Spin-wave utilization in a quantum computer

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We propose a quantum computer scheme using spin waves for quantum-information exchange. We demonstrate that spin waves in the antiferromagnetic layer grown on silicon may be used to perform single-qubit unitary transformations together with two-qubit operations during the cycle of computation. The most attractive feature of the proposed scheme is the possibility of random access to any qubit and, consequently, the ability to recognize two qubit gates between any two distant qubits. Also, spin waves allow us to eliminate the use of a strong external magnetic field and microwave pulses. By estimate, the proposed scheme has as high as 10^4 ratio between quantum system coherence time and the time of a single computational step.

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I. INTRODUCTION

The progress in nanostructure fabrication stimulates a great deal of interest in the solid-state implementation of the quantum computer, which will have a potential of overcoming the principal limits of a classical one [1]. The whole concept of quantum computer is based on the two ideas: (i) utilization of an atomic scale object as a qubit (two-level system), and (ii) using quantum state superposition instead of the pure $|0\rangle$ and $|1\rangle$ states in the classical computation process. Recent articles devoted to the quantum computer have discussed possibilities of using nuclei spin [2], electron spin [3], or electron state [4] as a qubit. Each of the proposed candidates has its own advantages and drawbacks, which are characterized by the coherence time, the energy, and the time of switching between $|0\rangle$ and $|1\rangle$ states, and their technological feasibility. In the recent works [3,5], the significant advantage of single-electron-spin usage as a qubit was shown. In this paper, adopting the above realization of qubit we suggest an operational scheme of quantum computer.

Quantum computational process requires the possibility of the following three steps: writing an initial state on the qubits, computing, and reading the results of computation. Computing in turn consists of a variety of unitary transformations of the single-qubit and two-qubit operations. In general, one may accomplish the above operations via different physical processes. [2–5]. For example, in Kane's well-known work [2], a high-magnetic field writes the initial state, unitary transformations accomplished via electron-spin-resonance microwave pulses, two-qubit operations are done with the help of the spin-spin exchange interaction and the readout is performed by current measurement in the field-effect transistor channel. One of the drawbacks of the approaches [2–5] is that they significantly complicate both the material structure and the operation procedure due to the variety of physical processes involved in the operation.

Another important issue is that quantum computation requires the possibility of an entanglement between any two qubits of the system. In the approaches [2–5], only the neighbor qubits can be coupled simultaneously. That means if one wants to entangle two distant qubits one must do it consequently by entangling two nearest neighbors in a bucket brigade fashion. This procedure implies more operations, and thus, an accumulation of error. In contrast, the possibility of simultaneous entanglement of two distant qubits, or random access entanglement, not only yields smaller error, but also enables a convenient implementation of simpler algorithms. Several quantum computers with random access entanglement capabilities have been proposed (for example, atoms or quantum dots in the quantum electrodynamics cavities [6,7] and ions trapped in the laser field [8]). However, these systems have another drawbacks: short coherent interaction time and limited scalability to large number of qubits, which is essential for the error correction.

The problems discussed above motivated us to propose a design of the quantum computer. In our model, all major quantum computing operations, including writing an initial state, computing, and readout are performed by spin waves. Our approach does not have many problems faced by other models. For instance, the proposed scheme alleviates the need of extremely difficult single-electron-spin detection. Spin wave provides random access entanglement capability with sufficient coherence time and scalability. By functional analogy with conventional computer, we term it *spin-wave bus*.

The rest of the paper is organized as follows. In Sec. II, we describe the material structure and the operation procedure. In Sec. III, we discuss the pros and cons of our model. Section IV summarizes the paper.

II. MATERIAL STRUCTURE AND OPERATION PROCEDURE

To provide media for spin-wave propagation, we suggest using a two-dimensional antiferromagnetic layer. In Fig. 1, we show schematically the material structure of the proposed quantum computer. It consists of the silicon layer of 300 Å wide and the antiferromagnetic layer (AFL) of a few atoms thick (~5 Å). This kind of structure can be fabricated by a wafer bonding technique or by direct ion-beam implantation

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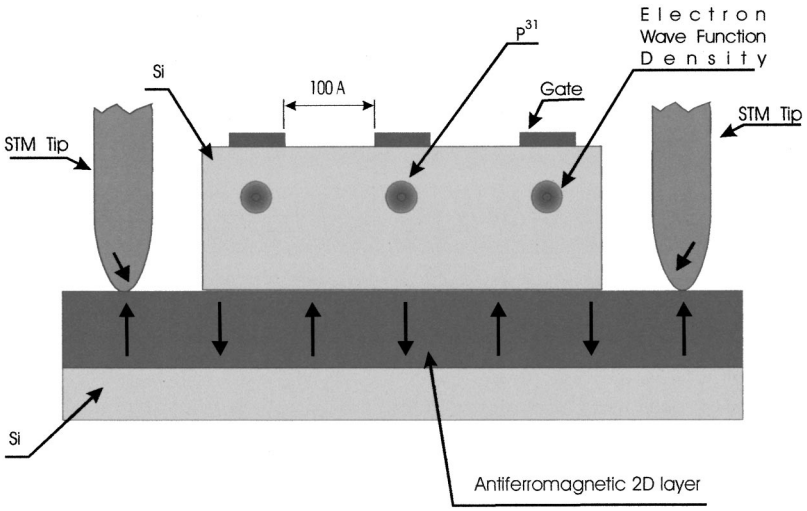

FIG. 1. Schematics of the material structure. There are an antiferromagnetic layer of a few atoms thick grown on silicon substrate, silicon layer with implanted ^{31}P atoms and metallic gate electrodes deposited on the top of the silicon layer. Spins of the ^{31}P atom bounded electrons are used as qubits. STM tips are shown at the edges.

of the antiferromagnetic atoms into silicon. The practical growing of the ferromagnetic materials on a silicon substrate has been already demonstrated [9,10] and exchange integrals of the composite materials were tabulated. In our system, ^{31}P atoms implanted in silicon serve as qubits. The atoms may be placed by implantation to 100 Å depth from the top of the silicon layer and separated by a distance of the order of 100 Å between each other. Each atom provides a weakly bounded electron (electron wave-function densities are depicted in Fig. 1 by the gray spheres), whose spin is used as a qubit. Metallic gate electrodes are deposited on the top of the silicon layer. Each electrode is placed over the one ^{31}P atom. Scanning-tunneling microscopy (STM) tips for magnetization detection are placed at the edges of this structure.

We suppose AFL belongs to the one antiferromagnetic domain. In this case, the planar dimensions of the whole structure are about $10^5 \times 10^5$ Å. In the absence of the external magnetic field, all spins in the AFL are polarized along with (or opposite to) one of the axes of the lightest magnetization. We will refer to this axis as the z direction hereafter.

In our scheme, each step in the computation process is accomplished via qubit interaction with the spin waves, which essentially are the collective oscillations of the spin projections $S_x S_y$ in the AFL. The AFL plays the key role for the information exchange between any qubits in the proposed structure. Being of the wave nature, the collective spin oscillations in the AFL present an excellent mechanism for the quantum information exchange. Two-dimensional (2D) antiferromagnetic layer with a number of spin waves transporting spin information between qubits is referred to as "*spin-wave bus*." Like any other bus, the spin-wave bus is characterized by the transmission speed, accuracy, and reliability. For our scheme, these characteristics become spin-wave group velocity, control of spin-wave excitation, and qubit information reliability with respect to the thermal spin-wave excitation. We devote the next section to discuss the above characteristics. Here, we note that all qubits are not in the permanent contact with the bus. In the absence of the gate voltage, an electron is far away from the layer so there is no interaction between the electron spin and the 2D AFL. Only wave functions of electrons under the negatively applied gate voltage shift to couple with the AFL. The coupling time of qubit-spin-wave bus does not exceed 10^{-10} s, which is much shorter then the total time of computation. Such small time of the qubit-bus coupling helps one to reduce the probability of the information loss caused by the thermally excited spin waves in the AFL, but it is still long enough to produce the coherent superposition of magnons (see below).

Another important element of the proposed structure are STM tips (shown in Fig. 1). Acting as a source of the local magnetic field, STM tips excite spin waves of large amplitudes for the initial qubit coding and after the computational process is finished, STM tips are used for the spin-wave detection in the readout procedure.

As mentioned above, all electron spins in the AFL are directed along with or opposite to the z axis. The neighbor spins in the AFL are bounded via exchange interaction, so the Hamiltonian of the system has the form:

$$H = -J \sum_{j\delta} S_j S_{j+\delta} - 2\mu_0 H_0 \sum_j S_{jz}, \tag{1}$$

where J has an energy dimensionality and describes exchange interaction, S_j and $S_{j+\delta}$ are electron-spin operators, S_{jz} is the spin projection along the z direction, the δ connects the nearest j atom in the Bravais lattice, μ is the magnetic moment, H_0 is the static strain of the external magnetic field. To couple some particular electron with the system of the AFL electrons, one has to apply a negative voltage onto the gate. The gate voltage shifts the electron toward the AFL, increasing electrons wave-function overlapping. In general, interaction between two electrons that belong to the different atoms is described in term of the exchange integral J' [11]

$$J' = \int \int U(r_1 - r_2)\psi_1(r_1)\psi_1^*(r_2)\psi_2(r_2)\psi_2^*(r_1)dV_1 dV_2, \tag{2}$$

where $\psi_1(r_1)$ and $\psi_2(r_2)$ are the wave functions and $U(r_1 - r_2)$ is the potential of interaction. The exchange integral depends strongly on the overlap of the wave functions $\psi_1(r_1)$ and $\psi_2(r_2)$. Approximately, J' can be rewritten as

$$J' \sim e^{-(n_1+n_2)|r_1-r_2|}, \quad \eta_1 = \tfrac{1}{\hbar}\sqrt{2m_1|E_1|}, \quad \eta_2 = \tfrac{1}{\hbar}\sqrt{2m_2|E_2|}, \tag{3}$$

where m_1 and m_2 are electron effective masses and E_1 and E_2 are the bound-electron energy levels corresponding to the wave-functions $\psi_1(r_1)$ and $\psi_2(r_2)$, respectively. The higher the gate voltage is applied, the bigger is the wave-function overlapping, and the stronger is the coupling between the qubit and the AFL.

As soon as coupling takes place, the spins of the AFL electrons start to rotate (whether the spin orientation of a ''qubit'' electron is different from that of the AFL). The Hamiltonian of the system AFL plus the ''qubit'' electron is written as

$$H = -J\sum_{j\delta} S_j S_{j+\delta} - \sum_j J'_{Jp} S_j S_p, \tag{4}$$

where p subscript denotes the ''qubit'' electron variable. The last term of Eq. (1) corresponding to the external magnetic field is replaced in Eq. (4) by the one corresponding to the ''effective'' magnetic field produced by the ''qubit'' electron. The rotation equations for the j spin in the AFL and for the ''qubit'' electron take the following forms:

$$\hbar \frac{d\vec{S}_j}{dt} = \vec{\mu}\times\vec{B}_j = J\sum_{\delta} \vec{S}_j \times \vec{S}_{j\delta} + J'[\vec{S}_j \times \vec{S}_p],$$

$$\hbar \frac{d\vec{S}_p}{dt} = \vec{\mu}\times\vec{B}_p = \sum_j J'[\vec{S}_p \times \vec{S}_j], \tag{5}$$

where μ is the magnetic moment, B_j and B_p are the effective magnetic fields, S_j and S_p are the spin operators of the j atom in the AFL and the p ''qubit'' electron, respectively.

In Fig. 2, we present the results of numerical simulation of the ''qubit'' electron-spin rotation while being coupled with the AFL. J' is not constant during the coupling but rather depends on the duration and the magnitude of the applied gate voltage. In our calculation, we took J' constant in order to make the results more clear. In Fig. 2(a), we depict the ''qubit'' electron-spin projections S_x, S_y for $J'/J = 1$. In Fig. 2(b), only the S_x projection is depicted for different values of J'/J. As it is seen from Fig. 2(a), the coupling results in the ''qubit''-spin projections oscillation. It is natural for us to choose the $|0\rangle$ qubit state along the plus x direction and $|1\rangle$ in the negative x. As it is apparent from the plot, by controlling the duration and strength of the coupling, it is possible to arbitrarily rotate a qubit, thus, one can define single-qubit unitary transformations.

Perturbation of the AFL spin system caused by the coupling with the ''qubit'' electron results in the collective spin oscillations. These oscillations are known as spin waves [11]. For a two-dimensional AFL, this oscillation has a form $S_{xj} = (u/\sqrt{R})e^{i(kR-\omega t+\varphi)}$, where u is the amplitude, R is the distance, k is the wave vector, ω is the fequency, and φ is the initial phase. A quantized spin wave is a quasiparticle named magnon. Like phonons, magnons are characterized by the wave vector and group velocity. Thus, for the long-

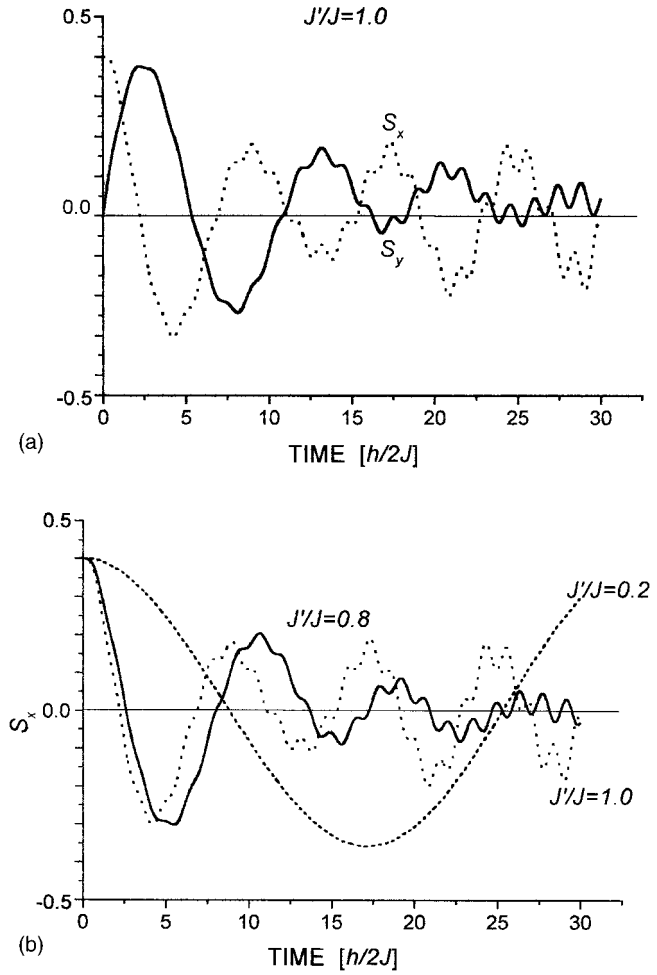

FIG. 2. Results of the numerical simulations. Time evolutions of the two-electron spin projections S_x,S_y during electron coupling with the AFL are shown in (a). The simulation was carried out for $J'/J = 1$. All AFL electrons have zero S_x,S_y projections in the beginning of coupling. In (b), the time evolution of the electron S_x projection is depicted. The simulation was carried out for three different ratios of the exchange integrals $J'/J - 0.2$, 0.8, and 1.0, respectively.

wavelength $ka \ll 1$, the spin-wave dispersion in a antiferromagnetic material is approximately linear [11]

$$\omega \approx \frac{4|J|S}{\hbar}|ka|. \tag{6}$$

The value of the exchange interaction J is usually about 1 meV and thus, the group velocity of spin waves is about 100 m/s. It is important to have the coupling between electron spins in the AFL much higher than interaction with the crystal ions so that the influence of the nuclei spins on the magnon propagation may be neglected. If the desired qubit is allowed to couple to the AFL for a sufficiently long time, then excited magnons will coherently form a cylindrical spin wave and we can justify our classical treatment. In this case, each electron in the 2D layer changes its spin adiabatically, (without phonon or photon emission) and no spin information is lost during the elementary spin exchange process.

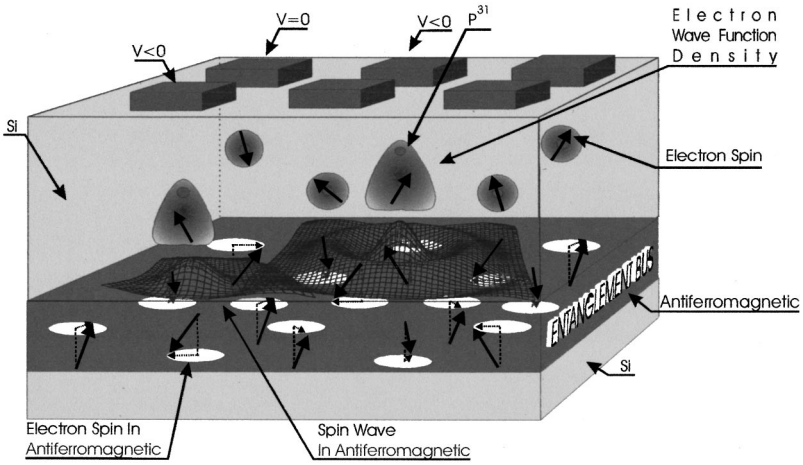

FIG. 3. Schematic representation of the two qubit interaction. Two qubits of interest couple to the AFL by the applied negative gate voltage. Each qubit generates cylindrical spin wave, which then propagates in the AFL. The interqubit interaction is provided by the spin-waves exchange.

So far, we have described single-qubit operations. As we underlined above, one needs to define two-qubits operations for computation as well, particularly, one should introduce the universal controlled-NOT (CNOT) operation. In order to achieve the CNOT gate, we need to perform a conjugate rotation of the ''target'' qubit with the ''control'' qubit. This can be readily achieved via coupling of two qubits of interest to the AFL (Fig. 3). By controlling the duration of coupling and the magnitude of the exchange integrals, we can obtain the necessary interaction via intermediate spin waves.

III. DISCUSSION

The main requirement for any quantum computer scheme is that the coherence time of the quantum system has to be larger than the time of computation. In the described scheme, the computation time is a sum of the qubit-AFL coupling time, spin-wave propagation time, and the spin-STM tip readout time. Assuming that exchange integral is about $J' \sim 0.5\,\mathrm{meV}$ for both the qubit-AFL, and AFL-STM tip couplings, the total interaction time is 10^{-9}–10^{-10} s. The spin-wave propagation time is the qubit-STM tip distance ($\sim$300 Å) divided by the spin-wave group velocity ($\sim$100 m/s), and thus, the same order of magnitude as the qubit coupling time. During this time, all qubits have to keep their information intact and spin waves excited from the qubits have to remain coherent. The coherence time for the electron spin in silicon at low temperature may be as high as microseconds [5]. As to the spin waves, there are several scattering processes that may destroy spin-wave coherence: scattering by phonons, spin waves, conduction electrons, and defects. We take into account only the strongest (spin-spin and spin-phonon) interactions, which define the coherence time of the spin wave in the defect-free antiferromagnetic at low temperature [12]. The corresponding relaxation times are

$$\tau_{mm} \approx \frac{\hbar}{\Theta_C}\left(\frac{\Theta_C}{T}\right)^4, \quad T \ll \Theta_C,$$

$$\tau_{mp} \approx \frac{\hbar}{\Theta_D}\left(\frac{\Theta_D}{T}\right)^5, \quad T \ll \Theta_D, \tag{7}$$

where T is the temperature and Θ_C is the Curie temperature, and Θ_D is the Debye temperature. These relaxation times have very strong temperature dependencies and are about microseconds at temperature of several K.

The effect of thermal excitations on the coherent spin-wave transport has a fundamental physical nature. Due to the low-energy fluctuations, there are no long-range order in the one- or two-dimensional antiferromagnetic layers. Only a limited number of the nearest spins may be kept in a coherent state. For example, in the one-dimensional spin chain, the number of spins N connected in a short order may be evaluated as [13]

$$N < \frac{9\pi^2|J|}{8kTS_z^2}\xi^{-1}, \tag{8}$$

where $\xi = \Sigma_{n=1}^{N} 1/n^2$. Thus, for $J' \sim 0.5\,\mathrm{meV}$ at $T = 1$ K, the N is about 3000. In order to achieve the long-range order, the effect of anisotropy may be used. The presence of the anisotropy energy leads to the energy gap in the spin-wave spectrum that suppresses thermal fluctuations. The critical value of the required anisotropy energy E_a is given by [13]

$$E_a > \frac{(kT)^2 S_z^3}{S^3(S+1)^3 J}. \tag{9}$$

As it is seen from Eq. (9), the required anisotropy energy decreases as the temperature square and $E_a < 10^7$ eV for the operational temperature $T = 1$ K.

In Table I, we summarize several characteristic times of the proposed scheme: electron-spin-relaxation time, spin-wave coherence time, qubit-AFL coupling time, and spin-wave propagation time. All above times are given for the temperature of 1 K. As seen in Table I simultaneous utilization of the single electron for information storage and spin

TABLE I. Characteristic times of proposed scheme.

System coherence time		Time of computation	
Electron-spin-relaxation time	Spin-wave coherence time	Qubit coupling time	Spin-wave propagation time
10^{-6} s	10^{-6} s	$10^{-9} \div 10^{-10}$ s	$10^{-9} \div 10^{-10}$ s

waves for the interqubit information exchange gives us favorable coherence and computation times.

The most attractive feature of the proposed scheme is the possibility of random access to *any* qubit and entanglement between *any* of them. The AFL acts as an information bus providing spin-wave transport. The capacity of this *spin wave bus* is limited only by the geometric size of the whole structure and maximum possible donor atom density. To minimize mutual interactions of the nearest qubits from simultaneously exciting spin waves, the coupling time should be decreased. The shorter is the coupling time, the lower is the probability for qubits to lose information via uncontrollable interactions. The speed of information exchange or the spin-wave group velocity may be increased by choosing proper antiferromagnetic materials with high values of the exchange integral *J*. The reliability of the *spin wave bus* depends strongly on the phonon-scattering rate. In the frame of the proposed scheme, the only way to suppress phonon excitation is to decrease temperature. Thus, parasitic thermal spin-wave excitations limit the operation temperature range to about 1–0.1 K.

In the proposed scheme, we escape the most difficult problem of the single-electron spin detection. Instead, we only have to measure the local magnetic field produced by tens or hundreds of electron spins bounded to the spin wave(s). At the same time, there are some problems to be solved in order to make the scheme robust. One of them is the attenuation of the spin-wave amplitude. As the spin wave propagates in the AFL, its energy is divided between many electron spins. At some critical distance, the wave amplitude becomes comparable with the noise level. Here, we come to the general problem of quantum noise effect on quantum computer performance that has a wide impact on most quantum computer schemes and requires additional future investigations.

IV. CONCLUSIONS

In conclusion, we have analyzed a quantum computer model, which utilizes spin waves in a 2D antiferromagnetic layer for information transport. Information exchange via the *spin wave bus* gives an attractive feature of random access interaction for quantum computation. The proposed scheme has a high (about 10^4) ratio between system coherence time and the time of a single computational step. The practical realization of the scheme may be done using a silicon structure bonded to an antiferromagnetic layer via the wafer bonding technique. One disadvantage is that spin-wave coherence time strongly depends on temperature. Thus, the operation temperature range is restricted to 0.1–1 K.

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