Existential contextuality and the models of Meyer, Kent, and Clifton

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It is shown that the models recently proposed by Meyer, Kent, and Clifton (MKC) exhibit a novel kind of contextuality, which we term existential contextuality. In this phenomenon it is not simply the pre-existing *value* but the actual *existence* of an observable which is context dependent. This result confirms the point made elsewhere, that the MKC models do not, as the authors claim, ''nullify'' the Kochen-Specker theorem. It may also be of some independent interest.

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I. INTRODUCTION

Meyer [1], Kent [2], and Clifton and Kent [3] (MKC in the sequel) have recently proposed a class of hidden variables models in which values are only assigned to a restricted subset of the set of all observables. MKC claim that their models ''nullify'' the Kochen-Specker theorem [4–8]. In Appleby [9], we showed that this claim is unfounded: the MKC models do not, in any way, invalidate the essential physical point of the Kochen-Specker theorem (for other critical discussions see Refs. [11–15]). Nevertheless, the MKC models are still of much interest. Together, with the models proposed by Pitowsky [16], they show that the physical interpretation of the Kochen-Specker theorem involves some important subtleties which, in the past, have not been sufficiently appreciated.

The purpose of this paper is to show that the MKC models exhibit a novel kind of contextuality, which has not previously been remarked in the literature, and which is even more strikingly at variance with classical assumptions than the usual kind of contextuality, featuring in the Kochen-Specker theorem. In the usual kind of contextuality, it is only the *value* assigned to an observable which is context dependent. In the MKC models, however, it is the very *existence* of an observable which is context dependent (its existence, that is, as a physical property whose value can be revealed by measurement). This phenomenon may be described as existential contextuality [29]. It confirms the point made in Ref. [9], that the MKC models do not, as MKC claim, provide a classical explanation for nonrelativistic quantum mechanics.

This paper was originally motivated by a seeming inconsistency in MKC's statement [2,3], that their models are both noncontextual *and* nonlocal. There do, of course, exist theories which have both these properties (Newtonian gravity, for example). However, in the framework of quantum mechanics, the phenomena of contextuality and nonlocality are closely connected, as has been stressed by Mermin [6] (also see Heywood and Redhead [17] and Basu *et al.* [10]). The discussion in Mermin [6] suggests that, if one were to examine the predictions the MKC models make regarding (for example) the Greenberger, Horne, and Zeilinger (GHZ) set up [20], then one might expect to find evidence that these models are not only nonlocal (as MKC state), but also contextual (which they deny). As we will see, this is in fact the case.

II. GHZ SETUP, WITH A LOCALITY ASSUMPTION

Consider Mermin's variant [6,21] of the GHZ setup [20], as illustrated in Fig. 1. This arrangement is usually regarded as a way of demonstrating nonlocality. As discussed in the Introduction, we will show that it can also be used to demonstrate a form of contextuality.

The system consists of three spin-half particles. Let $\hat{\boldsymbol{\sigma}}^{(r)}$ denote the Pauli spin vector for particle r, and let $\mathcal{H}^{(r)}$ be the two-dimensional Hilbert space on which it acts. The spin state is

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|1,1,1\rangle - |-1,-1,-1\rangle), \qquad (2.1)$$

where $|s_1,s_2,s_3\rangle$ denotes the joint eigenstate of $\hat{\sigma}_z^{(1)},\hat{\sigma}_z^{(2)},\hat{\sigma}_z^{(3)}$ with eigenvalues s_1,s_2,s_3. The particles emerge from a source and pass through three spacelike sepa-

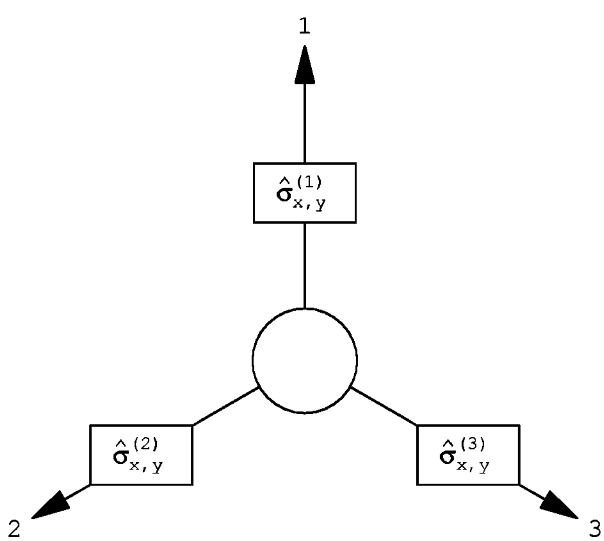

FIG. 1. Setup considered in Mermin's variant of the GHZ argument. For each r, the rth detector is set to measure one of the two target observables $\hat{\sigma}_x^{(r)}$ or $\hat{\sigma}_y^{(r)}$.

*Electronic address: D.M.Appleby@qmw.ac.uk rated detectors (see Fig. 1). For each r, the corresponding detector measures one of the two observables $\hat{\sigma}_x^{(r)}$ or $\hat{\sigma}_y^{(r)}$. One has [21]

$$\hat{\sigma}_x^{(1)}\hat{\sigma}_x^{(2)}\hat{\sigma}_x^{(3)}|\psi\rangle = -|\psi\rangle \tag{2.2}$$

and

$$\hat{\sigma}_x^{(1)}\hat{\sigma}_y^{(2)}\hat{\sigma}_y^{(3)}|\psi\rangle = \hat{\sigma}_y^{(1)}\hat{\sigma}_x^{(2)}\hat{\sigma}_y^{(3)}|\psi\rangle = \hat{\sigma}_y^{(1)}\hat{\sigma}_y^{(2)}\hat{\sigma}_x^{(3)}|\psi\rangle = |\psi\rangle. \tag{2.3}$$

Consequently, if the detectors are strictly ideal, and if they are set precisely at the combination xxx, then the product of measured values must necessarily be -1. Similarly, if the detectors are strictly ideal, and if they are set precisely at one of the combinations xyy, yxy, yyx, then the product of measured values must necessarily be $+1$.

MKC argue that it would not, in practice, be possible to align the detectors with infinite precision, implying that the detectors, instead of performing ideal measurements [30] of the observables $\hat{\sigma}_{j_1}^{(1)}, \hat{\sigma}_{j_2}^{(2)}, \hat{\sigma}_{j_3}^{(3)}$ (with $j_r = x$ or y), may actually perform ideal measurements of a slightly different set of commuting observables $\hat{\tau}^{(1)}, \hat{\tau}^{(2)}, \hat{\tau}^{(3)}$. They postulate that the observables $\hat{\tau}^{(1)}, \hat{\tau}^{(2)}, \hat{\tau}^{(3)}$ are always such that their joint spectral resolution is a subset of a countable set $\mathcal{P}_d$ (in the notation of Clifton and Kent [3]), which is dense in the space of all projections on $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$. For each r detector r reveals the pre-existing value of the observable $\hat{\tau}^{(r)}$ which it does in fact ideally measure.

The fact that the observables $\hat{\tau}^{(r)}$ may not precisely coincide with the observables $\hat{\sigma}_{j_r}^{(r)}$ means that there may be a small, nonzero probability of obtaining the "wrong" measurement outcome (i.e., 1 for the combination xxx, and -1 for the combinations xyy, yxy, yyx). This is consistent with the unavoidable imprecision of real, laboratory measurements.

In this paper, we are, for simplicity, confining ourselves to the kind of measurement envisaged by MKC, in which the imprecision is entirely due to the detectors not being aligned precisely in the directions specified. It should be stressed that such measurements are still highly idealized. MKC assume that there is always *some* observable which a detector ideally measures. They overlook the fact that a real, laboratory instrument does not, typically, perform an ideal measurement of anything: neither the nominal observable, which the experimenter records as having been measured, nor any other observable either. We discuss this point further in Appleby [9].

In their published papers, MKC take the view that the difference between $\hat{\tau}^{(r)}$ and $\hat{\sigma}_{j_r}^{(r)}$ is due to detector r not being aligned with infinite precision. On this view, $\hat{\tau}^{(r)}$ must be a local observable of the form $\hat{\tau}^{(r)} = \mathbf{n}_r \cdot \hat{\boldsymbol{\sigma}}^{(r)}$, where $\mathbf{n}_r$ is a unit vector close to the unit vector in the j_r direction, representing the actual alignment [31] of detector r. Kent's [22] subsequent suggestion, that $\hat{\tau}^{(r)}$ may be a nonlocal admixture of observables pertaining to more than one particle, will be discussed in the next section.

Given an arbitrary triplet of unit vectors $(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3)$, define projections

$$\hat{P}_{s_1 s_2 s_3} = \frac{1}{8}(1 + s_1 \mathbf{n}_1 \cdot \hat{\boldsymbol{\sigma}}^{(1)})(1 + s_2 \mathbf{n}_2 \cdot \hat{\boldsymbol{\sigma}}^{(2)})(1 + s_3 \mathbf{n}_3 \cdot \hat{\boldsymbol{\sigma}}^{(3)}), \tag{2.4}$$

where, for each r, $s_r = \pm 1$. These projections constitute the joint spectral resolution for the operators $\mathbf{n}_1 \cdot \hat{\boldsymbol{\sigma}}^{(1)}$, $\mathbf{n}_2 \cdot \hat{\boldsymbol{\sigma}}^{(2)}$, $\mathbf{n}_3 \cdot \hat{\boldsymbol{\sigma}}^{(3)}$. Define S_6' to be the set of vector triplets $(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3)$ for which the corresponding projections $\hat{P}_{s_1 s_2 s_3}$ all $\in \mathcal{P}_d$ (where, in the notation of Clifton and Kent [3], $\mathcal{P}_d$ is the countable set of projections on which the MKC valuations are defined). S_6' is a countable, dense subset of $S_2 \times S_2 \times S_2$ (where S_2 is the unit two sphere). Its significance is that $(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3)$ represents a possible set of alignments for the three detectors if and only if $(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3) \in S_6'$.

We will now show that S_6' cannot be a Cartesian product of the form $S_2'^{(1)} \times S_2'^{(2)} \times S_2'^{(3)}$, with $S_2'^{(1)}, S_2'^{(2)}, S_2'^{(3)} \subset S_2$. We will then use this to show that the MKC models exhibit a novel kind of contextuality.

In order to establish this result, suppose that S_6' *is* of the form $S_2'^{(1)} \times S_2'^{(2)} \times S_2'^{(3)}$. We will show that this assumption leads to a contradiction.

For each r, let $\mathbf{n}_{rx}, \mathbf{n}_{ry}$ be a fixed pair of vectors $\in S_2'^{(r)}$ such that $\mathbf{n}_{rx}$ (respectively, $\mathbf{n}_{ry}$) is close to $\mathbf{e}_x$ (respectively, $\mathbf{e}_y$), the unit vector in the x (respectively, y) direction. Then

$$\langle\psi|(\mathbf{n}_{1x} \cdot \hat{\boldsymbol{\sigma}}^{(1)})(\mathbf{n}_{2x} \cdot \hat{\boldsymbol{\sigma}}^{(2)})(\mathbf{n}_{3x} \cdot \hat{\boldsymbol{\sigma}}^{(3)})|\psi\rangle = -(1 - \epsilon_0), \tag{2.5a}$$

$$\langle\psi|(\mathbf{n}_{1x} \cdot \hat{\boldsymbol{\sigma}}^{(1)})(\mathbf{n}_{2y} \cdot \hat{\boldsymbol{\sigma}}^{(2)})(\mathbf{n}_{3y} \cdot \hat{\boldsymbol{\sigma}}^{(3)})|\psi\rangle = (1 - \epsilon_1), \tag{2.5b}$$

$$\langle\psi|(\mathbf{n}_{1y} \cdot \hat{\boldsymbol{\sigma}}^{(1)})(\mathbf{n}_{2x} \cdot \hat{\boldsymbol{\sigma}}^{(2)})(\mathbf{n}_{3y} \cdot \hat{\boldsymbol{\sigma}}^{(3)})|\psi\rangle = (1 - \epsilon_2), \tag{2.5c}$$

$$\langle\psi|(\mathbf{n}_{1y} \cdot \hat{\boldsymbol{\sigma}}^{(1)})(\mathbf{n}_{2y} \cdot \hat{\boldsymbol{\sigma}}^{(2)})(\mathbf{n}_{3x} \cdot \hat{\boldsymbol{\sigma}}^{(3)})|\psi\rangle = (1 - \epsilon_3), \tag{2.5d}$$

where $\epsilon_a \geq 0$ for each a. Let $\epsilon = \max(\epsilon_0, \epsilon_1, \epsilon_2, \epsilon_3)$. It follows from Eqs. (2.2) and (2.3) and the continuity of the expectation values that $\epsilon \to 0$ as $\mathbf{n}_{rx} \to \mathbf{e}_x$, $\mathbf{n}_{ry} \to \mathbf{e}_y$ for $r = 1,2,3$. The fact that S_6' is dense in $S_2 \times S_2 \times S_2$ means that $S_2'^{(r)}$ is dense in S_2 for $r = 1,2,3$. It follows that the vectors $\mathbf{n}_{rj}$ can be chosen so as to make ϵ arbitrarily small.

Let Λ be the hidden state space, and for each $\lambda \in \Lambda$, let $s_{rj}(\lambda)$ be the corresponding valuation of $\mathbf{n}_{rj} \cdot \hat{\boldsymbol{\sigma}}^{(r)}$. We have $s_{rj}(\lambda) = \pm 1$ for all r, j. Define

$$f_0(\lambda) = -s_{1x}(\lambda) s_{2x}(\lambda) s_{3x}(\lambda), \tag{2.6a}$$

$$f_1(\lambda) = s_{1x}(\lambda) s_{2y}(\lambda) s_{3y}(\lambda), \tag{2.6b}$$

$$f_2(\lambda)=s_{1y}(\lambda)s_{2x}(\lambda)s_{3y}(\lambda), \tag{2.6c}$$

$$f_3(\lambda)=s_{1y}(\lambda)s_{2y}(\lambda)s_{3x}(\lambda). \tag{2.6d}$$

Then, $f_a(\lambda)=\pm 1$ for all a,λ. Also

$$\begin{aligned} f_0(\lambda)f_1(\lambda)f_2(\lambda)f_3(\lambda) &= -[s_{1x}(\lambda)s_{2x}(\lambda)s_{3x}(\lambda) \\ &\quad \times s_{1y}(\lambda)s_{2y}(\lambda)s_{3y}(\lambda)]^2 \\ &= -1 \end{aligned} \tag{2.7}$$

for all λ.

Let μ be the probability measure on Λ corresponding to the state $|\psi\rangle$. The assumption that $S_6'=S_2'^{(1)}\times S_2'^{(2)}\times S_2'^{(3)}$ implies that

$$1-\epsilon\leqslant 1-\epsilon_a=\int f_a(\lambda)d\mu\leqslant 1 \tag{2.8}$$

for all a. For each a, let A_a be the set

$$A_a=\{\lambda\in\Lambda : f_a(\lambda)=1\}. \tag{2.9}$$

Then, it follows from inequality (2.8) that

$$1-\epsilon\leqslant \int f_a(\lambda)d\mu = 2\mu(A_a)-1 \tag{2.10}$$

for all a. It follows that $\mu(A_a)\geqslant 1-\epsilon/2$ for all a and, consequently, that $\mu(A_0\cap A_1\cap A_2\cap A_3)\geqslant 1-2\epsilon$. We noted above that, with a suitable choice of the vectors $\mathbf{n}_{rj}$, ϵ can be made arbitrarily small. It follows that there exist vectors $\mathbf{n}_{rj}$ such that $\mu(A_0\cap A_1\cap A_2\cap A_3)>0$ (in fact, there exist vectors $\mathbf{n}_{rj}$ such that $\mu(A_0\cap A_1\cap A_2\cap A_3)\approx 1$). On the other hand, it follows from Eq. (2.7) that $\mu(A_0\cap A_1\cap A_2\cap A_3)=0$ for every choice of $\mathbf{n}_{rj}$—which is a contradiction.

We have thus shown that the set S_6' does not have the form of a Cartesian product for any model of MKC type. This has important consequences: for it implies that it must, in general, happen that a change in the alignment of one detector forces a change in the alignment of at least one of the other two detectors. This represents a form of nonlocality. However, the point which concerns us here is that it also represents a form of contextuality.

It is a particularly striking form of contextuality. Let $S_2'^{(r)}$ be the set of possible alignments for detector r. In the usual kind of contextuality, $S_2'^{(r)}$ is fixed, and it is only the values assigned to the members of this set which depend on the measurement context. However, in the MKC models it is the set $S_2'^{(r)}$ itself which depends on the measurement context. In other words, it is not simply the *value*, but the very *existence* of an observable which is context dependent (its existence, that is, as a physical property whose value can be revealed by measurement).

III. GHZ SETUP, WITH NONLOCAL DETECTORS

In the last section, we assumed that detector r reveals the value of a local observable, defined on the state space of particle r. Kent [22] has objected to this assumption. He suggests, instead, that, on the level of the hidden variables, the detectors may function as nonlocal devices, which reveal the values of nonlocal admixtures of observables pertaining to more than one particle.

Let us begin by noting that this suggestion involves a significant departure from the view taken in MKC's published papers. In their published papers, MKC argue that the observable $\hat{\tau}^{(r)}$, whose value is revealed by detector r, is also the observable which detector r ideally measures [32], the discrepancy between $\hat{\tau}^{(r)}$ and $\hat{\sigma}^{(r)}_{j_r}$ being entirely attributable to an inaccuracy in the alignment of detector r. Clearly, detector r can only perform ideal quantum measurements of local observables pertaining to particle r. Consequently, the position adopted in MKC's published papers implies that $\hat{\tau}^{(r)}$ must be a local observable pertaining to particle r—as we assumed in the last section.

If a detector reveals the pre-existing value of some nonlocal observable then, on the level of the hidden variables, it must be interacting nonlocally with more than one particle. This interaction would represent a further element of nonlocality in the theory, additional to the nonlocality required by the standard arguments (Bell, GHZ, etc.). A model of this kind would thus be even more strongly nonclassical than the models originally proposed in MKC's published papers.

Nevertheless, the fact that the observables $\hat{\tau}^{(r)}$ may be assumed to be arbitrarily close to local observables of the form $\mathbf{n}_r\cdot\hat{\boldsymbol{\sigma}}^{(r)}$ means that a model of the kind indicated will still be consistent with the empirical predictions of conventional quantum mechanics. The question consequently arises whether the phenomenon of existential contextuality, discussed in the last section, also occurs in models of this more general kind. It is easily seen that the answer to this question is in the affirmative.

Let $\mathcal{P}$ be the set of all projection operators on $\mathcal{H}_1\otimes\mathcal{H}_2\otimes\mathcal{H}_3$, and let $\mathcal{P}_d$ be the countable, dense subset of $\mathcal{P}$ on which the MKC truth functions are defined (where we are employing the notation of Clifton and Kent [3], as before). Let $\overline{\mathcal{P}}_d$ be the set of self-adjoint operators on $\mathcal{H}_1\otimes\mathcal{H}_2\otimes\mathcal{H}_3$ whose spectral resolutions are contained in $\mathcal{P}_d$. The triplet $(\hat{\tau}^{(1)},\hat{\tau}^{(2)},\hat{\tau}^{(3)})$ of commuting observables whose values are revealed by the three detectors must $\in\overline{\mathcal{P}}_d\times\overline{\mathcal{P}}_d\times\overline{\mathcal{P}}_d$. It is determined by the hidden state of the three detectors. Let $T\subseteq\overline{\mathcal{P}}_d\times\overline{\mathcal{P}}_d\times\overline{\mathcal{P}}_d$ be the set of all possible triplets $(\hat{\tau}^{(1)},\hat{\tau}^{(2)},\hat{\tau}^{(3)})$, as determined by the set of all possible hidden detector states. The set T is the analogue, in the more general setting of this section, of the set S_6' defined in the last section.

We may now show, using a straightforward modification of the argument in the last section, that T is not a Cartesian product. In fact, suppose that T *was* of the form $T^{(1)}\times T^{(2)}\times T^{(3)}$ (with $T^{(r)}\subseteq\overline{\mathcal{P}}_d$ for $r=1,2,3$). We could then choose, for each $r=1,2,3$ and $j=x,y$, operators $\hat{\tau}^{(r)}_j\in T^{(r)}$ such that $\hat{\tau}^{(r)}_j\approx\hat{\sigma}^{(r)}_j$ for all r,j. This would imply

$$1-\epsilon\leqslant -\langle\psi|\,\hat{\tau}^{(1)}_x\hat{\tau}^{(2)}_x\hat{\tau}^{(3)}_x\,|\psi\rangle\leqslant 1, \tag{3.1a}$$

$$1-\epsilon \leqslant \langle \psi | \hat{\tau}_x^{(1)} \hat{\tau}_y^{(2)} \hat{\tau}_y^{(3)} | \psi \rangle \leqslant 1, \tag{3.1b}$$

$$1-\epsilon \leqslant \langle \psi | \hat{\tau}_y^{(1)} \hat{\tau}_x^{(2)} \hat{\tau}_y^{(3)} | \psi \rangle \leqslant 1, \tag{3.1c}$$

$$1-\epsilon \leqslant \langle \psi | \hat{\tau}_y^{(1)} \hat{\tau}_y^{(2)} \hat{\tau}_x^{(3)} | \psi \rangle \leqslant 1, \tag{3.1d}$$

where the positive constant ϵ can be chosen arbitrarily small [compare Eqs. (2.5) in the last section]. If $\epsilon < 1/2$, we can show that these inequalities lead to a contradiction, by an argument which is essentially the same as the argument following Eqs. (2.5) in the last section. It follows that T is not a Cartesian product.

We conclude that the MKC models still exhibit the phenomenon of existential contextuality described in the last section, even on the assumption that the detectors may reveal the pre-existing values of nonlocal observables.

IV. CONCLUSION

In this paper, we have argued that the MKC models are contextual. It follows that they do not, as MKC claim, provide a classical explanation for the empirically verifiable predictions of nonrelativistic quantum mechanics. This confirms the conclusion reached in Appleby [9], on the basis of a different, completely independent argument.

We would, however, stress that, notwithstanding these criticisms, it appears to us that the work of MKC is deeply interesting, and important. We have argued that MKC's attempt to explain nonrelativistic quantum mechanics in classical terms is misconceived. Nevertheless, their work is still valuable because, together with the earlier work of Pitowsky [16], it shows that the physical interpretation of the Kochen-Specker theorem is a great deal more subtle than may superficially appear. It consequently leads to a deeper understanding of the conceptual implications of quantum mechanics.

The work of MKC and Pitowsky is also interesting because it enlarges the scope of the hidden-variable concept in an imaginative way. In the past, hidden-variable theories have primarily been motivated by purely philosophical considerations. The emphasis has been largely (though not entirely—see Valentini [23,24]) on constructing alternative interpretations of conventional quantum mechanics. Recently, however, 't Hooft [25,26] (in one way) and Faraggi and Matone [27] and Bertoldi *et al.* [28] (in another way) have speculated that Planck scale physics may most appropriately be described in terms of a hidden-variable theory which is *not* equivalent to conventional quantum mechanics. A theory of this kind, if it could be constructed, would be *empirically* significant.

In this connection, it may be worth noting that 't Hooft [26] has argued that, on the level of Planck scale variables, it may not be possible to rotate a detector at will so as to measure either the x or y components of a particle's spin, and that this may provide a way of circumventing the Bell theorem. This proposal is similar to MKC's attempt to circumvent the Kochen-Specker theorem. Our analysis of the MKC models would consequently seem to indicate that one cannot restore classicality in the manner 't Hooft suggests.

On the other hand, there is no evident reason why one should demand noncontextuality and locality in respect of a theory of the kind proposed by 't Hooft. Such a theory must, by definition, restore the concept of a world of objective facts. However, this concept is by no means exclusive to classical physics. In other respects, the theory might be highly nonclassical. Indeed, it might be even more highly nonclassical than conventional quantum mechanics. The aim is to understand the actual constitution of the physical universe. There is no clear reason to exclude, at the outset, the possibility that the world actually is contextual and nonlocal.

The ideas of 't Hooft and Faraggi *et al.* are admittedly speculative. They do, however, provide an additional motive for investigating more imaginative implementations of the hidden-variable hypothesis.

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[29] It should be noted that the concept of existential contextuality introduced here is completely unrelated to the concept of ontological contextuality discussed by Heywood and Redhead [17], Redhead [18], and Pagonis and Clifton [19].
[30] It should be stressed that the detectors still perform *nonideal* measurements of $\hat{\sigma}^{(1)}_{j_1}, \hat{\sigma}^{(2)}_{j_2}, \hat{\sigma}^{(3)}_{j_3}$ (see Appleby [9]).
[31] We are assuming ideal detectors, so the concept ''actual alignment of detector r'' is unambiguous. In the case of nonideal detectors, this concept may not be sharply defined (see Appleby [9]).
[32] MKC only consider ideal detectors. Their assumption, that a measurement reveals the pre-existing value of the observable which is ideally measured, is obviously not applicable in the case of nonideal detectors (see Appleby [9]).