Effects of phase fluctuations of a laser on the dynamics of an atom in metallic cavities

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We study the effects of phase fluctuations of a laser on the dynamics of a two-level atom. We assume that the atom is in a cavity and consider two cases in which the atom is either sandwiched between two parallel metal plates or placed inside a rectangular metallic waveguide. We use the well-known master equation and utilize the phase diffusion model for our analysis. We derive expressions for the dipolar and dissipative forces acting on the atom. We then show that the laser fluctuations produce an increase in the magnitude of the dissipative force and a reduction in the dipolar force. We illustrate that the atom-laser interaction gives rise to a potential, which binds the atom to the central part of the cavity. We also show that the atomic velocity produces significant changes on the potential and on the force acting on the atom. The combined effects of the velocity and laser fluctuations on the atomic motion are discussed.

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I. INTRODUCTION

There is considerable interest in the study of the atomic motion induced by the field of a laser. The interest stems from the recent developments in the utilization of the radiation field to achieve cooling and trapping of atoms [1–3]. The atom optics [4] has become a major field of research activity dealing with the interaction of an atom with radiation. It is desirable in this field to find ways to maneuver atoms in some specific direction [5]. Al-Awfi and Babiker [6,7] have achieved significant progress in the study of atom dynamics induced by a laser radiation when the atom is inside two specific cavities: one formed by two parallel metal plates [6] and the other by a rectangular waveguide [7]. They have calculated forces acting on the atom and have shown that atoms can be trapped in the central part of the cavities in a potential well generated by the interaction of the atom with the radiation field.

Perfectly conducting parallel plates represents a relatively simple system that displays the effects of confinement in quantum electrodynamics. An atom, enclosed in the system, is attracted to the surface by the van der Waals force. However, a relatively stable state of the atom can be realized in the vicinity of the center of the cavity. In this region, the atom is under the influence of two almost equal forces in opposite directions. This fact was used by Sandogdhar *et al.* [8] for the first experimental demonstration of the van der Waals interaction between an atom located at the center of the gap and the metal surfaces. More recently the problem was tackled quantum mechanically by treating the change in the energy of the atom as a Lamb shift arising from excitations and de-excitations of the surface plasmons [9,10]. A metallic waveguide is another arrangement in which an atom can be guided along its axis by using a laser field as the driving force.

In the present paper, we consider the effects of phase fluctuations of a laser on the dynamics of an atom, enclosed in a cavity. Although the lasers are usually assumed to be coherent, there is significant presence of fluctuations both in the amplitude and in the phase angle. This is particularly true in the case of low power gas lasers in which collision broadening can be substantial. Since low power lasers are used in study of the atom dynamics, it is useful to examine the effect of the fluctuations on the atomic motion. We show in this paper that these effects can indeed be significant.

Lawande and Panat [11] have studied forces acting on an atom in free space by laser fields produced by a Laguerre-Gaussian, or by an ideal Bessel beam. The case of an atom in a cavity, which is the subject of the present investigations, differs substantially from the free-space situation. For an atom in a cavity, the electric fields are determined by the geometry of the cavity while in free space the fields are determined by the laser characteristics. Moreover the spontaneous decay coefficients are drastically changed when metallic boundaries are present. They also depend on the position of the atom within the cavity. The atomic transition rates are also affected to some extent by the van der Waals interaction between the atom and the metal plates. This interaction is, however, very weak everywhere except near the surface. We, therefore, consider the effects of the radiation field only and neglect the effects of the van der Waals interaction.

In this paper, we derive the exact master equation in which the effects of phase fluctuations are fully taken into account. We define the reduced density operator W for the system using the standard master equation under the rotating-wave approximation. The reduced density operator is averaged over the distribution of the phases following the procedure proposed by Lawande *et al.* [11]. The optical forces on the atom are then obtained by taking the trace of the gradient of the laser-atom interaction over the phase-averaged density operator $\overline{W}$.

The paper is organized as follows. In Sec. II, we give the basic formalism leading to the appropriate phase-averaged

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reduced atomic density operator $\overline{W}$. The density operator $\overline{W}$ is then used to derive the optical forces on the atom. Section III is devoted to the application of the formalism to study, in detail, the atomic motion in both the parallel plate and the rectangular waveguide systems. The effect of velocity on the atomic dynamics is presented in Sec. IV. Numerical results are discussed in Sec. V. Concluding remarks are made in Sec. VI.

II. FORMALISM

Consider a two-level atom in a cavity. We denote the ground state of the atom by $|1\rangle$ and the excited state by $|2\rangle$. The energy difference between the two levels is $\hbar\omega_0$. The atom, while inside the cavity, interacts with an externally tunable laser having a frequency ω_L. The frequency is selected so that it is in resonance with one of the modes of the cavity. We assume that the laser radiation propagates as a plane wave parallel to the plates (in the two geometrical configurations). The amplitude of the wave has the space dependence, consistent with the geometry of the cavity. With $\vec{P}$ and $\vec{R}$ as the momentum and position of the atom, the Hamiltonian of the system in the dipolar approximation [6,7] is given by

$$H=\frac{P^2}{2M}+\hbar\omega_0\Pi_{22}-i\hbar[\varepsilon(t)g(\vec{R})\exp(-i\omega_L t)\Pi_{21}-\text{H.c.}]$$
$$\equiv H_0+H_1. \tag{1}$$

Here $|i\rangle\langle j|=\Pi_{i,j}$ are the atomic operators obeying the commutation relation

$$[\Pi_{ij},\Pi_{kl}]=\Pi_{il}\delta_{jk}-\Pi_{kj}\delta_{il}, \tag{2}$$

where i, j, k, and l assume the values 1 and 2. $g(\vec{R})$ gives the variation of the electric field over the cross section of the cavity. The dynamics of the atom is described by the reduced atomic density operator ρ, which satisfies the master equation

$$i\hbar\frac{\partial\rho}{\partial t}=[H,\rho]+i\hbar R\rho, \tag{3}$$

where $R\rho$ is the dissipation term resulting from the coupling between the atom and the radiation vacuum of the cavity. It is given by [11]

$$R\rho=\Gamma[2\Pi_{12}\rho\Pi_{21}-\Pi_{22}\rho-\rho\Pi_{22}]. \tag{4}$$

Here 2Γ is Einstein's coefficient, which is appropriately modified by the cavity. The quantity $\varepsilon(t)$ is related to the electric field $\vec{E}(t)$ and the dipole moment $\vec{d}$ according to

$$\varepsilon(t)=\vec{d}_{12}\cdot\vec{E}(t)/\hbar. \tag{5}$$

The field $\vec{E}(t)$ will depend upon a particular mode of the cavity and can be written as

$$\vec{E}(t)=\vec{E}_0 e^{i\Theta+i\psi(t)}, \tag{6}$$

where Θ is $\vec{k}_g\cdot\vec{r}_\parallel-\omega_L t$ and $\psi(t)$ is the stochastic phase factor of the mode. Here $\vec{k}_g$ is a wave propagation vector and $\vec{r}_\parallel$ is the vector component of $\vec{r}$ parallel to the plates. Allowed values of the laser frequency ω_L depend on the geometry of the cavity. The dissipation constant Γ is altered by the cavity from its free-space value Γ_0 as a result of the changes in the density of states [6,7]. The Rabi frequency, given by $\Omega=|c(0)g(R)|$, where $c(0)=|\vec{d}_{12}\cdot\vec{E}_0/\hbar|$, depends on the mode function $\vec{E}_0$ and field profile function $g(\vec{R})$.

We describe the phase fluctuations in terms of a phase diffusion model where $\mu(t)=\dot{\psi}(t)$ is a Gaussian white noise such that

$$\overline{\mu}(t)=0 \quad \text{and} \quad \overline{\mu(t)\mu(t')}=2\Gamma_c\delta(t-t'). \tag{7}$$

Here the bar means ensemble average. Γ_c is the bandwidth of the phase fluctuations. Unlike Γ, the spontaneous emission rate, Γ_c does not change with the position of the atom. This is a consequence of the fact that Γ_c results from intrinsic laser fluctuations. The calculation of the optical forces on the atom requires obtaining the traces [11] of appropriate matrices such as $\text{Tr}(\rho\Pi_{21}e^{i\Theta+i\psi(t)})$ and $\text{Tr}(\rho\Pi_{12}e^{-i\Theta-i\psi(t)})$ averaged over the distributions of the phase fluctuations. To evaluate these quantities we make a unitary transformation of ρ:

$$W=\exp\{-i[\psi(t)+\omega_L t]\Pi_{22}\}\rho\exp\{i[\psi(t)+\omega_L t]\Pi_{22}\}. \tag{8}$$

Following the definition of W, we are able to express

$$\text{Tr}(\rho e^{i\psi(t)+i\Theta}\Pi_{21})=\text{Tr}(We^{i\Theta}\Pi_{21}) \tag{9}$$

and

$$\text{Tr}(\rho e^{-i\psi(t)-i\Theta}\Pi_{12})=\text{Tr}(We^{-i\Theta}\Pi_{12}). \tag{10}$$

Using Eqs. (3), (4), and (8), we obtain the master equation

$$\frac{dW}{dt}=[A_0-i\dot{\psi}(t)A_1]W, \tag{11}$$

where

$$A_0W=-i\Delta[\Pi_{22},W]-(\Omega e^{i\Theta}[\Pi_{21},W]-\text{H.c.})+\Gamma RW, \tag{12}$$

$$A_1W=[\Pi_{22},W], \tag{13}$$

and $\Delta_0=\omega_L-\omega_0$ is a detuning parameter. Equation (11) is the Langevin equation but with a difference that the operators A_0 and A_1 do not commute. Fortunately, it can be averaged over the phases using the delta function correlation given by Eq. (7). Thus, the phase averaged $\overline{W}$ satisfies

$$\frac{d\overline{W}}{dt}=(A_0-\Gamma_c A_1^2)\overline{W}. \tag{14}$$

Equation (14) can be solved in a steady state, i.e., $d\overline{W/dt}=0$. Various matrix elements of $\overline{W}$, given in [11], can be written as

$$(\overline{W})_{11}=\frac{|\Omega|^2(\Gamma+\Gamma_c)+\Gamma(\Gamma+\Gamma_c)^2+\Gamma\Delta^2}{D}, \tag{15}$$

$$(\overline{W})_{22}=1-(\overline{W})_{11}, \tag{16}$$

$$(\overline{W})_{12}=-\frac{\Omega\Gamma(\Gamma+\Gamma_c+i\Delta)e^{-i\Theta}}{D}, \tag{17}$$

and

$$(\overline{W})_{21}=(\overline{W})_{12}^{*}, \tag{18}$$

where Δ, the detuning factor, which includes the effect of atomic velocity V, according to Al-Awfi and Babiker [6,7] is given by

$$\Delta=\Delta_0-k_g V. \tag{19}$$

and

$$D=2|\Omega|^2(\Gamma+\Gamma_c)+\Gamma\Delta^2+\Gamma(\Gamma+\Gamma_c)^2. \tag{20}$$

When we take the average of the gradient of H_1, we will get two terms: one involving $\vec{\nabla}\Theta$ and the other involving $\vec{\nabla}\Omega$. The former is defined as the dissipative force and the later as the dipolar force. After evaluating the $\mathrm{Tr}(\overline{W}e^{i\Theta}\Pi_{21})$ and $\mathrm{Tr}(\overline{W}e^{-i\Theta}\Pi_{12})$ using expressions (15)–(18), and after some algebra we get the average dipolar force as

$$\langle F_{\mathrm{dipolar}}\rangle=-\frac{2\hbar\Gamma(\Delta)\Omega(\vec{\nabla}\Omega)}{(\Gamma+\Gamma_c)\left[2|\Omega|^2+\Gamma(\Gamma+\Gamma_c)+\left(\frac{\Gamma}{\Gamma+\Gamma_c}\right)\Delta^2\right]}. \tag{21}$$

The dipolar force is conservative and generates the expression for the potential energy U:

$$U=\frac{\hbar}{2}\left(\frac{\Gamma}{\Gamma+\Gamma_c}\right)\Delta\ln\left[1+\frac{2|\Omega|^2}{\Gamma(\Gamma+\Gamma_c)+\frac{\Gamma}{\Gamma+\Gamma_c}\Delta^2}\right]. \tag{22}$$

Similarly, the average dissipative force is given by

$$\langle F_{\mathrm{dissipative}}\rangle=\frac{2\hbar\Gamma|\Omega|^2\vec{\nabla}\Theta}{\left[2|\Omega|^2+\Gamma(\Gamma+\Gamma_c)+\frac{\Gamma}{\Gamma+\Gamma_c}\Delta^2\right]}. \tag{23}$$

The origin of the dissipative force arises from the absorption and spontaneous emission cycles. The dipolar force on the other hand arises from the absorption and stimulated emission cycles. The phase fluctuations are expected to affect the two forces differently. This will be reflected more clearly in the following analysis.

We take the ratio of the dipolar forces with and without fluctuations and find the condition for the ratio to be greater than unity. We separate the space dependent parts from Γ and Ω by introducing the definitions $\Gamma=\Gamma_0 f$ and $\Omega=\Omega_0 g$. The ratio of the dipolar forces with fluctuations present and when absent, using Eq. (21) can be written as

$$[\langle F_{\mathrm{dipolar}}\rangle_{\beta\neq 0}/\langle F_{\mathrm{dipolar}}\rangle_{\beta=0}]=\frac{f[2g^2+f^2(\Gamma_0/\Omega_0)^2+(\Delta/\Omega_0)^2]}{(f+\beta)[2g^2+f(f+\beta)(\Gamma_0/\Omega_0)^2+\{f/(f+\beta)\}(\Delta/\Omega_0)^2]}, \tag{24}$$

where $\beta=(\Gamma_c/\Gamma_0)$.

The ratio is greater than unity provided the following condition is satisfied:

$$0>[2g^2+f\beta(\Gamma_0/\Omega_0)^2+2f^2(\Gamma_0/\Omega_0)^2]. \tag{25}$$

When f is positive the above condition is never satisfied. We will find later that for the two cavities considered in this paper f is always positive. Hence, the dipolar force with fluctuations cannot exceed the force when the fluctuations are absent. The corresponding condition for the dissipative force requires that

$$(\Delta/\Omega_0)^2>f(f+\beta). \tag{26}$$

The condition (26) is satisfied easily for a range of values of Δ and Ω_0. Consequently the dissipative force can be greater with fluctuations present than when they are absent provided condition (26) is satisfied.

III. ATOMIC MOTION IN CAVITIES

In this section, we will define quantities required in Eqs. (21)–(23) and apply them for the motion of an atom enclosed within two parallel plates.

In a Cartesian coordinate system, we assume that the two metal plates are parallel to the $x-y$ plane and are located at $z=0$ and at $z=L$. The electric field for the TE mode is expressed by [6]

$$\vec{E}(\vec{k}_g,n,\vec{r},t)=C(\vec{k}_g,n)(\hat{k}_g\times\hat{z})\sin(n\pi z/L)\times\exp\{i[\vec{k}_g\cdot\vec{\rho}-\omega(k_g,n)t]\}. \quad (27)$$

The electric field for the TM mode is given by

$$\vec{E}(\vec{k}_g,n,\vec{r},t)=\frac{-iC(\vec{k}_g,n)}{\omega(k_g,n)}\left[\frac{i\vec{k}_g}{|\vec{k}_g|}\left(\frac{n\pi}{L}\right)\sin\left(\frac{n\pi z}{L}\right)-\hat{z}|\vec{k}_g|\cos\left(\frac{n\pi z}{L}\right)\right]\times\exp\{i[\vec{k}_g\cdot\vec{\rho}-\omega(k_g,n)t]\}. \quad (28)$$

Here the mode frequency ω is given by

$$\omega^2(\vec{k}_g,n)=c^2\left[k_g^2+\frac{n^2\pi}{L^2}\right] \quad (29)$$

and

$$C(k_g,n)=\left[\frac{\hbar\omega(k_g,n)}{\varepsilon_0 AL f_n}\right]^{1/2}, \quad (30)$$

where $f_0=2$ and $f_n=1$ for $n>0$, $\vec{r}\equiv(\vec{\rho},z)$ and A is the surface area of the plates.

The free-space spontaneous transition rate is given by

$$\Gamma_0=\frac{\omega_0^3|d_{12}|^2}{3\pi\hbar\varepsilon_0 c^3} \quad (31)$$

as demonstrated by Al-Awfi and Babiker [6]. For an atom moving within parallel conducting plates two situations must be considered. The first is when the dipole of the atom is oriented parallel to the plates, while the second is when the atomic dipole is oriented perpendicular to the plates. For the case where the dipole is parallel to the plates, the emission rate for $1<2L/\lambda<2$ is given by [6]

$$\Gamma_\parallel=\Gamma_0\left(\frac{3\lambda}{4L}\right)\left\{1+\frac{\lambda^2}{4L^2}\right\}\sin^2\left(\frac{\pi z}{L}\right), \quad (32)$$

$$=\Gamma_0 f_\parallel(z).$$

Clearly $f_\parallel$ is positive and hence the conclusion from Eq. (25) that dipolar force with fluctuations is always smaller than the dipolar force without fluctuations. The corresponding Rabi frequency Ω is

$$\Omega_\parallel=\Omega_0\sqrt{2}\left(\frac{\lambda}{2L}\right)\sin\left(\frac{\pi z}{L}\right)=\Omega_0 g_\parallel(z). \quad (33)$$

If I is the intensity of the laser used to excite the mode, the free-space Rabi frequency is

$$\Omega_0=\frac{|\vec{d}_{12}|\sqrt{I}}{\hbar\sqrt{2\varepsilon_0 c}}. \quad (34)$$

When the atomic dipole is perpendicular to the metal plates,

$$\Gamma_\perp=\Gamma_0\left[\frac{3\lambda}{4L}\left\{1+2\left(1-\frac{\lambda^2}{4L^2}\right)\cos^2\left(\frac{\pi z}{L}\right)\right\}\right]=\Gamma_0 f_\perp(z). \quad (35)$$

and

$$\Omega_\perp=\Omega_0\sqrt{2\left(1-\frac{\lambda^2}{4L^2}\right)}\cos\left(\frac{\pi z}{L}\right)=\Omega_0 g_\perp(z). \quad (36)$$

B. Atomic motion in a rectangular waveguide

We now consider a rectangular waveguide. We select a coordinate-axis scheme different from the one used in the previous section. We assume that the axis of the waveguide is oriented along the z axis and the cross section is assumed to be parallel to the $x-y$ plane. The cross section has the dimensions of a and b along the x axis and the y axis, respectively. According to Al-Awfi and Babiker [7], the electric fields for the TE mode is

$$\vec{E}(\vec{r},t)=\sqrt{\frac{2\hbar\omega(Q)}{AL\varepsilon_0[\kappa_m^2+\kappa_n^2]f_{mn}}}[\hat{e}_x\kappa_m\cos(\kappa_n x)\sin(\kappa_m y)-\vec{e}_y\kappa_n\sin(\kappa_n x)\cos(\kappa_m y)]\exp[i\{k_g z-\omega(Q)\}t], \quad (37)$$

and for TM modes, the electric fields are

$$\vec{E}(\vec{r},t)=\sqrt{\frac{2\hbar c^2}{AL\varepsilon_0\omega(Q)[\kappa_m^2+\kappa_n^2]}}[\hat{e}_x\kappa_n k_g\cos(\kappa_n x)\times\sin(\kappa_m y)+\vec{e}_y\kappa_m k_g\sin(\kappa_n x)\cos(\kappa_m y)+i\vec{e}_z(\kappa_n^2+\kappa_m^2)\sin(\kappa_n x)\sin(\kappa_m y)]\times\exp[i\{k_g z-\omega(Q)t\}], \quad (38)$$

where

$$\omega^2(Q)=c^2\{k_g^2+\kappa_m^2+\kappa_n^2\},\quad \kappa_n=n\pi/a,\quad \text{and}\quad \kappa_m=m\pi/b. \quad (39)$$

A is the cross-sectional area of the guide, L is its (large) length, $f_{01}=f_{10}=(1/2)$, and $f_{mn}=1$ for $m\geqslant 1$ and $n\geqslant 1$. We define $\Gamma_\parallel$ as the decay rate when the atomic dipole is parallel to the z axis and $\Gamma_\perp$ the rate when the dipole is in the y direction. We assume that $a=b=L$ and $1<(\lambda/L)<2$. Since the main aim of this paper is to examine the effects of phase fluctuations, these assumptions will not seriously restrict the conclusions of this paper. Interchange of x and y will provide the results when the dipole is in the x direction. The simplicity arises because of our assumption that $a=b=L$. The results for various parameters are obtained from the work of Al-Awfi and Babiker [7]. They are given as follows:

$$\frac{\Gamma_{\|}}{\Gamma_0}=f_{\|}=(3/4)(\lambda^4/L^4)\frac{1}{\sqrt{1-\lambda^2/2L^2}}\sin^2(\pi x/L)\sin^2(\pi y/L) \tag{40}$$

and

$$\frac{\Gamma_{\perp}}{\Gamma_0}=f_{\perp}=(3/\pi)\frac{\lambda^2}{L^2}\left[\frac{\sin^2(\pi x/L)}{\sqrt{1-\lambda^2/4L^2}}+\frac{\cos^2(\pi y/L)\sin^2(\pi x/L)}{4\sqrt{1-\lambda^2/2L^2}}\left(2-\frac{\lambda^2}{2L^2}\right)\right]. \tag{41}$$

The Rabi frequencies for the dipole parallel to the z axis and parallel to the y axis are given, respectively, by [7]

$$\frac{\Omega_{\|}}{\Omega_0}=g_{\|}=\sqrt{2}\left(\frac{\lambda}{L}\right)\sin(\pi x/L)\sin(\pi y/L) \tag{42}$$

and

$$\frac{\Omega_{\perp}}{\Omega_0}=g_{\perp}=\sqrt{2\left(1-\frac{\lambda^2}{2L^2}\right)}\cos(\pi y/L)\sin(\pi x/L). \tag{43}$$

IV. TIME-DEPENDENT EFFECTS

In this section, we discuss the time-dependent effects associated with atomic motion in the confined geometries. Equations (21)–(23) show that the force and the potential acting on the atom depends on the atomic velocity. The dependence arises through the term Δ defined by Eq. (19). To evaluate these changes, we use the equation of motion to determine the atomic velocity as a function of time t using the relation

$$M\frac{dV}{dt}=\frac{2\hbar\Gamma|\Omega|^2\vec{\nabla}\Theta}{\left[2|\Omega|^2+\Gamma(\Gamma+\Gamma_c)+\frac{\Gamma}{\Gamma+\Gamma_c}\Delta^2\right]}, \tag{44}$$

where we have used the expression for the dissipative force, which acts in the direction of laser propagation. The solution can be expressed as

$$t=a_1V^3+a_2V^2+a_3V, \tag{45}$$

where

$$a_1=\frac{k_gM}{6\hbar\Omega_0^2fg^2}\left[\frac{f}{f+\beta}\right],\quad a_2=\frac{\Delta_0M}{2\hbar\Omega_0^2fg^2}\left[\frac{f}{f+\beta}\right],$$

$$a_3=\left[2\Omega_0^2g^2+\left(\frac{f}{f+\beta}\right)\Delta_0^2+\Gamma_0^2f(f+\beta)\right]\times\frac{M}{2\hbar\Omega_0^2k_gfg^2}, \tag{46}$$

and we have assumed that the laser excitation was initiated at time $t=0$.

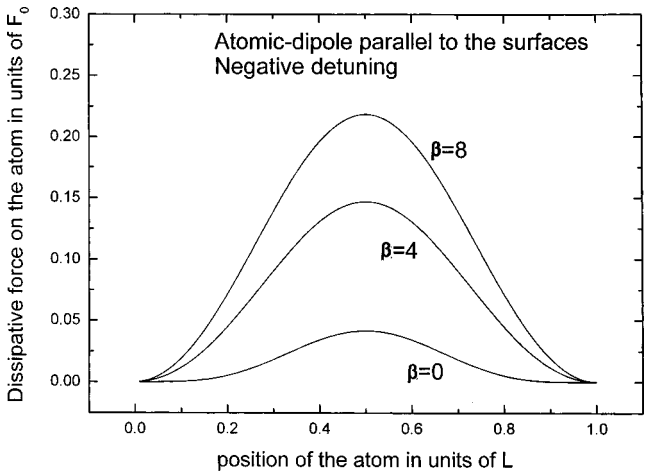

FIG. 1. Variations of the dissipative force on the atom over the gap distance in units of F_0 when the atom is situated between two parallel plates. We assume that the atomic velocity is zero. The effect of phase fluctuations on the force is denoted by three values of the parameter β.

From the knowledge of V as a function of time, it is possible to evaluate the time development of the dissipative force or the potential energy using Eqs. (23) and (22), respectively. The results are obtained numerically and are discussed in the following section for the two cavities.

V. RESULTS AND DISCUSSION

A. Atom between two parallel plates

We obtain quantitative results for the effect of fluctuations on the atomic motion using Eqs. (22) and (23) and using the results given in Sec. III A for the case in which the atom is situated between two parallel plates. The width between the planes is normalized to unity so that z/L varies from 0 to 1. We use the same values for the parameters as those used in the work of Al-Awfi and Babiker [6,7]. The energy difference between the ground state and the excited state of our two-level atom is defined by the sodium line $\lambda=5890$ Å. We assume that the intensity of the laser radiation is given by $I=10^7\ \mathrm{W\,m^{-2}}$ and that $|\vec{d}_{12}|=2.6ea_0$ (e is the electron charge and a_0 is the Bohr radius). Using Eq. (33), we get $\Omega_0=8.56\times10^9\ \mathrm{s^{-1}}$. The width between the parallel metal plates is assumed to be 5000 Å, which is smaller than λ for reasons discussed in the work of Al-Awfi and Babiker [6,7]. We use the value for $\Gamma_0=6.13\times10^7\ \mathrm{s^{-1}}$. The detuning factor, defined by $\Delta_0=\omega-\omega_0$, is selected to be $\Delta_0=\pm6\times10^2\Gamma_0$ where the negative value implies the negative detuning. The atom is subjected to a laser beam, which contains a phase-fluctuating component. The component is defined in terms of Γ_c, which is the bandwidth [see Eq. (7)]. The numerical estimates are obtained in terms of a dimensionless parameter $\beta=(\Gamma_c/\Gamma_0)$, which is the measure of the phase fluctuations of the laser beam.

In Fig. 1 we have shown the magnitude of the dissipative force expressed in units of F_0 when the dipole is oriented parallel to the plates. We define the force unit F_0 in conformity with Al-Awfi and Babiker [6,7] according to

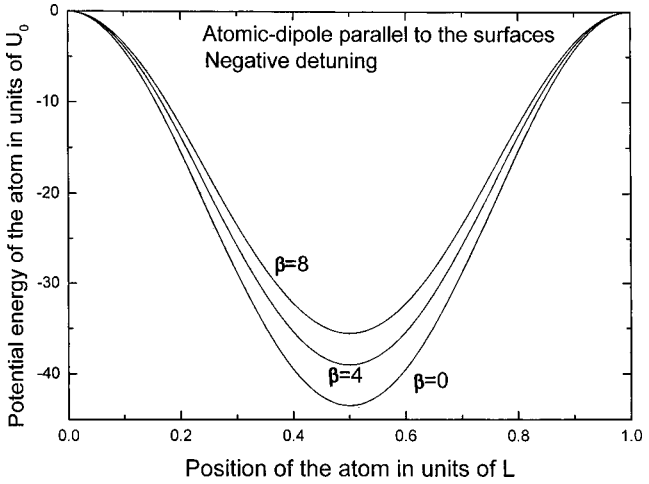

FIG. 2. Variation of the potential energy of the atom over the gap distance in units of U_0 when the atom is situated between two parallel plates. We assume that the atomic velocity is zero. The effect of the phase fluctuations on the potential energy is denoted by the three values of the parameter β.

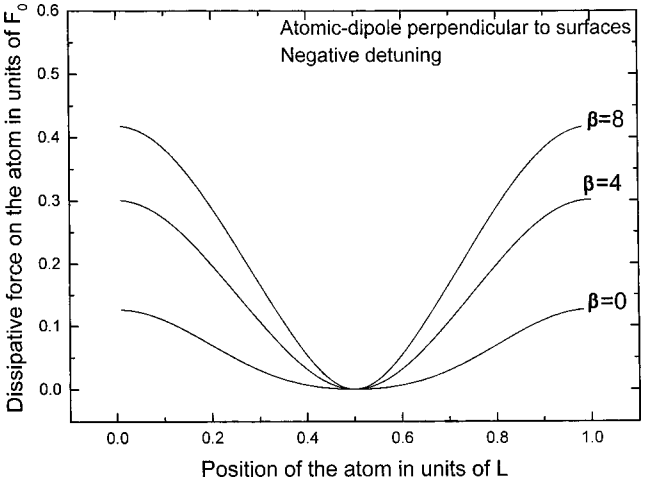

FIG. 3. Variation of the dissipative force on the atom over the gap distance in units of F_0 when the atom is situated between two parallel plates. We assume that the atomic velocity is zero. The effect of the phase fluctuations on the force is denoted by three values of the parameter β.

$$F_0 = 2\hbar\Gamma_0 = 2.33\times 10^{-20}\ \text{N}. \tag{47}$$

The force is along the direction of laser propagation. From Fig. 1, we conclude that the force is at its maximum value when the atom is at the center of the gap. It reduces to zero at the plates. The force near the center increases with the increase in the fluctuations (i.e., as β increases). This result is consistent with predictions of Eq. (26). With the increase in the fluctuations, the increase in the force is spread over greater area of the cross section of the waveguide. The fluctuations have virtually no effect on the atomic force at or near the surface. In Fig. 2, we have shown the dependence of the potential energy of the atom expressed in units of U_0, as a function of position within the gap, when the dipole is parallel to the plates. The unit of the potential energy U_0 [6,7] is given by

$$U_0 = (1/2)\hbar\Gamma_0 = 3.23\times 10^{-27}\ \text{J}. \tag{48}$$

The potential energy, as seen from Fig. 2, provides a binding potential for the atom, restricting the movement of the atom to the central region when negative detuning is applied. The potential depth as well its width of the potential, decreases as β increases. In summary, the phase fluctuations produce different effects on the atomic motion; the fluctuations increase in the magnitude of the force but reduce in the binding. Both these factors can be tailored to achieve maximization of atomic channeling.

In Fig. 3 we have sketched the magnitude of the dissipative force when the atomic dipole is oriented normal to the plates. The force is zero at the center of the gap and attains a large value as the atom approaches the plates. The fluctuations increase the force as the atom comes closer to the surface. Fluctuations have no effect on the force at the center. In Fig. 4, we have shown the variation in potential energy across the gap when negative detuning is applied. The potential energy is zero at the center and decreases towards the plates. The atom, therefore, is repelled away from the central region and attracted by the plates. If positive detuning is applied, the potential energy changes its sign and the repulsive potential at the center becomes attractive. The force, however, does not change in sign with the change in the sign of the detuning. The force although zero at the center, increases and attains a sufficiently large value in the region in-between the center of the gap and the plates. Thus effective channeling can be achieved with positive detuning when the dipole is perpendicular to the plates. The fluctuations reduce the binding potential when positive detuning is applied.

In Figs. 5(a) and 5(b) we have sketched the potential energy across the width of the two metal plates, as the velocity

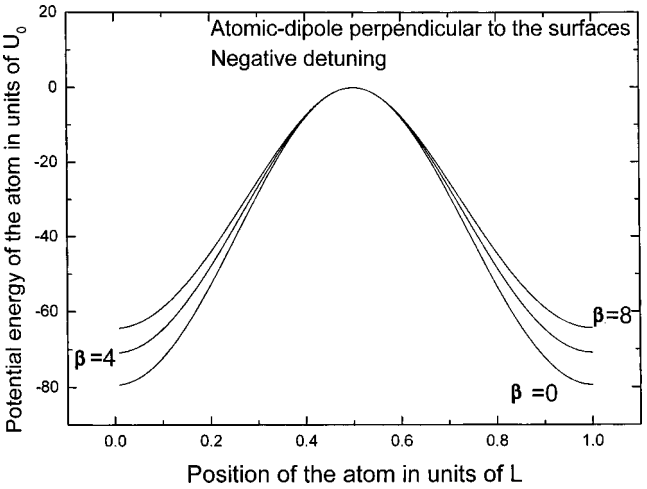

FIG. 4. Variation of the potential energy of the atom over the gap distance in units of U_0 when the atom is situated between two parallel plates. We assume that the atomic velocity is zero. The effect of phase fluctuations on the potential energy is denoted by three values of the parameter β.

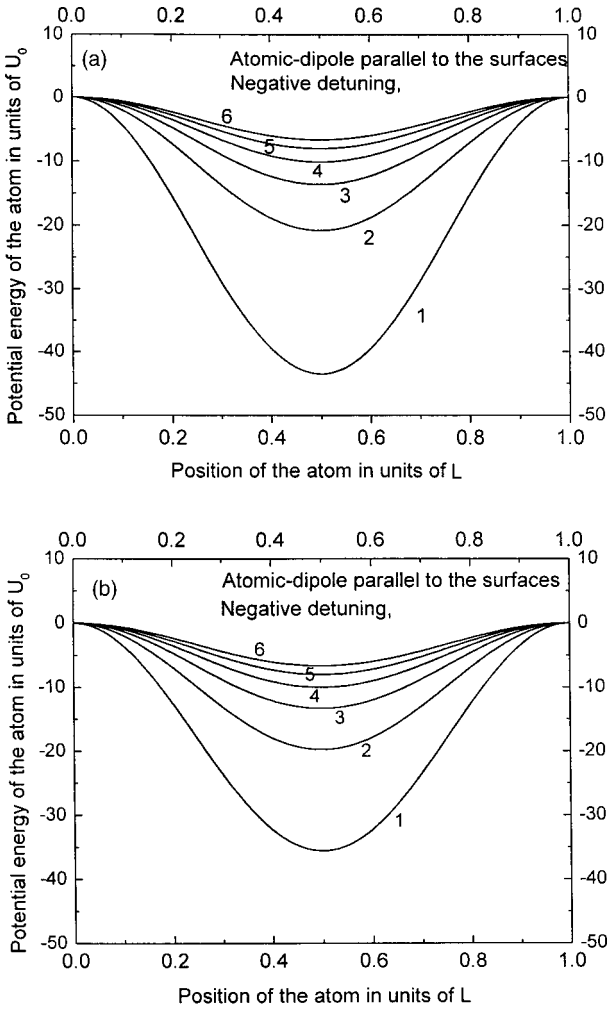

FIG. 5. Variation of the potential energy of the atom over the gap distance in units of U_0 for various values of the atomic velocity. The labels 1–6 stand for atomic velocities $V=0.0$, 0.4, 0.8, 1.2, 1.6, and 2.0×10^4 ms^{-1}, respectively. The fluctuation parameter is $\beta=0$ in (a) and $\beta=8$ in (b).

of the atom increases. The results are shown in Fig. 5(a) when fluctuation parameter $\beta=0$ and in Fig. 5(b) we show the results for $\beta=8$. Figures 5(a) and 5(b) show that as the atomic velocity increases the binding potential becomes weaker. For both the cases, we expect that for the atomic velocity in excess of $V=2.0\times10^4$ m/s the potential energy may be too shallow to bind the atom. Figures 6 and 7 show the time variations of the atomic velocity and the atomic force for three values of the fluctuation parameter β. Since the force increases with fluctuations, the atomic velocity is greater for the larger values of the parameter β. Figure 7 shows that the magnitude of the force decreases with time and approaches zero asymptotically.

The two parallel plates under the action of a laser can act as an atomic separator between the atoms having the dipole moment directed parallel and perpendicular to the surfaces. If negative detuning is used, the atoms with dipoles parallel to the plates experience a force in the direction of the beam and as a consequence the atom can be effectively channeled through the central region of the gap. The atoms with dipole

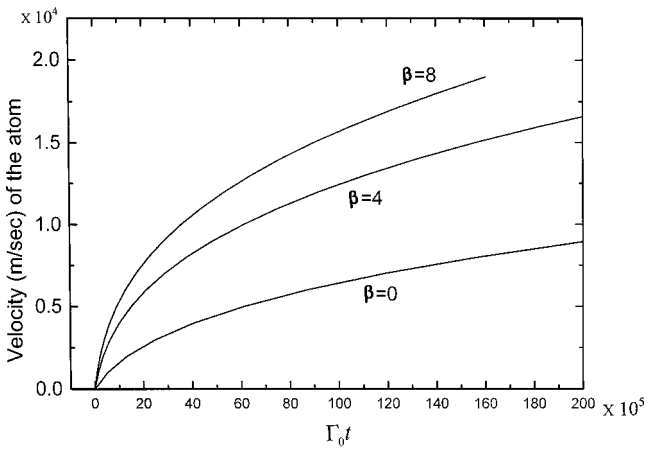

FIG. 6. Variation of the atomic velocity V with $\Gamma_0 t$ when the atomic dipole is parallel to the surfaces for three values (0,4,8) for the phase-fluctuation parameter β. The atom is assumed to be on the midpoint plane between the gap.

perpendicular to the plates are, however, pushed out from the central region and are attracted by the surface. For a positive detuning a reverse situation occurs. Dipoles parallel to the plates are pushed away from the central region while dipoles normal to the plates can be effectively channeled.

B. Atom in a rectangular waveguide

In this section we obtain numerical results for the motion of an atom in a rectangular waveguide. Our focus is, as before, on the effect of the phase fluctuations on the atomic motion. We use the same values for various parameters used by Al-Awfi and Babiker [6,7]. The cross section in normalized to unity so that x/L and y/L vary from 0 to 1. In Figs. 8 and 9, we have shown the dissipative force acting on the atom when the dipole is directed along the axis of the waveguide. In Fig. 8, we assume that the laser fluctuations are zero, i.e., $\beta=0$, while in Fig. 9, we assume that the fluctuations are present and are given by the parameter $\beta=8$. The

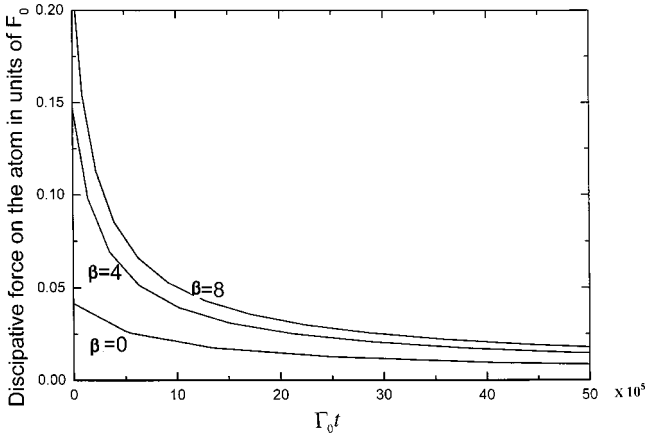

FIG. 7. Variation of the dissipative force on the atom with $\Gamma_0 t$ when the atomic dipole is parallel to the surfaces for three values (0,4,8) for the phase-fluctuation parameter β. The atom is assumed to be on the midpoint plane between the gap.

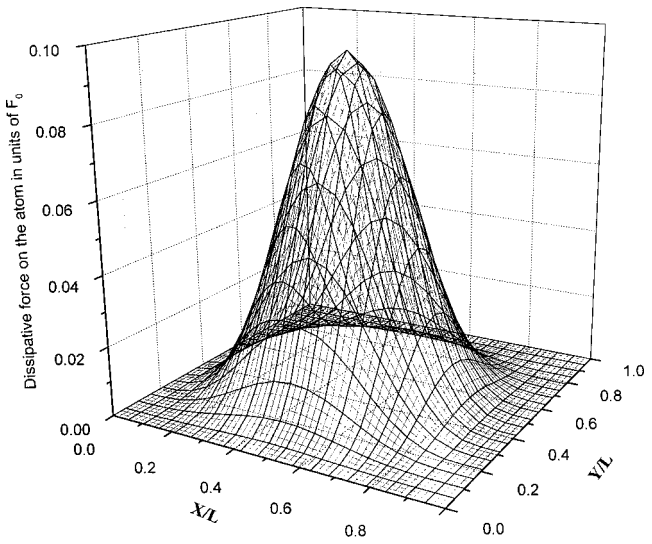

FIG. 8. Variation of the dissipative force on the atom over the cross section of a waveguide in units of F_0 when the atom is situated within a rectangular waveguide. The atomic dipole is assumed parallel to the axis of the waveguide. The phase-fluctuation parameter β is assumed to be zero.

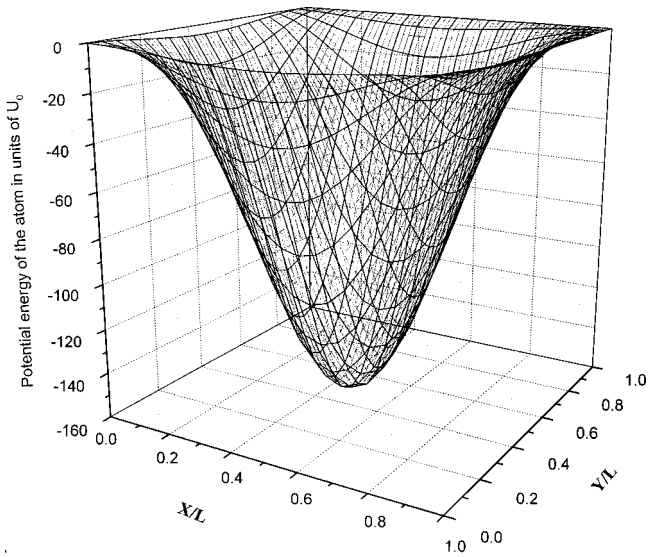

FIG. 10. Variation of the potential energy of the atom expressed in units of U_0 over the cross section of a waveguide when the atom is situated within a rectangular waveguide. The atomic dipole is assumed parallel to the axis of the waveguide. The phase-fluctuation parameter β is assumed to be zero.

figures clearly show that the force is large at the center of the waveguide and falls to zero at the surfaces. The comparison between Figs. 8 and 9 shows that the force increases as the fluctuations increase. This is consistent with the predictions of Eq. (26). The force at the center is about 3 times larger at $\beta=8$ than at $\beta=0$. While the force is relatively localized near the center of the guide when $\beta=0$, the spread of the force is broadened to a larger fraction of the cross section when $\beta=8$. Comparison between Figs. 1, 8, and 9 reveals that the maximum force on the atom is greater when the atom is within the waveguide than when the atom is in between two parallel plates. When negative detuning is applied, the interaction between the atom and the radiation field produces a binding potential for the atom when the dipole is parallel to the axis. As the fluctuations increase, the binding potential is weakened as seen from Figs. 10 and 11.

When the dipole is normal to the surfaces the force and the potential acting on the atom is different from the case when the atomic dipole is parallel to the axis. We discuss these results briefly without displaying them in diagrams.

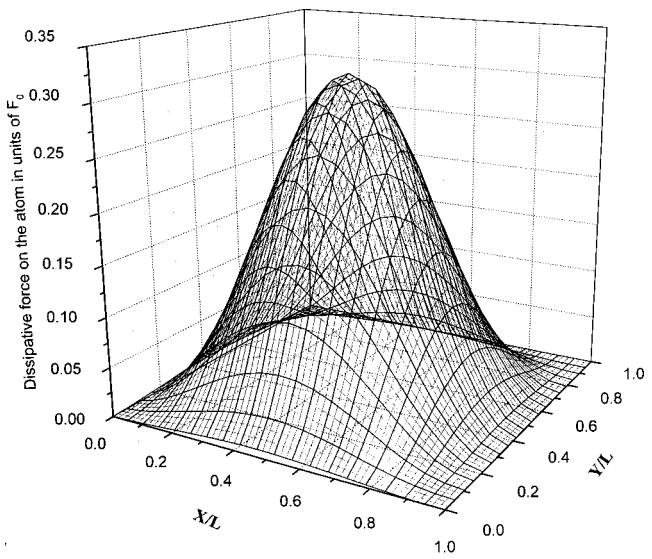

FIG. 9. Variation of the dissipative force on the atom over the cross section of a waveguide in units of F_0 when the atom is situated within a rectangular waveguide. The atomic dipole is assumed parallel to the axis of the waveguide. The effect of phase fluctuations on the force is denoted by the parameter $\beta=8$.

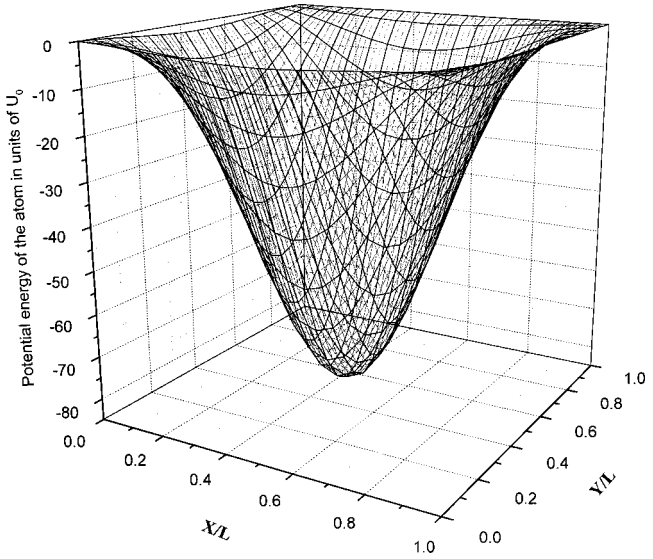

FIG. 11. Variation of the potential energy of the atom over the cross section of a waveguide in units of U_0 when the atom is situated within a rectangular waveguide. The atomic dipole is assumed parallel to the axis of the waveguide. The effect of phase fluctuations on the energy is denoted by the parameter $\beta=8$.

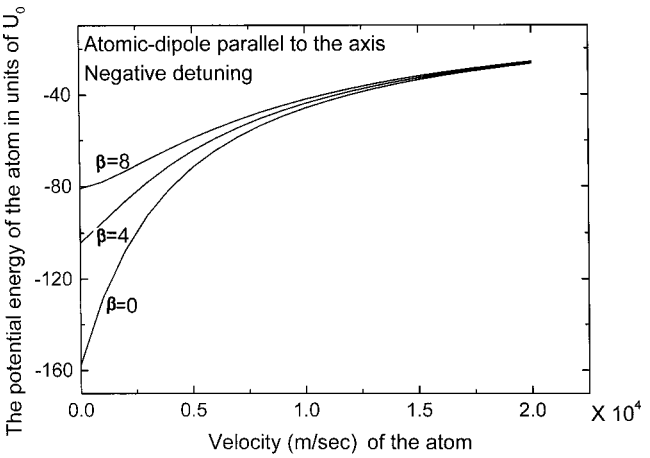

FIG. 12. Variation of the potential energy of the atom with atomic velocity V, when the atom is located on the axis of the waveguide for three values (0,4,8) of the phase-fluctuation parameter β. The atomic dipole is assumed to be parallel to axis of the waveguide.

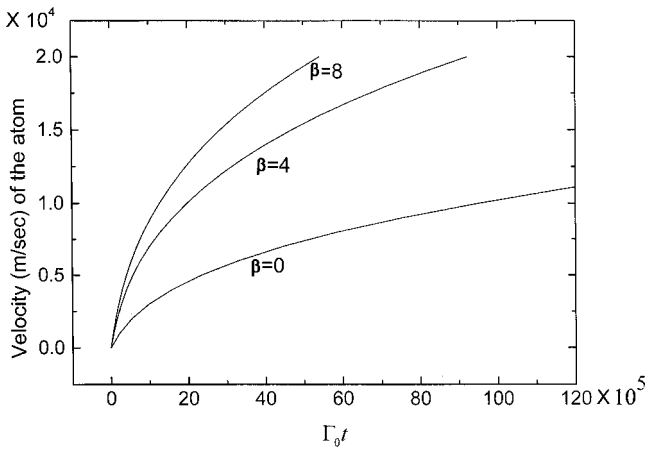

FIG. 13. Variation of the atomic velocity V as a function of $\Gamma_0 t$ when the atom is located on the axis of the waveguide. The atomic dipole is assumed parallel to the axis of the waveguide. The effect of phase fluctuations is denoted by three values (0,4,8) of the parameter β.

The force on the atom at the center of the waveguide is zero. It increases as we approach the $y=0$ and $y=1$ planes. As the fluctuations increase from $\beta=0$ to $\beta=8$, the force increases near the $y=0$ or $y=1$ planes by about a factor of 3. The force on the atom near planes $x=0$ and $x=1$ remains close to zero. The potential energy is zero at the center of waveguide and along the $y=0.5$ plane. It decreases to about $-35\,U_0$ as we approach the $y=0$ and $y=1$ planes. The potential would push the atom away from the center and the atom is attracted towards the $y=0$ and $y=1$ planes. The effect of fluctuations is to reduce the potential energy by about 10%–15%. When positive detuning is applied the potential energy reverses its sign, and instead of a repulsive potential it changes to a binding potential along $y=0.5$ plane.

In Fig. 12, we have displayed the variation of the potential energy of the atom at the center of waveguide as a function of velocity when the atomic dipole is parallel to the axis of the waveguide and when negative detuning is applied. As was the case of parallel plates, the depth of the potential energy well decreases as the velocity increases. The time variation of the atomic velocity is shown in Fig. 13 for three values of the fluctuation parameter β. As expected the atomic velocity increases with time and the increase is greater when the value of the parameter β is larger. The force acting on the atom decreases to a small value with time as can be seen from Fig. 14. At time $t=0$ the force acting on the atom is greater for larger values of β. The values are consistent with the force evaluated and shown in Figs. 8 and 9 for $\beta=0$ and $\beta=8$, respectively.

VI. CONCLUDING REMARKS

So far, we have not included effects of van der Waals interaction on the atomic motion. When the laser field is absent, the force between an atom and the surface is often entirely due to the van der Waals interaction. In view of its intrinsic importance, we would like to make comments on the role of the van der Waals to interaction on the conclusions of this paper. A long time ago, Zaremba and Kohn [12] made an exhaustive study of the van der Waals interaction between a metal surface and rare-gas atoms. The main result of their study shows that the interaction energy can be expressed in terms of a functional relation $-A/(z-z_0)$ where z is the separation of the atom from the metal surface and z_0 is the distance from the metal surface of a plane defined as a ''reference plane.'' A is a constant, derived in a classic paper by Lifshitz [13]. Zaremba and Kohn [12] have expressed the binding energy of the atom bound to the metal surface using the expression for the van der Waals interaction energy and the equilibrium position of the atom. The binding energy varies for different metal surfaces. It attains the lowest value of 0.13 meV for a He atom bound to a Cs surface. The value corresponds to the energy level of about $-6000\,U_0$, in terms of the energy units used in this paper and defined in Eq. (48).

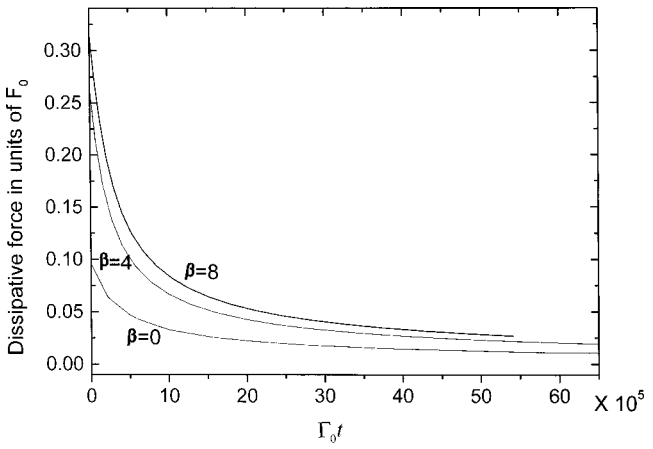

FIG. 14. Variation of the dissipative force on the atom in units of F_0 as a function of $\Gamma_0 t$ when the atom is situated on the axis of the waveguide. The atomic dipole is assumed parallel to the axis of the waveguide. The effect of phase fluctuations is denoted by three values (0,4,8) of the parameter β.

Neglecting the kinetic energy of the atom, the energy level of the atom induced by the laser field is [see Fig. 7] $-160\,U_0$ when the atomic dipole is parallel to the axis of the waveguide. The laser-induced energy level is thus much higher than the energy level of the atom bound to the surface [12]. We would expect the atom bound to the central potential to tunnel to one of the excited states of the atom bound to the surface. The atom would ultimately reach the ground state of the surface potential by emission of single-particle excitations of the metal electrons. The effective channeling is, therefore, possible only during the lifetime of the atom in the binding potential produced by the radiation field. A more detailed study is required in order to make quantitative predictions of lifetime and the associated atomic channeling.

ACKNOWLEDGMENTS

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