Magneto-optic drift of ions

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Light-induced drift of optically active atoms in a buffer gas is a consequence of the unequal diffusive frictions suffered by the excited and the ground-state atoms. This drift can be used to create an ''optical piston,'' in which the active atoms are pushed forward by light through the semipermeable membrane of the buffer gas. Normally, optical piston effect is studied when the active atoms are neutral in a confined one-dimensional situation. We present a detailed theory of this phenomenon when the active atoms are charged and a magnetic field is applied for ''tuning'' the direction as well as the magnitude of the drift, thus removing the necessity of confinement. Our study is in different geometries of the light beam and the magnetic field, first in the weak collision model, and then for the strong collision and the Boltzmann-Lorentz models.

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I. INTRODUCTION

The effect of light-induced drift (LID) of optically active two-level atoms embedded in a dense buffer gas was initially predicted by Gel'mukhanov and Shalagin [1]; its first unintentional observation was by Bjorkholm *et al.* [2], and the first detailed experimental reports on LID were given by Antsygin and coworkers [3]. LID occurs when (i) the excitation of the absorbing atoms is velocity selective because of the Doppler effect, and (ii) the rate of collisions with the buffer gas is state dependent, i.e., the interaction potential with the buffer gas atoms is not the same for the excited and the ground-state atoms. When the active atoms are excited by a narrow-band radiation field with a midfrequency red-detuned with respect to the Doppler-broadened absorption line center of the atoms, the traveling laser beam excites the ground-state atoms with a velocity component opposite to the light beam as they get Doppler-shifted into resonance. The excited atoms acquire a velocity component in the direction of the light beam and the ground-state atoms have an average velocity component in the opposite direction. These two fluxes would cancel if there were no buffer gas. In the presence of a buffer gas, if the rate of collisions with the buffer gas is state dependent, e.g., if an excited atom has a larger collisional cross section than a ground-state atom, the excited atoms suffer a stronger collisional damping of speeds than the ground-state atoms. Thus the active atoms acquire a net drift velocity opposite to the mean velocity of the excited atoms and the atoms are pushed forward by the light. Based on this drift, an ''optical piston effect'' (OPE) [4] can be demonstrated in a long gas cell in an optically dense system, which would otherwise not allow penetration of light beyond a few optical depths—but now, light can penetrate deep inside, sweeping the active atoms to the dark end of the cell. This mechanical action of light on atoms is different from the well-known radiation pressure; the LID pressure is much larger than the radiation pressure, since in the former, atomic momenta instead of photon momenta are transferred. In case of a blue-detuned laser, the direction of the drift is reversed. LID has found important applications in isotope separation [5], particularly in the astrophysical context [6].

Nienhuis [7] has theoretically investigated the LID and the OPE, treating the diffusion and drift of the atomic density in only one space dimension, viz., along the axis of the gas cell. In reality, it is not quite appropriate to ignore atomic motion in directions transverse to the cell axis and treat the system as one dimensional. Recently we have presented a variant of the OPE called the ''magneto-optic piston effect'' (MOPE) [8], in which the active atoms are taken to be electrically charged and are subjected to a large external magnetic field $\vec{B}$ along the light beam, which inhibits diffusion in the plane normal to the magnetic field. Thus, it is the magnetic field that causes a ''dimensional reduction'' and the optical piston action is ''confined'' to one dimension even though the system is three dimensional. For MOPE to work, the incident light intensity should be uniform over the cross section of the Landau orbit of the charged atoms in the applied magnetic field, and it can be ensured by taking a well-collimated beam.

We have also proposed an effect different from MOPE when the direction of the wave vector $\vec{k}$ of the light beam, instead of being parallel, is perpendicular to the direction of $\vec{B}$. It has an interesting consequence that we call the ''optical Hall effect'' (OHE) [9] in which the drift velocity of the ions is in a direction perpendicular to both $\vec{k}$ and $\vec{B}$, as in the usual Hall effect in solid state physics, but now in the absence of

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an electric field. We have generalized the results for various geometries of the light and the magnetic field.

The present paper is organized as follows. In Sec. II, we reproduce the basic equations governing the drift and the diffusion of active ions in the presence of a magnetic field. Our earlier results [8,9] applied only to the case of *weak* collisions, i.e., when active atoms/ions are heavier than the buffer atoms so that the velocity changes occur in small steps. Here in Sec. III, we elaborate on these results and present the details of our previous short communications for MOPE and OHE, in the weak-collision limit. In Sec. IV, new results are obtained for the LID when collisions are described by the *strong-collision* model (SCM) and the *Boltzmann-Lorentz* model (BLM). In the SCM, the active atoms are assumed to be lighter than the buffer atoms so that the effect of collisions is "strong," i.e., it washes out the memory of the precollision value of the velocity. The rate of collisions is taken as an average rate given by the inverse of the mean free time between collisions. The BLM is similar to the SCM, except that here the speed of atoms is taken to be constant in between collisions, only the direction of the velocity is randomized by the collisions. Also, the rate of collisions in the BLM is taken to be a dynamical variable, which depends on the instantaneous velocity of the absorber. Of course, in the end, the velocity is averaged over the Maxwellian distribution. Finally in Sec. V, we summarize our results that are testable in experiments and should lead to related applications.

II. DRIFT AND DIFFUSION OF ACTIVE TWO-LEVEL ATOMS

We follow the treatment of Nienhuis [7] in which the active atoms or ions are taken to have just *two* levels $|g\rangle$ and $|e\rangle$ with a frequency separation of ω_0. A more realistic analysis would have to take into account the multiplet structure of the atomic levels [10,11]. In our simplified picture, the Hamiltonian for the coupled atom and field in the rotating-wave approximation is

$$H=E_e|e\rangle\langle e|+E_g|g\rangle\langle g| -\frac{\hbar}{2}\Omega(\vec{r},t)\exp[i(\vec{k}\cdot\vec{r}-\omega_L t)]|e\rangle\langle g|+\text{H.c.}, \tag{1}$$

where Ω is the space-time-dependent Rabi frequency, $\vec{k}$ is the wave vector, and ω_L the frequency of the light beam. The time evolution of the atomic density matrix $\rho(\vec{r},\vec{v},t)$, where $\vec{r}$ and $\vec{v}$ are the position and velocity of the active atom, is governed by the quantum Bloch equations arising from the atom-field couplings (expressed by the commutator of ρ with H), as well as the classical stochastic motion of the atoms due to velocity-changing collisions with the buffer gas. The fast oscillations with the optical frequency ω_L, and the position-dependent phase $\vec{k}\cdot\vec{r}$ can be eliminated by the usual transformations

$$\rho_{ee}(\vec{r},\vec{v},t)=\sigma_{ee}(\vec{r},\vec{v},t), \tag{2}$$

$$\rho_{gg}(\vec{r},\vec{v},t)=\sigma_{gg}(\vec{r},\vec{v},t), \tag{3}$$

$$\rho_{eg}(\vec{r},\vec{v},t)=\sigma_{eg}(\vec{r},\vec{v},t)\exp[i(\vec{k}\cdot\vec{r}-\omega_L t)], \tag{4}$$

$$\rho_{ge}(\vec{r},\vec{v},t)=\sigma_{ge}(\vec{r},\vec{v},t)\exp[-i(\vec{k}\cdot\vec{r}-\omega_L t)]. \tag{5}$$

The resulting evolution equations for the transformed density matrix σ are

$$\frac{\partial}{\partial t}\sigma_{ee}(\vec{r},\vec{v},t)=-A\sigma_{ee}(\vec{r},\vec{v},t)+\frac{i}{2}[\Omega(\vec{r},t)\sigma_{ge}(\vec{r},\vec{v},t) -\Omega^*(\vec{r},t)\sigma_{eg}(\vec{r},\vec{v},t)]-\vec{v}\cdot\frac{\partial}{\partial\vec{r}}\sigma_{ee}(\vec{r},\vec{v},t) +\mathcal{L}_e[\sigma_{ee}(\vec{r},\vec{v},t)], \tag{6}$$

$$\frac{\partial}{\partial t}\sigma_{gg}(\vec{r},\vec{v},t)=A\sigma_{ee}(\vec{r},\vec{v},t)+\frac{i}{2}[\Omega^*(\vec{r},t)\sigma_{eg}(\vec{r},\vec{v},t) -\Omega(\vec{r},t)\sigma_{ge}(\vec{r},\vec{v},t)]-\vec{v}\cdot\frac{\partial}{\partial\vec{r}}\sigma_{gg}(\vec{r},\vec{v},t) +\mathcal{L}_g[\sigma_{gg}(\vec{r},\vec{v},t)], \tag{7}$$

$$\frac{\partial}{\partial t}\sigma_{eg}(\vec{r},\vec{v},t)=i(\omega_L-\omega_0-\vec{k}\cdot\vec{v})\sigma_{eg}(\vec{r},\vec{v},t)-\frac{1}{2}A\sigma_{eg}(\vec{r},\vec{v},t) +\frac{i}{2}\Omega(\vec{r},t)[\sigma_{gg}(\vec{r},\vec{v},t)-\sigma_{ee}(\vec{r},\vec{v},t)] -\vec{v}\cdot\frac{\partial}{\partial\vec{r}}\sigma_{eg}(\vec{r},\vec{v},t)-\gamma\sigma_{eg}(\vec{r},\vec{v},t), \tag{8}$$

$$\frac{\partial}{\partial t}\sigma_{ge}(\vec{r},\vec{v},t)=\frac{\partial}{\partial t}\sigma^*_{eg}(\vec{r},\vec{v},t). \tag{9}$$

Here A is the spontaneous decay rate, $\mathcal{L}_e$ and $\mathcal{L}_g$ are the operators describing the rate of change of the velocity distribution due to velocity-changing collisions, $(\vec{k}\cdot\vec{v})$ is the Doppler shift in frequency, and γ is the rate of collisional damping of the optical coherence due to phase-interrupting and velocity-changing collisions.

If $\vec{v}(t)$ represents a stationary Markov process having an underlying probability function $g(\vec{v},t)$, then $g(\vec{v},t)$ obeys the master equation

$$\mathcal{L}[g(\vec{v},t)]=\left[\frac{\partial g(\vec{v},t)}{\partial t}\right]_{coll} =-\int d\vec{v}\,'\,W(\vec{v}\,',\vec{v})g(\vec{v},t) +\int d\vec{v}\,'\,W(\vec{v},\vec{v}\,')g(\vec{v}\,',t), \tag{10}$$

where the collision kernel $W(\vec{v},\vec{v}\,')$ gives the probability per unit time that the initial velocity $\vec{v}\,'$ changes (instantaneously) to the final velocity $\vec{v}$. The general solution for

$g(\vec{v},t)$ is not known except in the two extreme cases: (i) the weak-collision model, where $\vec{v}(t)$ is assumed to describe Brownian motion, and (ii) the Boltzmann-Lorentz or strong-collision model, where $\vec{v}(t)$ is a jump process. The first term on the right-hand side (RHS) of Eq. (10) is generally written in terms of a rate of velocity-changing collision, $-\kappa(v)g(\vec{v},t)$. The total collision rate $\kappa(v)$ of atoms with speed v is related to the collision kernel W by the sum rule

$$\int d\vec{v}\, W(\vec{v},\vec{v}\,')=\kappa(v'). \tag{11}$$

If an external magnetic field $\vec{B}$ is applied to atoms with charge q as in the case under study, the effect due to the Lorentz force $q(\vec{v}\times\vec{B})$ is to be included in Eqs. (6)–(9) as well.

The coupled atom and field problem can be simplified using the fact that the quantum processes involving radiative transitions occur on time scales much faster than those for the free flow and the diffusionlike collisional processes. The implied time-scale separation allows one to first average $\sigma(t)$ over the quantum fluctuations, and then consider the phase-space dynamics of the averaged $\sigma(t)$. Following Nienhuis [7], one obtains the quasistationary population distributions, $\overline{\sigma}_{ee}$ and $\overline{\sigma}_{gg}$, as

$$\overline{\sigma}_{ee}(\vec{r},\vec{v},t)=\frac{s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}f(\vec{r},\vec{v},t), \tag{12}$$

$$\overline{\sigma}_{gg}(\vec{r},\vec{v},t)=\frac{1+s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}f(\vec{r},\vec{v},t), \tag{13}$$

where $f(\vec{r},\vec{v},t)d\vec{v}$ is the density of atoms at position $\vec{r}$ with velocity between $\vec{v}$ and $\vec{v}+d\vec{v}$, and s is the so-called effective saturation parameter

$$s(\vec{r},\vec{v},t)=\frac{\mathcal{B}}{A}\int d\omega\, I(\omega;\vec{r},t)P(\omega-\vec{k}\cdot\vec{v}). \tag{14}$$

Here $\mathcal{B}$ is the Einstein coefficient for stimulated radiative transitions, A is the spontaneous decay rate as before, I is the spectral intensity, and $P(\omega)$ is the collision-broadened absorption line shape, given by

$$P(\omega)=\frac{1}{\pi}\mathrm{Re}\left[\gamma+\frac{A}{2}-i(\omega-\omega_0)\right]^{-1}. \tag{15}$$

Now, restoring the free flow and the diffusion terms, the evolution of the atomic distribution function $f(\vec{r},\vec{v},t)\equiv\sigma_{ee}(\vec{r},\vec{v},t)+\sigma_{gg}(\vec{r},\vec{v},t)$ can be written as

$$\frac{\partial}{\partial t}f(\vec{r},\vec{v},t)=-\vec{v}\cdot\frac{\partial}{\partial\vec{r}}f(\vec{r},\vec{v},t)-\frac{q}{m}(\vec{v}\times\vec{B})\cdot\frac{\partial}{\partial\vec{v}}f(\vec{r},\vec{v},t)+\mathcal{L}_e\left\{\frac{s(\vec{r},\vec{v},t)f(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}\right\}+\mathcal{L}_g\left\{\frac{[1+s(\vec{r},\vec{v},t)]f(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}\right\}. \tag{16}$$

Equation (16) is a highly nonlinear equation, since the evolution of $f(\vec{r},\vec{v},t)$ depends on $s(\vec{r},\vec{v},t)$ which is proportional to the local instantaneous field intensity $I(\vec{r},t)$, which in turn depends on $f(\vec{r},\vec{v},t)$.

III. MOPE AND OHE IN THE WEAK COLLISION LIMIT

In considering the phase-space dynamics of the averaged atomic density matrix, we first assume the weak-collision model in which the active atoms are viewed heavier than the buffer atoms so that the velocity changes occur in small steps. The collision kernel $W(\vec{v},\vec{v}\,')$ in Eq. (10) can then be assumed to be nonzero mainly for small values of the velocity change $\vec{v}-\vec{v}\,'$, and it can be approximated by a second-order Kramers-Moyal expansion in velocity moments, yielding a Fokker-Planck form [12,13]. Thus,

$$\mathcal{L}[g(\vec{v})]=\left\{\zeta\frac{\partial}{\partial\vec{v}}\cdot\left[\vec{v}+\frac{k_BT}{m}\frac{\partial}{\partial\vec{v}}\right]\right\}g(\vec{v}), \tag{17}$$

where ζ's are the velocity-damping (diffusive friction) coefficients, m is the mass of the active atom, T is the temperature of the gas, and k_B is the Boltzmann constant.

Equation (16) for the atomic distribution function $f(\vec{r},\vec{v},t)$ then becomes

$$\frac{\partial}{\partial t}f(\vec{r},\vec{v},t)=-\vec{v}\cdot\frac{\partial}{\partial\vec{r}}f(\vec{r},\vec{v},t)-\frac{q}{m}(\vec{v}\times\vec{B})\cdot\frac{\partial f(\vec{r},\vec{v},t)}{\partial\vec{v}}+\zeta_g\frac{\partial}{\partial\vec{v}}\cdot\left[\vec{v}+\frac{k_BT}{m}\frac{\partial}{\partial\vec{v}}\right]f(\vec{r},\vec{v},t)+(\zeta_e-\zeta_g)\frac{\partial}{\partial\vec{v}}\cdot\left[\vec{v}+\frac{k_BT}{m}\frac{\partial}{\partial\vec{v}}\right]\times\left\{\frac{s(\vec{r},\vec{v},t)f(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}\right\}. \tag{18}$$

On the right-hand side of Eq. (18), the first term represents "free flow," while the second term arises from the Lorentz force due to the magnetic field $\vec{B}$. The last term contains the nonlinear atom-field contribution characterized by the saturation parameter $s(\vec{r},\vec{v},t)$. If $\zeta_e=\zeta_g$, Eq. (18) reduces to the usual equation of motion for a charge in a magnetic field [12]. In general, however, ζ_e exceeds ζ_g for neutral active atoms because excited atoms are bigger than the ground-state atoms. Even for active *ions*, the same holds true [8]. There may be examples in which $\zeta_g > \zeta_e$, as in the case of alkali atoms interacting with neon, in which case the direction of the drift velocity will reverse. The active ions (e.g., Na^+ or Ca^+) are expected to be surrounded by negatively charged electrons forming a neutral plasma. Normally these two components of the plasma have a strong Coulomb coupling giving rise to "ambipolar" diffusion. Here, however, we consider a *dilute* gas of active ions wherein the only important collisions are those with a neutral buffer gas (e.g., Ca^+ with buffer H). Besides, the ambipolar diffusion depends on the characteristic spatial scale of the problem. Because we have a weakly ionized plasma, and the effective range of Coulomb interaction is reduced due to finite "screening length," ambipolar diffusion can be neglected.

We introduce the usual definitions of the atomic density $n(\vec{r},t)$ and the corresponding current density (flux) $\vec{j}(\vec{r},t)$ as

$$n(\vec{r},t) = \int d\vec{v}\, f(\vec{r},\vec{v},t), \tag{19}$$

$$\vec{j}(\vec{r},t) = \int d\vec{v}\, \vec{v} f(\vec{r},\vec{v},t). \tag{20}$$

Since the magnetic field does not change the thermal velocity distribution [12], and we expect the drift velocity to be much slower than the average thermal velocity, we can factorize the atomic distribution function

$$f(\vec{r},\vec{v},t) = n(\vec{r},t) w(\vec{v}), \tag{21}$$

$w(\vec{v})$ being a Maxwellian

$$w(\vec{v}) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left[-\left(\frac{m}{2k_B T}\right)\vec{v}^2\right]. \tag{22}$$

That the velocity distribution of the active atoms or ions differs only slightly from a Maxwellian is an assumption that is valid in most experiments [10]. For times $t \gg \zeta_g^{-1}$, we can derive

$$\vec{j}(\vec{r},t) = -D_g \frac{\partial}{\partial \vec{r}} n(\vec{r},t) + n(\vec{r},t)\vec{u}(\vec{r},t) + \frac{q}{m\zeta_g}[\vec{j}(\vec{r},t)\times\vec{B}], \tag{23}$$

where the D_i's are the diffusion coefficients given by

$$D_i = \frac{k_B T}{m\zeta_i} \quad (i=g,e), \tag{24}$$

and $\vec{u}(\vec{r},t)$ is the "drift-velocity"

$$\vec{u}(\vec{r},t) = -\frac{(D_g - D_e)}{D_e} \int d\vec{v}\, \vec{v} w(\vec{v}) \frac{s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}. \tag{25}$$

The last term can be combined with the left-hand side to yield the three Cartesian components of $\vec{j}$. Assuming $\vec{B}$ to be along the z axis for definiteness, we obtain from Eq. (23)

$$\frac{\partial}{\partial t} n = D_g \frac{\partial^2 n}{\partial z^2} + D_B \Delta_\perp n - \frac{\partial}{\partial z}(n\, u_z) - \frac{D_B}{D_g}\left[\frac{\partial}{\partial x}(u_x + \omega_c \tau u_y) n + \frac{\partial}{\partial y}(u_y - \omega_c \tau u_x) n\right], \tag{26}$$

where $n = n(\vec{r},t)$, $\vec{u} = \vec{u}(\vec{r},t)$, $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $D_B = D_g/[1+(\omega_c\tau)^2]$, $\tau = \zeta_g^{-1}$ is the meantime between collisions, and $\omega_c = qB/m$ is the cyclotron frequency. Since $D_B < D_g$, the diffusion of the ions in the cross (x,y) directions is less than that in zero magnetic field, i.e., there is a transverse confinement of the LID of ions due to the suppression of cross diffusion in the presence of a magnetic field.

Equation (26) may be recast as

$$\frac{\partial}{\partial t} n = D_g \frac{\partial^2 n}{\partial z^2} + D_B \Delta_\perp n - \frac{\partial}{\partial z}(n\, u_z) - \frac{\partial}{\partial \vec{r}} \cdot \vec{u}' n, \tag{27}$$

where $\vec{u}'$ is the magnetic field-modified drift velocity given as

$$\vec{u}' = \frac{1}{1+\beta^2}[(u_x + \beta u_y)\hat{x} + (u_y - \beta u_x)\hat{y}] + u_z \hat{z}, \tag{28}$$

$\vec{u}$ being in the direction of the wave vector $\vec{k}$, i.e., $\vec{u} = u\vec{k}/k$, $\hat{x},\hat{y},\hat{z}$ are the unit vectors along the coordinate axes, and

$$\beta = \frac{\omega_c}{\zeta_g} = \omega_c \tau \tag{29}$$

is a dimensionless parameter. For the general case when $\vec{k}$ is in any arbitrary direction, the angle θ between $\vec{k}$ and $\vec{u}'$ is given as

$$\cos\theta = \frac{1}{k}\sqrt{\frac{k_x^2 + k_y^2}{1+\beta^2} + k_z^2}. \tag{30}$$

The following two interesting cases arise from Eq. (28).

Case I. The direction of propagation of the laser beam is along the $\vec{B}$ field [8]. In that case, in Eq. (14), $P(\omega - \vec{k}\cdot\vec{v})$ depends only on v_z and hence s also depends only on v_z. Because the Maxwellian distribution $w(\vec{v})$ factors into three independent, symmetric velocity distributions along the three Cartesian axes, Eq. (25) implies that $u_x(\vec{r},t) = u_y(\vec{r},t) = 0$. Additionally, by choosing a well-collimated beam, the intensity of which is uniform in the x-y plane normal to the beam direction, we can take the saturation parameter to be $s(z,v_z,t)$. This makes the drift velocity, which is now along the direction of propagation of the light beam, also a function of z only. Taking the limit of a large magnetic field for which $\omega_c \gg \zeta_g$, i.e., β in Eq. (29) is much larger than 1, we recover the *one-dimensional* diffusion equation for the density $n(z,t)$ of the active atoms [7],

$$\frac{\partial}{\partial t}n(z,t)=D_g\frac{\partial^2}{\partial z^2}n(z,t)-\frac{\partial}{\partial z}[u(z,t)n(z,t)], \quad (31)$$

where D_g is the diffusion coefficient (24) for the ground-state atoms, $u(z,t)$ is the z component of the drift velocity obtained from Eq. (25) that, as mentioned earlier, is proportional to $(\zeta_e-\zeta_g)$, and $n(z,t)$ is defined as

$$n(z,t)=\int\int dx\, dy\, n(\vec{r},t). \quad (32)$$

The observation of the OPE requires a long (capillary) cell geometry and depends on other assumptions about the uniformity of the beam intensity over the cross section of the cell [7], and negligible interaction of the active atoms with the capillary walls [1]. As demonstrated above, the application of a suitable coaxial magnetic field allows us to bypass such requirements and leads naturally to the one-dimensional diffusion equation (31) signifying the magneto-optic piston effect [8].

The intensity profile of the optical radiation changes during propagation through the system as the saturation effects give rise to different effective absorption coefficients for different frequencies within the bandwidth of the field. This leads to a spatially nonuniform drift. We assume that the Doppler width $k\overline{v}$ (k being the wave number and $\overline{v}$ the mean thermal velocity) is much larger than the homogeneous linewidth $(\gamma+A/2)$ so that the spread in the selected velocity v is small compared with thermal velocities (the Doppler limit). Then the saturation parameter $s(z,v_z,t)$ is zero unless the velocity v_z is close to the selected velocity $v_L=(\omega_L-\omega_0)/k$, ω_L being the laser frequency. The evolution of the light intensity $I(z,t)$ is then determined by [7]

$$\frac{\partial I(z,t)}{\partial z}=\frac{-\sigma_a n(z,t)I(z,t)}{[1+I(z,t)/I_{sat}]^{1/2}}. \quad (33)$$

In Eq. (33) σ_a is the unsaturated inhomogeneous (Doppler) absorption cross section proportional to the one-dimensional Maxwellian $w(v_L)$; $I_{sat}=\hbar\omega_L A/(2\sigma_h)$ is the saturation intensity, σ_h is the homogeneous absorption cross section on resonance. The drift term $u(z,t)n(z,t)$ in Eq. (31) is given by

$$u(z,t)n(z,t)=\frac{(D_g-D_e)}{D_e}\frac{v_L}{\hbar\omega_L(A+K_g)}\frac{\partial I(z,t)}{\partial z}, \quad (34)$$

where K_g is the thermalization rate for the ground-state atoms. Substituting Eq. (33) into Eq. (34) yields the drift velocity

$$u(z,t)=\frac{(D_g-D_e)}{D_e}\frac{v_L}{\hbar\omega_L(A+K_g)}\sigma_a\frac{I(z,t)}{[1+I(z,t)/I_{sat}]^{1/2}}. \quad (35)$$

For the stationary case of an optically dense system in a closed cell of length L, optical piston action occurs when the incident intensity $J_0=\int d\omega\, I(\omega;z=0,t)$ is larger than the transition value $N/(Lb)$, where N is the total number of active atoms per unit cross section of the cell, and $b=-v_L(D_g-D_e)/(D_gD_e\hbar\omega_L A)$, v_L being the axial velocity of the active ions as before. The ions are swept inside the cell and are driven along till the density reaches the asymptotic value (bJ_0). For excitation in the red Doppler wing, v_L is negative and b is positive, and the drift velocity u is proportional to $-v_L w(v_L)$ that is maximum for $v_L=-(k_BT/m)^{1/2}$.

Case II. If $\vec{k}\perp\vec{B}$, e.g., $\vec{k}$ is along the x axis, the drift is not only in the x direction but also in the y direction. From Eq. (28) we see that if $\vec{k}$ is confined to the x-y plane and $\beta\gg 1$ (strong magnetic field), then $\vec{k}$, $\vec{u}'$, and $\vec{B}$ are all at right angles to each other. In particular, when $\beta\gg 1$, if $\vec{k}$ is along the y direction, the drift velocity $\vec{u}'$ is along the x direction. This is the optical Hall effect [9].

Let us consider the ions in a closed box with an incident light beam with wave vector $\vec{k}$ along the y axis, and a magnetic field $\vec{B}$ along the z axis as before. Then Eq. (27) for the atomic density reduces to

$$\frac{\partial}{\partial t}n=D_g\frac{\partial^2 n}{\partial z^2}+D_B\Delta_\perp n-\frac{D_B}{D_g}\left[\frac{\partial}{\partial y}u_y n-\beta\frac{\partial}{\partial x}u_y n\right]. \quad (36)$$

It can be shown that

$$nu_y=\frac{(D_g-D_e)}{D_e}\frac{v_{Ly}}{\hbar\omega_0 A}\frac{\partial J}{\partial y}, \quad (37)$$

where $J(\vec{r},t)=\int d\omega\, I(\omega,\vec{r},t)$, and v_{Ly} is the y component of the selected velocity. Hence from Eq. (37), one can write using the continuity equation,

$$\vec{j}(\vec{r},t)=-\left[D_B\left(\frac{\partial n}{\partial x}+\beta b'\frac{\partial J}{\partial y}\right)\hat{x}+D_B\left(\frac{\partial n}{\partial y}+b'\frac{\partial J}{\partial y}\right)\hat{y}+D_g\frac{\partial n}{\partial z}\hat{z}\right], \quad (38)$$

where

$$b'=-\frac{(D_g-D_e)}{D_gD_e}\frac{v_{Ly}}{\hbar\omega_0 A}. \quad (39)$$

Since the box is closed, for stationary solutions, each component of $\vec{j}$ is equated to zero. Thus n becomes a function of $(y+\beta x)$, and we get

$$n(x,y)=n(y+\beta x)=F+b'J(y+\beta x), \quad (40)$$

where F is an arbitrary constant.

For the case without saturation, we obtain the following intensity and atomic density profiles:

$$J(x,y)=\frac{FJ_0}{bJ_0+(F+bJ_0)\exp[aF(y+\beta x)]}, \quad (41)$$

$$n(x,y)=\frac{F(F+bJ_0)}{(F+bJ_0)+bJ_0\exp[-aF(y+\beta x)]}, \qquad (42)$$

where J_0 is the constant intensity along the plane $y=-\beta x$. The above equations reduce to those of Nienhuis in the absence of magnetic field, i.e., when $\beta=0$. From the above equations we also note that the intensity (as well as the atomic density) is constant along the planes $y+\beta x=$const, intensity being maximum (equal to J_0) along the plane $y+\beta x=0$. The drift velocity, given as

$$\vec{u}'=u\left[\frac{\beta}{1+\beta^2}\hat{x}+\frac{1}{1+\beta^2}\hat{y}\right], \qquad (43)$$

is perpendicular to these planes of constant intensity, $y+\beta x=$const. These planes of constant intensity move from $y=$const for $\beta=0$ to $x=$const for β tending to ∞. When β tends to ∞, the drift velocity must be in the x direction, i.e., perpendicular to the planes $x=$const. This is consistent with Eq. (27), in that for β tending to ∞, the magnetic field, the incident light, and the drift velocity are all at right angles to each other.

In order to observe the above atomic density and light intensity distributions, the light intensity J has to be maintained constant along the plane $y=-\beta x$. In a given magnetic field, the stationary solution will be obtained for a particular direction of the incident light, depending on the value of the field strength β. For demonstration of the optical Hall effect in general, the condition of stationarity is not a requirement.

IV. THE STRONG-COLLISION AND THE BOLTZMANN-LORENTZ MODELS

As mentioned earlier, the probability function $g(\vec{v},t)$, associated with a stationary Markov process $\vec{v}(t)$, obeys the Chapman-Kolmogorov-Smoluchowski equation (10). The general solution for $g(\vec{v},t)$ is not known in analytically tractable forms, except in two cases. One such case, the so-called weak collision or Brownian motion model has already been introduced, and its implication for the magneto-optic drift assessed, in Sec. III. We now discuss the other case, viz., the SCM in which the active atom completely loses the memory of its precollision velocity. This situation is expected to apply when the mass of the active atom is much smaller than that of the perturbing buffer gas atom. A variant of the SCM is the BLM, borrowed from the classical kinetic theory in which it is assumed that the effect of each collision is to *randomize* the direction of the velocity but its magnitude remains constant [14].

Strong collision model. Our first task is to determine the structure of the collision operators $\mathcal{L}_e$ and $\mathcal{L}_g$. In doing this, we note that the collision kernel in Eq. (10) is given in the SCM by

$$W(\vec{v}\,',\vec{v})=\gamma w(\vec{v}), \qquad (44)$$

where γ is the rate of collision and $w(\vec{v})$ is the Maxwellian velocity distribution (22). Note that under the stipulation that the LHS of Eq. (44) is independent of the initial velocity $\vec{v}\,'$, the RHS is the only allowed form, consistent with the detailed balance of transitions

$$w(\vec{v}\,')W(\vec{v},\vec{v}\,')=w(\vec{v})W(\vec{v}\,',\vec{v}) \qquad (45)$$

and the conservation of probability

$$\frac{1}{\gamma}\int W(\vec{v},\vec{v}\,')d\vec{v}=1. \qquad (46)$$

Then, from Eq. (10), we get

$$\mathcal{L}_{SCM}[g(\vec{v},t)]=-\gamma g(\vec{v},t)+\gamma w(\vec{v})\int g(\vec{v}\,',t)d\vec{v}\,'. \qquad (47)$$

Following our discussion preceding Eq. (16), the evolution of the atomic distribution function obeys the following equation:

$$\begin{aligned}\frac{\partial}{\partial t}f(\vec{r},\vec{v},t)&+\vec{v}\cdot\frac{\partial}{\partial\vec{r}}f(\vec{r},\vec{v},t)+\frac{q}{m}(\vec{v}\times\vec{B})\cdot\frac{\partial f(\vec{r},\vec{v},t)}{\partial\vec{v}}\\
&=-\gamma_e\left[\frac{s(\vec{r},\vec{v},t)f(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}-w(\vec{v})\int d\vec{v}\,'\frac{s(\vec{r},\vec{v}\,',t)f(\vec{r},\vec{v}\,',t)}{1+2s(\vec{r},\vec{v}\,',t)}\right]\\
&\quad-\gamma_g\left[\frac{1+s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}f(\vec{r},\vec{v},t)-w(\vec{v})\int d\vec{v}\,'\frac{1+s(\vec{r},\vec{v}\,',t)}{1+2s(\vec{r},\vec{v}\,',t)}f(\vec{r},\vec{v}\,',t)\right]\\
&=-\gamma_g\left[f(\vec{r},\vec{v},t)-w(\vec{v})\int d\vec{v}\,'f(\vec{r},\vec{v}\,',t)\right]-(\gamma_e-\gamma_g)\\
&\quad\times\left[\frac{s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}f(\vec{r},\vec{v},t)-w(\vec{v})\int d\vec{v}\,'\frac{s(\vec{r},\vec{v}\,',t)}{1+2s(\vec{r},\vec{v}\,',t)}f(\vec{r},\vec{v}\,',t)\right]. \qquad (48)\end{aligned}$$

Thus, if γ_e equals γ_g, the principal cause of the light-induced drift disappears; then the nonlinear contribution of the atom-field coupling vanishes and the equation above reduces to the standard kinetic equation for magnetohydrodynamics [15].

We are now ready to write down the equation of motion for the current density defined in Eq. (20), following the arguments preceding Eq. (23). In the SCM

$$\frac{\partial}{\partial t}\vec{j}(\vec{r},t)+\frac{k_BT}{m}\frac{\partial}{\partial\vec{r}}n(\vec{r},t)-\frac{q}{m}[\vec{j}(\vec{r},t)\times\vec{B}] = -\gamma_g\vec{j}(\vec{r},t)-(\gamma_e-\gamma_g)\int d\vec{v}\,\vec{v}\,w(\vec{v}) \times\frac{s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}\,n(\vec{r},t), \tag{49}$$

since

$$\int w(\vec{v})\vec{v}\,d\vec{v}=0. \tag{50}$$

Therefore, again in the diffusive limit ($t\gg\gamma_g^{-1}$), we get

$$\vec{j}(\vec{r},t)=-\frac{k_BT}{m\gamma_g}\frac{\partial}{\partial\vec{r}}n(\vec{r},t)+\frac{q}{m\gamma_g}[\vec{j}(\vec{r},t)\times\vec{B}] -\frac{(\gamma_e-\gamma_g)}{\gamma_g}n(\vec{r},t)\int\frac{s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}\,w(\vec{v})\vec{v}\,d\vec{v}. \tag{51}$$

Comparing with Eq. (23) we find that we have an identical equation (and concomitant analysis) if we identify the collision rate γ in the SCM with the friction coefficient ζ of the weak-collision model.

Boltzmann-Lorentz model. The case in the BLM follows along similar lines, but now, in a collision, the velocity does not change in magnitude, only its orientation specified by the Euler angle Ω changes. Thus the equation corresponding to Eq. (44) reads

$$W(\{v,\Omega\},\{v,\Omega'\})=\frac{\gamma(v)}{4\pi}. \tag{52}$$

Note that in this case the effective rate of collisions γ is a function of the instantaneous velocity of the active atom. Hence,

$$\mathcal{L}_{BLM}[g(\vec{v},t)]=-\gamma(v)g(\vec{v},t)+\frac{\gamma(v)}{4\pi}\int g(v,\Omega',t)d\Omega'. \tag{53}$$

The evolution of the atomic distribution function is given by

$$\frac{\partial}{\partial t}f(\vec{r},\vec{v},t)+\vec{v}\cdot\frac{\partial}{\partial\vec{r}}f(\vec{r},\vec{v},t)+\frac{q}{m}(\vec{v}\times\vec{B})\cdot\frac{\partial f(\vec{r},\vec{v},t)}{\partial\vec{v}} = -\gamma_g(v)\left[f(\vec{r},\vec{v},t)-\frac{1}{4\pi}\int d\Omega'\,f(\vec{r},v,\Omega',t)\right] -[\gamma_e(v)-\gamma_g(v)]\left[\frac{s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}\,f(\vec{r},\vec{v}',t) -\frac{1}{4\pi}\int d\Omega'\frac{s(\vec{r},v,\Omega',t)}{1+2s(\vec{r},v,\Omega',t)}\,f(\vec{r},v,\Omega',t)\right]. \tag{54}$$

The equation of motion for the current density (20) in the BLM is

$$\frac{\partial}{\partial t}\vec{j}(\vec{r},t)+\frac{k_BT}{m}\frac{\partial}{\partial\vec{r}}n(\vec{r},t)-\frac{q}{m}[\vec{j}(\vec{r},t)\times\vec{B}] = -\int d\vec{v}\,\vec{v}\,\gamma_g(v)f(\vec{r},\vec{v},t) -\int d\vec{v}\,\vec{v}[\gamma_e(v)-\gamma_g(v)]w(\vec{v}) \times\frac{s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)}\,f(\vec{r},\vec{v},t), \tag{55}$$

again using the fact that

$$\int\gamma_g(v)f(\vec{r},\vec{v},t)\vec{v}\,d\vec{v}=0. \tag{56}$$

But, now we need to express the first term on the right-hand side of Eq. (55) in relation to the current density. Note that the collision rate in the BLM is given by [14]

$$\gamma_i(v)=\pi a_i^2 n_p v \quad (i=g,e), \tag{57}$$

where a is an effective scattering radius and n_p is the number of scatterers (perturbers) per unit volume. Therefore,

$$\int d\vec{v}\,\vec{v}\,\gamma_g(v)f(\vec{r},\vec{v},t) = \pi a_g^2 n_p\int dv\,v^2\int d\Omega\,v^2\hat{v}(\Omega)f(\vec{r},v,\Omega,t), \tag{58}$$

where $\hat{v}(\Omega)$ is the unit vector in the direction of $\vec{v}$. Comparing with the definition of the current density in Eq. (20), we may then rewrite Eq. (58) as

$$\int d\vec{v}\,\vec{v}\,\gamma_g(v)f(\vec{r},\vec{v},t)=\pi a_g^2 n_p\frac{\langle v^4\rangle}{\langle v^3\rangle}\,\vec{j}(\vec{r},t), \tag{59}$$

where $\langle\cdots\rangle$ denotes average over the Maxwellian in Eq. (22). Thus,

$$\int d\vec{v}\,\vec{v}\,\gamma_g(v)f(\vec{r},\vec{v},t)=\lambda_g\vec{j}(\vec{r},t), \tag{60}$$

where

$$\lambda_g=\frac{3}{4}\pi a_g^2 n_p\sqrt{\frac{2\pi k_B T}{m}}. \tag{61}$$

Hence, in the diffusive limit ($t\gg\lambda_g^{-1}$),

$$\vec{j}(\vec{r},t)=-\frac{k_BT}{m\lambda_g}\frac{\partial}{\partial\vec{r}}n(\vec{r},t)+\frac{q}{m\lambda_g}[\vec{j}(\vec{r},t)\times\vec{B}] -\frac{4}{3}\frac{(a_e^2-a_g^2)}{a_g^2}\sqrt{\frac{m}{2\pi k_BT}}\int\frac{s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)} \times f(\vec{r},\vec{v},t)v\vec{v}\,d\vec{v}. \tag{62}$$

Keeping in mind Eq. (21), Eq. (62) is very similar to Eq. (23) but now we have a new definition for the "drift velocity" [cf. Eq. (25)],

$$\vec{u}(\vec{r},t)=-\frac{4}{3}\frac{(a_e^2-a_g^2)}{a_g^2}\sqrt{\frac{m}{2\pi k_BT}}\int\frac{s(\vec{r},\vec{v},t)}{1+2s(\vec{r},\vec{v},t)} \times w(v)v\vec{v}\,d\vec{v}. \tag{63}$$

From the knowledge of the experimentally observed drift velocity, one can determine the ratio of the scattering radii a_g/a_e. Further, in this classical model of collisions and the stochastic motion, if we assume that the scattering radius is given as

$$a_i=r+r_i \quad (i=e,g), \tag{64}$$

where r is the known radius of the buffer atoms and r_i that of the active atom in state i one can experimentally determine the ratio r_e/r_g of the active atoms.

V. DISCUSSIONS AND SUMMARY

The OPE has been observed in Na-noble gas mixtures contained in a narrow capillary [3,10,11,16], and there are discrepancies between the predicted and observed values, which are attributed to the nonuniformity of the laser beam over the cross section of the capillary and the effect of adsorption and desorption of atoms at the capillary surface. For the observation of drifts of ions in a magnetic field, one may consider Ca in a buffer gas—the single-electron ionization energy for Ca is 6.113 eV (or 589.84 kJ/mol). For ionization of the active atoms, a radio-frequency discharge may be used. An intense laser beam, tuned above a resonance in the ions (say, the Zeeman-split $4^2\mathrm{S}\leftrightarrow4^2\mathrm{P}$ of Ca^+ subject to the selection rule $\Delta F=0,\ \pm1$) then produces velocity-selective saturation of the ions due to the Doppler effect.

The drift velocity for ions is substantial only at high temperatures ~5000 K (as in a plasma or in stellar atmospheres) [6,17]. From the calculation of the interaction potential between ions of Ca and *neutral* H [18], the relative difference of the collision cross section of the excited state and the ground state is shown to be as high as 0.25. At the high temperature considered above, at most half of the buffer (say, hydrogen) will also be ionized, and still, the opposing fluxes of excited and unexcited Ca ions will not compensate each other.

A magnetic field B of about 2.5 T will produce a cyclotron frequency $\omega_c\approx6\times10^6\ \mathrm{s}^{-1}$ of the Ca^+ ions. The MOPE will be seen for a low-to-moderate rate of velocity-changing collisions, with a $\zeta_g\approx5\times10^5\ \mathrm{s}^{-1}$ or lower. For a cyclotron frequency of $\omega_c\approx10\zeta_g$ we get $D_B/D_g\approx0.01$, i.e., the "leakage" in the transverse directions is substantially reduced. The Ca^+ ions at a high temperature T of 5000 K will have an rms. speed of $v=1.76$ km/s. Then for our chosen value of $\omega_c=6\times10^6\ \mathrm{s}^{-1}$, the radius r_B of the transverse Landau orbit of an active ion is $v/\omega_c=0.29$ mm. For the transverse confinement to work efficiently, we need this $r_B\leqslant$ the radius of the laser beam causing excitation of the ions. This condition is easily satisfied in a realistic situation, typical focussed-beam radius being ~1 mm. It may be difficult to observe the proposed magneto-optic drift of ions over and above their large thermal speed at a temperature of about 5000 K in the MOPE geometry. The geometry of the OHE may be more amenable to observations in a laboratory.

In summary, we have presented a complete analysis of the magneto-optic drift effects of optically active ions, incorporating buffer-induced collision mechanisms within the Markov limit. It has been shown that a combination of the cyclotron motion of charged active atoms in an applied coaxial magnetic field and the usual light-induced drift due to velocity-changing collisions with a buffer gas yields an effective way to confine the optical piston action to a one-dimensional motion. The magnetic field inhibits diffusion of the active atoms in the plane normal to the field and thus facilitates the observation of the magneto-optic piston effect. It has also been shown that when the incident light is perpendicular to the coaxial magnetic field, the drift of the ions gives rise to the optical Hall effect. We have treated the problem in the weak-collision, the strong-collision, and the Boltzmann-Lorentz models. From the experimentally measured drift velocity in the Boltzmann-Lorentz model, one can extract the ratio of the radii r_e/r_g of the active atoms in their excited and ground states. The magneto-optic drift is believed to be responsible for the observed chemical "anomalies" and the surface inhomogeneity in certain stars. Also, it is important to assess the (negative) impact of this drift on the workings of ion traps, since the vacuum in a trap can never be perfect. It should be possible to observe the drift in the OHE geometry in a laboratory plasma. We hope our results will stimulate studies in other suitable samples, e.g., in optically excitable charged polymers exhibiting diffusion.

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