Microscopic model of semiconductor laser without inversion

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From a microscopic set of equations which takes into account spontaneous emission into lasing mode, we derive a macroscopic quantum model of low-threshold semiconductor lasers that includes the parabolic band structure, Pauli blocking of the injection current, and the carrier distribution dependence on the temperature. This model confirms the predictions of lasing without inversion and inversionless intensity squeezing, that were previously made by Yamamoto and co-authors on the basis of a semiclassical two-level approach. In addition, our analysis demonstrates the existence of an optimum temperature value that minimizes the injection current necessary to obtain lasing and intensity squeezing.

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I. INTRODUCTION

The concept of lasing without inversion (LWI) has often been proposed in connection with three- or four-level atomic systems involving a field trapped state that interacts with external coherent radiation as well as with an incoherent pump [1]. This approach, however, seems to be in disagreement with the fundamental idea that lasing must develop as a self-organizing process that creates a coherent field starting from an incoherent energy pump, without the need of any external source of coherence [2,3]. The following question therefore arises: In the absence of any external coherence source, is inversion indeed necessary for lasing? If the answer is positive there will be no way to obtain laser generation in all those materials that do not allow for population inversion. In the opposite case it will be possible, at least in certain conditions, to achieve lasing even from inversionless media.

In [4] it was pointed out that LWI may be obtained from a semiconductor device, under the constraint that a sufficient fraction of spontaneous emission enters the lasing mode. A close relation between lasing without inversion and low-threshold lasing was also demonstrated. Low-threshold lasing has been intensively studied in recent years both theoretically [5] and experimentally [6] for several kinds of laser, such as vertical cavity surface emitting semiconductor lasers (VCSELs), heterostructure diode lasers, microdroplets, high-Q Fabry-Pérot microcavity lasers, and microsphere lasers. Threshold reduction in semiconductor lasers is very important for practical applications, since it helps to decrease the injection current and thus to reduce energy losses and thermal heating. Also, the reduction of intensity noise in the laser beam would be highly desirable for the purposes of information transmission and elaboration in optical networks, but it is well known that intensity noise in quietly pumped semiconductor lasers can be suppressed below the shot noise level (intensity squeezing) only well above the laser threshold [7–9]. This high-pump condition contrasts with the requirement of small losses and heating, but the trade-off disappears if inversionless lasing is accompanied by intensity squeezing at low pump level. This possibility was confirmed by the well-known analysis of low-threshold semiconductor lasers carried out in [4]. The research was based on a semiclassical two-level rate equation model, which deals with the noise by using classical Langevin forces and takes into account spontaneous emission into the lasing mode by means of the phenomenological parameter β [10], which is the ratio of the spontaneous emission rate into the lasing mode to that into free space. Such a model, while providing a clear understanding of the physical nature of thresholdless lasing, unavoidably misses several quantum aspects of the problem such as, for example, the pump blocking induced by the Pauli exclusion principle. In addition, it is unable to take into account some important features of semiconductor lasers, such as the band structure and the dependence of the carrier distribution on the temperature. Moreover, the results described in [4] deviate from the semiclassical limit of a rigorous quantum-mechanical model of thresholdless lasing in a system of two-level atoms [11].

All of these considerations make the generalization of the thresholdless semiconductor laser model originally presented in [4] a topical theoretical problem. However, a complete quantum-mechanical analysis unavoidably presents a level of complication much higher than that of the basic two-level model. In particular, it is quite difficult to carry out a microscopic analysis of the thresholdless semiconductor laser keeping all its features and, in the meanwhile, presenting the results in a reasonably compact form, so that they can be employed in practical applications. The aim of this paper is to present research that consolidates and extends the results of [4], with an effort to keep the basic quantum equations as simple as possible. It is well known that a considerable source of difficulties when dealing with a quantum-

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mechanical description of semiconductor lasers is carrier-carrier interaction. Technically, this interaction can be taken into account by means of a hierarchy of coupled equations [12], an approach unavoidable when one deals with many-body effects like nonlinear excitonic emission or secondary emission after very short pulse excitation [13]. On the other hand, the influence of carrier-carrier interactions on the noise properties of semiconductor lasers is still an open question: summarizing and commenting on the results published in several papers ([14]), the authors of [15] point out the difficulty of handling the memory effects arising from the nonequilibrium Green function that solves the many-body problem, and admit that these effects are usually neglected. In [16–18] carrier scattering was taken into account by means of a damping term in the microscopic equation for the carrier distribution, and it was concluded that it does not affect either the intensity or the phase noise spectrum. In a very recent paper the standard macroscopic Yamamoto approach was employed very fruitfully to analyze the possible coexistence of squeezing and thermal noise [19]. Our research and that described in [19] are in many respects complementary, since they study the impact on squeezing of carrier temperature and of thermal noise, respectively. Another basic assumption of the present paper is the spatial homogeneity of the laser under investigation. As a first approximation such a limitation holds for small area devices, but it prevents the study of the spatial coherence of spontaneous emission, whose impact has been clarified by the research reported in [15].

In this paper we present a quantum analysis of thresholdless semiconductor laser starting from the microscopic (i.e., resolved in the electronic wave vector k) set of quantum equations presented and studied in [16–18], and proceed to the formal derivation of quantum rate equations by taking into account spontaneous emission into the laser mode, which was previously neglected. The derivation of the quantum macroscopic rate equations is shown in Sec. II, and in Sec. III the dependence of the coupling efficiency β on the laser parameters is described. In the following Sec. IV the laser stationary state is determined, and in Sec. V the zero-frequency intensity noise is found. The results are summarized and discussed in the final Sec. VI. Our estimations are made for parameter values typical of microcavity VCSELs, because of the great importance of these lasers for information technology applications.

II. THE MACROSCOPIC RATE EQUATIONS

In this section we derive the operatorial rate equations that describe a single-mode semiconductor laser, taking into account the contribution of spontaneous emission into the lasing mode. Our starting point is the set of microscopic operator equations introduced in [16–18] for the laser field $\hat{A}$, the material polarization $\hat{\sigma}_k$, the electron and hole distributions $\hat{n}_{\alpha k}$ $(\alpha=e,h)$, and the total carrier population $\hat{N}_\alpha=\Sigma_k\hat{n}_{\alpha k}$. To each one of these operators is associated a Langevin noise term $\hat{F}_{\hat{O}}$, $\hat{O}=\hat{A},\hat{\sigma}_k,\hat{n}_{\alpha k},\hat{N}_\alpha$, so that the resulting set of equations reads

$$\frac{d\hat{A}}{dt}=-[\kappa+i(\Omega-\nu)]\hat{A}-i\sum_k g_k^*\hat{\sigma}_k+\hat{F}_{\hat{A}}, \tag{1a}$$

$$\frac{d\hat{\sigma}_k}{dt}=-[\gamma+i(\omega_k-\nu)]\hat{\sigma}_k+ig_k\hat{A}(\hat{n}_{ek}+\hat{n}_{hk}-1)+\hat{F}_{\hat{\sigma}_k}, \tag{1b}$$

$$\frac{d\hat{n}_{\alpha k}}{dt}=\gamma_p\frac{f_{pk}}{N_p}(1-\hat{n}_{ek})(1-\hat{n}_{hk})-\left(\frac{1}{\tau_{nr}}+\frac{1}{\tau_{sp}^{om}}\right)\hat{n}_{ek}\hat{n}_{hk} -\gamma_\alpha(\hat{n}_{\alpha k}-\hat{f}_{\alpha k})+i(g_k^*\hat{A}^\dagger\hat{\sigma}_k-g_k\hat{\sigma}_k^\dagger\hat{A})+\hat{F}_{\hat{n}_{\alpha k}}, \tag{1c}$$

$$\frac{d\hat{N}_\alpha}{dt}=\gamma_p\sum_k\frac{f_{pk}}{N_p}(1-\hat{n}_{ek})(1-\hat{n}_{hk})-\left(\frac{1}{\tau_{nr}}+\frac{1}{\tau_{sp}^{om}}\right)\times\sum_k\hat{n}_{ek}\hat{n}_{hk}+i\sum_k(g_k^*\hat{A}^\dagger\hat{\sigma}_k-g_k\hat{\sigma}_k^\dagger\hat{A})+\hat{F}_{\hat{N}_\alpha}. \tag{1d}$$

In Eqs. (1) κ, γ, and γ_α are the cavity field decay rate, the polarization relaxation rate, and the relaxation rate of carriers toward the Fermi-Dirac quasiequilibrium distribution $\hat{f}_{\alpha k}$, respectively; Ω is the empty cavity mode frequency, ν is the lasing mode frequency, ω_k is the frequency associated with the radiative recombination of a k wave vector electron-hole pair, and g_k is the light-material coupling constant, with the dimensions of 1/time; f_{pk} and $N_p=\Sigma_k f_{pk}$ represent the distribution and the total number of carriers in the pump source, respectively, while the pump parameter γ_p is determined by the macroscopic parameters of the electric circuit that drives the laser.

The term $\Sigma_k\hat{n}_{ek}\hat{n}_{hk}/\tau_{nr}$ represents nonradiative recombination, while the term $\Sigma_k\hat{n}_{ek}\hat{n}_{hk}/\tau_{sp}^{om}$ represents spontaneous radiative recombination of carriers into all the field modes except the lasing one. We stress that the term $\Sigma_k\hat{n}_{ek}\hat{n}_{hk}/\tau_{sp}^{om}$ does not include the spontaneous emission into the lasing mode, because this particular contribution is taken into account by the Hamiltonian part of Eqs. (1). In the previous analysis [16–18] spontaneous emission into the lasing mode was neglected, because the focus was on the system stationary and noise behavior well above the lasing threshold. Now we refine that model by considering spontaneous emission into the lasing mode and investigating its effects. With this aim we determine the time derivative of the cavity photon number operator $\hat{I}\equiv\hat{A}^\dagger\hat{A}$,

$$\frac{d\hat{I}}{dt}=\frac{d\hat{A}^\dagger}{dt}\hat{A}+\hat{A}^\dagger\frac{d\hat{A}}{dt}, \tag{2}$$

and use Eqs. (1a,d) to obtain the macroscopic dynamical equations that control $\hat{I}$ and the carrier number in the active volume:

$$\frac{d\hat{I}}{dt}=-2\kappa\hat{I}+i\sum_{k}(\hat{R}_k-\hat{R}_k^\dagger)+\hat{F}_{\hat{I}},\tag{3a}$$

$$\frac{d\hat{N}_\alpha}{dt}=\gamma_p\sum_k\frac{f_{pk}}{N_p}(1-\hat{n}_{ek})(1-\hat{n}_{hk})-\left(\frac{1}{\tau_{nr}}+\frac{1}{\tau_{sp}^{om}}\right)\times\sum_k\hat{n}_{ek}\hat{n}_{hk}-i\sum_k(\hat{R}_k-\hat{R}_k^\dagger)+\hat{F}_{\hat{N}_\alpha},\tag{3b}$$

where we have defined the photon number noise operator

$$\hat{F}_{\hat{I}}\equiv\hat{F}_{\hat{A}^\dagger}\hat{A}+\hat{A}^\dagger\hat{F}_{\hat{A}}\tag{4}$$

and the field-medium coupling operator

$$\hat{R}_k\equiv g_k\hat{\sigma}_k^\dagger\hat{A}.\tag{5}$$

We remark that, due to the conservation of total carrier number $\Sigma_k\hat{n}_{\alpha k}=\Sigma_k\hat{f}_{\alpha k}$, the scattering term proportional to γ_α is not present in Eq. (3b).

Note that the set of Eqs. (3) is incomplete, since the dynamical evolution of $\hat{R}_k$ depends on the material polarization $\hat{\sigma}_k$, which in turn depends on the carrier distribution $\hat{n}_{\alpha k}$. To obtain a closed set of equations, we take the time derivative of Eq. (5),

$$\frac{d\hat{R}_k}{dt}=g_k\left(\frac{d\hat{\sigma}_k^\dagger}{dt}\hat{A}+\hat{\sigma}_k^\dagger\frac{d\hat{A}}{dt}\right),\tag{6}$$

and use Eqs. (1a,b) to obtain

$$\frac{d\hat{R}_k}{dt}=-(\kappa+\gamma+i\,\delta_k)\hat{R}_k-i|g_k|^2\hat{I}(\hat{n}_{ek}+\hat{n}_{hk}-1)-ig_k\sum_{k'}g_{k'}^*\hat{\sigma}_k^\dagger\hat{\sigma}_{k'}+\hat{F}_{\hat{R}_k}.\tag{7}$$

In the above equation, we have defined

$$\delta_k=\Omega-\omega_k\tag{8}$$

and introduced the noise operator

$$\hat{F}_{\hat{R}_k}=g_k(\hat{F}_{\hat{\sigma}_k}^\dagger\hat{A}+\hat{\sigma}_k^\dagger\hat{F}_{\hat{A}}).\tag{9}$$

Let us now examine the term $\Sigma_{k'}g_{k'}^*\hat{\sigma}_k^\dagger\hat{\sigma}_{k'}$ that appears in Eq. (7), and note that the relative phase of the operators $\hat{\sigma}_k^\dagger$ and $\hat{\sigma}_{k'}$ with $k'\neq k$ changes randomly with k and k', so that the contribution to the overall sum arising from the terms like $\hat{\sigma}_k^\dagger\hat{\sigma}_{k'\neq k}$ averages to zero. We therefore obtain

$$\sum_{k'}g_{k'}^*\hat{\sigma}_k^\dagger\hat{\sigma}_{k'}\approx g_k^*\hat{\sigma}_k^\dagger\hat{\sigma}_k=g_k^*\hat{n}_{ek}\hat{n}_{hk},\tag{10}$$

where the operatorial identity $\hat{\sigma}_k^\dagger\hat{\sigma}_k\equiv\hat{n}_{ek}\hat{n}_{hk}$ has been employed. By using Eq. (10), Eq. (7) takes the form

$$\frac{d\hat{R}_k}{dt}=-(\kappa+\gamma+i\,\delta_k)\hat{R}_k-i|g_k|^2\hat{I}(\hat{n}_{ek}+\hat{n}_{hk}-1)-i|g_k|^2\hat{n}_{ek}\hat{n}_{hk}+\hat{F}_{\hat{R}_k},\tag{11}$$

which makes it apparent that $\hat{R}_k$ evolves at the rate $\kappa+\gamma$. We now recall that in semiconductor lasers in the cw regime the polarization decay rate γ and the carrier scattering rates γ_α are by far larger than all the other rates, namely, the field decay rate, the pump rate, and the nonradiative, spontaneous, and stimulated emission rates. This allows us both to eliminate $\hat{R}_k$ adiabatically and to assume intraband quasiequilibrium, so that from Eq. (11) we obtain, neglecting $\kappa\ll\gamma$,

$$\hat{R}_k=\frac{-i\hat{I}|g_k|^2(\hat{f}_{ek}+\hat{f}_{hk}-1)-i|g_k|^2\hat{f}_{ek}\hat{f}_{hk}+\hat{F}_{\hat{R}_k}}{\gamma+i\,\delta_k}.\tag{12}$$

The intraband quasiequilibrium carrier distribution follows Fermi-Dirac statistics and is determined by

$$\hat{f}_{\alpha k}(\hat{N}_\alpha)=\frac{1}{1+e^{[E_{\alpha k}-\mu_\alpha(\hat{N}_\alpha)]/(k_BT)}},\tag{13}$$

where T is the carrier temperature, k_B the Boltzmann constant, and $E_{\alpha k}=\hbar^2k^2/(2m_\alpha)$, m_α being the effective carrier mass. The chemical potential $\mu_\alpha(\hat{N}_\alpha)$ is determined by the relation

$$\hat{N}_\alpha=\sum_k\hat{f}_{\alpha k}.\tag{14}$$

Substituting Eq. (12) in Eqs. (3), we obtain a set of equations that involves only the macroscopic operators $\hat{I}$ and $\hat{N}_\alpha$:

$$\frac{d\hat{I}}{dt}=-2\kappa\hat{I}+\sum_k\frac{2\gamma}{\gamma^2+\delta_k^2}|g_k|^2\hat{I}(\hat{f}_{ek}+\hat{f}_{hk}-1)+\sum_k\frac{2\gamma}{\gamma^2+\delta_k^2}|g_k|^2\hat{f}_{ek}\hat{f}_{hk}+\hat{F}_{\hat{I}}+\hat{F}_{\hat{R}},\tag{15a}$$

$$\frac{d\hat{N}_\alpha}{dt}=\gamma_p\sum_k\frac{f_{pk}}{N_p}(1-\hat{f}_{ek})(1-\hat{f}_{hk})-\left(\frac{1}{\tau_{nr}}+\frac{1}{\tau_{sp}^{om}}\right)\sum_k\hat{f}_{ek}\hat{f}_{hk}-\sum_k\frac{2\gamma}{\gamma^2+\delta_k^2}|g_k|^2\hat{f}_{ek}\hat{f}_{hk}-\sum_k\frac{2\gamma}{\gamma^2+\delta_k^2}|g_k|^2\times\hat{I}(\hat{f}_{ek}+\hat{f}_{hk}-1)+\hat{F}_{\hat{N}_\alpha}-\hat{F}_{\hat{R}},\tag{15b}$$

where we introduced the macroscopic noise operator

$$\hat{F}_{\hat{R}}\equiv i\sum_k\frac{\hat{F}_{\hat{R}_k}}{\gamma+i\,\delta_k}-i\sum_k\frac{\hat{F}_{\hat{R}_k^\dagger}}{\gamma-i\,\delta_k}.\tag{16}$$

We remark that the macroscopic set of Eqs. (15) is closed, since the carrier distribution $\hat{f}_{\alpha k}$ is determined by the total carrier number $\hat{N}_\alpha$ via Eqs. (13) and (14).

For the sake of simplicity, from now on we will assume that electrons and holes have the same mass m, given by the average

$$m=(m_e+m_h)/2, \tag{17}$$

and define the total carrier number operator and the relative noise operator as

$$\hat{N}\equiv(\hat{N}_e+\hat{N}_h)/2, \tag{18a}$$

$$\hat{F}_{\hat{N}}\equiv(\hat{F}_{\hat{N}_e}+\hat{F}_{\hat{N}_h})/2. \tag{18b}$$

We therefore obtain from Eqs. (15)

$$\frac{d\hat{I}}{dt}=-2\kappa\hat{I}+\hat{g}\hat{I}+\hat{B}^{lm}+\hat{F}_{\hat{I}}+\hat{F}_{\hat{R}}, \tag{19a}$$

$$\frac{d\hat{N}}{dt}=\hat{\Lambda}-\hat{B}^{nr}-\hat{B}^{om}-\hat{B}^{lm}-\hat{g}\hat{I}+\hat{F}_{\hat{N}}-\hat{F}_{\hat{R}}, \tag{19b}$$

where the following definitions have been introduced:

$$\hat{\Lambda}\equiv\gamma_p\sum_k\frac{f_{pk}}{N_p}(1-\hat{f}_{ek})(1-\hat{f}_{hk}), \tag{20a}$$

$$\hat{B}^{nr}\equiv\frac{1}{\tau_{nr}}\sum_k\hat{f}_{ek}\hat{f}_{hk}, \tag{20b}$$

$$\hat{B}^{om}\equiv\frac{1}{\tau_{sp}^{om}}\sum_k\hat{f}_{ek}\hat{f}_{hk}, \tag{20c}$$

$$\hat{B}^{lm}\equiv\sum_k\frac{2\gamma}{\gamma^2+\delta_k^2}|g_k|^2\hat{f}_{ek}\hat{f}_{hk}, \tag{20d}$$

$$\hat{g}\equiv\sum_k\frac{2\gamma}{\gamma^2+\delta_k^2}|g_k|^2(\hat{f}_{ek}+\hat{f}_{hk}-1). \tag{20e}$$

Let us summarize the meaning of every symbol: $\hat{\Lambda}$ represents the pump, $\hat{B}^{nr}$ the nonradiative recombination, $\hat{B}^{om}$ the spontaneous emission in all the other field modes except the lasing one, $\hat{B}^{lm}$ the spontaneous emission in the lasing mode, and $\hat{g}$ the laser gain.

III. THE CONTROL PARAMETER β

The expectation values of the operators relative to the photon and the carrier number are, respectively,

$$I\equiv\langle\hat{I}\rangle, \tag{21a}$$

$$N\equiv\langle\hat{N}\rangle. \tag{21b}$$

Under the assumption expressed by Eq. (17) we obtain

$$\langle\hat{f}_{ek}\rangle\equiv\langle\hat{f}_{hk}\rangle\equiv f_k(N)\equiv\frac{1}{1+e^{[E_k-\mu(N)]/(k_BT)}}, \tag{22}$$

where $E_k=\hbar^2k^2/(2m)$.

Let us now define τ_{sp} as the total spontaneous emission time in the semiconductor material, which spans a typical range of 1–3 ns [4]. It can easily be verified that the variation of $|g_k|^2$ with k is much smoother than the variation of the Fermi-Dirac distribution, so that in the summations appearing in Eqs. (20d,e) we can substitute $|g_k|^2$ by $|g_0|^2$, where g_0 is the light-material coupling coefficient for a transition from the bottom of the conduction band to the top of the valence band, i.e., for $k=0$. In agreement with [20], the coupling efficiency β of spontaneous emission into the lasing mode is given by

$$\beta\equiv\frac{2|g_0|^2\tau_{sp}}{\gamma}. \tag{23}$$

Thus, we can rewrite the spontaneous emission time rate in the collection of all the field modes except the lasing one in terms of β and τ_{sp}:

$$\frac{1}{\tau_{sp}^{om}}=\frac{1-\beta}{\tau_{sp}}. \tag{24}$$

By means of Eq. (23) and Eq. (24), we obtain from Eqs. (20) the expectation values

$$\Lambda\equiv\langle\hat{\Lambda}\rangle=\gamma_p\sum_k\frac{f_{pk}}{N_p}(1-f_k)^2, \tag{25a}$$

$$B^{nr}\equiv\langle\hat{B}^{nr}\rangle=\frac{1}{\tau_{nr}}\sum_k f_k^2, \tag{25b}$$

$$B^{om}\equiv\langle\hat{B}^{om}\rangle=\frac{1-\beta}{\tau_{sp}}\sum_k f_k^2, \tag{25c}$$

$$B^{lm}\equiv\langle\hat{B}^{lm}\rangle=\frac{\beta}{\tau_{sp}}\sum_k L_kf_k^2, \tag{25d}$$

$$g\equiv\langle\hat{g}\rangle=\frac{\beta}{\tau_{sp}}\sum_k L_k(2f_k-1), \tag{25e}$$

where

$$L_k\equiv\frac{\gamma^2}{\gamma^2+\delta_k^2} \tag{26}$$

is the dimensionless Lorentzian function of δ_k.

IV. THE STATIONARY STATE

The stationary values of the photon number in the laser cavity and of the carrier number in the active volume are determined by the stationary solution of the semiclassical system of equations

$$\frac{dI}{dt}=-2\kappa I+gI+B^{lm}, \tag{27a}$$

$$\frac{dN}{dt}=\Lambda-B^{nr}-B^{om}-B^{lm}-gI, \tag{27b}$$

which was obtained from Eqs. (19) by taking the expectation values of the operators and factorizing them. The stationary value N_{st} of the carrier number can be obtain numerically, by solving the nonlinear equation

$$(\Lambda-B^{nr}-B^{om}-B^{lm})(2\kappa-g)-B^{lm}g=0, \tag{28}$$

which follows from Eqs. (27a,b) by setting the time derivatives to zero and eliminating the photon number. Once N_{st} and therefore $f_k(N_{st})$ is known, the stationary photon number is given by

$$I_{st}=\frac{\Lambda-B^{nr}-B^{om}-B^{lm}}{g}. \tag{29}$$

The value Λ_{th} of the pumping threshold, as determined by semiclassical laser theory (“semiclassical” threshold), can be found by setting $I=0$ in the stationary Eq. (27b):

$$\Lambda_{th}=B^{nr}+B^{om}+B^{lm}. \tag{30}$$

In order to better characterize the deviations from semiclassical behavior we introduce the ratio

$$r\equiv\frac{\Lambda}{\Lambda_{th}}=\frac{\Lambda}{B^{nr}+B^{om}+B^{lm}}. \tag{31}$$

The injection current is given by

$$J=e\Lambda, \tag{32}$$

where e is the electron charge.

In Fig. 1(a) we have plotted on log-log scale I_{st} as a function of the injection current J, for a temperature of 300 K and three different values of β, and the other parameter values typical of VCSELs [21]. The solid lines correspond to negligible nonradiative recombinations ($\tau_{nr}\to\infty$), while the dashed lines correspond to $\tau_{nr}=20\tau_{sp}$. This figure is similar to Fig. 1 of [4], and the intersections with the line $I_{st}=1$ correspond to the lasing thresholds, according to the definition adopted in that paper. Figure 1(b) shows the temperature dependence of these thresholds, in the case $\tau_{nr}=20\tau_{sp}$. One can observe that for $\beta=0.1$ there is an increase of the threshold for small temperatures, and a minimum that indicates the optimum temperature ($\sim$80 K) to achieve lasing with the lowest possible injection current. We found that a minimum exists also for $0.1<\beta<0.9$, while for $\beta>0.9$ the lasing threshold becomes almost independent of the temperature.

In Fig. 2(a) we have represented the stationary photon number versus the normalized pump r defined in Eq. (31). The parameters are the same as those employed in Fig. 1(a),

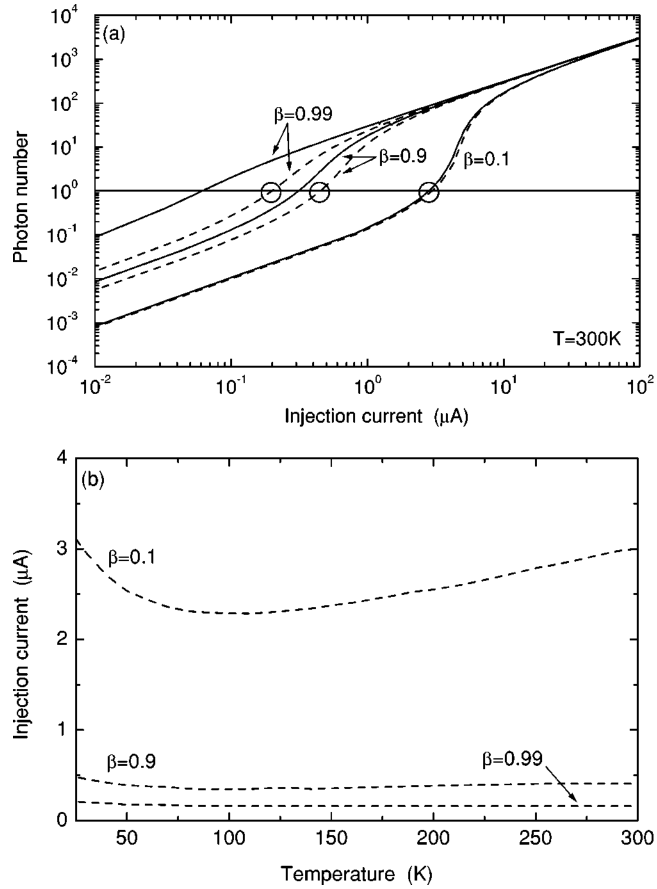

FIG. 1. (a) Stationary photon number for $\tau_{sp}=3$ ns, average mass $m=0.09m_0$, m_0 being the free electron mass, active volume 2.5×10^{-13} cm^3, $\gamma=10^{13}$ s^{-1}, $N_p=N$, $f_{pk}=f_k$, $\kappa=10^{11}$ s^{-1}. The circles indicate the threshold determined according to the condition $I_{st}=1$. (b) Temperature dependence of the lasing thresholds indicated by the circles in (a).

and the plot is on a linear scale. By inspecting Fig. 2(a) one can clearly identify a critical point, i.e., a value of r for which the slope of $I_{st}(r)$ abruptly changes. The critical point is always well defined, even when β is close to 1; it approaches 0 as $\beta\to1$, and 1 as $\beta\to0$. This abrupt change in the laser behavior may be identified as a phase transition taking place as the parameter r is varied.

Looking back at Fig. 1(a) and comparing it with Fig. 2(a) we can see that the definition of threshold according to [4] suffers from two major drawbacks: the first one is that it depends on the arbitrary assumption $I_{st}=1$, and the second one is that as β approaches 1 it does not mark any appreciable change in the laser behavior. Both these problems are removed if one refers to the pumping parameter r instead of the injection current J.

Figure 2(b) displays the normalized inversion $g/(2\kappa)$ as a function of r. One can see that for $r<1$ the inversion is negative, for $r=1$ it is zero, and only for $r>1$ does it become positive, for every value of β. The gain clamping condition $g=2\kappa$ is satisfied for $r\to\infty$. The fact that $r=1$ marks the zero inversion point is another advantage of introducing the parameter r. Comparing Fig. 2(a) with Fig. 2(b) we conclude that the critical value of r is situated in the region

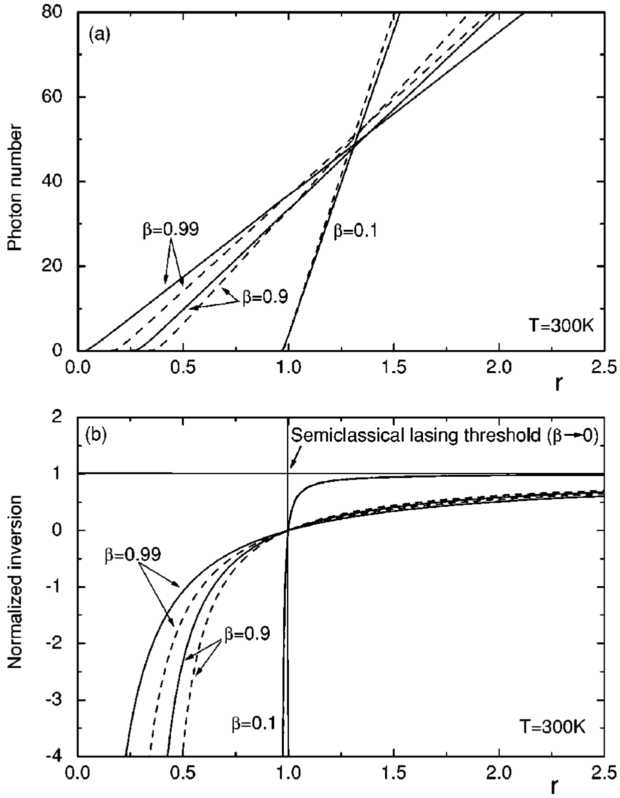

FIG. 2. (a) Stationary photon number I_{st} as a function of the normalized pump parameter r. The notation and the parameter values are the same as in Fig. 1(a). (b) Normalized inversion $g/(2\kappa)$ as a function of the normalized pump r. The value $r=1$ corresponds to zero inversion for any value of β.

without inversion, even though for $\beta=0.1$ the critical value of r is very close to 1.

However, we recognize that even though the laser may exhibit a phase transition at the critical value of r, this is of no help in the experimental determination of the lasing threshold as β approaches 1. Indeed, while r can easily be derived from the theoretical model, it cannot be extracted directly from the experimental data. To help in the experimental identification of the threshold also for lasing without inversion, we propose the approach described in Fig. 3(a), which was also independently adopted in [19]. Let us prolong the linear behavior of $I_{st}(J)$, which is characteristic of $I_{st}\gg 1$, to the intersection with $I_{st}=0$, and define this point as the lasing threshold. The number of photons at this threshold is given by the intersection of the vertical segment with the I_{st} curve. One can see that the threshold, according to this definition, is close to the end of the intermediate region of the $I_{st}(J)$ curve between the nonlasing region (small slope) and the "full-strength" lasing region (large slope). On the contrary, the threshold defined by the condition $I_{st}=1$ and indicated in Fig. 3(a) by the squares, corresponds to the very beginning of the intermediate region. The definition adopted in [19] and here has the advantage that it can be easily derived from the experimental data, it corresponds to $I_{st}>1$, and thus it is less affected by noise. It fails only in the limiting case of negligible nonradiative processes $\tau_{nr}\to\infty$ and β very close to 1, when $I_{st}(J)$ becomes practically a straight line starting from 0.

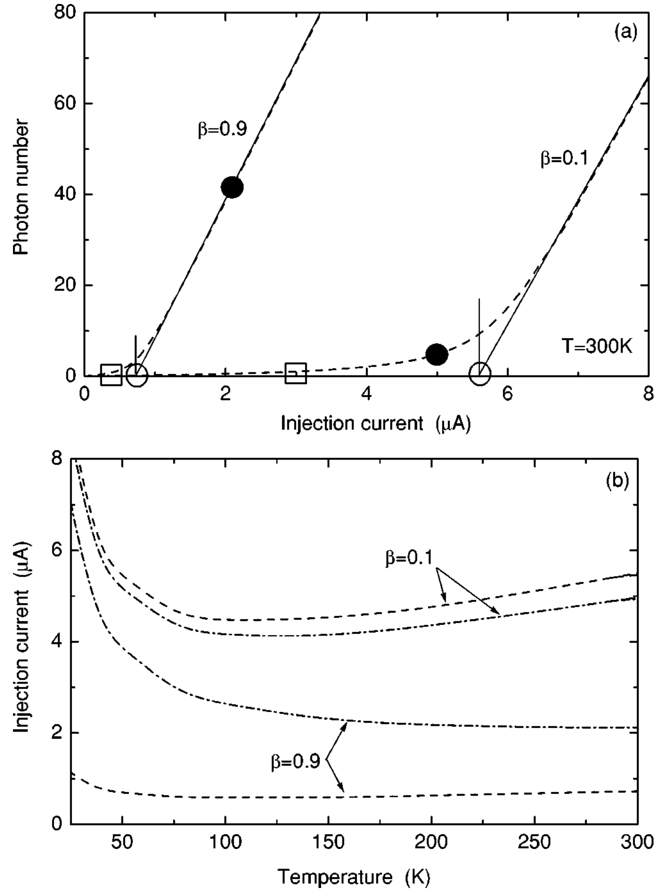

FIG. 3. (a) Stationary photon number I_{st} as a function of the injection current J for $\tau_{nr}=20\tau_{sp}$ and two different values of β. All the other parameters are as in Fig. 1(a). The points corresponding to $I_{st}=1$ are indicated by boxes, those corresponding to our definition of the laser threshold by empty circles, and those corresponding to zero inversion by full circles. (b) Temperature dependence of the lasing threshold indicated by the empty circles in (a) (dashed lines) and of the injection current corresponding to zero inversion, indicated by the full circles in (a) (dash-dotted lines).

Figure 3(b) shows the temperature dependence of the threshold determined according to Fig. 3(a) (dashed lines) and the injection current at zero inversion (dot-dashed lines). The temperature dependence of the injection current at zero inversion, not shown in Fig. 2(b) or Fig. 4(b) below, is the same as in Fig. 3(b). With the lasing threshold defined according to Fig. 3(a), we do not have lasing without inversion for $\beta=0.1$ in the whole temperature range examined, while we have it for $\beta=0.9$. This in contrast with Fig. 1(b), which predicts lasing without inversion even for $\beta=0.1$.

For the purpose of comparing our model with that of [4], we note that Eqs. (27) can be formally reduced to equations similar to Eqs. (2) and (3) of [4] by setting in Eqs. (27) $f_k^2\approx f_k$ and $L_k\approx 1$. Such approximations are valid at least in the low-temperature limit and for low enough threshold current. Indeed, for low temperature $f_k^2\approx f_k$, and nonzero contributions to the sum $\Sigma_k L_k f_k$ come only from $k\leqslant k_F$, where k_F is the wave vector corresponding to the quasi-Fermi level. If we now take into account that the cavity mode frequency Ω is bounded by $\Omega_g\leqslant\Omega<\Omega_g+\omega_{k_F}$, where Ω_g is the band gap

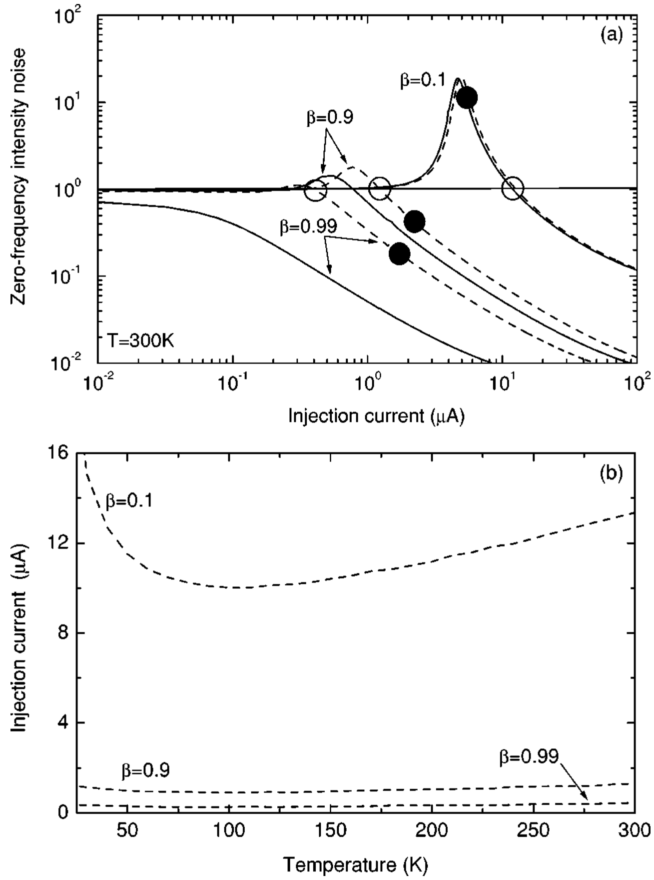

FIG. 4. (a) Zero-frequency intensity noise as a function of the injection current J. Full circles correspond to zero inversion, empty circles to squeezing threshold. (b) Temperature dependence of the squeezing thresholds indicated in (a) as empty circles.

frequency, one can estimate $\delta_k^2 \leqslant \delta_F^2 \equiv (\Omega_g - \omega_{k_F})^2$. The smaller the threshold current, the smaller will be the population in the conduction band, the quasi-Fermi level, and consequently the term $(\Omega_g - \omega_{k_F})^2$. Therefore one can, in principle, satisfy the condition $\delta_k^2 \leqslant \delta_F^2 \ll \gamma^2$, and accordingly set $L_k \approx 1$ in a laser with small enough threshold current. Supposing this is the case, and neglecting pump blocking, we obtain

$$\Lambda = \gamma_p , \tag{33a}$$

$$B^{nr} \approx N/\tau^{nr}, \tag{33b}$$

$$B^{om} \approx (1-\beta)N/\tau^{sp}, \tag{33c}$$

$$B^{lm} \approx \beta N/\tau^{sp}, \tag{33d}$$

$$g \approx (2\beta/\tau^{sp})(N-N_t), \tag{33e}$$

where $N = \Sigma_k f_k$ and $2N_t = \Sigma_k L_k$. We therefore obtain from Eqs. (27)

$$\frac{dI}{dt} = -2\kappa I + \frac{2\beta}{\tau^{sp}}(N-N_t)I + \frac{\beta}{\tau^{sp}}N, \tag{34a}$$

$$\frac{dN}{dt} = \gamma_p - \frac{N}{\tau^{nr}} - \frac{N}{\tau^{sp}} - \frac{2\beta}{\tau^{sp}}(N-N_t)I. \tag{34b}$$

To conclude, we have shown that, using the approximations $f_k^2 \approx f_k$ and $L_k \approx 1$, the microscopic Eqs. (27) can be replaced by a set of equations very similar to Eqs. (2) and (3) of [4], with the only difference of an extra factor 2 in the stimulated emission term. A set of equations identical to Eqs. (34) is reported in [19].

V. THE ZERO-FREQUENCY INTENSITY NOISE

To calculate the zero-frequency intensity noise we go back to Eqs. (19), and separate the operators $\hat{I}$ and $\hat{N}$ into the mean value and a fluctuating part:

$$\hat{I} = I + \delta\hat{I}, \tag{35a}$$

$$\hat{N} = N + \delta\hat{N}. \tag{35b}$$

Then we linearize Eqs. (19) with respect to $\delta\hat{I}$ and $\delta\hat{N}$ and obtain

$$\frac{d\,\delta\hat{I}}{dt} = -(2\kappa - g)\,\delta\hat{I} + (B_N^{lm} + I g_N)\,\delta\hat{N} + \hat{F}_{\hat{I}} + \hat{F}_{\hat{R}}, \tag{36a}$$

$$\frac{d\,\delta\hat{N}}{dt} = -g\,\delta\hat{I} - (-\Lambda_N + B_N^{nr} + B_N^{om} + B_N^{lm} + I g_N) \times \delta\hat{N} + \hat{F}_{\hat{N}} - \hat{F}_{\hat{R}}, \tag{36b}$$

where Q_N means $\partial Q/\partial N$. The field intensity fluctuations $\delta\hat{I}^{out}$ in the output laser field are linked to the photon number fluctuations inside the cavity by the relation

$$\delta\hat{I}^{out} = 2\kappa\,\delta\hat{I} - \hat{F}_{\hat{I}}. \tag{37}$$

Taking the Fourier transform of Eqs. (36), setting the Fourier component frequency ω to zero, resolving the resulting set of linear algebraic equations with respect to the zero-frequency Fourier component of $\delta\hat{I}$, and inserting the result into Eq. (37), we obtain

$$\delta\hat{I}^{out}_{\omega=0} = [(2\kappa - g)(-\Lambda_N + B_N^{nr} + B_N^{om} + B_N^{lm} + I g_N) + g(I g_N + B_N^{lm})]^{-1}[g(-\Lambda_N + B_N^{nr} + B_N^{om})\hat{F}_{\hat{I}} + 2\kappa(-\Lambda_N + B_N^{nr} + B_N^{om})\hat{F}_{\hat{R}} + 2\kappa(I g_N + B_N^{lm})\hat{F}_{\hat{N}}]. \tag{38}$$

From the stationary state solution of Eqs. (27), it can be noted that if spontaneous emission in the lasing mode is neglected then g becomes equal to 2κ, and consequently in Eq. (38) 2κ disappears from all the coefficients that multiply the noise operators.

If thermal photons are neglected (otherwise see [19]) the only significant noise diffusion coefficients take the forms (see Appendix and [17])

$$2D_{\hat{I}\hat{I}}=2\kappa I, \tag{39a}$$

$$2D_{\hat{R}\hat{R}}=\frac{\beta}{\tau_{sp}}\sum_k L_k[I(1-2f_k+2f_k^2)+f_k^2], \tag{39b}$$

$$2D_{\hat{N}\hat{N}}=\xi\Lambda+B^{nr}+B^{om}, \tag{39c}$$

where ξ must be set equal to zero under the assumption of a quiet pump. With the help of Eqs. (38) and (39) we find that the zero-frequency noise, normalized to the shot noise level $2\kappa I$, is

$$\begin{aligned}S(\omega=0)&=\frac{\langle\delta\hat{I}^{out}_{\omega=0}\delta\hat{I}^{out}_{\omega=0}\rangle}{2\kappa I}\\&=[(2\kappa-g)(-\Lambda_N+B_N^{nr}+B_N^{om}+B_N^{lm}+Ig_N)\\&\quad+g(Ig_N+B_N^{lm})]^{-2}\times\Bigg[g^2(-\Lambda_N+B_N^{nr}+B_N^{om})^2\\&\quad+\frac{2\kappa\beta}{\tau_{sp}}(-\Lambda_N+B_N^{nr}+B_N^{om})^2\\&\quad\times\sum_k L_k\left(2f_k^2-2f_k+1+\frac{f_k^2}{I}\right)\\&\quad+\frac{2\kappa}{I}(Ig_N+B_N^{lm})^2(\xi\Lambda+B^{nr}+B^{om})\Bigg].\end{aligned} \tag{40}$$

Figure 4(a) displays the zero-frequency intensity noise determined by Eq. (40) as a function of the injection current, under the assumption of a quiet pump. The values of all the parameters and the notation are the same as in Fig. 1(a). For the case $\tau_{nr}=20\tau_{sp}$ the empty circles indicate the squeezing threshold, i.e., the value of the injection current at which the noise goes below the shot noise level. The full circles mark zero inversion points. It can be seen that squeezing takes place for positive inversion when $\beta=0.1$, while there is squeezing without inversion for $\beta=0.9$ and $\beta=0.99$. The temperature dependence of the squeezing thresholds is plotted in Fig. 4(b). A comparison between Fig. 1(b) and Fig. 4(b) reveals that the temperature behavior of the squeezing threshold is similar to that of the lasing threshold. In particular, we find that, if β is not too big, there is an optimum temperature at which squeezing can be obtained for a minimum value of the injection current. As β approaches 1, the squeezing threshold becames essentially independent of the carrier temperature.

VI. CONCLUSIONS

In this study we derived the macroscopic dynamical equations of a VCSEL, starting from a microscopic model that takes into account spontaneous emission into the lasing mode, pump blocking, and the temperature dependence of carrier distribution. Our analysis confirms that a low-threshold VCSEL may operate without inversion [4]. We used the fraction β of spontaneous emission that enters the lasing mode as a control parameter, and fixed the total spontaneous emission time τ_{sp} to a value in the range commonly reported in the literature. The specific values of β and τ_{sp} depend on the particular configuration of the VCSEL cavity, and they can be calculated by taking into account the density of the cavity field modes and the solid angle available for spontaneous emission [22,23].

We investigated the temperature dependence of the laser threshold, determined according to [4] as the injection current corresponding to $I_{st}=1$. We found that there exists a particular temperature that minimizes the lasing threshold [see Fig. 1(a)], at least if β is not too close to 1. For $\beta\approx1$ the threshold essentially does not depend on the temperature.

As mentioned in [4], the lasing threshold can be defined only approximately, because the intermediate region that separates the nonlasing from the lasing regime enlarges as β approaches 1. However, as shown in Fig. 2(a), a clear-cut transition point can be identified if instead of the injection current we use the dimensionless pump parameter r, defined as the ratio between the pump rate and the semiclassical threshold. Another advantage of introducing the normalized pump r is that the value $r=1$ separates, for all the parameter values, the regions with negative ($r<1$) and positive ($r>1$) inversion; see Fig. 2(b). We can therefore suggest that, as soon as β is different from zero, the laser undergoes a phase transition without inversion for a well-defined critical value of r. Further investigations of this inversionless phase transition would be very interesting.

Since the pump parameter r cannot be extracted directly from experiments, it is not possible to use it in practice for a definition of the lasing threshold. On the other hand, we remark that the threshold defined in [4] corresponds to the very beginning of the intermediate region [see Fig. 3(a)], and that it may be difficult to identify precisely the injection current corresponding to $I_{st}=1$, because of the noise in the lasing mode. This is why we proposed, in accordance with [19], to identify the lasing threshold as explained in Fig. 3(a). Such a threshold corresponds to $I_{st}>1$, so that it is less affected by noise and can easily be found from experimental data. We calculated the temperature dependence of this threshold and found again the possibility of lasing without inversion; see Fig. 3(b).

For the purpose of comparison of our microscopic model and the model studied in [4] we considered the low-temperature limit. In such a case the equations that follow from the microscopic model can be reduced to a form very similar to the equations of [4], and identical to the equations of [19].

We also studied the zero-frequency intensity noise for the case of a noiseless pump. Our model confirms that there can be intensity squeezing even in the case of lasing without inversion, if only the value of β is sufficiently close to 1; see Fig. 4(a). We calculated the temperature dependence of the injection current that corresponds to the shot noise level, and we found that there exists an optimum temperature at which squeezing takes place for a minimum of the injection current, as shown in Fig. 4(b). In connection with the results displayed in Fig. 4, we recall that because of pump blocking the zero-frequency intensity noise does not decrease down to zero for diverging injection current, but reaches an asymptotic value determined by several laser parameters [16–18]. However, the noise-increasing effect of pump blocking has been found to be negligibly small for current values as small as those corresponding to the circles in Fig. 4. We can therefore conclude that the temperature dependence of the squeezing threshold evidenced in Fig. 4(b) does not depend on pump blocking.

In a forthcoming paper our microscopic model will be generalized for the study of the noise properties of multimode semiconductor lasers, on the basis of the approach shown in [24].

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APPENDIX: THE NOISE DIFFUSION COEFFICIENTS

The diffusion coefficients of the field operators are determined by the noise correlation functions

$$\langle \hat{F}_{\hat{A}^\dagger}(t)\hat{F}_{\hat{A}}(t')\rangle = 2D_{\hat{A}^\dagger\hat{A}}\delta(t-t'), \tag{A1a}$$

$$\langle \hat{F}_{\hat{A}}(t)\hat{F}_{\hat{A}^\dagger}(t')\rangle = 2D_{\hat{A}\hat{A}^\dagger}\delta(t-t'), \tag{A1b}$$

and neglecting thermal photons they are [3]

$$2D_{\hat{A}^\dagger\hat{A}} = 2\kappa, \tag{A2a}$$

$$2D_{\hat{A}\hat{A}^\dagger} = 0. \tag{A2b}$$

For an analysis that does not neglect the noise effects of thermal photons, the reader is referred to [19]. As shown in [17], in a semiconductor laser the only other significant noise diffusion coefficients are $2D_{\hat{\sigma}_k^\dagger\hat{\sigma}_k}$, $2D_{\hat{\sigma}_k\hat{\sigma}_k^\dagger}$, and $2D_{\hat{N}_\alpha\hat{N}_{\alpha'}}$, which are related to the noise correlation functions

$$\langle \hat{F}_{\hat{\sigma}_k^\dagger}(t)\hat{F}_{\hat{\sigma}_k}(t')\rangle = 2D_{\hat{\sigma}_k^\dagger\hat{\sigma}_k}\delta(t-t'), \tag{A3a}$$

$$\langle \hat{F}_{\hat{\sigma}_k}(t)\hat{F}_{\hat{\sigma}_k^\dagger}(t')\rangle = 2D_{\hat{\sigma}_k\hat{\sigma}_k^\dagger}\delta(t-t'), \tag{A3b}$$

$$\langle \hat{F}_{\hat{N}_\alpha}(t)\hat{F}_{\hat{N}_{\alpha'}}(t')\rangle = 2D_{\hat{N}_\alpha\hat{N}_{\alpha'}}\delta(t-t'). \tag{A3c}$$

The procedure outlined in [17] leads to

$$2D_{\hat{\sigma}_k^\dagger\hat{\sigma}_k} = 2\gamma f_{ek}f_{hk} + \Lambda_k - B_k f_{ek}f_{hk} \approx 2\gamma f_{ek}f_{hk}, \tag{A4a}$$

$$2D_{\hat{\sigma}_k\hat{\sigma}_k^\dagger} = 2\gamma(1-f_{ek})(1-f_{hk}) - \Lambda_k + B_k f_{ek}f_{hk} \approx 2\gamma(1-f_{ek})(1-f_{hk}), \tag{A4b}$$

where the terms proportional to Λ_k and B_k can be neglected with respect to those proportional to γ, and intraband quasi-equilibrium has been assumed. The diffusion coefficients $2D_{\hat{I}\hat{I}}$ and $2D_{\hat{R}\hat{R}}$, linked to the correlation functions

$$\langle \hat{F}_{\hat{I}}(t)\hat{F}_{\hat{I}}(t')\rangle = 2D_{\hat{I}\hat{I}}\delta(t-t'), \tag{A5a}$$

$$\langle \hat{F}_{\hat{R}}(t)\hat{F}_{\hat{R}}(t')\rangle = 2D_{\hat{R}\hat{R}}\delta(t-t'), \tag{A5b}$$

can be found from the definitions of $\hat{F}_{\hat{I}}$ and $\hat{F}_{\hat{R}}$ by using the expressions of the diffusion coefficients $2D_{\hat{A}^\dagger\hat{A}}$, $2D_{\hat{A}\hat{A}^\dagger}$, $2D_{\hat{\sigma}_k^\dagger\hat{\sigma}_k}$, and $2D_{\hat{\sigma}_k\hat{\sigma}_k^\dagger}$. In the end, the only important noise diffusion coefficients turn out to be

$$2D_{\hat{I}\hat{I}} = 2\kappa I, \tag{A6a}$$

$$2D_{\hat{R}\hat{R}} = \sum_k \frac{2\gamma}{\gamma^2+\delta_k^2}|g_k|^2[I(1-f_{ek}-f_{hk}+2f_{ek}f_{hk})+f_{ek}f_{hk}], \tag{A6b}$$

$$2D_{\hat{N}\hat{N}} = \xi\gamma_p\sum_k \frac{f_{pk}}{N_p}(1-f_{ek})(1-f_{hk}) + \left(\frac{1}{\tau_{nr}}+\frac{1}{\tau_{sp}^{om}}\right) \times \sum_k f_{ek}f_{hk} = \xi\Lambda + B^{nr} + B^{om}. \tag{A6c}$$

The parameter ξ has been added *ex post facto* and takes the values $\xi=1$ and $\xi=0$ in the cases of Poissonian pump noise and a noiseless pump, respectively. The diffusion coefficients $2D_{\hat{I}\hat{R}}$ and $2D_{\hat{N}\hat{R}}$, respectively determined by the correlations

$$\langle \hat{F}_{\hat{I}}(t)\hat{F}_{\hat{R}}(t')\rangle = 2D_{IR}\delta(t-t'), \tag{A7a}$$

$$\langle \hat{F}_{\hat{N}}(t)\hat{F}_{\hat{R}}(t')\rangle = 2D_{NR}\delta(t-t'), \tag{A7b}$$

are proportional to $\kappa/\gamma \ll 1$ and can be neglected.

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