A}I_B^{\dagger}+qC_B\rho_{\text{in}}^{A}C_B^{\dagger}+q_1D_B\rho_{\text{in}}^{A}D_B^{\dagger}. \tag{6}$$

This density matrix can be written as

$$\begin{aligned}\overset{A,B}{\rho_f}=&(1-p-p_1)(1-q-q_1)\{I_A\otimes I_B\rho_{\text{in}}I_A^{\dagger}\otimes I_B^{\dagger}\}\\&+p(1-q-q_1)\{C_A\otimes I_B\rho_{\text{in}}C_A^{\dagger}\otimes I_B^{\dagger}\}+p_1(1-q-q_1)\\&\times\{D_A\otimes I_B\rho_{\text{in}}D_A^{\dagger}\otimes I_B^{\dagger}\}+(1-p-p_1)q\{I_A\otimes C_B\rho_{\text{in}}I_A^{\dagger}\\&\otimes C_B^{\dagger}\}+pq\{C_A\otimes C_B\rho_{\text{in}}C_A^{\dagger}\otimes C_B^{\dagger}\}+p_1q\{D_A\otimes C_B\rho_{\text{in}}D_A^{\dagger}\\&\otimes C_B^{\dagger}\}+(1-p-p_1)q_1\{I_A\otimes D_B\rho_{\text{in}}I_A^{\dagger}\otimes D_B^{\dagger}\}+pq_1\{C_A\\&\otimes D_B\rho_{\text{in}}C_A^{\dagger}\otimes D_B^{\dagger}\}+p_1q_1\{D_A\otimes D_B\rho_{\text{in}}D_A^{\dagger}\otimes D_B^{\dagger}\}.\end{aligned} \tag{7}$$

The basis vectors of initial quantum state with three pure classical strategies are $|11\rangle$, $|12\rangle$, $|13\rangle$, $|21\rangle$, $|22\rangle$, $|23\rangle$, $|31\rangle$, $|32\rangle$, and $|33\rangle$. Setting the initial quantum state to the following general form:

$$\begin{aligned}|\psi_{\text{in}}\rangle=&c_{11}|11\rangle+c_{12}|12\rangle+c_{13}|13\rangle+c_{21}|21\rangle+c_{22}|22\rangle\\&+c_{23}|23\rangle+c_{31}|31\rangle+c_{32}|32\rangle+c_{33}|33\rangle\end{aligned} \tag{8}$$

with normalization

$$\begin{aligned}&|c_{11}|^2+|c_{12}|^2+|c_{13}|^2+|c_{21}|^2+|c_{22}|^2+|c_{23}|^2+|c_{31}|^2+|c_{32}|^2\\&\quad+|c_{33}|^2=1\end{aligned} \tag{9}$$

and writing payoff operators for Alice and Bob as [7]

$$\begin{aligned}(P_{A,B})_{\text{oper.}}=&(\alpha,\beta)_{11}|11\rangle\langle 11|+(\alpha,\beta)_{12}|12\rangle\langle 12|\\&+(\alpha,\beta)_{13}|13\rangle\langle 13|+(\alpha,\beta)_{21}|21\rangle\langle 21|\\&+(\alpha,\beta)_{22}|22\rangle\langle 22|+(\alpha,\beta)_{23}|23\rangle\langle 23|\\&+(\alpha,\beta)_{31}|31\rangle\langle 31|+(\alpha,\beta)_{32}|32\rangle\langle 32|\\&+(\alpha,\beta)_{33}|33\rangle\langle 33|;\end{aligned} \tag{10}$$

the payoffs to Alice or Bob can be obtained by taking a trace of $[(P_{A,B})_{\text{oper.}}]\overset{A,B}{\rho_f}$, i.e. [7],

$$P_{A,B}=\text{tr}\{(P_{A,B})_{\text{oper.}}\}\overset{A,B}{\rho_f}\}. \tag{11}$$

Payoff to Alice, for example, can be written as

$$P_A=\Phi\Omega\Upsilon^{T}, \tag{12}$$

where T is for transpose and the matrices Φ, Ω, and Υ are

$$\Phi=[(1-p-p_1)(1-q-q_1) \quad p(1-q-q_1) \quad p_1(1-q-q_1) \quad (1-p-p_1)q \quad pq \quad p_1q \quad (1-p-p_1)q_1 \quad pq_1 \quad p_1q_1],$$

$$\Upsilon=[\alpha_{11} \quad \alpha_{12} \quad \alpha_{13} \quad \alpha_{21} \quad \alpha_{22} \quad \alpha_{23} \quad \alpha_{31} \quad \alpha_{32} \quad \alpha_{33}],$$

$$\Omega=\begin{bmatrix} |c_{11}|^2 & |c_{12}|^2 & |c_{13}|^2 & |c_{21}|^2 & |c_{22}|^2 & |c_{23}|^2 & |c_{31}|^2 & |c_{32}|^2 & |c_{33}|^2 \\ |c_{31}|^2 & |c_{32}|^2 & |c_{33}|^2 & |c_{21}|^2 & |c_{22}|^2 & |c_{23}|^2 & |c_{11}|^2 & |c_{12}|^2 & |c_{13}|^2 \\ |c_{21}|^2 & |c_{22}|^2 & |c_{23}|^2 & |c_{11}|^2 & |c_{12}|^2 & |c_{13}|^2 & |c_{31}|^2 & |c_{32}|^2 & |c_{33}|^2 \\ |c_{13}|^2 & |c_{12}|^2 & |c_{11}|^2 & |c_{23}|^2 & |c_{22}|^2 & |c_{21}|^2 & |c_{33}|^2 & |c_{32}|^2 & |c_{31}|^2 \\ |c_{33}|^2 & |c_{32}|^2 & |c_{31}|^2 & |c_{23}|^2 & |c_{22}|^2 & |c_{21}|^2 & |c_{13}|^2 & |c_{12}|^2 & |c_{11}|^2 \\ |c_{23}|^2 & |c_{22}|^2 & |c_{21}|^2 & |c_{13}|^2 & |c_{12}|^2 & |c_{11}|^2 & |c_{33}|^2 & |c_{32}|^2 & |c_{31}|^2 \\ |c_{12}|^2 & |c_{11}|^2 & |c_{13}|^2 & |c_{22}|^2 & |c_{21}|^2 & |c_{23}|^2 & |c_{32}|^2 & |c_{31}|^2 & |c_{33}|^2 \\ |c_{32}|^2 & |c_{31}|^2 & |c_{33}|^2 & |c_{22}|^2 & |c_{21}|^2 & |c_{23}|^2 & |c_{12}|^2 & |c_{11}|^2 & |c_{13}|^2 \\ |c_{22}|^2 & |c_{21}|^2 & |c_{23}|^2 & |c_{12}|^2 & |c_{11}|^2 & |c_{13}|^2 & |c_{32}|^2 & |c_{31}|^2 & |c_{33}|^2 \end{bmatrix}. \tag{13}$$

This payoff is for the general matrix given in Eq. (4). In case an exchange of strategies by Alice and Bob also exchanges their respective payoffs, the game is said to be symmetric. The idea of evolutionary stability in mathematical biology is generally considered in symmetric contests. In a symmetric contest, payoff to a player is then defined by his strategy and not by his identity. Payoffs in a classical mixed strategy game can be obtained from Eq. (11) when the initial state is $|\psi_{\text{in}}\rangle=|11\rangle$ and the game is symmetric when $\alpha_{ij}=\beta_{ji}$ in the matrix (4). Similarly the quantum game played using the general quantum state of Eq. (8) becomes symmetric when $|c_{ij}|^2=|c_{ji}|^2$ for all constants c_{ij} in the initial quantum state of Eq. (8). This condition should hold along with the requirement $\alpha_{ij}=\beta_{ji}$ on the matrix (4). The payoff to Alice or Bob, i.e., P_A, P_B then not need a subscript and we can use only P.

We now come to the question of evolutionary stability in this quantized version of the RSP game.

III. EVOLUTIONARY STABILITY IN THE QUANTIZED RSP GAME

We define a strategy by a pair of numbers (p,p_1) when players are playing the quantized RSP game. It is understood that the identity operator is then, applied with probability $1-p-p_1$. Similar to the requirements in Eqs. (2) and (3), the conditions for making a strategy (p^*,p_1^*) an ESS can now be written [2,3] as

$$(1) \quad P\{(p^*,p_1^*),(p^*,p_1^*)\}>P\{(p,p_1),(p^*,p_1^*)\},$$

$$(2) \quad \text{if } P\{(p^*,p_1^*),(p^*,p_1^*)\}=P\{(p,p_1),(p^*,p_1^*)\}$$

$$\text{then } P\{(p^*,p_1^*),(p,p_1)\}>P\{(p,p_1),(p,p_1)\}. \tag{14}$$

Suppose (p^*,p_1^*) is a mixed NE, then

$$\left\{ \frac{\partial P}{\partial p}\bigg|_{\substack{p=q=p^* \\ p_1=q_1=p_1^*}}(p^*-p)+\frac{\partial P}{\partial p_1}\bigg|_{\substack{p=q=p^* \\ p_1=q_1=p_1^*}}(p_1^*-p_1)\right\}\geqslant 0. \tag{15}$$

Using substitutions

$$\begin{array}{ll} |c_{11}|^2-|c_{31}|^2=\Delta_1, & |c_{21}|^2-|c_{11}|^2=\Delta'_1 \\ |c_{13}|^2-|c_{33}|^2=\Delta_2, & |c_{22}|^2-|c_{12}|^2=\Delta'_2 \\ |c_{12}|^2-|c_{32}|^2=\Delta_3, & |c_{23}|^2-|c_{13}|^2=\Delta'_3 \end{array} \tag{16}$$

we get

$$\begin{aligned} \frac{\partial P}{\partial p}\bigg|_{\substack{p=q=p^* \\ p_1=q_1=p_2^*}} &= p^*(\Delta_1-\Delta_2)\{(\alpha_{11}+\alpha_{33})-(\alpha_{13}+\alpha_{31})\} \\ &\quad +p_1^*(\Delta_1-\Delta_3)\{(\alpha_{11}+\alpha_{32})-(\alpha_{12}+\alpha_{31})\} \\ &\quad -\Delta_1(\alpha_{11}-\alpha_{31})-\Delta_2(\alpha_{13}-\alpha_{33}) \\ &\quad -\Delta_3(\alpha_{12}-\alpha_{32}) \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\partial P}{\partial p}\bigg|_{\substack{p=q=p^* \\ p_1=q_1=p_1^*}} &= p^*(\Delta'_3-\Delta'_1)\{(\alpha_{11}+\alpha_{23})-(\alpha_{13}+\alpha_{21})\} \\ &\quad +p_1^*(\Delta'_2-\Delta'_1)\{(\alpha_{11}+\alpha_{22})-(\alpha_{12}+\alpha_{21})\} \\ &\quad +\Delta'_1(\alpha_{11}-\alpha_{21}+\Delta'_2(\alpha_{12}-\alpha_{22}) \\ &\quad +\Delta'_3(\alpha_{13}-\alpha_{23}). \end{aligned} \tag{18}$$

For the matrix (1) the above equations can be written as

$$\begin{aligned} \frac{\partial P}{\partial p_1}\bigg|_{\substack{p=q=p^* \\ p_1=q_1=p_1^*}} &= \Delta_1\{-2\epsilon p^*-(3+\epsilon)p_1^*+(1+\epsilon)\} \\ &\quad +\Delta_2\{2\epsilon p^*+(1-\epsilon)\}+\Delta_3\{(3+\epsilon)p_1^*-2\}, \end{aligned} \tag{19}$$

$$\left.\frac{\partial P}{\partial p_1}\right|_{\substack{p=q=p^* \\ p_1=q_1=p_1^*}} = \Delta_1'\{-p^*(3-\epsilon)+2\epsilon p_1^*+(1-\epsilon)\} - \Delta_2'\{2\epsilon p_1^*-(1+\epsilon)\}+\Delta_3'\{(3-\epsilon)p^*-2\} \tag{20}$$

Also the payoff difference in the second condition of an ESS given in Eq. (14) reduces to

$$\begin{aligned} &P\{p^*,p_1^*),(p,p_1)\}-P\{(p,p_1),(p,p_1)\} \\ &=(p^*-p)[-\Delta_1\{2\epsilon p+(3+\epsilon)p_1-(1+\epsilon)\} \\ &\quad+\Delta_2\{2\epsilon p+(1-\epsilon)\}+\Delta_3\{(3+\epsilon)p_1-2\}] \\ &\quad+(p_1^*-p_1)[-\Delta_1'\{(3-\epsilon)p,-2\epsilon p_1-1-\epsilon)\} \\ &\quad-\Delta_2'\{2\epsilon p_1-(1+\epsilon)\}+\Delta_3'\{(3-\epsilon)p-2\}]. \end{aligned} \tag{21}$$

With the substitutions $p^*-p=x$ and $p_1^*-p_1=y$, the above payoff difference is

$$\begin{aligned} &P\{(p^*,p_1^*),(p,p_1)\}-P\{(p,p_1),(p,p_1)\} \\ &=\Delta_1 x\{2\epsilon x+(3+\epsilon)y\}-\Delta_2(2\epsilon x^2)-\Delta_3 xy(3+\epsilon) \\ &-\Delta_1' y\{2\epsilon y-(3-\epsilon)x\}+\Delta_2'(2\epsilon y^2)-\Delta_3' xy(3-\epsilon) \end{aligned} \tag{22}$$

provided

$$\left.\frac{\partial P}{\partial p}\right|_{\substack{p=q=p^* \\ p_1=q_1=p_1^*}}=0, \quad \left.\frac{\partial P}{\partial p_1}\right|_{\substack{p=q=p^* \\ p_1=q_1=p_1^*}}=0. \tag{23}$$

The conditions in Eq. (23) together define the mixed NE (p^*,p_1^*). Consider now the modified RSP game in classical form obtained by setting $|c_{11}|^2=1$ and all the rest of the constants to zero. The Eqs. (23) now become

$$\begin{aligned} &-2\epsilon p^*-(\epsilon+3)p_1^*+(\epsilon+1)=0, \\ &(-\epsilon+3)p^*-2\epsilon p_1^*+(\epsilon-1)=0, \end{aligned} \tag{24}$$

and $p^*=p_1^*=\frac{1}{3}$ is obtained as a mixed NE for the whole range $-1<\epsilon<0$. From Eq. (22) we get

$$\begin{aligned} &P\{(p^*,p_1^*),(p,p_1)\}-P\{(p,p_1),(p,p_1)\} \\ &=2\epsilon(x^2+y^2+xy) \\ &=\epsilon\{(x+y)^2+(x^2+y^2)\}\leqslant 0. \end{aligned} \tag{25}$$

In the classical form of the RSP game, therefore, the mixed NE $p^*=p_1^*=\frac{1}{3}$ is a NE but not an ESS because the second condition of ESS given in Eq. (14) does not hold.

Define now a new initial state as follows:

$$|\psi_{\mathrm{in}}\rangle=\tfrac{1}{2}\{|12\rangle+|21\rangle+|13\rangle+|31\rangle\} \tag{26}$$

and use it to play the game instead of the classical game obtained from $|\psi_{\mathrm{in}}\rangle=|11\rangle$. The strategy $p^*=p_1^*=\frac{1}{3}$ still forms a mixed NE because the conditions given by Eq. (23) hold true for it. However the payoff difference of Eq. (22) is now given below when $-1<\epsilon<0$ and $x, y\neq 0$,

$$\begin{aligned} &P\{(p^*,p_1^*),(p,p_1)\}-P\{(p,p_1),(p,p_1)\} \\ &=-\epsilon\{(x+y)^2+(x^2+y^2)\}>0. \end{aligned} \tag{27}$$

Therefore, the mixed Nash equilibrium $p^*=p_1^*=\frac{1}{3}$ not existing as an ESS in the classical form of this modified RSP game becomes an ESS when the game is quantized and played using the initial entangled quantum state given by Eq. (26).

Note that from Eq. (11) the payoff sum to Alice and Bob P_A+P_B can be obtained for both the classical mixed-strategy game (i.e., $|\psi_{\mathrm{in}}\rangle=|11\rangle$) and the quantum game played using the quantum state of Eq. (26). For the matrix (1) we write these sums as $(P_A+P_B)_{\mathrm{cl}}$ and $(P_A+P_B)_{\mathrm{qu}}$ for classical mixed-strategy and quantum games, respectively, and find

$$(P_A+P_B)_{\mathrm{cl}}=-2\epsilon\{(1-p-p_1)(1-q-q_1)+p_1q_1+pq\} \tag{28}$$

and

$$(P_A+P_B)_{\mathrm{qu}}=-\{\tfrac{1}{2}(P_A+P_B)_{\mathrm{cl}}+\epsilon\}. \tag{29}$$

In case $\epsilon=0$ both the classical and quantum games are clearly zero sum. For our slightly modified version of the RSP game we have $-1<\epsilon<0$ and both versions of the game become nonzero sum.

IV. DISCUSSION

Game-theoretical modeling of interactions between living organisms in the natural world has been developed mostly during the last three decades. Use of matrix games is quite common in areas such as theoretical and mathematical biology. The RSP game that we investigate in the present paper is also played in nature like many other games. Lizards in the coast range of California play this game using three alternative male strategies locked in an ecological never-ending process from which there seems little escape. On the other hand, the recently developed quantum game theory has been shown to find applications in quantum information [12]. Though there is no evidence yet, the possibility of quantum games being played at molecular level was hinted by Dawkins [13]. Trying to find the relevance of ideas from population biology in quantum settings is something that we call an inspiration from Dawkins's ideas.

The possibility of quantum mechanics playing a more direct role in life than binding together atoms has attracted much attention [14,15]. Quantum mechanics "fast tracking" a chemical soup to states that are biological and complex is an idea about which physicists from many areas have expressed opinions and the debate still continues. Supersymmetry in particle physics giving a unified description of fermions and bosons has also been suggested to provide an explanation of coding assignments in genetic code [16]. Patel's idea of quantum dynamics having a role in the DNA replication is another interesting suggestion [17]. Quantum game theory [18,18] can also have possibly interesting contributions to make towards attempts to understand the role of quantum mechanics in life.

Mathematical biologists have successfully developed mathematical models of evolution, especially, after attention was diverted to game-theoretical models of evolution [3], and the idea of an ESS became central in evolutionary game theory. The central idea of evolution, i.e., survival of the fittest is formulated as a mathematical algorithm known as a replicator dynamic. We suggest that recent progress in quantum game theory allows evolutionary ideas to enter and have a role in situations generally believed to lie in the domain of quantum mechanics. This combination of evolutionary ideas in quantum settings is interesting from several perspectives. Quantum considerations in the evolution of genetic code and genetic algorithms in which replicators receive their payoffs via quantum strategies are two cases [6] where evolutionary ideas can be incorporated in quantum gamelike situations. Another possible relevance is the competing chemical reactions in life molecules treated as players in a game. A winning chemical reaction corresponding to life hints a role of quantum mechanics because quantum strategies have been recently shown to be more effective than their classical counterparts [7, 8].

The population approach borrowed from evolutionary game theory with its central idea of an ESS combined with recent developments in quantum game theory provides a new approach to certain questions relating to role of quantum mechanics in life. The analysis of the RSP game from the evolutionary point of view is an example where ''stability'' comes to a classical NE when players revert to quantum strategies. The ''stability'' is with respect to an invasion by mutants appearing in small numbers. This stability of NE coming out of quantization can have a relevance in all the three situations indicated above.

V. CONCLUSION

We explored evolutionary stability in a modified rock-scissors-paper quantum game. We showed that a mixed-strategy NE, not an ESS in the classical version of the game, can be made an ESS when the two players play instead a quantum game by using a selected form of the initial quantum state on which they apply unitary operators in their possession. Quantum mechanics, thus, gives stability to a classical mixed NE against invasion by mutants. Stability against mutants for a mixed classical NE can be made to disappear in certain types of three-player symmetric games when players decide to resort to quantum strategies [6]. Stability against mutants in pairwise contests coming as a result of quantum strategies have been shown a possibility for pure strategies in certain types of symmetric games [4]. Our results imply that the selected method of quantization [7] can bring stability against mutants to a classical mixed NE in pairwise symmetric contests when the classically available number of pure strategies to a player is increased to three from two. A behavior of mixed NE different from pure NE is also observed in relation to quantization.

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