Space-time formulation of quantum transitions

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In a previous paper we have studied dressed excited states in the Friedrichs model, which describes a two-level atom interacting with radiation. In our approach, excited states are distributions (or generalized functions) in the Liouville space. These states decay in a strictly exponential way. In contrast, the states one may construct in the Hilbert space of wave functions always present deviations from exponential decay. We have considered the momentum representation, which is applicable to global quantities (trace, energy transfer). Here we study the space-time description of local quantities associated with dressed unstable states, such as, the intensity of the photon field. In this situation the excited states become factorized in Gamow states. To go from local quantities to global quantities, we have to proceed to an integration over space, which is far from trivial. There are various elements that appear in the space-time evolution of the system: the unstable cloud that surrounds the bare atom, the emitted real photons and the "Zeno photons," which are associated with deviations from exponential decay. We consider a Hilbert space approximation to our dressed excited state. This approximation leads already to decay close to exponential in the field surrounding the atom, and to a line shape different from the Lorentzian line shape. Our results are compared with numerical simulations. We show that the time evolution of an unstable state satisfies a Boltzmann-like $\mathcal{H}$ theorem. This is applied to emission and absorption as well as scattering. The existence of a microscopic $\mathcal{H}$ theorem is not astonishing. The excited states are "nonequilibrium" states and their time evolution leads to the emission of photons, which distributes the energy of the unstable state among the field modes.

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I. INTRODUCTION

As is well known the decay of excited states or unstable particles leads in the framework of quantum mechanics to deviations from exponential decay [1]. This effect, while small, leads to some puzzles. Schwinger has written "... with the failure of the simple exponential decay law we have reached, not merely the point at which some approximation ceases to be valid, but rather the limit of physical meaningfulness of the very concept of unstable particle" [2]. Wigner has gone so far as to limit the idea of elementary particles to stable particles [3].

We have presented a solution to this problem in a recent paper [4], in the framework of our extension of quantum mechanics to density matrices outside the Hilbert space. In this extension we have complex spectral representations of the Liouville–von Neumann operator (or Liouvillian) $L_H = [H, \ ]$ that allow us to rigorously disentangle the exponential and nonexponential components of the evolution of a given initial condition. The exponential component corresponds to the dressed excited state or unstable particle, and dressed photons. The nonexponential component corresponds to dressed correlations. The dressed states and correlations are given by nonfactorizable density matrices outside the Liouville-Hilbert space. They are related to the bare density matrices through a transformation Λ. This transformation is star unitary [4,5], which corresponds to a generalization of unitary transformations to unstable systems.

In our recent paper we considered global quantities, such as, the trace and total energy. To obtain these global quantities we used the momentum representation. Now we consider local quantities using the space-time representation. As will be discussed in Sec. III, the transition from the momentum representation to the space representation is far from trivial because of the singularities associated with states outside the Hilbert space.

In Sec. II we briefly summarize our previous paper. For simplicity we consider the Friedrichs model in the rotating wave approximation and in one-dimensional space. We consider in succession stable and unstable excited states. We describe the Gamow vectors, which correspond to the complex spectral representation of the Hamiltonian. We describe as well the complex spectral representation of L_H in the extended Liouville space that includes distributions. Starting from this representation, we formulate the dressed unstable state $|\rho_1^0\rangle\rangle$ as well as the dressed photon states and correlations.

In Sec. III we consider the space-time representation of the decay, starting from the bare excited state. We obtain a closed form for the field intensity $I(x,t)$ at time t.

In our previous paper we have shown that the time evolution, starting from the bare excited state, may be split into two parts: a slow one (Markovian), corresponding to the exponential decay and emission of the dressed excited state [cf. Eq. (40)], and a rapid one (non-Markovian) associated with the dressing of the bare state, leading to nonexponential effects. In the local field intensity $I(x,t)$ we may also distinguish these two parts. Note that to obtain causality (the vanishing of the emitted field outside the light cone) we have to combine both the Markovian and non-Markovian components of the field. We may not isolate either component as this would lead to noncausal behavior.

The evolution law of the non-Markovian component depends on the initial conditions. In contrast, the decay law of the dressed excited state is "universal." In this way we retain the indiscernibility of excited states or unstable particles. In addition, our decomposition in subdynamics permits us to identify in $I(x,t)$ various contributions corresponding to the dressing cloud, the decay products, and the Zeno photons. It is remarkable that in the calculation of the photon intensity only the factorizable part of $|\rho_1^0\rangle\rangle$ plays a role. To obtain global quantities such as the transfer of energy from matter to photons we have to integrate over the whole space.

In Sec. IV we give an example of a state in the Hilbert space that comes closer to an exponential decay of the field (inside the light cone), as compared with the bare state. The essential feature of this state is that it already has a cloud at $t=0$ (as is the case for ground states). This leads to a line shape closer to the line shape of $|\rho_1^0\rangle\rangle$ than to the Lorentzian line shape. Hence this state offers an approximate scheme in the Hilbert space of our non-Hilbertian unstable state.

In Sec. V we come to an important point: the description of emission and absorption in terms of an $\mathcal{H}$ function, which is a microscopic analog of Boltzmann's $\mathcal{H}$ function in statistical mechanics. In our earlier work [3,6,7] we have shown that if there exists a microscopic entropy it must be an operator. In the Friedrichs model and in the simplest case dealing with nonsingular states, we can construct an $\mathcal{H}$ function in terms of the operator [6,8]

$$\mathcal{H}=|\tilde{\phi}_1\rangle\langle\tilde{\phi}_1|, \tag{1}$$

where $|\tilde{\phi}_1\rangle$ is a Gamow vector. The $\mathcal{H}$ function is, therefore, outside the Hilbert space. We do not give here to a detailed discussion on entropy. Let us only notice that there have always been two points of view: the point of view of Planck, relating entropy to dynamics and the point of view of Boltzmann, relating entropy to probabilities (ignorance) [9]. We understand now that Planck could not realize his program as he worked in the usual representation of dynamics, equivalent to a Hilbert-space representation.

The Heisenberg evolution of Eq. (1) is given by

$$\mathcal{H}(t)=e^{iHt}\mathcal{H}e^{-iHt}=e^{-2\gamma t}\mathcal{H}. \tag{2}$$

The physical meaning of $\mathcal{H}$ is very simple. It decreases as the energy of the excited state is transferred to the field modes.

In Ref. [8] we have discussed the effect of momentum inversion on $\mathcal{H}$. Momentum inversion leads to a jump of $\mathcal{H}$ corresponding to an "injection" of negative "entropy." Here appears the basic distinction between precollisional and postcollisional correlations. The emission of photons corresponds to a postcollisional correlation decreasing the $\mathcal{H}$ function (the photons escape from the particle). The momentum inversion leads to a precollisional correlation increasing the $\mathcal{H}$ function. This observation was made many years ago in the framework of statistical mechanics [5].

In Sec. V we show that this idea also applies to excited states. We consider the scattering of a wave packet by the particle (or atom) in terms of $\mathcal{H}$. We show that $\mathcal{H}$ changes drastically depending on the direction of the initial wave packet. If the wave packet moves towards the particle, $\mathcal{H}$ is large. Conversely, if the wave packet moves away from the particle, $\mathcal{H}$ is small. The initial precollisional correlations are of long range and are dominated by resonance effects. In contrast, postcollisional correlations are of short range and are due only to off-resonance effects. We define space-time-dependent Lyapounov functions $h^{\pm}(x,t)$ that measure the postcollisional and precollisional correlations between the field and the particle at each location in space, respectively. As we shall show, the distinction between the functions $h^{+}(x,t)$ and $h^{-}(x,t)$ is related to the positivity of energy, which is associated with nonlocal effects in the scattering of photons [10].

In the appendices we discuss a few miscellaneous topics that include a simplified derivation of the form of our dressed particle state, some properties of the Hilbert-space state presented in Sec. IV and a study of resonance scattering.

Processes involving matter-radiation interactions remain interesting as ever. They have been the starting point of quantum theory and now they provide tests for our proposed extension of the framework of quantum mechanics. Hamiltonian physics leads to a description of independent noninteracting entities. For unstable particles or excited states we need a different description. The units should be interacting as energy is transferred between matter and field.

II. DECAYING QUANTUM STATES

First we summarize our previous results. For more details see Ref. [4]. We consider the Friedrichs model in one-dimensional space. The Hamiltonian of this model is given by[1]

$$H=H_0+\lambda V=\omega_1|1\rangle\langle 1|+\sum_k \omega_k|k\rangle\langle k| +\lambda\sum_k V_k(|k\rangle\langle 1|+|1\rangle\langle k|). \tag{3}$$

The state $|1\rangle$ represents a bare particle or atom in its excited level and no field present, while the state $|k\rangle$ represents a bare-field mode of momentum k together with the particle in its ground state. Hereafter we shall refer to these states as "particle" and "photon" states, respectively. For $\alpha,\beta=1,k$ we have

$$\langle\alpha|\beta\rangle=\delta_{\alpha,\beta}, \qquad \sum_{\alpha=1,k}|\alpha\rangle\langle\alpha|=1. \tag{4}$$

The energy of the ground state is chosen to be zero; ω_1 is the bare energy of the excited level, and $\omega_k\equiv|k|$ is the photon energy with a unit $c=1$. The coupling constant λ is dimensionless. As usual, we assume periodic boundary conditions. We put the system in a "box" of size L and eventually take

[1] Here we neglect virtual transitions. In a recent publication [11] we solved the Friedrichs model including virtual processes.

the limit $L\to\infty$. For L finite, the momenta k are discrete. In the limit $L\to\infty$ they become continuous, i.e.,

$$\sum_k \to \frac{L}{2\pi}\int dk. \tag{5}$$

The summation sign is written with the understanding that the limit Eq. (5) is taken at the end. The potential V_k is of order $L^{-1/2}$. To indicate this we write

$$V_k=(2\pi/L)^{1/2}v_k, \tag{6}$$

where v_k is of order one in the continuous spectrum limit $L\to\infty$. As a specific example we shall assume that v_k is of the form

$$v_k\equiv v(\omega_k)=(2\omega_k)^{1/2}u(\omega_k),$$

$$u(\omega_k)\equiv\frac{1}{[1+(\omega_k/\omega_M)^2]^n} \tag{7}$$

with n a positive integer. This form appears, for example, in a two-level model of the hydrogen atom [12]. The constant ω_M^{-1} determines the range of the interaction. We shall assume that the interaction is of short range, i.e., $\omega_M\gg\omega_1$.

The state $|1\rangle$ is either unstable or stable depending on whether its energy ω_1 is above or below a threshold, respectively [see Eq. (2.6) of Ref. [4]].

A. Stable case

For the stable case, one can construct dressed states $|\bar{\phi}_\alpha\rangle$ that are eigenstates of H. In the limit $L\to\infty$ we have

$$H|\bar{\phi}_1\rangle=\bar{\omega}_1|\bar{\phi}_1\rangle,\quad H|\bar{\phi}_k\rangle=\omega_k|\bar{\phi}_k\rangle, \tag{8}$$

where $\bar{\omega}_1$ is the (real) shifted energy of the discrete state. We use the bars to refer to the stable case. Note that there is a one-to-one correspondence between the dressed and bare states as $\lim_{\lambda\to 0}|\bar{\phi}_\alpha\rangle=|\alpha\rangle$ and $\lim_{\lambda\to 0}\bar{\omega}_1=\omega_1$. The explicit forms of the dressed states are

$$|\bar{\phi}_1\rangle=\bar{N}_1^{1/2}\left[|1\rangle+\sum_k |k\rangle\frac{\lambda V_k}{\bar{\omega}_1-\omega_k}\right], \tag{9}$$

$$|\bar{\phi}_k\rangle=|k\rangle+\frac{\lambda V_k}{\eta^+(\omega_k)}\left[|1\rangle+\sum_p |p\rangle\frac{\lambda V_p}{\omega_k-\omega_p+i\epsilon}\right]. \tag{10}$$

Here $\bar{N}_1$ is a normalization constant and

$$\eta^+(\omega)\equiv\omega-\omega_1-\sum_l \frac{\lambda^2V_l^2}{(z-\omega_l)^+_\omega}. \tag{11}$$

The $+$ (or $-$) superscript indicates analytic continuation of z from the upper (or lower) half plane to $z=\omega$ (this continuation will play a role in the unstable case below). The shifted energy of the discrete state is given by the solution of the equation $\eta^\pm(\bar{\omega}_1)=0$.

From the eigenstates of H one can construct the density operators (see Ref. [4])

$$|\bar{\rho}^0_\alpha\rangle\rangle\equiv|\bar{\phi}_\alpha;\bar{\phi}_\alpha\rangle\rangle,\quad \alpha=1,k \tag{12}$$

$$|\bar{\rho}^{\alpha\beta}\rangle\rangle\equiv|\bar{\phi}_\alpha;\bar{\phi}_\beta\rangle\rangle,\quad \alpha\neq\beta,$$

which are eigenstates of the Liouvillian as

$$L_H|\bar{\rho}^0_\alpha\rangle\rangle=0,\quad L_H|\bar{\rho}^{\alpha\beta}\rangle\rangle=(\bar{\omega}_\alpha-\bar{\omega}_\beta)|\bar{\rho}^{\alpha\beta}\rangle\rangle, \tag{13}$$

where $\bar{\omega}_k=\omega_k$.

The density operators $|\bar{\rho}^0_1\rangle\rangle$ and $|\bar{\rho}^0_k\rangle\rangle$ correspond to dressed particle and photon states, respectively, and are invariants of motion. The states $|\bar{\rho}^{\alpha\beta}\rangle\rangle$ represent dressed correlations, which oscillate in time. The dressed states are related to the bare states through a unitary transformation

$$|\bar{\rho}^0_\alpha\rangle\rangle=U^{-1}|\alpha;\alpha\rangle\rangle,\quad \alpha=1,k \tag{14}$$

$$|\bar{\rho}^{\alpha\beta}\rangle\rangle=U^{-1}|\alpha;\beta\rangle\rangle,\quad \alpha\neq\beta.$$

B. Unstable case

In the unstable case, one can also construct eigenstates of H that are in one-to-one correspondence with the unperturbed states [8,13–15]. This requires, however, to go outside the Hilbert space, as the discrete state has a complex eigenvalue

$$H|\phi_1\rangle=z_1|\phi_1\rangle,\quad H|\phi_k\rangle=\omega_k|\phi_k\rangle. \tag{15}$$

Here,

$$z_1\equiv\tilde{\omega}_1-i\gamma \tag{16}$$

is the pole of Green's function that is given as a solution of $\eta^+(z_1)=0$. The negative imaginary part of z_1 describes decay for $t>0$. Note that for $\mathrm{Im}(\omega)<0$ the function $\eta^+(\omega)$ in Eq. (11) is evaluated with $z=\omega$ on the second Riemann sheet [4,8].

The state $|\phi_1\rangle$ is called Gamow vector. The left eigenstates of H that belong to the same eigenvalues are different from the right eigenstates

$$\langle\tilde{\phi}_1|H=\langle\tilde{\phi}_1|z_1,\quad \langle\tilde{\phi}_k|H=\langle\tilde{\phi}_k|\omega_k. \tag{17}$$

The right and left eigenstates of H are given by [4,8]

$$|\phi_1\rangle=N_1^{1/2}\left[|1\rangle+\sum_k |k\rangle\frac{\lambda V_k}{(z-\omega_k)^+_{z_1}}\right], \tag{18}$$

$$|\tilde{\phi}_1\rangle=[N_1^{\mathrm{c.c}}]^{1/2}\left[|1\rangle+\sum_k |k\rangle\frac{\lambda V_k}{(z-\omega_k)^-_{z_1^{\mathrm{c.c.}}}}\right],$$

$$|\phi_k\rangle=|k\rangle+\frac{\lambda V_k}{\eta_d^+(\omega_k)}\left[|1\rangle+\sum_p |p\rangle\frac{\lambda V_p}{\omega_k-\omega_p+i\epsilon}\right],$$

$$|\tilde{\phi}_k\rangle = |k\rangle + \frac{\lambda V_k}{\eta^+(\omega_k)}\left[|1\rangle + \sum_p |p\rangle \frac{\lambda V_p}{\omega_k - \omega_p + i\epsilon}\right].$$

Here

$$\frac{1}{\eta_d^+(\omega_k)} \equiv \frac{1}{\eta^+(\omega_k)} \frac{z_1 - \omega_k}{(z-\omega_k)_{z1}^+}. \tag{19}$$

Note that the difference between unstable and the stable discrete states lies in the need for analytic continuation for the unstable state.

The eigenstates of H form a bicomplete biorthonormal set in the wave-function space as

$$\langle \tilde{\phi}_\alpha | \phi_\beta \rangle = \delta_{\alpha,\beta}, \quad \sum_{\alpha=1,k} |\phi_\alpha\rangle\langle\tilde{\phi}_\alpha| = 1. \tag{20}$$

A first possibility to define a dressed particle state in the unstable case is through factorized density operators constructed with Gamow vectors. As we shall see, for certain types of observables these operators give a correct description of the unstable particle. However, as we have shown in Ref. [4], if we want to describe fundamental properties, such as, the transfer of energy from the particle (or excited state) to the field, then the factorized Gamow density operators are not adequate. For example, the density operator $|\phi_1;\phi_1\rangle\rangle$ is traceless and has no energy, while $|\phi_1;\tilde{\phi}_1\rangle\rangle$ cannot decay since this is an invariant of motion. Still, dyads of Gamow vectors play an essential role in our formulation.

C. Complex spectral representation of L_H

In the Liouville space one can construct complex spectral representations of the Liouvillian that are not reducible to a product of representations of the Hamiltonian [4,7,16]. In these representations the right eigenstate $|F_j^\nu\rangle\rangle$ for a given eigenvalue $z_j^{(\nu)}$ is different from the Hermitian conjugate of the left eigenstate $\langle\langle\tilde{F}_j^\nu|$, as is the case for Gamow vectors [see Eq. (17)],

$$L_H|F_j^\nu\rangle\rangle = z_j^{(\nu)}|F_j^\nu\rangle\rangle, \quad \langle\langle\tilde{F}_j^\nu|L_H = \langle\langle\tilde{F}_j^\nu|z_j^{(\nu)}, \tag{21}$$

where j together with ν specify the eigenstates. In Ref. [4] we have solved the eigenvalue problem of the Liouville operator for the Friedrichs model and shown that the set of eigenstates consists of a purely decaying state and invariant states [see Eq. (4.27) in Ref. [4]]

$$L_H|F_1^0\rangle\rangle = -2i\gamma|F_1^0\rangle\rangle, \quad L_H|F_k^0\rangle\rangle = O(1/L) \to 0, \tag{22}$$

where

$$|F_1^0\rangle\rangle \equiv |\phi_1;\phi_1\rangle\rangle,$$

$$|F_k^0\rangle\rangle \equiv |\tilde{\phi}_k;\tilde{\phi}_k\rangle\rangle, \tag{23}$$

as well as dressed correlation states

$$L_H|F^{\alpha\beta}\rangle\rangle = (z_\alpha - z_\beta)|F^{\alpha\beta}\rangle\rangle, \quad \alpha \neq \beta, \tag{24}$$

where $z_k \equiv \omega_k$ and

$$|F^{kk'}\rangle\rangle = |\phi_k;\phi_{k'}\rangle\rangle, \tag{25}$$

$$|F^{1k}\rangle\rangle = |(F^{1k})^\dagger\rangle\rangle = |\phi_1;\phi_k\rangle\rangle - \sum_l |l,l\rangle\rangle \times [\langle\langle l,l|\phi_1;\phi_k\rangle\rangle - F(k,l)],$$

with

$$F(k,l) \equiv -N_1^{1/2} \frac{\lambda V_k}{\eta_d^-(\omega_k)} \frac{\lambda^2 V_l^2}{z_1 - \omega_k} \times \left[\frac{1}{\omega_l - \omega_k + i\epsilon} + \frac{1}{(z-\omega_l)_{z_1}^+}\right]. \tag{26}$$

Note that eigenstates, such as $|F_1^0\rangle\rangle = |\phi_1\rangle\langle\phi_1|$ are still factorizable, while other states such as $|F^{1k}\rangle\rangle$ are not factorizable. The explicit forms of the left eigenstates are given in Appendix B of Ref. [4].

The eigenstates of L_H form a bicomplete and biorthonormal set in the Liouville space. The states $|F_\alpha^0\rangle\rangle$ span the so-called "vacuum of correlations" subspace that is defined by the projection operator

$$\Pi^{(0)} \equiv \sum_{\alpha=1,k} |F_\alpha^0\rangle\rangle\langle\langle\tilde{F}_\alpha^0|. \tag{27}$$

The other states define the particle-field and field-field dressed correlation subspaces

$$\Pi^{(1)} \equiv \sum_l [|F^{1l}\rangle\rangle\langle\langle\tilde{F}^{1l}| + |F^{l1}\rangle\rangle\langle\langle\tilde{F}^{l1}|], \tag{28}$$

$$\Pi^{(2)} \equiv \sum_{l,l'}{}' |F^{ll'}\rangle\rangle\langle\langle\tilde{F}^{ll'}|,$$

respectively, where the prime in the summation sign denotes the restriction $l \neq l'$. The index d in the projectors $\Pi^{(d)}$ indicates the "degree of correlation" [4] of each subspace. Due to the completeness and orthogonality of the eigenstates of L_H, the projectors are also complete

$$\sum_{d=0}^{2} \Pi^{(d)} = 1, \tag{29}$$

and orthogonal

$$\Pi^{(d)}\Pi^{(d')} = \delta_{d,d'}\Pi^{(d)}. \tag{30}$$

The projectors commute with L_H. We note that the projectors are not Hermitian in the Liouville space, i.e.,

$$[\Pi^{(d)}]^\dagger \neq \Pi^{(d)}. \tag{31}$$

D. Dressed density operators

In the unstable case we express the dressed density operators as linear combinations of the eigenstates of L_H. To obtain their specific forms we construct a transformation Λ that maps dressed states to bare states (see [4,5] for a detailed discussion on Λ). The Λ transformation is obtained by analytic continuation of the corresponding unitary transformation U in the stable case. Λ is no longer unitary, but it has a new symmetry property called star unitarity, which is an extension of unitarity [5]. In terms of the eigenstates $|\alpha;\beta\rangle\rangle$ of L_0, the dressed states and their duals are defined as

$$|\rho^0_\alpha\rangle\rangle \equiv \Lambda^{-1}|\alpha;\alpha\rangle\rangle, \quad \langle\langle\tilde{\rho}^0_\alpha| \equiv \langle\langle\alpha;\alpha|\Lambda, \tag{32}$$

and

$$|\rho^{\alpha\beta}\rangle\rangle \equiv \Lambda^{-1}|\alpha;\beta\rangle\rangle, \quad \langle\langle\tilde{\rho}^{\alpha\beta}| \equiv \langle\langle\alpha;\beta|\Lambda, \tag{33}$$

for $\alpha \neq \beta$. The states $|\rho^0_1\rangle\rangle$ and $|\rho^0_k\rangle\rangle$ are the dressed excited state and dressed photon states, respectively, and $|\rho^{\alpha\beta}\rangle\rangle$ are dressed correlation states. In the limit $\lambda \to 0$ the Λ transformation reduces to the unit operator, and the dressed states reduce to the bare states. Moreover, in the stable case the dressed states reduce to the states in Eq. (12).

In terms of the eigenstates of L_H the dressed states are given by [4]

$$|\rho^0_1\rangle\rangle = |F^0_1\rangle\rangle + \sum_k b_k |F^0_k\rangle\rangle, \tag{34}$$

$$|\rho^0_k\rangle\rangle = |F^0_k\rangle\rangle - b_k |F^0_1\rangle\rangle, \tag{35}$$

$$|\rho^{\alpha\beta}\rangle\rangle = |F^{\alpha\beta}\rangle\rangle, \tag{36}$$

where

$$b_k \equiv \frac{\lambda^2}{|1+\Sigma_k c_k^2|}[(r c_k^2 + \text{c.c.}) - c_k c_k^{\text{c.c.}}] \tag{37}$$

with

$$c_k \equiv \frac{V_k}{(z-\omega_k)^+_{z_1}} \tag{38}$$

and r given in Eq. (A9). The distribution b_k satisfies

$$\sum_k b_k = 1. \tag{39}$$

The dressed excited state evolves as

$$e^{-iL_H t}|\rho^0_1\rangle\rangle = e^{-2\gamma t}|\rho^0_1\rangle\rangle + (1-e^{-2\gamma t})\sum_k b_k|\rho^0_k\rangle\rangle. \tag{40}$$

This shows that b_k is the line shape of emitted photons. For small coupling constant $\lambda \ll 1$, b_k is approximated by

$$b_k \approx \left(\frac{2\pi}{L}\right)\frac{1}{\pi}\frac{(\lambda^2\gamma_2)^3}{[(\omega_k-\tilde{\omega}_1)^2+\lambda^4\gamma_2^2]^2}, \tag{41}$$

where $\lambda^2\gamma_2$ is the lowest-order approximation of γ. This differs drastically from the Lorentzian line shape, as it leads to no divergence for the fluctuation of energy. For the left states we have analogous expressions to Eqs. (34)–(36) [see Eqs. (B23)–(B26) of Ref. [4]].

The state $|\rho^0_1\rangle\rangle$ may be also expressed in terms of dyads of Gamow vectors as [4] as (see Appendix A)

$$|\rho^0_1\rangle\rangle = Q^{(0)}|\phi_1;\phi_1\rangle\rangle + P^{(0)}[r^{\text{c.c.}}|\phi_1;\tilde{\phi}_1\rangle\rangle + r|\tilde{\phi}_1;\phi_1\rangle\rangle], \tag{42}$$

where $P^{(0)}$ and $Q^{(0)}$ are projectors to the diagonal and off-diagonal components of density matrices, respectively,

$$P^{(0)} = 1 - Q^{(0)} = \sum_{\alpha=1,k} |\alpha;\alpha\rangle\rangle\langle\langle\alpha;\alpha|, \tag{43}$$

and r is a numerical coefficient. This means that we have

$$\langle\langle k;k|\rho^0_1\rangle\rangle = r^{\text{c.c.}}\langle\langle k;k|\phi_1;\tilde{\phi}_1\rangle\rangle + r\langle\langle k;k|\tilde{\phi}_1;\phi_1\rangle\rangle$$
$$\langle\langle k;k'|\rho^0_1\rangle\rangle = \langle\langle k;k'|\phi_1;\phi_1\rangle\rangle, \quad k \neq k'. \tag{44}$$

The states $|\rho^0_\alpha\rangle\rangle$ and their duals generate the $\Pi^{(0)}$ subspace

$$\Pi^{(0)} = \sum_\alpha |\rho^0_\alpha\rangle\rangle\langle\langle\tilde{\rho}^0_\alpha|, \tag{45}$$

while the states $|\rho^{(\alpha\beta)}\rangle\rangle$ and their duals generate the correlation subspaces [see Eqs. (36) and (28)]. Hence, the dressed states form a complete set in the Liouville space

$$\sum_\alpha |\rho^0_\alpha\rangle\rangle\langle\langle\tilde{\rho}^0_\alpha| + \sum_{\alpha\neq\beta}|\rho^{\alpha\beta}\rangle\rangle\langle\langle\tilde{\rho}^{\alpha\beta}| = 1. \tag{46}$$

Note that the replacement of the unitary transformation U in the stable case by Λ in the unstable case implies radical changes. U is distributive as $U(AB)=(UA)(UB)$ while Λ is not. Therefore, irreversibility implies a deep change in the mathematics of quantum mechanics.

E. Diagonal singularities and observables

We come now to an important point. We have to classify observables according to whether they have or they do not have a diagonal singularity in momentum representation (see also [17,18]). To each observable corresponds an operator, say A. By definition an operator A has a diagonal singularity if the diagonal element $\langle k|A|k\rangle$ is at least as important (in orders of magnitude of the volume L) as the sum of the off-diagonal elements $\langle k|A|k'\rangle$ with $k \neq k'$, i.e.,

$$\langle k|A|k\rangle \sim \sum_{k'}{}' \langle k|A|k'\rangle. \tag{47}$$

Examples are functions of the Hamiltonian H, such as, H^2, for which we have

$$\langle k|H^2|k\rangle=\omega_k^2+O(1/L), \tag{48}$$

$$\sum_{k'}{}' \langle k|H^2|k'\rangle=\sum_{k'}{}' \lambda^2 V_k V_{k'}\Rightarrow\int dk\lambda^2 v_k v_{k'} .$$

Both quantities are of the same order L^0 in volume. The diagonal singularities play an important role in our formulation as they are associated with the components of the dressed states that are nonfactorizable into a product of wave functions [see Eq. (42)]. These components are essential to obtain important physical features of the unstable particle state $|\rho_1^0\rangle\rangle$. For example, thanks to the nonfactorizable components of $|\rho_1^0\rangle\rangle$, the average energy $\langle\langle H|\rho_1^0\rangle\rangle$ is real and positive. In addition the state $|\rho_1^0\rangle\rangle$ has a unit trace as $\Sigma_{\alpha=1,k}\langle\langle\alpha;\alpha|\rho_1^0\rangle\rangle=1$, and the time evolution is given by the Markovian equation (40).

In our classification, the second class of observables are the ones with no diagonal singularities. For these observables the diagonal elements in momentum representation are $O(1/L)$ smaller than the sum of the off-diagonal elements. For example for the "field-intensity" operator $|x\rangle\langle x|$, with the $|x\rangle$ kets defined by[2]

$$|x\rangle\equiv\sum_k |k\rangle(2\omega_k L)^{-1/2}e^{-ikx}, \tag{49}$$

we have

$$\langle k|x\rangle\langle x|k\rangle\sim 1/L, \tag{50}$$

$$\sum_{k'}{}' \langle k|x\rangle\langle x|k'\rangle=\int\frac{dk'}{2\pi}(4\omega_k\omega_{k'})^{-1/2}e^{ix(k-k')}.$$

The first quantity is $O(1/L)$ smaller than the second.

For any operator A with no diagonal singularity, the spectral representation of L_H takes a factorized form and we have

$$\langle\langle A|\rho_1^0\rangle\rangle=\langle\langle A|F_1^0\rangle\rangle=\langle\langle A|\phi_1;\phi_1\rangle\rangle+O(1/L),$$

$$\langle\langle A|\rho_k^0\rangle\rangle\sim\langle\langle A|F_k^0\rangle\rangle\sim O(1/L), \tag{51}$$

$$\langle\langle A|\rho^{\alpha\beta}\rangle\rangle=\langle\langle A|F^{\alpha\beta}\rangle\rangle=\langle\langle A|\phi_\alpha;\phi_\beta\rangle\rangle+O(1/L),$$

where we have neglected the diagonal components, such as $\langle\langle A|k;k\rangle\rangle\langle\langle k;k|\rho_1^0\rangle\rangle$, which vanish as $O(1/L)$ in the continuous limit $L\rightarrow\infty$. We have analogous expressions for the left states, e.g. $\langle\langle\tilde{F}^{\alpha\beta}|A\rangle\rangle=\langle\langle\tilde{\phi}_\alpha;\tilde{\phi}_\beta|A\rangle\rangle+O(1/L)$.

[2] In the second quantization formalism we have $|k\rangle=a_k^\dagger|0\rangle$ where $|0\rangle$ is the vacuum and $[a_k,a_{k'}^\dagger]=\delta_{kk'}$ as usual. The Friedrichs model is limited to the one-particle sector. In this sector we have $\langle x|\rho|x\rangle=\mathrm{Tr}[\phi^2(x)\rho]$ where $\phi(x)\equiv\Sigma_k(2\omega_k L)^{-1/2}(a_k^\dagger e^{-ikx}+a_k e^{ikx})$. Hence $|x\rangle\langle x|$ corresponds to the field intensity in the one-particle sector.

An example of nonsingular observable is the photon field intensity we study in the following section. The dressed states can then be reduced to a product of wave amplitudes $|\phi_\alpha\rangle\langle\phi_\alpha|$ in the extended space of wave functions. The question of going from local observables to global observables involves an integration over space (see Sec. III). This means that important features, such as, the transfer of energy from matter to radiation between dressed states cannot be observed by local measurements (except, of course, to the lowest-order approximation when we deal with bare particles). The $1/L$ contributions in Eq. (51) have to be kept to be able to make the transition to observables with diagonal singularities.

III. SPACE-TIME DESCRIPTION OF THE EMISSION PROCESS

We shall first calculate the intensity of the field emitted by the bare excited state $|\rho(0)\rangle\rangle=|1;1\rangle\rangle$. Our aim is to separate the non-Markovian part of the field associated with the preparation of the unstable particle from the Markovian part coming from the decay process.

The field intensity is defined by

$$I(x,t)\equiv\langle\langle x;x|e^{-iL_H t}|1;1\rangle\rangle=|\langle x|e^{-iHt}|1\rangle|^2. \tag{52}$$

In our model the bare particle is "located" at $x=0$ (this is where the effective interaction between the particle or atom and the field occurs. For example, for a two-level model of the hydrogen atom in the dipole approximation, $x=0$ is assumed to be the average position of the dipole).

A. Calculation of the amplitude

As the dressed states will take the factorized form in Eq. (51), we start by evaluating the amplitude in Eq. (52)

$$f(x,t)\equiv\langle x|e^{-iHt}|1\rangle. \tag{53}$$

For simplicity we shall consider the case of weak coupling $\lambda\ll 1$. Hence we have $\tilde{\omega}_1\gg\gamma$, where $\tilde{\omega}_1\sim O(\lambda^0)$ while $\gamma\sim O(\lambda^2)$. Inserting the bicomplete set of eigenstates of the Hamiltonian in Eq. (20) we have

$$f(x,t)=\sum_{\alpha=1,k}\langle x|e^{-iHt}|\phi_\alpha\rangle\langle\tilde{\phi}_\alpha|1\rangle. \tag{54}$$

Defining [see Eq. (15)]

$$f_0(x,t)\equiv e^{-iz_1 t}\langle x|\phi_1\rangle\langle\tilde{\phi}_1|1\rangle, \tag{55}$$

$$f_1(x,t)\equiv-\sum_k e^{-i\omega_k t}\langle x|\phi_k\rangle\langle\tilde{\phi}_k|1\rangle,$$

we have

$$f(x,t)=f_0(x,t)-f_1(x,t). \tag{56}$$

In the limit $L\rightarrow\infty$ the first term is given by [cf. Eq. (6) and Eq. (18)]

$$f_0(x,t)=N_1e^{-iz_1t}\int_{-\infty}^{\infty}\frac{dk}{\sqrt{2\pi}}e^{ikx}\frac{\lambda u(\omega_k)}{(z-\omega_k)_{z_1}^{+}}, \tag{57}$$

while the second term is given by

$$f_1(x,t)=-\int_{-\infty}^{\infty}\frac{dk}{\sqrt{2\pi}}e^{ikx}\lambda u(\omega_k)\left[\frac{e^{-i\omega_k t}}{\eta^-(\omega_k)}+\int_{-\infty}^{\infty}dl\times\frac{\lambda^2 v^2(\omega_l)}{\eta_d^+(\omega_l)\eta^-(\omega_l)}\frac{e^{-i\omega_l t}}{\omega_l-\omega_k+i\epsilon}\right]. \tag{58}$$

Using the relation

$$\frac{\lambda^2 v^2(\omega_l)}{\eta^+(\omega_l)\eta^-(\omega_l)}=\frac{-1}{4\pi i}\left[\frac{1}{\eta^+(\omega_l)}-\frac{1}{\eta^-(\omega_l)}\right], \tag{59}$$

we get

$$f_1(x,t)=-\int_{-\infty}^{\infty}\frac{dk}{\sqrt{2\pi}}e^{ikx}\lambda u(\omega_k)\left[\frac{e^{-i\omega_k t}}{\eta^-(\omega_k)}-\int_0^{\infty}\frac{dl}{2\pi i}\times\left(\frac{1}{\eta_d^+(l)}-\frac{1}{\eta^-(l)}\right)\frac{e^{-ilt}}{l-\omega_k+i\epsilon}\right]. \tag{60}$$

For $l<0$, the integrand in the second term vanishes. Hence we may extend the l integration from $-\infty$ to ∞. Approximating $\eta_d^+(l)=-1/(z-l)_{z_1}^{+}+O(\lambda^2)$ and taking the residue at $l=\omega_k$ we obtain

$$f_1(x,t)\approx\int_{-\infty}^{\infty}\frac{dk}{\sqrt{2\pi}}e^{i(kx-\omega_k t)}\frac{\lambda u(\omega_k)}{(z-\omega_k)_{z_1}^{+}}. \tag{61}$$

Then, using the relations $\omega_k=|k|$ and $N_1=1+O(\lambda^2)$ we have (for $a=0,1$)

$$f_a(x,t)\approx\int_0^{\infty}dk g_a(k,x,t), \tag{62}$$

where

$$g_0(k,x,t)\equiv\frac{1}{\sqrt{2\pi}}e^{-iz_1t}\frac{\lambda u(k)}{(z-k)_{z_1}^{+}}[e^{ikx}+e^{-ikx}],$$

$$g_1(k,x,t)\equiv\frac{1}{\sqrt{2\pi}}e^{-ikt}\frac{\lambda u(k)}{(z-k)_{z_1}^{+}}[e^{ikx}+e^{-ikx}]. \tag{63}$$

Now we extract the resonance pole and "cut" contributions by adding and subtracting integrations from $-\infty$ to zero

$$f_a(x,t)\approx p_a(x,t)+c_a(x,t), \tag{64}$$

where

$$p_a(x,t)\equiv\int_{-\infty}^{\infty}dk\, g_a(k,x,t),$$

$$c_a(x,t)\equiv-\int_{-\infty}^{0}dk\, g_a(k,x,t). \tag{65}$$

The functions p_a may be evaluated by closing the contour in the upper or lower infinite semicircle of the k complex plane, depending on whether $\pm x-at$ is positive or negative, respectively. This leads to the pole contributions at $k=z_1$ as

$$p_a(x,t)\approx-\sqrt{2\pi}i\lambda u(z_1)e^{i(|x|-t)z_1}\theta(|x|-at) \tag{66}$$

for $t>0$, where θ is the step function. The poles of $u(k)$ also contribute [see Eq. (7)], but for distances much larger than $1/\omega_M$ the contributions are negligible. Henceforth we shall restrict x to the regions $|x|\gg 1/\omega_M$ and neglect the poles of $u(k)$.

Note that the pole contributions $p_a(x,t)$ increase exponentially with the distance $|x|$ as $p_a(x,t)\sim e^{iz_1|x|}\sim e^{\gamma|x|}$. However, in Eq. (56) these contributions cancel outside the light cone $|x|>t$ as we have

$$p_1(x,t)=\theta(|x|-t)p_0(x,t). \tag{67}$$

For the "cut" contributions c_a in Eq. (65) we change k to $-k$ to obtain

$$c_0(x,t)=-e^{-iz_1t}\int_0^{\infty}\frac{dk}{\sqrt{2\pi}}[e^{ikx}+e^{-ikx}]\frac{\lambda u(k)}{k+z_1},$$

$$c_1(x,t)=-\int_0^{\infty}\frac{dk}{\sqrt{2\pi}}[e^{ik(x+t)}+e^{ik(-x+t)}]\frac{\lambda u(k)}{k+z_1}. \tag{68}$$

The cut contributions appear because of the positivity of energy, which leads to a branch-point singularity in the resolvent of the Hamiltonian in the complex energy plane, at $\omega=0$.

Using Eq. (67) in Eq. (64) we get

$$f(x,t)\approx c_0(x,t)-c_1(x,t)+\theta(t-|x|)p_0(x,t). \tag{69}$$

The field intensity is

$$I(x,t)=|f(x,t)|^2. \tag{70}$$

As we shall see, $c_0(x,t)$ will be associated with the cloud surrounding the particle, $p_0(x,t)$ with the decay products and $c_1(x,t)$ with the "Zeno photons."

B. Relation with the dressed states

Now we write the field intensity [Eq. (52)] in terms of the dressed states in the Liouville space. As the dressed states form a complete set in the Liouville space, we may express the bare particle state as

$$|1;1\rangle\rangle=\sum_{\alpha=1,k}|\rho_\alpha^0\rangle\rangle\langle\langle\tilde{\rho}_\alpha^0|1;1\rangle\rangle+\sum_{\alpha,\beta}{}'\,|\rho^{\alpha\beta}\rangle\rangle\langle\langle\tilde{\rho}^{\alpha\beta}|1;1\rangle\rangle. \tag{71}$$

Then we can decompose the field intensity $I(x,t)$ into its dressed particle and dressed photon components,

$$I_\alpha^{(0)}(x,t) \equiv \langle\langle x;x|e^{-iL_H t}|\rho_\alpha^0\rangle\rangle\langle\langle\tilde{\rho}_\alpha^0|1;1\rangle\rangle, \qquad (72)$$

for $\alpha = 1,k$, respectively, and its dressed correlation components

$$I^{(1)}(x,t) \equiv \sum_k [I^{(1k)}(x,t) + I^{(k1)}(x,t)], \qquad (73)$$

$$I^{(2)}(x,t) \equiv \sum_{k,k'}{}' I^{(kk')}(x,t),$$

where

$$I^{(\alpha\beta)}(x,t) \equiv \langle\langle x;x|e^{-iL_H t}|\rho^{\alpha\beta}\rangle\rangle\langle\langle\tilde{\rho}^{\alpha\beta}|1;1\rangle\rangle. \qquad (74)$$

We have

$$I(x,t) = \sum_{d=0}^{2} I^{(d)}(x,t), \qquad (75)$$

where

$$I^{(0)}(x,t) \equiv \sum_{\alpha=1,k} I_\alpha^{(0)}(x,t). \qquad (76)$$

The superscript d corresponds to the subdynamics projections

$$I^{(d)}(x,t) = \langle\langle x;x|e^{-iL_H t}\Pi^{(d)}|1;1\rangle\rangle. \qquad (77)$$

Using the explicit forms of the dressed states in terms of the eigenstates of L_H and neglecting terms of $O(1/L)$ we get [see Eqs. (34) and (35)]

$$I_1^{(0)}(x,t) = \left[e^{-2\gamma t}\langle\langle x;x|F_1^{(0)}\rangle\rangle + \sum_k b_k \langle\langle x;x|F_k^{(0)}\rangle\rangle \right] \times \langle\langle \tilde{F}_1^{(0)}|1;1\rangle\rangle, \qquad (78)$$

$$I_k^{(0)}(x,t) = \langle\langle x;x|F_k^{(0)}\rangle\rangle[\langle\langle\tilde{F}_k^{(0)}|1;1\rangle\rangle - b_k\langle\langle\tilde{F}_1^{(0)}|1;1\rangle\rangle].$$

Then, using Eq. (51) we obtain the factorizable expressions

$$I_1^{(0)}(x,t) = e^{-2\gamma t}\langle\langle x;x|\phi_1;\phi_1\rangle\rangle\langle\langle\tilde{\phi}_1;\tilde{\phi}_1|1;1\rangle\rangle + O(1/L),$$

$$I_k^{(0)}(x,t) = O(1/L), \qquad (79)$$

as well as

$$I^{(1)}(x,t) = \sum_k \exp[-i(z_1-\omega_k)t]\langle\langle x;x|\phi_1;\phi_k\rangle\rangle \times \langle\langle\tilde{\phi}_1;\tilde{\phi}_k|1;1\rangle\rangle + \text{c.c.}, \qquad (80)$$

$$I^{(2)}(x,t) = \sum_{k,k'}{}' \exp[-i(\omega_k-\omega_{k'})t]\langle\langle x;x|\phi_k;\phi_{k'}\rangle\rangle \times \langle\langle\tilde{\phi}_k;\tilde{\phi}_{k'}|1;1\rangle\rangle.$$

Neglecting $O(1/L)$ this leads to [cf. Eq. (55)]

$$I_1^{(0)}(x,t) = |f_0(x,t)|^2, \qquad (81)$$

$$I^{(1)}(x,t) = -[f_0(x,t)f_1(x,t)^{\text{c.c.}} + \text{c.c.}],$$

$$I^{(2)}(x,t) = |f_1(x,t)|^2.$$

For the dressed particle component we have [see Eqs. (64) and (66)]

$$I_1^{(0)}(x,t) \approx |c_0(x,t) + \sqrt{2\pi} i u(z_1) e^{i(|x|-t)z_1}|^2. \qquad (82)$$

For large x this grows in space as $e^{2\gamma|x|}$. However, adding the correlation components $I^{(1)}$ and $I^{(2)}$ in Eq. (81) the exponential growth is canceled outside the light cone $|x|>t$, as then we recover the square of the amplitude Eq. (56), which leads to Eq. (69).

C. Separation of the cloud and decay products

Next we discuss the interpretation of our decomposition [Eq. (75)] of the field intensity. Using the expression for $f_0(x,t)$ in Eq. (57) together with Eq. (81), it follows that the dressed particle component $I_1^{(0)}$ decays in a purely exponential way as

$$I_1^{(0)}(x,t) = e^{-2\gamma t} I_1^{(0)}(x,0). \qquad (83)$$

This corresponds to a Markovian evolution. On the other hand, the correlation components $I^{(d)}(x,t)$ (with $d>0$) have a non-Markovian evolution associated with memory effects (the preparation). Therefore, with our decomposition, we separate the exponential and nonexponential elements of the time evolution of the field. These elements, in turn, contain both "cut" and pole contributions, which we describe now.

1. The cloud

The function $|c_0(x,t)|^2$ corresponds to the cut contribution of the dressed particle component. It gives the cloud of photons surrounding the particle. Indeed, as seen in Eq. (68), c_0 is given by an oscillatory integrand in k. In x space, c_0 has a maximum at $x=0$, where the particle is located, and decreases with a power law of $|x|$ for large $|x|$.[3] The function $|c_0(x,t)|^2$ is represented by the dotted line in Fig. 1 (we shall give more details on the numerical simulation below). A cloud analogous to $c_0(x,t)$ also exists in the stable case [19]. The difference is that in the stable case there is no decay, i.e., we have $\gamma=0$, while in the unstable case we have $\gamma>0$. Thus for the unstable case the cloud decays exponentially as $|c_0(x,t)|^2 = \exp(-2\gamma t)|c_0(x,0)|^2$.

2. The Zeno photons

The function $|c_1(x,t)|^2$ [see Eq. (68)] corresponds to the cut contribution of the correlation components. This function represents two wave packets that move away from the particle (see Fig. 1). They have peaks centered at $x=t$ and

[3]We are considering a massless scalar field. If the field were massive, the cloud would decrease exponentially in space.

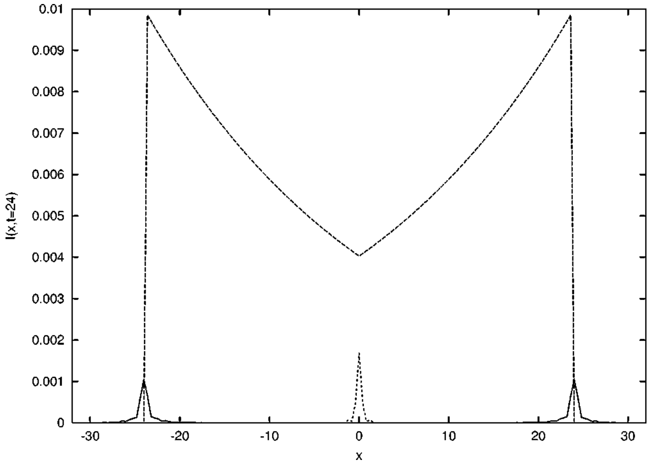

FIG. 1. Cloud (dotted line), Zeno photons (solid line), and decay products (dashed line) emitted by the bare-excited particle. The cloud centered around the particle at $x=0$ decays while the Zeno photons and the decay products run away from the particle. In this and the subsequent figures we use units with $\hbar=1$ and $c=1$. Both x and t are measured in units of the inverse frequency $\omega_1^{-1}=1$ of the unstable state. The field intensity is dimensionless.

$x=-t$, respectively. As explained below, the wave packets play an important role in the deviations from exponential decay of the survival probability of the excited state. This includes the Zeno effect [20] and thus we call the wave packets ''Zeno photons.''

Similar to the cloud, the Zeno photons are nonlocalized in space and have long tails that decrease with an inverse power law of $\pm x-t$ for large $|\pm x-t|$. Although these tails are not confined to the light cone of the particle, as we have shown in Ref. [19], this does not violate causality because the Zeno photons move with the speed of light.[4]

The cloud and the Zeno photons lead to a ''curtain'' effect that describes the dressing process [10,19]. At early times the Zeno photons interfere destructively with the cloud [at $t=0$ we have $c_0(x,0)=c_1(x,0)$]. In other words the cloud is hidden behind a curtain made by the Zeno photons. As the Zeno photons move away, the cloud emerges.

3. The decay products

The pole contributions $p_0(x,t)$ and $p_1(x,t)$ associated with the dressed particle and correlation components, respectively, lead to the decay products

$$|p_0(x,t)-p_1(x,t)|^2=\theta(t-|x|)|p_0(x,t)|^2 =2\pi|u(z_1)|^2\theta(t-|x|)e^{2\gamma(|x|-t)}, \tag{84}$$

which stay inside the light cone. The exponential growth of the decay products in space within the light cone is directly related to a ''history'' of the exponential decay of the bare particle. For early times the particle is more likely to emit than for later times. Therefore the field further away from the particle (which was emitted earlier) is stronger than the field closer to the particle. Since the emission of photons started at $t=0$, the wave fronts of the decay products end at $x=\pm t$ (see Fig. 1).

Neglecting the cut contributions, one can write the emitted field as

$$I(x,t)\propto\theta(t-|x|)e^{2\gamma(|x|-t)}, \tag{85}$$

which for given x decays exponentially inside the light cone, $t>|x|$. The deviations from exponential decay are due to the Zeno photons as well as the interference effects involving the Zeno photons, the cloud and the decay products. The nonexponential contributions are strongest within the early non-Markovian time scale

$$t_Z\sim|\omega_1|^{-1}\ll\gamma^{-1}, \tag{86}$$

which is proportional to the characteristic size of the cloud and Zeno photons [19].

The deviations from exponential decay are manifest also in the survival probability of the bare state. The time scale t_Z gives an upper bound of the so-called Zeno time [20,22,23]. Equation (86) agrees with an estimation of the time scale for the onset of exponential decay given by Peres [24], based on the rigorous solution of Schrödinger's wave equation for general Hamiltonian systems.[5]

After the time scale t_Z, when the Zeno photons move away from the cloud (as in Fig. 1), the survival probability of the bare particle decays in an approximately exponential way in time with a small disturbance due to the long tails of the Zeno photons in space. For much longer times $t\gg\gamma^{-1}$, the cloud sticking around the particle disappears exponentially. On the other hand the effect of the long tails of the Zeno photons remains since the tails decay slowly in space, following a power law. This corresponds to the well-known power-law decay in time of the survival probability of the bare particle for extremely long times [1,24].

D. Numerical plots

The components involved in the dressing and decay process can be visualized with the help of numerical simulations.

In Fig. 1, we show the cloud $|c_0(x,t)|^2$, the Zeno photons $|c_1(x,t)|^2$ and the decay products $|\theta(t-|x|)p_0(x,t)|^2$. We have chosen the parameters as $\omega_1=1$, $\lambda=0.1$, and $t=24$. For the numerical plots, the discreteness of the space coordinate introduces a natural cutoff $k_{\max}$ for the momenta. We have $k_{\max}=\pi/\Delta x$ where Δx is the spacing between the discrete coordinates x. The potential is $V_k=(2\omega_k/L)^{1/2}\theta(k_{\max}$

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[4]If we include virtual processes in the Hamiltonian, then the field intensity created by the bare particle is strictly confined inside the light cone [21] as the tails of the cloud and the Zeno photons then cancel for $|x|>t$. On the other hand for partially dressed states the long tails may lead to nonlocal effects that simulate superluminality [19].

[5]Equation (86) agrees with Eqs. (50) and (52) of Ref. [24] applied to the Friedrichs model with the form factor in Eq. (7).

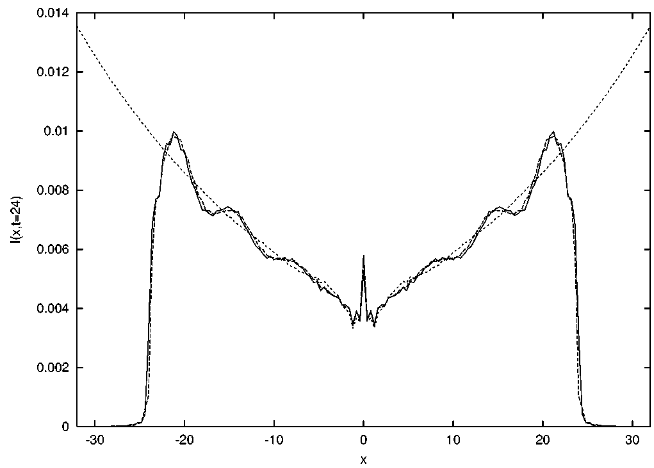

FIG. 2. Field intensity $I(x,t)$ for the unstable case: solution of the Schrödinger equation (solid line), and theoretical estimation using Eq. (70). Also plotted is the field intensity $I_1^{(0)}(x,t)$ of the dressed unstable state (dotted line).

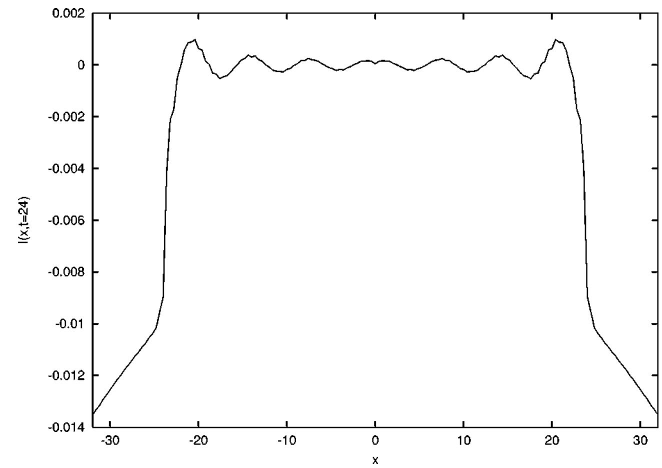

FIG. 3. Non-Markovian field $I(x,t)-I_1^{(0)}(x,t)$. It shows fluctuations due to the preparation inside the light cone, and counterfields that cancel the exponential growth of $I_1^{(0)}(x,t)$ outside the light cone.

$-|k|$) with $L=1250$ and $k_{\max}=2\pi$. The plots were obtained through a direct numerical calculation of our theoretical formulas. We solved numerically the equation $\eta^+(z_1)=0$ to obtain $z_1\approx 0.95-i0.02$.

In Fig. 1 we have $\gamma t\approx 0.48$. The cloud at the center is decaying. The decay products and the Zeno photons move away from the particle at a speed $c=1$.

The theoretical field intensity $I(x,t)$ obtained by combining the above contributions as well as their interference terms is shown by the dashed line in Fig. 2. The solid line shows the result obtained through the numerical solution of Schrödinger's equation, obtained by the diagonalization of the discretized Hamiltonian matrix (see Ref. [19] for a description of the numerical method; we have used a 2500×2500 Hamiltonian matrix). Both numerical plots are in good agreement.

For reference, we have also plotted the theoretical contribution from the dressed particle alone given by $I_1^{(0)}(x,t)$ (the dotted line in Fig. 2). This is the Markovian component of the field. Note that outside the light cone $I_1^{(0)}(x,t)$ continues to grow exponentially in space. This exponential growth is canceled by the dressed correlations (non-Markovian) component of the field shown in Fig. 3.

E. Global quantities

So far we have considered the local-field operator $\langle\langle x;x|$ for which the dressed states could be factorized in terms of Gamow states, e.g.,

$$\langle\langle x;x|\rho_1^0\rangle\rangle=\langle\langle x;x|\phi_1;\phi_1\rangle\rangle. \tag{87}$$

Now we consider an example where this equivalence no longer holds. Our example is the global observable

$$\hat{p}_F\equiv\sum_k |k\rangle\langle k|=\sum_k \langle\langle k;k|. \tag{88}$$

The expectation value of this observable gives the total emission probability. The operator $\hat{p}_F$ is a component of the trace. Indeed, the trace is the expectation value of the unit operator, which may be written as

$$1=\hat{p}_F+\langle\langle 1;1|. \tag{89}$$

As shown in Ref. [4] we have $\mathrm{Tr}[\exp(-iL_H t)\rho_1^0]=1$. Hence

$$\langle\langle\hat{p}_F|e^{-iL_H t}|\rho_1^0\rangle\rangle=1-\langle\langle 1;1|e^{-iL_H t}|\rho_1^0\rangle\rangle=1-|N_1|e^{-2\gamma t}, \tag{90}$$

where we have used Eq. (40). In contrast, for the dyad $|\phi_1;\phi_1\rangle\rangle$ we have [8]

$$\langle\langle\hat{p}_F|e^{-iL_H t}|\phi_1;\phi_1\rangle\rangle=-|N_1|e^{-2\gamma t}. \tag{91}$$

Equations (90) and (91) clearly show the distinction between $|\rho_1^0\rangle\rangle$ and $|\phi_1;\phi_1\rangle\rangle$ for observables with diagonal singularities in momentum. In general, the two states have different expectation values for observables that include terms of the form $\hat{f}=\Sigma_k f_k\langle\langle k;k|$, where f_k is a function independent of the volume L. An example of such observable is the Hamiltonian H.

For $|\rho(t)\rangle\rangle=\exp(-iL_H t)|1;1\rangle\rangle$, the expectation value of $\hat{p}_F$ may be written as an integration over space,

$$\langle\langle\hat{p}_F|\rho(t)\rangle\rangle=\int_{-\infty}^{\infty}dx\,\rho(x,t), \tag{92}$$

where

$$\rho(x,t)\equiv\sum_{k,k'}\exp[i(k-k')x]\langle\langle k;k'|\rho(t)\rangle\rangle \tag{93}$$

is the photon density emitted by the bare particle. Similar to the field intensity in Eq. (75), we can decompose the photon density into its dressed state components as

$$\rho(x,t)=\sum_{d=0}^{2}\rho^{(d)}(x,t), \tag{94}$$

where

$$\rho^{(d)}(x,t)=\sum_{k,k'}e^{i(k-k')x}\langle\langle k;k'|\Pi^{(d)}|\rho(t)\rangle\rangle. \tag{95}$$

Then we have

$$\langle\langle\hat{p}_F|\rho(t)\rangle\rangle=\int_{-\infty}^{\infty}dx\sum_{d=0}^{2}\rho^{(d)}(x,t). \tag{96}$$

Note that the summation $\Sigma_{d=0}^{2}$ has to be taken before the integration over x, in order to cancel the exponential growth in space of the separate components. Under the integration over x, the relation (87) no longer holds. We have to use the complete expression [cf. Eq. (34)]

$$|\rho_1^0\rangle\rangle=|\phi_1;\phi_1\rangle\rangle+\sum_k b_k|\tilde{\phi}_k;\tilde{\phi}_k\rangle\rangle \tag{97}$$

in order to get a consistent decomposition in terms of our sybdynamics. The second term in Eq. (97) is essential to preserve the trace within the $\Pi^{(0)}$ Markovian component.

IV. HILBERT-SPACE APPROXIMATION TO THE DRESSED UNSTABLE STATE

As mentioned in the preceding section, the evolution of the unperturbed excited state $|1;1\rangle\rangle$ deviates from the exponential law in a short-time scale (the quantum Zeno effect) and in a long-time scale (the long-time tail). This is a direct consequence of the well-known fact that any state in the Hilbert space cannot decay in a strictly exponential way. In contrast the dressed excited state $|\rho_1^0\rangle\rangle$ can decay in a strictly exponential way because it does not belong to the Hilbert space. However, since $\langle\langle x;x|\rho_1^0\rangle\rangle=I_1^{(0)}(x,0)/\langle\langle\tilde{\rho}_1^0|1;1\rangle\rangle$ diverges exponentially for $x\to\infty$, the non-Hilbertian state $|\rho_1^0\rangle\rangle$ cannot be prepared as an isolated state.

We now show that one can construct a dressed excited state in the Hilbert space whose emitted field inside the light cone $|x|<t$ is close to the field of $|\rho_1^0\rangle\rangle$. In this sense this new Hilbert-space state is a better approximation to our dressed excited state $|\rho_1^0\rangle\rangle$ than the unperturbed excited state $|1;1\rangle\rangle$. Since this new state is localized in space, we may prepare it in isolation, which is in contrast to $|\rho_1^0\rangle\rangle$.

A. Approximation of the emitted field

We introduce the state[6]

$$|\psi_1\rangle\equiv n_1^{1/2}\left[|1\rangle+\sum_k|k\rangle\frac{\lambda V_k}{z_1^{\text{c.c.}}-\omega_k}\right], \tag{98}$$

[6]A state close to $|\psi_1\rangle$ has been proposed by Passante [25].

where n_1 is the normalization constant (see Appendix B). The state $|\psi_1\rangle$ consists of the bare state $|1\rangle$ plus a cloud. Note that this is not an eigenstate of H. To $|\psi_1\rangle$ we associate the density operator $|\psi_1;\psi_1\rangle\rangle$, which we shall compare with $|\rho_1^0\rangle\rangle$.

For weak coupling the approximate time evolution of the amplitude $\langle x|\psi_1\rangle$ is given by

$$\langle x|\psi_1(t)\rangle\approx n_1^{1/2}\Bigg[c_0(x,t)+\theta(t-|x|)p_0(x,t)-c_1(x,t)+\int_{-\infty}^{\infty}\frac{dk}{\sqrt{2\pi}}\frac{\lambda u(\omega_k)}{z_1^{\text{c.c.}}-\omega_k}e^{i(kx-\omega_k t)}\Bigg], \tag{99}$$

where

$$|\psi_1(t)\rangle=e^{-iHt}|\psi_1\rangle. \tag{100}$$

The first line in Eq. (99) gives the evolution of the bare particle [cf. Eq. (69)], and the second line gives the evolution of the cloud of $|\psi_1\rangle$ in the lowest-order approximation. The "cut" part of this cloud coincides with the cut part of $f_1(x,t)$ in Eq. (61), i.e., with the Zeno photons $c_1(x,t)$. Hence, the Zeno photons are canceled in this approximation. The remaining field amplitude is given by

$$\langle x|\psi_1(t)\rangle\approx n_1^{1/2}[c_0(x,t)+\theta(t-|x|)p_0(x,t)-\theta(|x|-t)\sqrt{2\pi}i\lambda u(\omega_1)e^{iz_1^{\text{c.c.}}(|x|-t)}]. \tag{101}$$

The field intensity $|\langle x|\psi_1(t)\rangle|^2$ includes the usual cloud $|c_0(x,t)|^2$, the decay products and an additional cloud, which comes from the pole contribution of the second line in Eq. (99) and is shown on the second line of Eq. (101). The additional cloud decreases as $\exp[-2\gamma(|x|-t)]$ for $|x|>t$. This additional cloud already exists at $t=0$. Therefore, the presence of a field outside the light cone does not imply any violation of causality. It simply means that the initial state includes an extended field.

Comparing Eq. (101) with Eq. (82) we see that (up to the normalization factors) inside the light cone the field intensities of $|\rho_1^0\rangle\rangle$ and $|\psi_1;\psi_1\rangle\rangle$ coincide in the lowest-order approximation.

In Fig. 4 we show a comparison of the field intensities $|\langle x|\psi(t)\rangle|^2$ for $|\psi\rangle=|\psi_1\rangle$ (dashed line) and $|\psi\rangle=|1\rangle$ (solid line) for fixed x. For easier comparison we have adjusted the normalization constant of $|\psi_1\rangle$ to $n_1=1$. We have obtained this graph through numerical calculations based on a diagonalization of the Hamiltonian for the same parameters described in Sec. III. The graph shows that inside the light cone, for $t>|x|$, the state $|\psi_1\rangle$ has smaller deviations from exponential than the bare state (note that the state $|\rho_1^0\rangle\rangle$ has an exactly exponential evolution for all t). In Fig. 4 one can also see the additional cloud of $|\psi_1\rangle$ for $t<|x|$.

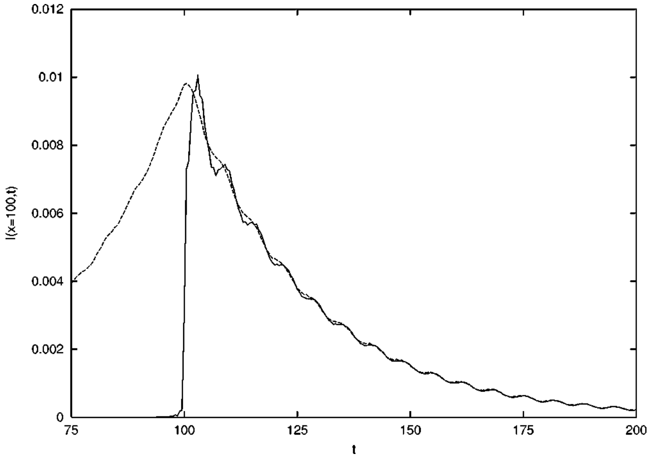

FIG. 4. Field intensity $I=|\langle x|\psi(t)\rangle|^2$ for $|\psi\rangle=|1\rangle$ (solid line) and $|\psi\rangle=|\psi_1\rangle$ (dashed line), for $x=100$. The dressed state $|\psi_1\rangle$ has smaller deviations from exponential decay than the bare state. For easier comparison we have normalized the state $|\psi_1\rangle$ with $n_1=1$.

B. Line shape and survival probability

Now we consider the line shape of emission of the state $|\psi_1\rangle$ and the survival probability. We start with a few general remarks.

The line shape of any state $|\psi;\psi\rangle\rangle$ is given by

$$h_k(\psi)\equiv|\lim_{t\to\infty}\langle k|\psi(t)\rangle|^2=|\langle\tilde{\phi}_k^-|\psi\rangle|^2, \tag{102}$$

where $|\tilde{\phi}_k^-\rangle$ is the "out" Möller scattering state

$$|\tilde{\phi}_k^-\rangle\equiv|\tilde{\phi}_k\rangle^{\text{c.c.}}. \tag{103}$$

The survival probability of $|\psi\rangle$ is given by

$$S_\psi(t)\equiv\langle\langle\psi;\psi|e^{-iL_Ht}|\psi;\psi\rangle\rangle=|\langle\psi|\exp(-iHt)|\psi\rangle|^2. \tag{104}$$

For short times we have

$$S_\psi(t)=1-(t\Delta E_\psi)^2+O(t^3), \tag{105}$$

where

$$\begin{aligned}(\Delta E_\psi)^2&\equiv\langle\psi|H^2|\psi\rangle-\langle\psi|H|\psi\rangle^2\\ &=\sum_k h_k(\psi)\omega_k^2-\left[\sum_k h_k(\psi)\omega_k\right]^2\end{aligned} \tag{106}$$

is the energy fluctuation. For example for the bare state $|\psi\rangle=|1\rangle$ we have

$$(\Delta E_1)^2=\int_{-\infty}^{\infty}dk\,\lambda^2 v^2(\omega_k). \tag{107}$$

The fluctuation ΔE_1 is large, of the order of the ultraviolet cutoff ω_M of the interaction.

For states belonging to the Hilbert space the line shape and the survival probability are closely connected. Indeed, using the completeness relation of the Möller states we have

$$S_\psi(t)=\left|\sum_k h_k(\psi)e^{-i\omega_k t}\right|^2. \tag{108}$$

As seen in Eq. (105) the energy fluctuation is associated with the early stage of the evolution of $S_\psi(t)$. This early stage for states in the Hilbert space typically corresponds to a dressing process, rather than a decay process. For example, for the bare state the fluctuation ΔE_1 is associated with the initial dressing time scale $t_{\text{dress}}\sim 1/\Delta E_1$, during which the Zeno effect [20] may take place.

In general, the energy fluctuation consists of two components, one due to the resonance pole and the other due to branch-point (or other) singularities of the line shape. Since the branch point is related to the dressing of the particle, this component gives the dressing time scale. On the other hand the fluctuation associated with the resonance gives the relaxation time. Peres [24] has shown that in order to have a well-defined exponential regime in the *survival probability* of Hilbert-space states $|\psi\rangle$, one should have $\Delta E_\psi\gg\gamma$. This means that the branch-point contribution of the energy fluctuation should dominate. This is indeed the case for $|1\rangle$, for which the energy fluctuation is entirely due to the branch-point contribution. We have a clear separation of time scales for dressing and decay: $\Delta E_1\gg\gamma$.

We now turn to the line shape and survival probability of the state $|\psi_1\rangle$. The overlap of this state with the Möller states gives (see Appendix B)

$$\begin{aligned}\langle\tilde{\phi}_k^-|\psi_1\rangle=&\frac{\lambda V_k}{\eta^+(\omega_k)}\frac{n_1^{1/2}}{z_1^{\text{c.c.}}-\omega_k}\times\lambda^2\int dl\,v^2(\omega_l)\\ &\times\left[\frac{1}{(z-\omega_l)^-_{z_1^{\text{c.c.}}}}-\frac{1}{z_1^{\text{c.c.}}-\omega_l}\right].\end{aligned} \tag{109}$$

In the lowest-order approximation we have

$$h_k(\psi_1)\approx\left(\frac{2\pi}{L}\right)\frac{1}{\pi}\frac{\gamma^3}{[(\omega_k-\tilde{\omega}_1)^2+\gamma^2]^2}\approx b_k, \tag{110}$$

where we have replaced $\lambda^2v^2(\omega_k)$ by $\lambda v^2(\omega_1)\approx\gamma/(2\pi)$, as the resonance at $\omega_k=z_1$ gives the dominant contribution. Therefore, the line shape of $|\psi_1;\psi_1\rangle\rangle$ coincides with the line shape of $|\rho_1^0\rangle\rangle$ in the lowest-order approximation. As shown in Ref. [4], the line shape in Eq. (110) leads to the energy fluctuation

$$(\Delta E_{\psi_1})^2\approx\gamma^2. \tag{111}$$

The resonance pole gives the dominant contribution to the energy fluctuation, while the branch-point (dressing) contribution is negligible in this approximation. This is due to the fact that the state $|\psi\rangle$ has already a dressing $t=0$.

In Appendix B we show that the average energies of $|\psi_1;\psi_1\rangle\rangle$ and $|\rho_1^0\rangle\rangle$ coincide up to the $O(\lambda^2)$ correction.

Our results, up to now, demonstrate that the suppression of dressing effects, the energy fluctuation and exponential decay of the field are linked together for states in the Hilbert space, such as $|\psi_1;\psi_1\rangle\rangle$. As compared with the bare state, the dressing effects in this state are significantly reduced due to the approximate elimination of the Zeno photons. The emitted field is closer to exponential decay and the energy fluctuation is much smaller, $\Delta E_{\psi_1} \ll \Delta E_1$.

So far we have pointed out the similarities between the states $|\psi_1;\psi_1\rangle\rangle$ and $|\rho_1^0\rangle\rangle$. Now we comment on the differences. For the survival probability, using Eq. (108) we obtain

$$|\langle\psi_1|\psi_1(t)\rangle|^2 \sim e^{-2\gamma t}(1+\gamma t)^2. \tag{112}$$

The polynomial factors in t are due to the existence of a double pole in the line shape $h_k(\psi_1)$.[7] On the other hand for $|\rho_1^0\rangle\rangle$ we have to define the survival probability in a generalized way, since $|\rho_1^0\rangle\rangle$ is outside the Hilbert space. We define a generalized survival probability as

$$\tilde{S}(t) \equiv \langle\langle\tilde{\rho}_1^0|e^{-iL_H t}|\rho_1^0\rangle\rangle = \exp(-2\gamma t). \tag{113}$$

In contrast to the survival probability of $|\psi_1;\psi_1\rangle\rangle$, this is strictly exponential, regardless of the line shape. Furthermore, in the local-field description the state $|\rho_1^0\rangle\rangle$ also decays exponentially regardless of the line shape. Therefore, the relation between time evolution and line shape of $|\rho_1^0\rangle\rangle$ is different than for Hilbert-space states. For $|\rho_1^0\rangle\rangle$ the line shape only appears in the global time evolution, where it determines the distribution of emitted field modes [cf. Eq. (40)].

Our discussion above states in a more precise way the relation between dressing and exponential decay for unstable states we have mentioned in our previous paper [4].

Let us finally note that a state close to $|\psi_1\rangle$ has been proposed in 1956 by Glaser and Källén [26]. They considered the Lee model. Translated into our present model their state corresponds to

$$|\psi_{GK}\rangle = n_{GK}^{1/2}\left[|1\rangle + \sum_k |k\rangle \frac{\lambda V_k}{\tilde{\omega}_1 - i\tilde{\gamma} - \omega_k}\right], \tag{114}$$

where n_{GK} is the normalization constant and $\tilde{\gamma}$ is a characteristic parameter, which should satisfy the condition $\tilde{\gamma} \gg \gamma$ for their case. For $\tilde{\gamma} \neq \gamma$ the double pole in the line shape is avoided, and one obtains a superposition of two exponential decays, one with a rate γ and the other with a rate $\tilde{\gamma}$. On the other hand, if we choose $\tilde{\gamma} = \gamma$, one can show that this state has exactly the same line shape as $|\psi_1\rangle$ (note that for $\tilde{\gamma} = \gamma$ we have $|\psi_{GK}\rangle = |\psi_1\rangle^{\text{c.c.}}$).

C. Possible experiments

Our study of the Markovian and non-Markovian components of the field emitted by the bare-excited state has led us to the Hilbert-space state $|\psi_1;\psi_1\rangle\rangle$ that, in the sense discussed above, comes close to the dressed unstable $|\rho_1^0\rangle\rangle$. The appearance of $|\psi_1;\psi_1\rangle\rangle$ (or a state close to it) in an experimental situation can be checked through the line shape. The line shape of this dressed state must be narrower than the Lorentzian and decay as ω_k^{-4} for large ω_k [see Eq. (110)].[8]

One possibility to observe line shapes narrower than the Lorentzian is to discard the measured field corresponding to the early stages of evolution of the initial state, after which the atom may be found in a dressed or partially dressed state.

In order to study the physical effects of the unstable clouds, one may consider the force between two excited atoms. This force depends on the overlapping of the fields surrounding each atom [21]. This involves not only the off-resonance cloud, but also the field due to resonance effects. To study this force one can consider the emission spectrum and the line shape of the two-atom system as a function of the distance between the atoms. This can then be compared with a theoretical estimation based on the dressed state outside the Hilbert space or its Hilbert-space approximation. A theoretical analysis on the forces between excited atoms will be presented elsewhere.

V. $\mathcal{H}$ FUNCTION

Because of the instability, which is due to the resonance effect, one can introduce a microscopic analog of Boltzmann's $\mathcal{H}$ theorem by constructing a Lyapounov operator that decays monotonically for all times [see Eqs. (1) and (2)]. As noticed in Sec. I this quantity is defined outside the Hilbert space. The expectation value of this operator is a Lyapounov function that depends on the initial state $|\xi\rangle$ of the system

$$\langle\mathcal{H}(t)\rangle \equiv \langle\xi|\mathcal{H}(t)|\xi\rangle = e^{-2\gamma t}\langle\mathcal{H}(0)\rangle = e^{-2\gamma t}\langle\xi|\tilde{\phi}_1\rangle\langle\tilde{\phi}_1|\xi\rangle. \tag{115}$$

As discussed in Ref. [8], the $\mathcal{H}$ function gives an indication of how far the system is from its final asymptotic state, when the dressed particle disappears. The $\mathcal{H}$ function decreases until it reaches its asymptotic value $\langle\mathcal{H}(\infty)\rangle = 0$. Most initial conditions, including ones giving rise to a temporary "backwards" evolution (e.g., after a momentum inversion) eventually end up with the particle relaxing to the ground state and all the field moving away from the particle to infinity.[9] This behavior justifies the interpretation of the $\mathcal{H}$ function as a measure of the distance to the asymptotic state: the larger the $\mathcal{H}$ function, the more one has to wait to reach the asymptotic state.

[7]For $|\psi\rangle = |\psi_1\rangle$, Peres's condition $\Delta E_\psi \gg \gamma$ [24] is not met, which is consistent with the nonexponential behavior in Eq. (112). Note that the deviations from exponential in Eq. (112) are due to the resonance-pole contribution, and not to branch-point contributions.

[8]In contrast, if the initial state is close to a bare state, then a line shape close to the Lorentzian should be observed. The infinite energy fluctuation of the Lorentzian shape (which approximates ΔE_1) is connected to the rapid dressing process that occurs during the Zeno period. Conversely, a suppression of the Zeno effect should lead to a narrower line shape.

[9]An exception is given by initial conditions that are the asymptotic states of a backwards time evolution. Such states take an infinite amount of time to reach the forward asymptotic state.

A microscopic $\mathcal{H}$ operator in the Liouville space was already introduced many years ago by one of the present authors (I.P.) as $M=\Lambda^{\dagger}\Lambda$ (see Ref. [5]). To connect this operator with our present $\mathcal{H}$ function, we note that for systems with many particles or field modes we may introduce a reduced Lyapounov operator

$$M_{\alpha}=\Lambda^{\dagger}|\hat{n}_{\alpha}\rangle\rangle\langle\langle\hat{n}_{\alpha}|\Lambda, \tag{116}$$

where $\hat{n}_{\alpha}$ is a one-particle observable corresponding to particle α. This is similar to a Gibbs entropy with the replacement of unitary transformations U by Λ (for unitary transformations the Gibbs entropy is an invariant of motion, while with Λ the entropy evolves monotonically). The Lyapounov function is the expectation value $\langle\langle M_{\alpha}(t)\rangle\rangle \equiv\langle\langle\rho(t)|M_{\alpha}|\rho(t)\rangle\rangle$, where $|\rho(t)\rangle\rangle\equiv\exp(-iL_H t)|\rho\rangle\rangle$. For states with no diagonal singularity in momentum representation, which were discussed at the end of Sec. II, there is a simple relation between this Lyapounov function M_{α} and the $\mathcal{H}$ operator in Eq. (115). For example, if ρ is a pure state $\rho=|\xi\rangle\langle\xi|$ normalized as $\mathrm{Tr}(\rho)=1$, we have for $|\hat{n}_1\rangle\rangle=|1;1\rangle\rangle$, [c.f. Eq. (51)]

$$\langle\langle M_1(t)\rangle\rangle\equiv\langle\langle\rho(t)|\tilde{\rho}_1^0\rangle\rangle\langle\langle\tilde{\rho}_1^0|\rho(t)\rangle\rangle. \tag{117}$$

Here the state $\langle\langle\rho(t)|$ plays the role of a test function with no diagonal singularity. This allows us to write $\langle\langle\rho(t)|\tilde{\rho}_1^0\rangle\rangle=\langle\langle\rho(t)|\tilde{\phi}_1;\tilde{\phi}_1\rangle\rangle$ and the corresponding relation for the dual states. Thus we obtain

$$\langle\langle M_1(t)\rangle\rangle=\langle\langle\rho(t)|\tilde{\phi}_1;\tilde{\phi}_1\rangle\rangle\langle\langle\tilde{\phi}_1;\tilde{\phi}_1|\rho(t)\rangle\rangle \tag{118}$$

or

$$\langle\langle M_1(t)\rangle\rangle=[\langle\mathcal{H}(t)\rangle]^2, \tag{119}$$

where $\langle\mathcal{H}(t)\rangle=\langle\xi|\mathcal{H}(t)|\xi\rangle$.

For states with diagonal singularities the M_1 operator is no longer factorizable in terms of Gamow states. States with diagonal singularities occur naturally in systems in the thermodynamic limit, e.g., for a particle coupled to a heat bath. This will be studied in a separate publication.

A. Scattering of a wave packet

To illustrate the connection between the $\mathcal{H}$ function and dynamics, we consider the scattering of a wave packet (i.e., a state with no diagonal singularities).

At the initial state the particle is in its ground state. We consider a rectangular wave packet of width b in x representation,

$$\langle x|\xi\rangle=\frac{e^{ixk_0}}{b^{1/2}}[\theta(x-x_0-b/2)-\theta(x-x_0+b/2)]. \tag{120}$$

The packet is centered at $x=x_0<0$, with $|x_0|\gg b$. The momentum representation is given by $\langle k|\xi\rangle=(2\pi/L)^{1/2}\xi_k$ with

$$\xi_k=-2\sqrt{\frac{2\omega_k}{W}}e^{-ix_0(k-k_0)}\frac{\sin[b(k-k_0)/2]}{(2\pi b)^{1/2}(k-k_0)}, \tag{121}$$

where W is a normalization factor. We decompose the wave packet into two components,

$$\xi_k=\theta(k)\xi_k+\theta(-k)\xi_k=\xi_k^{+}+\xi_k^{-}. \tag{122}$$

This decomposition appears naturally due to the positivity of the energy $\omega_k=|k|$ [10]. Under free motion, the two components move undistorted in opposite directions,

$$\langle x|e^{-iH_0t}|\xi^{\pm}\rangle=\langle(x\mp t)|\xi^{\pm}\rangle. \tag{123}$$

Both components are nonlocal, i.e., they have long tails in space. At $t=0$ the long tails cancel to obtain the rectangular shape of $|\xi\rangle$. For $t>0$, as the two components move away from each other, the long tails no more longer cancel (we have called this the "curtain" effect [10]). Thus, in principle, the particle may be excited immediately after $t=0$. This nonlocal effect does not violate causality, because the components move with the finite speed $c=1$ ($|\xi^{-}\rangle$ to the left and $|\xi^{+}\rangle$ to the right). We note that nonlocal tail effects appear even if the initial wave packet is not strictly localized. For example, if it is a Gaussian wave packet, there will appear tails that extend over a much larger range than the Gaussian tails. Our main focus here is, however, not the study of nonlocal effects. We shall neglect the interaction with the tails as we have assumed that the interaction between the particle and the photons is of short range. Hence we approximate $H\approx H_0$ for $t\ll t_1$ where $t_1\equiv-(x_0+b/2)$. Around $t=t_1$ the interaction between the wave packet $|\xi_k^{+}\rangle$ and the particle is no more negligible as the body of the wave packet comes in contact with the particle. Around $t=t_2\equiv-(x_0-b/2)$, the body of the wave packet finishes passing through the particle, and only the tail effects remain. Therefore, neglecting the tail effects we may separate the evolution in three periods: $t<t_1$ (before the collision of $|\xi_k^{+}\rangle$ with the bare particle), $t_1<t<t_2$ (during the collision) and $t>t_2$ (after the collision).

The relative sizes of the two components $|\xi_k^{\pm}\rangle$ in x representation depend on the sign of the initial momentum k_0. For $k_0>0$ the component $|\xi_k^{+}\rangle$ is large while $|\xi_k^{-}\rangle$ is small, and the opposite is true for $k_0<0$. The intensity of the scattering depends on how far the energy ω_{k_0} is from the energy of the particle. For $\omega_{k_0}\approx\tilde{\omega}_1$ the intensity is largest and we have a strong resonance scattering.

In Figs. 5–7 we show numerical plots of the field intensity. The numerical plots are obtained by solving the Schrödinger equation through diagonalization of the Hamiltonian matrix, with the same parameters described in Sec. III. The wave packet $|\xi_k\rangle$ at $t=0$ is shown in Fig. 5. We have chosen $k_0=\tilde{\omega}_1$. For these parameters the component $|\xi_k^{-}\rangle$ is very small. This component has two peaks at the edges [10]. In Fig. 6 these two peaks correspond to the two small peaks on the left hand side. The central part of the wave packet $|\xi_k^{-}\rangle$ is too small to be seen in this figure.

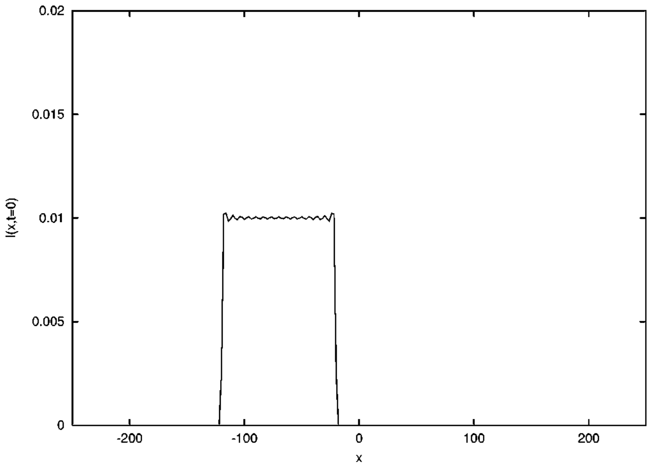

FIG. 5. Field intensity $I(x,t)$ for a rectangular wave packet at $t=0$. The wave packet is approaching the particle located at $x=0$ from the left-hand side. Parameters are $b=100$, $x_0=-70$, and $k_0=\tilde{\omega}_1=0.95$.

In Fig. 6 one can also see the interaction between the wave packet $|\xi_k^+\rangle$ and the bare particle at an intermediate time $t_1<t<t_2$ when the wave packet is passing through the particle. During this period the particle is excited (see Appendix C for detailed calculations). The interference pattern to the left of the bare particle is due to the interference between the incident wave packet and the emitted photons. To the right of the particle we have the part of the incident wave packet that has been transmitted. It presents a dip towards the origin, as part of it has been absorbed by the particle. For $t>t_2$ (Fig. 7) the particle decays, emitting the decay products. The transmitted wave packet is seen to the right of the particle (it is no more a rectangular wave packet, as it has been partially ''eaten'' by the particle). The distant profile to the left of the particle (in the region $x<-Q$, where $Q\equiv t-t_2$) represents the photons that were emitted as soon as they were absorbed (i.e., the reflected photons). Around the particle at $x=0$ we see the cloud and the decay products emitted after $t=t_2$ in the region $|x|<Q$. These are the same cloud and

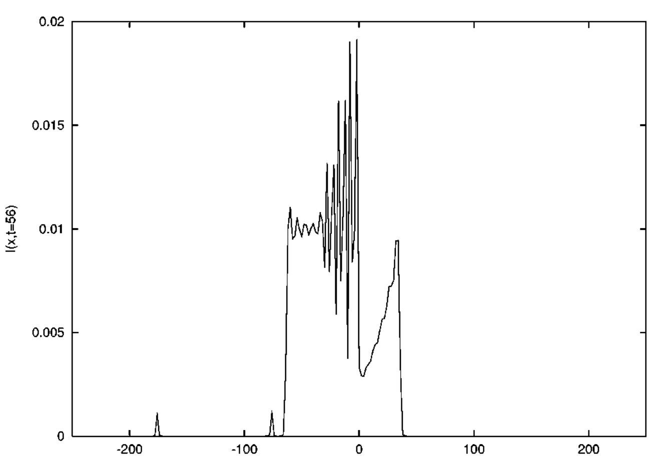

FIG. 6. Wave packet during the collision with the particle at $t=56$.

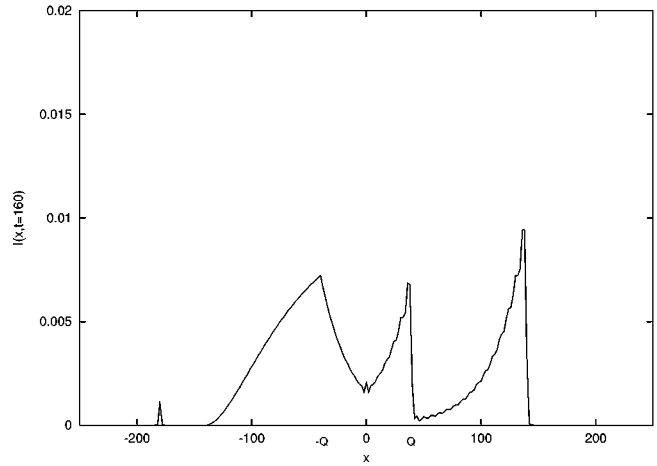

FIG. 7. Field intensity after the collision with the particle at $t=160$. The field emitted after the excitation of the particle appears in the region $|x|<Q\equiv t-t_2$. The transmitted wave packet is in the region $x>Q$ and the reflected wave packet is in the region $x<-Q$.

decay products that we have identified in Sec. III, when we studied the evolution of the bare-excited state. This was a different initial condition than the one we are considering here. The fact that the same components appear for two different initial conditions underlines the ''universal'' character of the dressed particle component.

We shall consider separately the $\mathcal{H}$ functions associated with the two components $|\xi^+\rangle$ and $|\xi^-\rangle$. They are given by

$$\langle\mathcal{H}^{\pm}(t)\rangle\equiv\langle\xi^{\pm}|\mathcal{H}(t)|\xi^{\pm}\rangle=e^{-2\gamma t}|\langle\tilde{\phi}_1|\xi^{\pm}\rangle|^2, \quad (124)$$

where

$$\langle\tilde{\phi}_1|\xi^{\pm}\rangle=\sum_k \langle\tilde{\phi}_1|k\rangle\langle k|\xi^{\pm}\rangle=\int_{-\infty}^{\infty}dk\,\frac{\lambda v(\omega_k)}{(z-\omega_k)^+_{z_1}}\,\xi_k^{\pm}$$

$$=\int_0^{\infty}dk\,\frac{\lambda v(k)}{(z-k)^+_{z_1}}\,\xi_{\pm k}. \quad (125)$$

As before we add and subtract an integral from $k=-\infty$ to 0, corresponding to the cut contribution

$$\langle\tilde{\phi}_1|\xi^{\pm}\rangle=\int_{-\infty}^{\infty}dk\,\frac{\lambda v(k)}{(z-k)^+_{z_1}}\,\xi_{\pm k}+\langle\tilde{\phi}_1|\xi^{\pm}\rangle_{\rm cut}. \quad (126)$$

After changing $k\Rightarrow -k$ we have

$$\langle\tilde{\phi}_1|\xi^{\pm}\rangle_{\rm cut}\equiv-\int_0^{\infty}dk\frac{\lambda v(-k)}{z_1+k}\,\xi_{\mp k}. \quad (127)$$

This cut contribution is due to the overlap between the cloud of the dressed particle and the wave packet. One can show that it decreases with an inverse power law of the initial distance x_0. On the other hand, the first term in Eq. (126) will give a much larger contribution at the pole $k=z_1$ if we can close the contour in the upper infinite semicircle of complex k [as in Sec. III we neglect the pole contributions of $v(k)$]. Considering the explicit form of ξ_k in Eq. (121) we see that closing the contour is possible only if

$$x_0 \pm b/2 < 0 \quad \text{for } \langle \tilde{\phi}_1 | \xi^+ \rangle,$$

$$x_0 \pm b/2 > 0 \quad \text{for } \langle \tilde{\phi}_1 | \xi^- \rangle. \tag{128}$$

As the wave packets are to the left of the particle at the initial time, we have $x_0 \pm b/2 < 0$ and, therefore, only the $|\xi^+\rangle$ component moving towards the particle gives a pole contribution. Thus, taking the residue at the pole in Eq. (126) we get

$$\langle \tilde{\phi}_1 | \xi^+ \rangle = 2\pi i \lambda v(z_1) \xi_{z_1} + \langle \tilde{\phi}_1 | \xi^+ \rangle_{\text{cut}},$$

$$\langle \tilde{\phi}_1 | \xi^- \rangle = \langle \tilde{\phi}_1 | \xi^- \rangle_{\text{cut}}. \tag{129}$$

The pole contribution dominates as we have $\xi_{z_1} \propto \exp(\gamma |x_0|)$. Therefore [see Eq. (124)]

$$\langle \mathcal{H}^+(t) \rangle \propto e^{2\gamma(|x_0| - t)} \tag{130}$$

and $\langle \mathcal{H}^-(t) \rangle$ is negligible as compared with $\langle \mathcal{H}^+(t) \rangle$,

$$\langle \mathcal{H}^-(t) \rangle \ll \langle \mathcal{H}^+(t) \rangle. \tag{131}$$

In short, the value of $\mathcal{H}$ changes drastically depending on the direction of the wave packet. It is large for the component moving towards the particle, and small for the component moving away from the particle.

To interpret this result we write

$$\begin{aligned}
\langle \mathcal{H}^\pm(t) \rangle &= \langle \langle \tilde{\phi}_1 ; \tilde{\phi}_1 | e^{-iL_H t} | \xi^\pm ; \xi^\pm \rangle \rangle \\
&= \langle \langle \tilde{\rho}_1^0 | e^{-iL_H t} | \xi^\pm ; \xi^\pm \rangle \rangle + O(1/L) \\
&= \sum_{\alpha,\beta} \langle \langle \tilde{\rho}_1^0 | \alpha ; \beta \rangle \rangle \langle \langle \alpha ; \beta | e^{-iL_H t} | \xi^\pm ; \xi^\pm \rangle \rangle.
\end{aligned} \tag{132}$$

The $\mathcal{H}$ function is expressed as the overlap between the dressed particle and the evolving wave packets. This overlap, in turn, is a superposition of the bare correlation components $\langle\langle \alpha;\beta | \exp(-iL_H t) | \xi^\pm ; \xi^\pm \rangle\rangle$. For the wave packet moving towards the particle the initial correlation components evaluated at the resonance $\omega_k = z_1$ give a large contribution that grows with the distance between the particle and the wave packet [cf. Eq. (130)]. The correlations are of long range due to the resonance. We may call them "precollisional" correlations, as they are associated with the wave packet moving towards the particle. The wave packet $|\xi^+\rangle$ is "far from equilibrium." As it approaches the particle, the correlations decay and the $\mathcal{H}$ function decreases. Eventually the particle is excited as it absorbs the wave packet and subsequently it decays emitting photons (see Figs. 5–7), which leads to the continued decrease of $\mathcal{H}$. Note that the emitted photons move away from the particle and hence they give a negligible contribution to the $\mathcal{H}$ function. In the final state, the particle is in its ground state and all the field is emitted away to infinity. Using the analogy of statistical mechanics, we may say that as a whole we go from a nonequilibrium to an equilibrium state.[10]

For the component $|\xi^-\rangle$ moving away from the particle the situation is different. The initial correlation components simply express the overlap of the tail of the wave packet with the the particle cloud (an off-resonance effect). These are "postcollisional" correlations. As the wave packet moves further away, the correlations become weaker. There is also a small effect due to the excitation and subsequent relaxation of the particle caused by the tail of $|\xi^-\rangle$. As a consequence of these two effects, the $\mathcal{H}$ function decreases. The final state is again the particle in its ground state and the field moving away to infinity.

Of course the distinction between "towards" and "away" from the particle requires knowledge of the position of the particle. In order to aim the wave packet towards the particle, we have to see the particle. This means that photons emitted from the particle must have reached us first. "Aiming" implies that the particle is in the future light cone of the wave packet. Therefore, the $\mathcal{H}$ function can be constructed only within timelike separations of the points it relates. Namely, the $\mathcal{H}$ function gives a global information that depends on the nonlocal correlation components of the dressed particle and the field. At any given time these components depend on the states of both the particle and the field at different locations [21], which have to be communicated in accordance with causality.

B. Momentum inversion

It is interesting to consider what happens when we perform momentum inversion [3,8]. This is achieved by the antilinear time-inversion operator T_I. Suppose that at time t we perform a momentum inversion. Then the state changes as

$$|\xi^\pm(t)\rangle \equiv e^{-iHt}|\xi^\pm\rangle \Rightarrow T_I e^{-iHt}|\xi^\pm\rangle = e^{+iHt}|\xi^\mp\rangle = |\xi^\mp(-t)\rangle. \tag{133}$$

As a consequence of the momentum inversion the expectation value of $\mathcal{H}$ "jumps" to a higher or lower value. This corresponds to a flow of "entropy" from the outside. As we shall see the direction of the jump depends on the time when one performs the momentum inversion. The ratio of the $\mathcal{H}$ function after the inversion to the one before the inversion is given by

$$j_r^\pm(t) \equiv \frac{\langle \xi^\mp(-t) | \mathcal{H} | \xi^\mp(-t) \rangle}{\langle \xi^\pm(t) | \mathcal{H} | \xi^\pm(t) \rangle} = \frac{\exp(2\gamma t) |\langle \tilde{\phi}_1 | \xi^\mp \rangle|^2}{\exp(-2\gamma t) |\langle \tilde{\phi}_1 | \xi^\pm \rangle|^2}. \tag{134}$$

Suppose that at $t=0$ we start with the component $|\xi^+\rangle$ alone. The wave packet moves to the right. We perform momentum inversion at a time when the wave packet has excited the particle and the particle is emitting the decay products, as in Fig. 7. The momentum inversion (point A in Fig. 8) causes

[10]Here "equilibrium" means the particle in the ground state plus the end decay products.

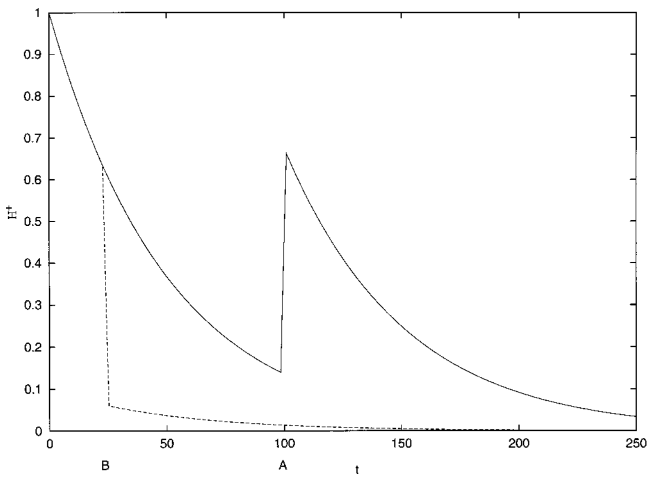

FIG. 8. Schematic plot of the Lyapounov function $\mathcal{H}^+$. The time inversion at A (before the wave packet collides with the particle) creates correlations; at B (after the wave packet collides with the particle) it destroys correlations. In this figure and in Fig. 9 time t is measured in units of the inverse frequency $\omega_1^{-1}=1$ of the unstable state and the Lyapounov function is dimensionless.

the $\mathcal{H}$ function to jump up, as the decay products start to move towards the particle (we have an increase of "order" or a flow of negative "entropy" due to the correlations coming from outside [8]). After the inversion, we have a "backwards" evolution and we follow the inverse sequence, from Fig. 7 to 5. The decay products move towards the particle that subsequently absorbs them. Eventually the field collects itself back into the initial wave packet, which then moves away from the particle. We interpret the continued decrease of $\mathcal{H}$ (solid line in Fig. 8) as due to the disappearance of the "anomalous" correlations that were fed to the system at point A. The $\mathcal{H}$ function continues to decrease as the system approaches its final relaxed state.

A different situation occurs if we perform the momentum inversion earlier, at a time $t \ll \gamma^{-1}$ when the wave packet is still far from the particle. The wave packet then changes to $|\xi^-\rangle$ and moves to the left. At the moment of inversion (point B in Fig. 8) we have [cf. Eq. (130)] $j_r(t) \propto \exp(-2\gamma|x_0|)$. The $\mathcal{H}$ function jumps down, due to the change of the direction of motion (dotted line in Fig. 8). The momentum reversal turns the precollisional correlations into postcollisional correlations, which corresponds to a flow of positive entropy into the system.

If we start at $t=0$ with the component $|\xi^-\rangle$ alone and perform a momentum inversion some time later, then the $\mathcal{H}$ function jumps up, because the wave packet, which was moving away from the particle, now moves towards the particle. After the inversion the $\mathcal{H}$ function decreases in time as described previously (see Fig. 9).

The examples discussed above illustrate the following point: if we perform momentum inversion when the entropy of the system is high, then the entropy will jump down. Conversely, if we perform the inversion when the entropy is low, the entropy will jump up.

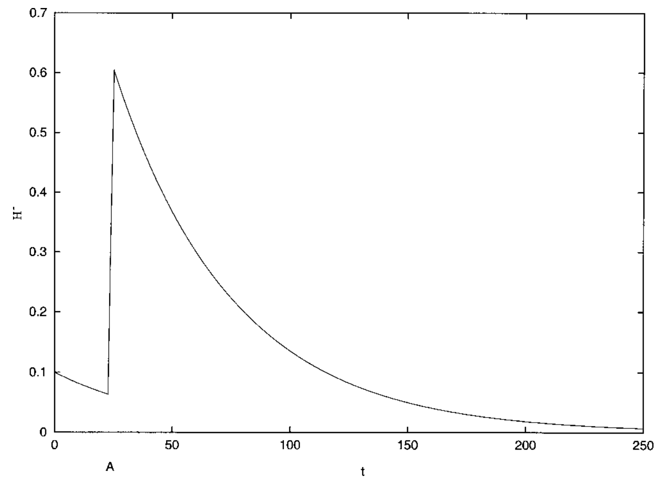

FIG. 9. Schematic plot of the Lyapounov function $\mathcal{H}^-$. The time inversion at A creates correlations and causes $\mathcal{H}^-$ to jump up.

C. Space dependence of $\mathcal{H}$

Our considerations above may also be applied to the state $|\xi\rangle = |x\rangle$. Then we can define space- and time-dependent Lyapounov functions as

$$h^{\pm}(x,t) \equiv \langle x|^{\pm}\mathcal{H}(t)|x^{\pm}\rangle. \tag{135}$$

We have [cf. Eq. (129)]

$$h^{+}(x,t) \propto e^{2\gamma(|x|-t)} \tag{136}$$

and

$$h^{-}(x,t)\rangle \propto e^{-2\gamma t}|\langle\tilde{\phi}_1|x^-\rangle_{\rm cut}|^2. \tag{137}$$

The function $h^+(x,t)$ decays in time at each point in space, and at a given instant it increases exponentially with the distance from the particle. This may be interpreted as a measure of the space-time dependence of the precollisional correlations of the field. Locations more distant from the particle are further away from their final asymptotic state, as they have to wait for more time to interact with the particle. The function $h^-(x,t)$ gives the space-time dependence of the postcollisional correlations. It decays exponentially in time, and it decreases with $|x|$, as a power law.

VI. CONCLUDING REMARKS

Using our states outside the Hilbert space we have separated preparation effects from decay effects in space representation. The two types of effects correspond to branch-point singularities and pole singularities of Green's function, respectively. Our unstable state $|\rho_1^0\rangle\rangle$ is not a solution of the Schrödinger or Heisenberg equations. It is a combination of eigenfunctions of the Liouville operator, which may also be written as a mixture involving dyads of Gamow vectors.

As commented at the beginning of Sec. IV, the state $|\rho_1^0\rangle\rangle$ may not be prepared in isolation. However, we can identify it within the components of any given evolving state. The space-time structure of the field associated with $|\rho_1^0\rangle\rangle$ is "universal" as it appears for different initial conditions (compare Figs. 2 and 7 around $x=0$). Furthermore we expect that the unstable cloud of $|\rho_1^0\rangle\rangle$ will play a role, e.g., in the interatomic forces between unstable atoms.

We are aware of the limitations of our model, and we hope to consider more realistic situations in the future. Our formulation is closely connected to the thermodynamic aspects of absorption and emission as well as the theory of "quasiparticles" in nonequilibrium field theory [27,28].

The $\mathcal{H}$ function we have studied is in a sense a microscopic realization of Boltzmann's $\mathcal{H}$ function. The $\mathcal{H}$ function distinguishes precollisional and postcollisional correlations, i.e., whether the particle is on target or off target. It is interesting to study the situation in two- or three-dimensional spaces. Here the precollisional and postcollisional fields correspond, respectively, to incoming and outgoing fields. In a later paper we shall study as well the distinction between real and virtual processes from the point of view of the $\mathcal{H}$ function [29].

We note that as our model is time reversal invariant (we have $[H,T_I]=0$), all our discussions are applicable if we exchange the roles of past and future. The main point is that there is a wide class of initial conditions that lead to asymptotic states, either on the distant future or the distant past, where the particle has decayed and the field is emitted away. This is reflected in the existence complex spectral representations of H or L_H. The components of these representations break time symmetry. Of course, if the system is coupled to the outside world, then for consistency we have to choose the representation that describes decay in the future.

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APPENDIX A: SIMPLE DERIVATION OF THE UNSTABLE STATE

In Ref. [4] we have obtained the dressed unstable state $|\rho_1^0\rangle\rangle$ and shown that it may be written in terms of Gamow-vector dyads as

$$|\rho_1^0\rangle\rangle = Q^{(0)}|\phi_1;\phi_1\rangle\rangle + P^{(0)}[r^{\text{c.c.}}|\phi_1;\tilde{\phi}_1\rangle\rangle + r|\tilde{\phi}_1;\phi_1\rangle\rangle]. \tag{A1}$$

To obtain this result we postulated that the energy fluctuation of $|\rho_1^0\rangle\rangle$ should be of the order of the inverse lifetime. Here we present an alternative derivation, where we invert the logic: we start by postulating the form (A1) and then we obtain the coefficient r using normalization conditions. This leads to the same results obtained in Ref. [4], including the energy fluctuation.

As explained below, to postulate the form (A1) we use two arguments: (1) that the state $|\rho_1^0\rangle\rangle$ reduces to the stable state $|\phi_1;\phi_1\rangle\rangle$ when there are no resonances and (2) that the state $|\rho_1^0\rangle\rangle$ is Hermitian, has a unit trace, and is analytic when the coupling constant vanishes at $\lambda=0$.

A simple form of unstable state one may postulate is the dyad of Gamow vectors $|\rho_1^G\rangle\rangle \equiv |\phi_1;\phi_1\rangle\rangle$. However, this dyad is both traceless and nonanalytic at $\lambda=0$ (see also Ref. [30]). The problem occurs in the $P^{(0)}$ component as

$$\text{Tr}(\rho_1^G) = \text{Tr}(P^{(0)}\rho_1^G) = 0. \tag{A2}$$

The $P^{(0)}$ component is nonanalytic in λ as we have $\lim_{\lambda\to 0} P^{(0)}\rho_1^G \neq |1\rangle\langle 1|$. In contrast, the $Q^{(0)}$ component is analytic at $\lambda=0$. This suggests that we retain the $Q^{(0)}$ component of $|\rho_1^G\rangle\rangle$ while modifying the $P^{(0)}$ component. The requirement (1) and the condition of Hermiticity then lead to the form in Eq. (A1). The coefficient r is chosen so that $r + r^{\text{c.c.}} \Rightarrow 1$ in the stable case. This ensures that in the stable case we recover the stable particle state as

$$\begin{aligned} |\rho_1^0\rangle\rangle &\Rightarrow Q^{(0)}|\bar{\phi}_1;\bar{\phi}_1\rangle\rangle + (r^{\text{c.c.}}+r)P^{(0)}|\bar{\phi}_1;\bar{\phi}_1\rangle\rangle \\ &= (Q^{(0)}+P^{(0)})|\bar{\phi}_1;\bar{\phi}_1\rangle\rangle = |\bar{\phi}_1;\bar{\phi}_1\rangle\rangle. \end{aligned} \tag{A3}$$

Moreover, from the condition $\text{Tr}(\rho_1^0)=1$ and the relations $\text{Tr}|\phi_1;\tilde{\phi}_1\rangle\rangle = \text{Tr}|\tilde{\phi}_1;\phi_1\rangle\rangle = 1$ we obtain

$$r + r^{\text{c.c.}} = 1 \tag{A4}$$

also in the unstable case. To find r we take the $\langle\langle 1;1|$ diagonal component in Eq. (A1) to obtain

$$\langle\langle 1;1|\rho_1^0\rangle\rangle = r^{\text{c.c.}}\langle\langle 1;1|\phi_1;\tilde{\phi}_1\rangle\rangle + r\langle\langle 1;1|\tilde{\phi}_1;\phi_1\rangle\rangle, \tag{A5}$$

where [c.f. Eq. (18)] we have

$$\langle\langle 1;1|\phi_1;\tilde{\phi}_1\rangle\rangle = N_1, \quad \langle\langle 1;1|\tilde{\phi}_1;\phi_1\rangle\rangle = N_1^{\text{c.c.}}. \tag{A6}$$

From the normalization condition $\langle\langle\tilde{\rho}_1^0|\rho_1^0\rangle\rangle = 1$, where $\langle\langle\tilde{\rho}_1^0| = \langle\langle\tilde{\phi}_1;\tilde{\phi}_1|Q^{(0)} + [\langle\langle\phi_1;\tilde{\phi}_1|r + \langle\langle\tilde{\phi}_1;\phi_1|r^{\text{c.c}}]P^{(0)}$, we obtain $\langle\langle 1;1|\rho_1^0\rangle\rangle = |N_1| + O(1/L)$. Equation (A5) now reduces to

$$|N_1| = r^{\text{c.c.}}N_1 + rN_1^{\text{c.c.}} \tag{A7}$$

or

$$r^{\text{c.c.}}e^{-i\text{a}} + re^{i\text{a}} = 1, \tag{A8}$$

where we have used polar coordinates $N_1 = |N_1|\exp(-i\text{a})$. This equation together with Eq. (A4) lead to the solution

$$r = \frac{\exp(-i\text{a}/2)}{2\cos(\text{a}/2)}, \tag{A9}$$

which coincides with the result given in (6.18) of Ref [4]. As shown in Ref. [4], this solution gives an energy fluctuation of $|\rho_1^0\rangle\rangle$ of the order of γ and also gives a small deviation of the average energy from Green's-function energy. It leads as well to the line shape b_k of our unstable state [see Eq. (41)].

APPENDIX B: PROPERTIES OF $|\psi_1\rangle$

We consider here some properties of the state $|\psi_1\rangle$ defined in Eq. (98). We note that in contrast to the Gamow vector [see Eq. (18)], $|\psi_1\rangle$ has no analytic continuation from the upper to the lower half plane and consequently it is not an eigenstate of the Hamiltonian. In the stable, case limit z_1 becomes real and the state $|\psi_1\rangle$ reduces to the stable particle state.

1. Normalization constant

From Eq. (98) we obtain

$$|n_1|=\left[1+\sum_k \frac{\lambda^2 V_k^2}{|z_1-\omega_k|^2}\right]^{-1}. \tag{B1}$$

In the limit $L\to\infty$ the second term is the integral

$$\int_{-\infty}^{\infty} dk \frac{\lambda_2 v^2(\omega_k)}{(\omega_k-\tilde{\omega}_1)^2+\gamma^2}=2\int_0^{\infty} dk \frac{\lambda_2 v^2(k)}{(k-\tilde{\omega}_1)^2+\gamma^2}$$
$$\approx 4\pi i \frac{\lambda^2 v^2(\omega_1)}{2i\gamma}\approx 1, \tag{B2}$$

where we have taken the dominant contribution at the resonance. This leads to $|n_1|=1/2+O(\lambda^2)$.

2. Proof of Eq. (109)

Using the explicit forms of $\langle\tilde{\phi}_k^-|$ and $|\psi_1\rangle$ we get

$$\langle\tilde{\phi}_k^-|\psi_1\rangle=n_1^{1/2}\left[\frac{\lambda V_k}{z_1^{\text{c.c.}}-\omega_k}+\frac{\lambda V_k}{\eta^+(\omega_k)}\right.$$
$$\left.+\frac{\lambda V_k}{\eta^+(\omega_k)}\sum_l \frac{\lambda V_l}{\omega_k-\omega_l+i\epsilon}\frac{\lambda V_l}{z_1^{\text{c.c.}}-\omega_l}\right]. \tag{B3}$$

Then, using the relations

$$\sum_l \frac{\lambda^2 V_l^2}{\omega_k-\omega_l+i\epsilon}=\omega_k-\omega_1-\eta^+(\omega_k),$$

$$\frac{1}{\omega_k-\omega_l+i\epsilon}\frac{1}{z_1^{\text{c.c.}}-\omega_l}=\left(\frac{1}{\omega_k-\omega_l+i\epsilon}-\frac{1}{z_1^{\text{c.c.}}-\omega_l}\right)\frac{1}{z_1^{\text{c.c.}}-\omega_k} \tag{B4}$$

as well as $\eta^-(z_1^{\text{c.c.}})=0$, we obtain Eq. (109).

3. Average energy

Up to second order in λ, the average energy of the dressed state $|\rho_1^0\rangle\rangle$ is

$$\langle\langle H|\rho_1^0\rangle\rangle\approx\tilde{\omega}_1\approx\omega_1+\int_{-\infty}^{\infty}dk\frac{\lambda^2 v^2(\omega_k)}{2}\left(\frac{1}{\omega_1-\omega_k+i\epsilon}+\text{c.c.}\right)$$
$$+O(\lambda^4), \tag{B5}$$

which coincides with the real part of Green's-function pole in this approximation [4]. On the other hand, the average energy of the state $|\psi_1\rangle$ is given by

$$\langle\psi_1|H|\psi_1\rangle=|n_1|\left[\omega_1+\int_{-\infty}^{\infty}dk\frac{\lambda^2 v_k^2}{|z_1-\omega_k|^2}\omega_k\right.$$
$$\left.-\int_{-\infty}^{\infty}dk\lambda^2 v_k^2\left(\frac{1}{z_1-\omega_k}+\text{c.c.}\right)\right]. \tag{B6}$$

With the approximation Eq. (B2) we may write the second term as

$$|n_1|\int_{-\infty}^{\infty}dk\frac{\lambda^2 v_k^2}{|z_1-\omega_k|^2}\omega_k$$
$$\approx\frac{\omega_1}{2}+\frac{1}{2}\int_{-\infty}^{\infty}dk\frac{\lambda^2 v_k^2}{|z_1-\omega_k|^2}(\omega_k-\omega_1)$$
$$=\frac{\omega_1}{2}+\frac{1}{2}\left[-\int_{-\infty}^{\infty}dk\frac{\lambda^2 v_k^2}{z_1^{\text{c.c.}}-\omega_k}+z_1-\omega_1\right]$$
$$\approx\frac{\omega_1}{2}+\frac{1}{2}\left[-\int_{-\infty}^{\infty}dk\frac{\lambda^2 v_k^2}{\omega_1+i\epsilon-\omega_k}+z_1-\omega_1\right]$$
$$=\frac{\omega_1}{2}+O(\lambda^4), \tag{B7}$$

where we have subtracted and added z_1 on the third line. This leads to

$$\langle\langle H|\psi_1;\psi_1\rangle\rangle=\omega_1+\int_{-\infty}^{\infty}dk\frac{\lambda^2 v^2(\omega_k)}{2}\left(\frac{1}{\omega_1-\omega_k+i\epsilon}+\text{c.c.}\right)$$
$$+O(\lambda^4), \tag{B8}$$

which coincides with the energy of the dressed unstable state up to $O(\lambda^2)$ [we note that similar to $|\rho_1^0\rangle\rangle$, the energy of the state $|\psi_1;\psi_1\rangle\rangle$ deviates from Green's-function energy $\tilde{\omega}_1$ starting from terms of $O(\lambda^4)$].

APPENDIX C: EXCITATION PROBABILITY IN RESONANCE SCATTERING

Here we consider the resonance scattering of the photon wave packet considered in Sec. V [see Eq. (120)]. The energy of the wave packet is chosen so that it resonates with the energy $\tilde{\omega}_1$, i.e., $\omega_{k0}=\tilde{\omega}_1$. In the following we estimate the excitation probability of the bare particle.

The excitation probability that the particle will be found in the bare-excited state after it absorbs the photon is given by

$$E(t)\equiv\langle\langle 1,1|e^{-iL_Ht}|\xi;\xi\rangle\rangle=|\langle 1|e^{-iHt}|\xi\rangle|^2. \tag{C1}$$

As in Sec. III we split $E(t)$ into its dressed state components as

$$E(t)=\sum_\alpha E_\alpha^{(0)}(t)+E^{(1)}(t)+E^{(2)}(t), \tag{C2}$$

where

$$E_\alpha^{(0)}(t)=\langle\langle 1,1|e^{-iL_Ht}|\rho_\alpha^0\rangle\rangle\langle\langle\tilde{\rho}_\alpha^0|\xi;\xi\rangle\rangle,$$

$$E^{(1)}(t)=\sum_k \langle\langle 1,1|e^{-iL_Ht}|\rho^{1k}\rangle\rangle\langle\langle\tilde{\rho}^{1k}|\xi;\xi\rangle\rangle+\text{c.c.},$$

$$E^{(2)}(t)=\sum_{k\neq k'} \langle\langle 1,1|e^{-iL_Ht}|\rho^{kk'}\rangle\rangle\langle\langle\tilde{\rho}^{kk'}|\xi;\xi\rangle\rangle. \tag{C3}$$

As $|\xi;\xi\rangle\rangle$ is a nonsingular operator we can use the factorized form of the dressed states [cf. Eq. (51)]. Defining

$$g_a(t)\equiv\int_{-\infty}^{\infty}dk\,\xi_k\frac{\lambda v(\omega_k)}{[\omega_k-z]_{z_1}^+}e^{-ia\omega_kt} \tag{C4}$$

and $g_0\equiv g_0(t)$, we obtain for $\lambda\ll 1$,

$$E_1^{(0)}(t)\approx|g_0|^2e^{-2\gamma t},$$

$$E^{(1)}(t)\approx -e^{-iz_1t}[g_0g_1(t)^{\text{c.c.}}+\text{c.c.}],$$

$$E^{(2)}(t)\approx|g_1(t)|^2. \tag{C5}$$

The contribution from the dressed photons $E_k^{(0)}(t)$ is negligible. Adding the terms in Eq. (C5) we have

$$E(t)\approx|e^{-iz_1t}g_0-g_1(t)|^2. \tag{C6}$$

Neglecting the singularities of $v(k)$ we may split Eq. (C4) into a pole and cut contribution, $g_a(t)\approx g_{a,p}(t)+g_{a,c}(t)$, where

$$g_{a,p}(t)=\int_{-\infty}^{\infty}dk(\xi_k+\xi_{-k})\frac{\lambda v(k)}{[k-\tilde{\omega}_1+i\gamma]_+}e^{-iakt}, \tag{C7}$$

$$g_{a,c}(t)=\int_0^{\infty}dk(\xi_k+\xi_{-k})\frac{\lambda v(-k)}{k+z_1}e^{iakt}. \tag{C8}$$

The terms ξ_k and ξ_{-k} correspond, respectively, to the components $|\xi^+\rangle$ and $|\xi^-\rangle$ we have introduced in Sec. V.

For the pole contributions we have

$$g_{0,p}\approx ig[e^{\gamma t_2}-e^{\gamma t_1}],$$

$$g_{1,p}(t)\approx ig\{\theta(t_2-t)[e^{-\gamma(t-t_2)}-1]-\theta(t_1-t)\times[e^{-\gamma(t-t_1)}-1]\}, \tag{C9}$$

where $t_{1,2}\equiv|x_0\pm b/2|$ and $g\equiv 2\omega_1\gamma^{-1}(2\pi/bW)^{1/2}$.

For $\tilde{\omega}_1\gg\gamma$ the cut contributions $g_{a,c}(t)$ are relatively small and, henceforth, we shall neglect them. Let us simply note that the cut contributions of $g_{1,c}(t)$ contain small peaks around $t=t_i$ and long tails proportional to inverse powers of $|t_i-t|$ for large values of $|t_i-at|$ (with $i=1,2$). This leads, respectively, to deviations from the exponential functions Eq. (C10) around the points $t=t_1$ and $t=t_2$, as well as the long-time tails for $t-t_2\gg\gamma^{-1}$ (see Refs. [10,19] for more on the tail effects). Another remark is that, due to the cut contributions, $E(t)$ is nonzero even before $t=t_1$ (i.e., before the moment when one may expect the rectangular wave packet will "touch" the bare particle). As mentioned in Sec. V, this nonlocal effect is associated with the tails (in space) of the components of the rectangular wave packet.

Neglecting the cut contributions we get

$$E(t)\approx\begin{cases}0 & \text{for } t<t_1,\\ g^2[1-e^{-\gamma(t-t_1)}]^2 & \text{for } t_1<t<t_2,\\ g^2e^{-2\gamma t}[e^{\gamma t_2}-e^{\gamma t_1}]^2 & \text{for } t>t_2.\end{cases} \tag{C10}$$

We note that each of the functions in Eq. (C9) contains exponentially decaying terms, and hence each dressed state component evolves irreversibly, even during the absorption period. The dressed particle component $E_1^{(0)}(t)$ decays exponentially even before the wave packet reaches the atom at $t=t_1$. However, for $t<t_1$ this component is canceled by the other components and we obtain the causal behavior of $E(t)$, in the pole approximation.

If we consider a large wave packet with $b\gg\gamma^{-1}$, then the bare particle will reach a stationary ("nonequilibrium") excited state. This state is maintained from the continued excitation due to the incoming field. In terms of our subdynamics, the stationary state is obtained through a balance between the decay process of the dressed particle (contained in the $\Pi^{(0)}$ component) and the creation of dressed correlations in the $\Pi^{(d)}$ components, with $d\neq 0$. Note that due to the normalization conditions of the wave packet, as $b\to\infty$, the excitation intensity $E(t)$ goes to zero. Therefore, to achieve the stationary state we have to give up the normalization condition. This corresponds to the existence of a background field (heat bath) in the thermodynamic limit.

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