One- and two-dimensional *N*-qubit systems in capacitively coupled quantum dots

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Coulomb blockade effects in capacitively coupled quantum dots can be utilized for constructing an *N*-qubit system with antiferromagnetic Ising interactions. Starting from the tunneling Hamiltonian, we theoretically show that the Hamiltonian for a weakly coupled quantum-dot array is reduced to that for nuclear-magnetic-resonance (NMR) spectroscopy. Quantum operations are carried out by applying only electrical pulse sequences. Thus various error-correction methods developed in NMR spectroscopy and NMR quantum computers are applicable without using magnetic fields. A possible measurement scheme in an *N*-qubit system is quantitatively discussed.

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Quantum computers have been widely investigated from many perspectives [1–12]. A quantum-dot array is a promising candidate for the basic element of a quantum computer from the viewpoint of technological feasibility [13,14]. It is noteworthy that controlling of lateral and vertical order in self-organized quantum-dot superlattices has been realized [15]. Although a spin-based quantum-dot computer has been intensively discussed [5,6], it seems that the quantum-dot system using charged states is more accessible because the latter will be constructed of various materials other than III-V group materials, such as GaAs. From this perspective, several authors investigated the quantum computer based on the charged states [10–12]. In Ref. [12], we discussed the quantum-dot computer in the limit of a free-electron approximation. The free-electron approximation will be valid only when the interdot tunneling is strong and Coulomb blockade is suppressed, or the size of the quantum dots is as small as that of artificial atoms. Generally speaking, however, given the current state of technology, it seems that it will not be possible to make quantum dots as small as atomic order in the near future. Moreover, controlled-NOT (CNOT) operation for a quantum computer based on charged states has been discussed only in the two neighboring qubits so far; a scheme for constructing more than two qubits on a device remains unclear.

In this paper we advance the analysis of the quantum computer based on charged states and show a general *N*-qubit scheme. Starting from tunneling Hamiltonian in the Coulomb blockade regime, we demonstrate that the Hamiltonians for one- and two-dimensional arrays of weakly coupled quantum dots are reduced to those for standard nuclear-magnetic-resonance (NMR) spectroscopy [16]. This enables any quantum computation to be described by electric pulse sequences. We assume that interdot tunneling is not so strong that the capacitance between two quantum dots is defined and we can use the Hamiltonian that allows the electrostatic energy of the quantum-dot structure to be described in terms of the capacitances of individual dots and gate voltages. This type of quantum-dot system was discussed experimentally by Livermore *et al.* [17]. The interaction between qubits is derived from the Coulomb interaction between the excess electrons in the quantum dots. We also numerically illustrate the measurement process using the field effect, which is considered to be a sensitive measurement method for electronic states [18]. We set $e=1$ and $k_B=1$.

The fundamental idea of the quantum-dot qubits based on charged states is as follows. A qubit is composed of two quantum dots coupled via a thin tunneling barrier and a gate electrode that is attached on a thick insulating material. This is a bistable well structure [19] where the electronic quantum state of the coupled dots is controlled by the gate bias. First, we consider the one-dimensional arrayed qubits (Fig. 1). The quantum dots are, e.g., Si nanocrystals [20–22] or GaAs dots [13,14]. The quantum dots are assumed to be sufficiently small for charging effects to be observed. $N_{\alpha i}$ and $N_{\beta i}$ are the numbers of excess electrons from the neutral states in two quantum dots. One excess charge is assumed to be inserted from a substrate first and to stay in the two-coupled

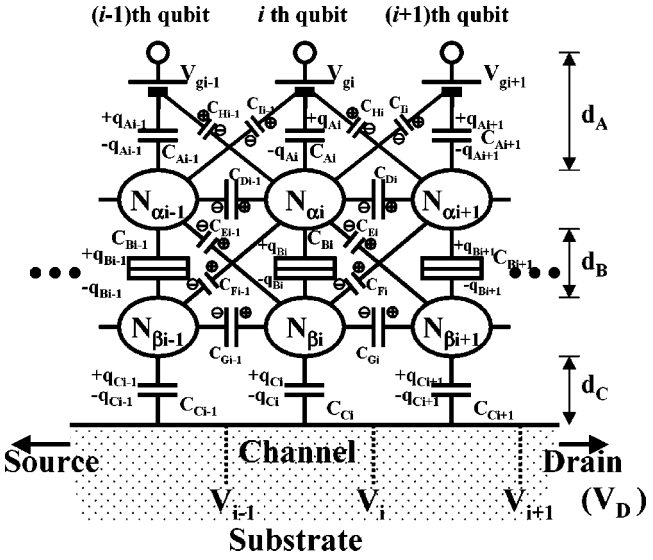

FIG. 1. Schematic of the capacitance network of a one-dimensionally coupled quantum-dot array. The capacitances C_{Ai}, C_{Bi}, and C_{Ci}, the gate V_{gi} and the two quantum dots constitute the *i*th qubit. The excess electron moves between the two quantum dots in a qubit via the tunneling barrier, and electron transfer between qubits is prohibited.

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dots ($C_{Bi}>C_{Ci}$). When the excess charge exists in the upper dot and lower dot, we call them $|0\rangle=|\uparrow\rangle$ state and $|1\rangle=|\downarrow\rangle$ state, respectively. The quantum logic gates are assumed to be operated in the region where the $|0\rangle$ and $|1\rangle$ states are near degenerate, then the system becomes a two-state system [7,8,23]. The qubits are arranged one dimensionally and are capacitively coupled.

Here we show the Hamiltonian of the one-dimensionally arrayed coupled quantum dots for quantum computation by starting from the tunneling Hamiltonian

$$H=\sum_{i=1}^{N}\left(t\hat{a}_i^{\dagger}\hat{b}_i+t^{*}\hat{b}_i^{\dagger}\hat{a}_i+\epsilon_{\alpha i}\hat{a}_i^{\dagger}\hat{a}_i+\epsilon_{\beta i}\hat{b}_i^{\dagger}\hat{b}_i\right)+H_{\mathrm{ch}}, \quad (1)$$

where $\hat{a}_i$ ($\hat{b}_i$) describes the annihilation operator when the excess electron exists in the upper (lower) dot, and $\epsilon_{\alpha i}$ ($\epsilon_{\beta i}$) shows the electronic energy of the upper (lower) dot. There is no restriction on the number of coupled quantum dots N. H_{ch} is the charging energy that includes the interaction between qubits. Because we consider the Coulomb blockade in the weak coupling region, the resistance of the interdot tunneling barrier should be larger than $R_K(=h/e^2\sim 25.8\ k\Omega)$ and the operational temperature be less than the charging energies. Thus, the operational speed should be less than the CR constant of the capacitance network so that the double-well potential profile generated by the charging energy is effective (adiabatic region). The criterion of the operation is discussed below in detail. As for the interaction between qubits, the distribution of the excess charge is considered to be antiferromagnetic due to the repulsive Coulomb interaction. We can show that this interaction between qubits is an Ising interaction by minimizing the general formulation of the charging energy

$$\begin{aligned}H_{\mathrm{ch}}=&\sum_{i=1}^{N}\left\{\frac{q_{Ai}^2}{2C_{Ai}}+\frac{q_{Bi}^2}{2C_{Bi}}+\frac{q_{Ci}^2}{2C_{Ci}}-q_{Ai}V_{gi}\right\}\\&+\sum_{i=1}^{N-1}\left\{\frac{q_{Di}^2}{2C_{Di}}+\frac{q_{Ei}^2}{2C_{Ei}}+\frac{q_{Fi}^2}{2C_{Fi}}+\frac{q_{\mathrm{G}i}^2}{2C_{\mathrm{G}i}}\right\}\\&+\sum_{i=2}^{N}\left\{\frac{q_{Hi}^2}{2C_{Hi}}-q_{Hi}V_{gi-1}\right\}\\&+\sum_{i=1}^{N-1}\left\{\frac{q_{Ii}^2}{2C_{Ii}}-q_{Ii}V_{gi+1}\right\},\end{aligned} \quad (2)$$

with constraints $-N_{\alpha i}=q_{Ai}-q_{Bi}+q_{Di}-q_{Di-1}+q_{Ei}-q_{Fi-1}+q_{Hi-1}+q_{Ii}$ and $-N_{\beta i}=q_{Bi}-q_{Ci}+q_{\mathrm{G}i}-q_{\mathrm{G}i-1}-q_{Ei-1}+q_{Fi}$ $(i=1,\ldots,N)$. The last line of Eq. (2) shows the effects of other gate electrodes on the ith qubit (cross talk). By using Lagrange multiplier constants, the energy of the system is obtained as a function of the relative excess charge $n_i\equiv N_{\alpha_i}-N_{\beta_i}$ and the total excess charge of the two quantum dots $N_i\equiv N_{\alpha_i}+N_{\beta_i}(=1)$,

$$\begin{aligned}H_{\mathrm{ch}}\cong&\sum_{i=1}^{N}\frac{C_{bi}}{2D_i}\left[n_i+\frac{C_{ci}}{C_{bi}}N_i+\left(1+\frac{C_{ci}}{C_{bi}}\right)Q_V\right]^2\\&+\sum_{i=2}^{N}\frac{2}{D_iD_{i-1}}(C_{bi}C_{bi-1}C_{ei-1}\\&+C_{ci}C_{ci-1}C_{di-1})n_in_{i-1},\end{aligned} \quad (3)$$

with $C_{ai}\equiv C_{Ai}+C_{Ci}+4C_{Bi}+2(C_{Di}+C_{Ei})+C_{Hi}+C_{Ii}$, $C_{bi}\equiv C_{Ai}+C_{Ci}+2(C_{Di}+C_{Ei})+2(C_{Di-1}+C_{Ei-1})+C_{Hi}+C_{Ii}$, $C_{ci}\equiv C_{Ci}-C_{Ai}+C_{Hi}+C_{Ii}$, $C_{di}\equiv C_{Di}+C_{Ei}$, $C_{ei}\equiv C_{Di}-C_{Ei}$ $(C_{Di}=C_{Gi},\ C_{Ei}=C_{Fi})$, and $D_i\equiv C_{ai}C_{bi}-C_{ci}^2$ and $Q_V\equiv C_{Ai}V_{gi}+C_{Hi-1}V_{gi-1}+C_{Ii}V_{gi+1}$. It is assumed that the coupling between qubits is smaller than that within a qubit and we neglect higher-order terms than $(C_{di}^2/D_i)^2(\ll 1)$. This assumption is valid when the distance between the quantum dots in different qubits is larger than that between quantum dots in a qubit. From Eq. (3), we define the characteristic charging energy of the system as $E_C\equiv C_b/(2D)$. We consider the gate voltage region, where the $n_i=-1$ state and $n_i=1$ state are near degenerate as in Refs. [7,8,23], and we obtain the Hamiltonian of the coupled quantum-dot system

$$H=\sum_{i=1}^{N}\left[t\hat{a}_i^{\dagger}\hat{b}_i+t^{*}\hat{b}_i^{\dagger}\hat{a}_i+\Omega_i\hat{I}_{iz}\right]+\sum_{i=1}^{N-1}J_{i,i+1}\hat{I}_{iz}\hat{I}_{i+1z}, \quad (4)$$

where $|\uparrow_i\rangle=|n_i=1\rangle$ and $|\downarrow_i\rangle=|n_i=-1\rangle$, $\hat{I}_{iz}=(\hat{a}_i^{\dagger}\hat{a}_i-\hat{b}_i^{\dagger}\hat{b}_i)/2$, and

$$\Omega_i=\frac{4C_{Ci}}{D_i}[Q_V-Q_V^{\mathrm{res}}], \quad (5)$$

$$J_{i,i+1}=\frac{1}{2D_iD_{i+1}}[C_{bi}C_{bi+1}C_{ei+1}+C_{ci}C_{ci+1}C_{di+1}]. \quad (6)$$

Q_V^{res} includes $\epsilon_{\alpha i}$ and $\epsilon_{\beta i}$ and shows the gate voltage when the $|0\rangle$ and $|1\rangle$ degenerate (on resonance). We control the time-dependent quantum states of the qubits in the vicinity of on resonant gate bias by applying a gate voltage, such as $V_i(\tau)=V_i^{\mathrm{res}}+v_i(\tau)$ where V_i^{res} is the gate bias of on resonance. In this on-resonant region, a transformation of the coordinate

$$\begin{pmatrix}\hat{\alpha}_{+i}\\ \hat{\alpha}_{-i}\end{pmatrix}\equiv U_0\begin{pmatrix}\hat{a}_i\\ \hat{b}_i\end{pmatrix},\quad U_0\equiv\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}, \quad (7)$$

is convenient (we neglect the phase of t_i for simplicity). Then the Hamiltonian Eq. (4) can be described as

$$H(\tau)=\sum_{i=1}^{N}\left[2t_i\hat{I}'_{zi}-\Delta_i(\tau)\hat{I}'_{xi}\right]+\sum_{i=1}^{N-1}J_{i,i+1}\hat{I}'_{xi}\hat{I}'_{xi+1}, \quad (8)$$

where

$$\hat{I}'_{xi} \equiv (\hat{\alpha}^{\dagger}_{+i}\hat{\alpha}_{-i} + \hat{\alpha}^{\dagger}_{-i}\hat{\alpha}_{+i})/2, \quad \hat{I}'_{yi} \equiv i(\hat{\alpha}^{\dagger}_{+i}\hat{\alpha}_{-i} - \hat{\alpha}^{\dagger}_{-i}\hat{\alpha}_{+i})/2,$$

$$\hat{I}'_{zi} \equiv (\hat{\alpha}^{\dagger}_{+i}\hat{\alpha}_{+i} - \hat{\alpha}^{\dagger}_{-i}\hat{\alpha}_{-i})/2, \tag{9}$$

and

$$\Delta_i(\tau) = \frac{4C_{Ai}C_{Ci}}{D_i}[v_i(\tau) + \delta v_{i,\mathrm{cr}}(\tau)]. \tag{10}$$

$\delta v_{i\mathrm{cr}}(\tau) \equiv [C_{Hi-1}v_{i-1}(\tau) + C_{Ii}v_{i+1}(\tau)]/C_{Ai}$ is a cross-talk term and is an effect of other gate electrodes. Hamiltonian (8) is an NMR Hamiltonian when we regard $\omega_i = 2t_i$ as a Zeeman energy and $\Delta_i(\tau)$ as a transverse magnetic field, if $\omega > \Delta_0 \gg J$. Thus, when $\Delta_i(\tau) = \Delta_{0i}\cos(\omega_i\tau + \delta_i)$ and we take a rotating wave approximation by the unitary transformation $U_{\mathrm{rwa}}(\tau) = \exp(-i\Sigma_{i=1}^{N}\omega_i\tau\hat{I}'_{iz})$, we have

$$H_{\mathrm{rwa}} = -\sum_{i=1}^{N}\frac{\Delta_{i0}}{2}[\hat{I}'_{xi}\cos\delta_i + \hat{I}'_{yi}\sin\delta_i] + \sum_{i=1}^{N-1}\frac{J_{i,i+1}}{2}[\hat{I}'_{xi}\hat{I}'_{xi+1} + \hat{I}'_{yi}\hat{I}'_{yi+1}]. \tag{11}$$

The pulse process is carried out when the oscillating electric field is applied ($\Delta_0 \neq 0$) where the interaction term is considered to be able to be neglected because $\Delta_0 \gg J$. For example, π_z pulse in the $(\hat{a}_i, \hat{b}_i)$ basis corresponds to π_x-pulse representation in this $(\hat{\alpha}_{+i}, \hat{\alpha}_{-i})$ basis and is carried out when $\delta = 0$ and $\Delta_0\tau/2 = \pi$. In general, rotation $R_\gamma(\theta) \equiv e^{i\theta I_\gamma}$ ($\gamma = x,y,z$) in the $(\hat{a}_i, \hat{b}_i)$ basis is interpreted by $R'_i(\theta) = U_0^{-1}R_i(\theta)U_0$ in the $(\hat{\alpha}_{+i}, \hat{\alpha}_{-i})$ basis. The evolution process is carried out when the oscillating voltage is not applied ($\Delta_0 = 0$). A two-qubit CNOT gate is described by a pulse sequence of the form $\hat{U}^{ij}_{CNOT} \propto U_0^{-1}R'_{ix}(\pi/2)R'_{jy}(\pi/2)R'_{jx}(\pi/2)R_{ij}(-\pi)R'_{jy} \times (-\pi/2)U_0$ [24,25]. The two-body interaction $R_{ij}(-\pi) = e^{-\pi\hat{I}_{iz}\hat{I}_{i+1z}}$ is obtained by using an average Hamiltonian theory [16]. Here we apply the "Carr-Purcell" sequence $\tau - \pi_y - 2\tau - \pi_y - \tau$ to the Hamiltonian Eq. (8) in order to average the effect of Zeeman term to zero. Thus, we can show that the Hamiltonian of the coupled quantum-dot system can be reduced to that of the weakly coupled system in NMR and quantum operations can be carried out in a manner similar to that of operations in NMR quantum computers.

Similarly, we can show that the Hamiltonian of the two-dimensionally arrayed coupled dots is reduced to that of an artificial Ising system controllable by electric fields. Here we consider the case where there are only nearest-neighbor capacitive couplings (C_E and C_F are neglected in the above formulation). If we express the coupling strength between the sites $\mathbf{i} \equiv (i,j)$ and $\mathbf{i} + \mathbf{x} \equiv (i+1,j)$ as $J^x_{\mathbf{i}}$ and that between the sites $\mathbf{i}$ and $\mathbf{i} + \mathbf{y} \equiv (i,j+1)$ as $J^y_{\mathbf{i}}$ ($1 \leqslant i \leqslant N_x$, $1 \leqslant j \leqslant N_y$: $N = N_xN_y$), we have a Hamiltonian similar to Eq. (4) where

$$J^x_{\mathbf{i}} = \frac{2}{D_{\mathbf{i}}D_{\mathbf{i}+\mathbf{x}}}(C_{b\mathbf{i}}C_{b\mathbf{i}+\mathbf{x}} + C_{c\mathbf{i}}C_{c\mathbf{i}+\mathbf{x}})C^x_{D\mathbf{i}}, \tag{12}$$

$$J^y_{\mathbf{i}} = \frac{2}{D_{\mathbf{i}}D_{\mathbf{i}+\mathbf{y}}}(C_{b\mathbf{i}}C_{b\mathbf{i}+\mathbf{y}} + C_{c\mathbf{i}}C_{c\mathbf{i}+\mathbf{y}})C^y_{D\mathbf{i}}, \tag{13}$$

with $D_{\mathbf{i}} \equiv C_{a\mathbf{i}}C_{b\mathbf{i}} - C^2_{c\mathbf{i}}$, $C_{a\mathbf{i}} \equiv C_{A\mathbf{i}} + C_{C\mathbf{i}} + 4C_{B\mathbf{i}}$, $C_{b\mathbf{i}} \equiv C_{A\mathbf{i}} + C_{C\mathbf{i}}$, $C_{c\mathbf{i}} \equiv C_{C\mathbf{i}} - C_{A\mathbf{i}}$. $C^x_{D\mathbf{i}} (= C^x_{G\mathbf{i}})$ and $C^y_{D\mathbf{i}}$ $(= C^y_{G\mathbf{i}})$ are capacitances between qubits in x direction and y direction, respectively.

Here we summarize the criterion for realizing the above-mentioned scheme. First of all, the Coulomb blockade should be effective at the operational temperature T and we have $T \ll E_C$. In view of time-dependent operation, the excess charge is assumed to be affected by the electronic potential generated by the capacitance network. Therefore, all quantities concerning time evolution should be smaller than the CR constant of the network. Here we take $CR = C_{\mathrm{int}}R_{\mathrm{int}}$ where $C_{\mathrm{int}} (\equiv D/C_b)$ and R_{int} are respectively, capacitance and resistance of the interdot tunneling barrier in a qubit. By including the condition for using the pulse sequence as mentioned above, we have a condition for the operation

$$T \ll J \ll \Delta_0 < t \ll (CR)^{-1}. \tag{14}$$

We can roughly estimate this criterion by taking typical values, $r_0 = 2.5$ nm (radius of a quantum dot), $d_A = 8$ nm, $d_B = 1.5$ nm, $d_C = 2.5$ nm, and the distance between qubits d_D is 12 nm [$\epsilon_{\mathrm{ox}} = 4$ (SiO_2) and $\epsilon_{\mathrm{Si}} = 12$], reflecting several experimental data [20–22]. Using relations, $C_{A,C} = 2\pi\epsilon_{\mathrm{ox}}r_0^2/[d_{A,C} + (\epsilon_{\mathrm{ox}}/\epsilon_{\mathrm{Si}})r_0]$ and $C_B = 2\pi\epsilon_{\mathrm{ox}}r_0^2/[d_B + 2(\epsilon_{\mathrm{ox}}/\epsilon_{\mathrm{Si}})r_0]$, we obtain $E_C \sim 13$ meV (150 K), $t \sim 0.4$ meV, and $J \sim 0.1$ meV. If R_{int} is of the order of MΩ, we obtain $(C_{\mathrm{int}}R_{\mathrm{int}})^{-1} \sim 3.1$ THz and greater than $t \sim 100$ GHz. Thus, the condition (14) is satisfied if the operational temperature should be much less than 1 K. If we could prepare $r_0 = 0.5$ nm, $d_B = 1.2$ nm, and $d_D = 2$ nm, we obtain $t \sim 120$ K and $J \sim 90$ K and quantum calculations can be expected to be carried out at around liquid-nitrogen temperature. The effects of cross talk are of the order of $C_{Hi}/C_{Ai} \sim d_{Ai}/d_{Di}$ and cannot be neglected even when gate electrodes are set closer to the corresponding quantum dots. However, we can control the cross-talk effects by adjusting $v_i(\tau)$ to obtain the required $\Delta_i(\tau)$.

At the near-degeneracy point, the system becomes a two-state system and the estimation of the decoherence (in order of μ sec) discussed in Ref. [12] may be applicable. Until now, we have not known of any corresponding experimental data for the decoherence time [26]. The point is that various methods developed in NMR spectroscopy, such as the composite-pulse method [16], can be utilized to reduce the imperfections of the pulse and coherence transfer, that is, errors that are brought about in the operations. In addition, if the speed of quantum computations can be increased to much more than the shortest decoherence time, $\tau_c \sim \omega_c^{-1}$ ($\sim 10^{-14}$ s), group-theoretic approaches [27,28] for decreasing the decoherence will be effective. In this case, we have to reduce the corresponding CR of the junction such that Eq. (14) holds and the cycle time of operations should be much less than τ_c. Thus, the errors and the effects of

decoherence of a coupled quantum-dot system will be reduced by developing many contrivances.

Hereafter we consider the one-dimensionally arrayed qubits for simplicity. Next, we quantitatively illustrate the *reading out* process based on a field-effect transistor (FET) structure (Fig. 1). Measurement is carried out, after quantum calculations, by applying a finite bias V_D between the source and the drain [12]. The detection mechanism is such that the change of the charge distribution in a qubit induces a threshold voltage shift $\Delta V_{\rm th}$ of the gate voltage-above which the channel current flows in the substrate. $\Delta V_{\rm th}$ is of the order of $ed_q/\epsilon_{\rm ox}$ (d_q is a distance between the centers of two quantum dots in a qubit) [21]. The effect of $\Delta V_{\rm th}$ differs depending on the position of the qubit, because the width of the depletion region in the substrate changes gradually from source to drain. A simplified model of a metal-oxide semiconductor field-effect-transistor including velocity saturation effects [Θ in Eq. (15)] [29] is used for the current that flows under the ith qubit

$$I_D^{(i)} = \Lambda \frac{[V_{gi} - V_{\rm thi}](V_i - V_{i-1}) - (1/2)\,\eta_i (V_i^2 - V_{i-1}^2)}{1 + \Theta(V_i - V_{i-1})}, \tag{15}$$

where $\Lambda \equiv Z\mu_0 C_0 / L_0$ (Z is the channel width, μ_0 is the mobility, L_0 is the channel length of one qubit, and C_0 is the capacitance of the gate insulator), $\eta_i \equiv 1 + \zeta_i$ where ζ_i is determined by the charge of the surface depletion region, and V_i is the voltage of ith qubit to be determined from $V_{\rm N} = V_D$ and $I_D^{(1)} = I_D^{(2)} = \cdots = I_D^{(N)}$. The detection should be carried out before the change of the quantum states of the qubits. For this purpose, Λ should be as large as possible [12]. The threshold voltage of ith qubit $V_{{\rm th}i}$ is given by $V_{{\rm th}i} = V_{\rm th}(\zeta_i) + \Delta V_{{\rm th}i}$. Figure 2(a) shows the ratio of the current change $|I_i\text{-}I_0|/I_0$ as a function of V_D in eight qubits with $\zeta_i = 0$, where I_0 is the initial current and I_i is the current when $V_{gi}\text{-}V_{{\rm th}i}$ of ith qubit changes by 10%. This ratio is largest for a qubit near the drain ($i=8$) because it has the narrowest inversion layer. To show how to distinguish qubits, we compare the current where only the ith qubit shifts its threshold voltage with that where only the $(i+1)$th qubit shifts its threshold voltage. Figure 2(b) shows the results for (i)$i=1$, (ii) $i=N/2$, and (iii) $i=N-1$, with the same voltage shift as in Fig. 2(a). The maximum allowable number of arrayed qubits depends on the sensitivity of the external circuit to the channel current. When the density of acceptor in the substrate is of the order of 10^{17} cm^{-3}, the number of acceptors below one qubit is less than one. If this effect is represented by ζ_i as a random number, the above ratios may increase or decrease and the overall features are similar to those in Fig. 2. To construct a large qubit array, additional dummy qubits are needed over the source and drain so that separated qubits on different FETs are connected.

In conclusion, we have theoretically shown that the Hamiltonian for a weakly coupled quantum-dot array in the Coulomb blockade regime is reduced to that for NMR spectroscopy. The flexible quantum information processing developed in the NMR quantum computer and a variety of error-correction methods in conventional NMR is applicable

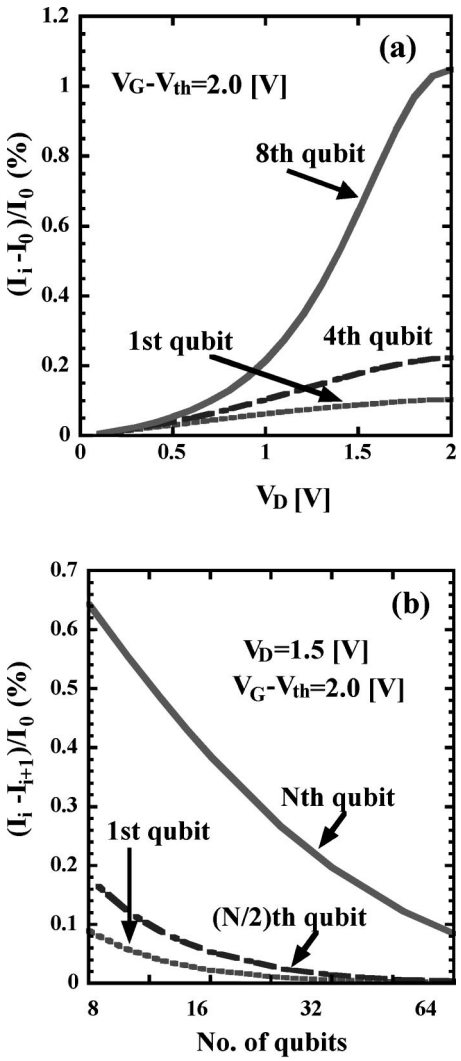

FIG. 2. I_i represents the current where only the ith qubit shifts its threshold voltage $V_{\rm th}(0) + \Delta V_{{\rm th}i}$. I_0 represents the current where all qubits have the same threshold voltage $V_{\rm th}(0)$. (a) The ratio of current change $|I_i - I_0|/I_0$ as a function of V_D in an eight-qubit quantum computer ($i=1$, $i=4$, and $i=8$). (b) The ratio of change $|I_i - I_{i+1}|/I_0$ as a function of the number of qubits, in the case $i=1$ (near source), $i=N/2$ (middle) and $i=N-1$ (near drain) at $V_D = 1.5$ V. The efficiency of detecting the quantum state of qubits is highest when the qubits near the drain change their states. In (a) and (b), $V_g\text{-}V_{\rm th} = 2$ V and $\Theta = 0.3$ V^{-1} in Eq. (7) in text. The threshold shift is 10% of $V_g\text{-}V_{\rm th}$.

to the quantum-dot system that is considered to be the most feasible system in view of the present technology. The difficulty of scalability in the NMR computer, namely, that the overlap of pulses restricts the number of qubits [2], is overcome in the operations by individual gate electrodes. The disadvantage of the short decoherence time is also compensated for by another advantage, namely, that the measurement procedure is compatible with classical circuits. It is expected that a transmitting loss of signals to classical circuits will decrease due to this compatibility. Recently, nanofabrication of two-dimensionally distributed self-aligned Si

doubly stacked dots has been successfully realized in the form of a nonvolatile memory device [22] and the detailed analysis of the behavior of electrons, such as, an artificial antiferromagnet, is expected to be performed. The *N*-qubit system composed of arrayed quantum dots is expected to be developed as a result of further advances in fabrication technologies.

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