# Quantum-information processing for a coherent superposition state via a mixed entangled coherent channel

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(Received 4 June 2001; published 10 October 2001)

An entangled two-mode coherent state is studied within the framework of $2\times2$-dimensional Hilbert space. An entanglement concentration scheme based on joint Bell-state measurements is worked out. When the entangled coherent state is embedded in vacuum environment, its entanglement is degraded but not totally lost. It is found that the larger the initial coherent amplitude, the faster entanglement decreases. We investigate a scheme to teleport a coherent superposition state while considering a mixed quantum channel. We find that the decohered entangled coherent state may be useless for quantum teleportation as it gives the optimal fidelity of teleportation less than the classical limit 2/3.

DOI: 10.1103/PhysRevA.64.052308 PACS number(s): 03.67.−a, 89.70.+c

## I. INTRODUCTION

Quantum entanglement and its remarkable features make it possible to realize quantum-information processing including quantum teleportation [1], cryptography [2], and quantum computation [3]. An entanglement of two systems in coherent states allows tests of local realism [4] and may be used as a quantum entangled channel for quantum-information transfer. Proposals to entangle fields in two spatially separated cavities exist [5]. Recently, entanglement of nonorthogonal states called quasi-Bell states and teleportation using them have been studied [6–8].

In quantum-information processing, the entangled coherent state is normally categorized into a two-mode-continuous-variable state. However, there was a suggestion to implement a logical qubit encoding by treating a coherent superposition state, a single-mode-continuous-variable state, as a qubit in two-dimensional Hilbert space [9]. In this paper, we study the entangled coherent state within the framework of $2\times2$-dimensional Hilbert space. We assess the entanglement of the evolved state and how useful it may be to transfer the quantum information when the entangled coherent state decoheres in the vacuum.

We first construct an orthogonal Bell basis set from nonorthogonal coherent states to reformulate the problem to $2\times2$-dimensional Hilbert space. We then investigate the Bell-state measurement scheme that works perfectly in the large amplitude limit. The measurement scheme composed of linear devices is proposed to use for entanglement concentration [10] and quantum teleportation. The teleportation scheme, in effect, reillustrates van Enk and Hirota's [6]. When the quantum system is open to the outside world, the initially prepared system decoheres and becomes mixed. Assuming the vacuum environment, we find how an entangled coherent state loses its initial entanglement as it interacts with the environment. We use the measure of entanglement [11] based on the partial transposition condition of entanglement [12]. We then consider optimal quantum teleportation via the mixed quantum channel. We find that even though the channel is always entangled under the influence of the vacuum environment, it becomes useless for teleportation at some point.

## II. CONSTRUCTION OF BELL BASIS WITH ENTANGLED COHERENT STATES

It is possible to consider an entangled coherent state in $C^2\otimes C^2$ Hilbert space. It makes the problem simpler because two-qubit entangled states have the simplest mathematical structure among entangled states.

Let us consider two kinds of entangled coherent states which have symmetry in phase space

$$|C_1\rangle=\frac{1}{\sqrt{N}}(|\alpha\rangle|\alpha\rangle+e^{i\varphi}|-\alpha^*\rangle|-\alpha^*\rangle), \tag{1}$$

$$|C_2\rangle=\frac{1}{\sqrt{N'}}(|\alpha\rangle|-\alpha^*\rangle+e^{i\varphi'}|-\alpha^*\rangle|\alpha\rangle), \tag{2}$$

where $|\alpha\rangle$ and $|-\alpha^*\rangle$ are coherent states of amplitudes $\alpha$ and $-\alpha^*$, $N$ and $N'$ are normalization factors, and $\varphi$ and $\varphi'$ are relative phase factors. It may be verified that any entangled coherent states in the form of $(|\beta\rangle|\beta\rangle+e^{i\varphi}|\gamma\rangle|\gamma\rangle)/\sqrt{\mathcal{N}}$ or $(|\beta\rangle|\gamma\rangle+e^{i\varphi'}|\gamma\rangle|\beta\rangle)/\sqrt{\mathcal{N}'}$, where $\beta$ and $\gamma$ are any complex amplitudes, may be converted, respectively, to $|C_1\rangle$ or $|C_2\rangle$ by applying local unitary operations [13]. A set of $|C_1\rangle$ for $\varphi=0,\pi$ and $|C_2\rangle$ for $\varphi'=0,\pi$ was studied as quasi-Bell states [7] but the four quasi-Bell states do not form a complete measurement set by themselves because they do not satisfy orthogonality and completeness.

By the Gram-Schmidt theorem, it is always possible to make orthonormal bases in $N$-dimensional vector space from any $N$ linear independent vectors. Suppose orthonormal bases by superposing nonorthogonal and linear independent two coherent states $|\alpha\rangle$ and $|-\alpha^*\rangle$:

 

$$|\psi_+\rangle = \frac{1}{\sqrt{N_\theta}}(\cos\theta e^{-(1/2)i\phi}|\alpha\rangle - \sin\theta e^{(1/2)i\phi}|-\alpha^*\rangle), \tag{3}$$

$$|\psi_-\rangle = \frac{1}{\sqrt{N_\theta}}(-\sin\theta e^{-(1/2)i\phi}|\alpha\rangle + \cos\theta e^{(1/2)i\phi}|-\alpha^*\rangle), \tag{4}$$

where $N_\theta = \cos^2 2\theta$ is a normalization factor and real parameters $\theta$ and $\phi$ are defined as

$$\sin 2\theta e^{-i\phi} = \langle\alpha|-\alpha^*\rangle = \exp\{-2\alpha_r^2 + 2i\alpha_r\alpha_i\} \tag{5}$$

with real $\alpha_r$ and imaginary $\alpha_i$ parts of $\alpha$.

We define four maximally entangled Bell states using the orthonormal bases in Eqs. (3) and (4)

$$|B_{1,2}\rangle = \frac{1}{\sqrt{2}}(|\psi_+\rangle|\psi_+\rangle \pm |\psi_-\rangle|\psi_-\rangle), \tag{6}$$

$$|B_{3,4}\rangle = \frac{1}{\sqrt{2}}(|\psi_+\rangle|\psi_-\rangle \pm |\psi_-\rangle|\psi_+\rangle). \tag{7}$$

They may be represented by $|\alpha\rangle$ and $|-\alpha^*\rangle$ as

$$|B_1\rangle = \frac{1}{\sqrt{2}N_\theta}\{e^{-i\phi}|\alpha\rangle|\alpha\rangle + e^{i\phi}|-\alpha^*\rangle|-\alpha^*\rangle - \sin 2\theta(|\alpha\rangle|-\alpha^*\rangle + |-\alpha^*\rangle|\alpha\rangle)\}, \tag{8}$$

$$|B_2\rangle = \frac{1}{\sqrt{2N_\theta}}(e^{-i\phi}|\alpha\rangle|\alpha\rangle - e^{i\phi}|-\alpha^*\rangle|-\alpha^*\rangle), \tag{9}$$

$$|B_3\rangle = \frac{1}{\sqrt{2}N_\theta}\{|\alpha\rangle|-\alpha^*\rangle + |-\alpha^*\rangle|\alpha\rangle - \sin 2\theta(e^{-i\phi}|\alpha\rangle|\alpha\rangle + e^{i\phi}|-\alpha^*\rangle|-\alpha^*\rangle)\}, \tag{10}$$

$$|B_4\rangle = \frac{1}{\sqrt{2N_\theta}}(|\alpha\rangle|-\alpha^*\rangle - |-\alpha^*\rangle|\alpha\rangle), \tag{11}$$

where we immediately recognize that $|B_2\rangle$ and $|B_4\rangle$ are in the form of entangled coherent states $|C_1\rangle$ and $|C_2\rangle$ while $|B_1\rangle$ and $|B_3\rangle$ become so as $|\alpha|\to\infty$.

Now we are ready to consider decoherence and teleportation with mixed entangled coherent states. For simplicity, we assume $\phi=0$, i.e., $\alpha$ is real, in the rest of the paper.

## III. TELEPORTATION VIA A PURE CHANNEL

There have been studies on the quantum teleportation of a coherent superposition state via an entangled coherent channel $|B_2\rangle$ [6]. Here, we suggest a scheme for the same purpose with the use of Bell bases (8), (9), (10), and (11). The scheme includes direct realization of Bell-state measurements. We also show that the Bell-state measurement method enables the entanglement concentration of partially entangled coherent states.

### A. Teleportation and Bell-state measurement

Suppose a coherent superposition state

$$|\Psi\rangle = \frac{1}{\sqrt{M_-}}(|\sqrt{2}\alpha\rangle - |-\sqrt{2}\alpha\rangle), \tag{12}$$

where $M_-$ is a normalization factor, is superposed on a vacuum $|0\rangle$ by a loss-less 50:50 beam. It can be shown that the output state is $|B_4\rangle$. It is possible to generate a superposition of the two coherent states $|\sqrt{2}\alpha\rangle$ and $|-\sqrt{2}\alpha\rangle$, from a coherent state $|\sqrt{2}\alpha\rangle$ propagating through a nonlinear medium [14].

Let us assume that Alice wants to teleport a coherent superposition state

$$|\psi\rangle_a = \mathcal{A}|\alpha\rangle_a + \mathcal{B}|-\alpha\rangle_a, \tag{13}$$

via the pure entangled coherent channel $|B_4\rangle_{bc}$, where the amplitudes $\mathcal{A}$ and $\mathcal{B}$ are unknown. The state (13) may be represented as

$$|\psi\rangle_a = \mathcal{A}'|\psi_+\rangle_a + \mathcal{B}'|\psi_-\rangle_a, \tag{14}$$

with $\mathcal{A}' = \mathcal{A}\cos\theta + \mathcal{B}\sin\theta$ and $\mathcal{B}' = \mathcal{A}\sin\theta + \mathcal{B}\cos\theta$. After sharing the quantum channel $|B_4\rangle_{bc}$, Alice performs a Bell-state measurement on her part of the quantum channel and the state (13) and sends the outcome to Bob. Bob accordingly chooses one of the unitary transformations $\{i\sigma_y, \sigma_x, -\sigma_z, \mathbb{1}\}$ to perform on his part of the quantum channel. Here, $\sigma$'s are Pauli operators and $\mathbb{1}$ is the identity operator and the correspondence between the measurement outcomes and the unitary operations are $B_1 \Rightarrow i\sigma_y$, $B_2 \Rightarrow \sigma_x$, $B_3 \Rightarrow -\sigma_z$, $B_4 \Rightarrow \mathbb{1}$. The acting of these operators on $|\alpha\rangle$ and $|-\alpha\rangle$ gives impacts as follows:

$$|\alpha\rangle \xrightarrow{i\sigma_y} \frac{1}{N_\theta}(\sin 2\theta|\alpha\rangle - |-\alpha\rangle), \tag{15}$$

$$|-\alpha\rangle \xrightarrow{i\sigma_y} \frac{1}{N_\theta}(|\alpha\rangle - \sin 2\theta|-\alpha\rangle), \tag{16}$$

$$|\alpha\rangle \overset{\sigma_x}{\leftrightarrow} |-\alpha\rangle, \tag{17}$$

$$|\alpha\rangle \xrightarrow{-\sigma_z} \frac{1}{N_\theta}(|\alpha\rangle - \sin 2\theta|-\alpha\rangle), \tag{18}$$

$$|-\alpha\rangle \xrightarrow{-\sigma_z} \frac{1}{N_\theta}(\sin 2\theta|\alpha\rangle - |-\alpha\rangle). \tag{19}$$

It is not a trivial problem to discriminate all four Bell states. In fact, it was shown that complete Bell-state measurements on a product Hilbert space of two two-level systems are not possible using linear elements [15]. We here

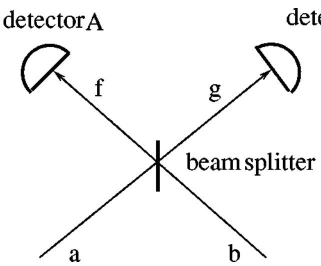


FIG. 1. Scheme to discriminate all four Bell states with an arbitrarily high precision using a 50:50 beam splitter and two photodetectors. If an odd number of photons is detected at detector $A$ for mode $f$, then we know that the entangled state incident on the measurement set up was $|B_2\rangle$. On the other hand, if an odd number of photons is detected at detector $B$ for mode $g$, then the incident entangled state was $|B_4\rangle$. For $\alpha \gg 1$, if a nonzero even number of photons is detected for mode $f$, the incident state was $|B_1\rangle$ and if a nonzero even number is detected for mode $g$, it was $|B_3\rangle$.

suggest an experimental setup as shown in Fig. 1 to discriminate Bell states constructed from entangled coherent states. Although perfect discrimination is not possible, arbitrarily high precision can be achieved when the amplitude of the coherent states becomes large. For simplicity, we shall assume that the 50:50 beam splitter imparts equal phase shifts to reflected and transmitted fields.

Suppose that each mode of the entangled state is incident on the beam splitter. After passing the beam splitter (bs), the Bell states become

$$|B_1\rangle_{ab} \xrightarrow{\text{bs}} \frac{1}{\sqrt{2N_\theta}}(|\text{even}\rangle_f|0\rangle_g - \sin 2\theta|0\rangle_f|\text{even}\rangle_g),$$

$$|B_2\rangle_{ab} \xrightarrow{\text{bs}} \frac{1}{\sqrt{2N_\theta}}|\text{odd}\rangle_f|0\rangle_g,$$

$$|B_3\rangle_{ab} \xrightarrow{\text{bs}} \frac{1}{\sqrt{2N_\theta}}(|0\rangle_f|\text{even}\rangle_g, -\sin 2\theta|\text{even}\rangle_f|0\rangle_g),$$

$$|B_4\rangle_{ab} \xrightarrow{\text{bs}} \frac{1}{\sqrt{2N_\theta}}|0\rangle_f|\text{odd}\rangle_g, \tag{20}$$

where $|\text{even}\rangle = |\sqrt{2}\alpha\rangle + |-\sqrt{2}\alpha\rangle$ has nonzero photon-number probabilities only for even numbers of photons and $|\text{odd}\rangle = |\sqrt{2}\alpha\rangle - |-\sqrt{2}\alpha\rangle$ has nonzero photon-number probabilities only for odd numbers of photons. Note that $|\text{even}\rangle$ and $|\text{odd}\rangle$ are not normalized. If an odd number of photons is detected at detector $A$ for mode $f$, then we know that the entangled state incident on the measurement set up was $|B_2\rangle$. On the other hand, if an odd number of photons is detected at detector $B$ for mode $g$, then the incident entangled state was $|B_4\rangle$. When even numbers of photons are measured, we cannot, in general, tell if the incident state was $|B_1\rangle$ or $|B_3\rangle$. However, for $\sin 2\theta (=\langle\alpha|-\alpha\rangle) \simeq 0$, i.e., $\alpha \gg 1$, if a nonzero even number of photons is detected for mode $f$, the incident state was $|B_1\rangle$, and if a nonzero even number is detected for mode $g$, it was $|B_3\rangle$. When $\sin 2\theta$ is not negligible, the probability of wrong estimation is

$$P_i(\alpha) = \frac{1}{2(1+e^{4\alpha^2})}. \tag{21}$$

For the limit of $\alpha \gg 1$, this probability approaches to zero and all the Bell states may be discriminated with arbitrarily high precision.

When the measurement outcome is $|B_2\rangle$, the receiver performs $|\alpha\rangle \leftrightarrow |-\alpha\rangle$ on $c$. Such a phase shift by $\pi$ may be done using a phase shifter whose action is described by $R(\varphi) = e^{i\varphi a^\dagger a}$:

$$R(\varphi) a R^\dagger(\varphi) = a e^{-i\varphi}, \tag{22}$$

where $a$ and $a^\dagger$ are the annihilation and creation operators. When the measurement outcome is $|B_4\rangle$, the receiver does nothing on $c$ as the required unitary transformation is only the identity operation $\mathbb{1}$. When the outcome is $|B_3\rangle$, an operator $(|\alpha\rangle\langle\alpha| - |-\alpha\rangle\langle-\alpha|)/N_\theta$ plays the corresponding role, which becomes a unitary operator for $\alpha \gg 1$. When the outcome is $|B_1\rangle$, $\sigma_x$ and $\sigma_z$ should be successively applied.

### B. Concentration of partial entanglement via entanglement swapping

If the initially prepared quantum channel is in a pure but not maximally entangled state, the channel may be distilled to a maximally entangled state before using it for quantum-information processing including teleportation. This process is known as the entanglement concentration protocol [16,17]. For an entangled coherent channel, it may be simply realized via entanglement swapping [10,18] using the Bell measurement proposed in Sec. III A.

Suppose an ensemble of a partially entangled pure state

$$|D_4\rangle = \frac{1}{\sqrt{N_\eta}}(\cos\eta|\alpha\rangle|-\alpha\rangle - \sin\eta|-\alpha\rangle|\alpha\rangle), \tag{23}$$

from which we want to distill a subensemble of a maximally entangled state. $N_\eta$ is a normalization factor and the real phase factor $\eta$, $0<\eta<\pi/2$, determines the degree of entanglement for $|D_4\rangle$. The state $|D_4\rangle$ in Eq. (23) is written in the orthonormal bases (3) and (4) as follows:

$$|D_4\rangle = \frac{1}{\sqrt{N_\eta}}\bigg\{\frac{1}{2}\sin 2\theta(\cos\eta - \sin\eta)(|\psi_+\rangle|\psi_+\rangle + |\psi_-\rangle|\psi_-\rangle) + (\cos^2\theta\cos\eta - \sin^2\theta\sin\eta)|\psi_+\rangle|\psi_-\rangle + (\sin^2\theta\cos\eta - \cos^2\theta\sin\eta)|\psi_-\rangle|\psi_+\rangle\bigg\}. \tag{24}$$

First, we consider the case when $\alpha$ is large. In this case, state $|D_4\rangle \simeq |E_4\rangle$ where

$$|E_4\rangle = \cos\eta|\psi_+\rangle|\psi_-\rangle - \sin\eta|\psi_-\rangle|\psi_+\rangle. \tag{25}$$

After sharing a quantum channel between Alice and Bob, Alice prepares a pair of particles that are in the same entangled state as the quantum channel. Alice then performs

Bell-state measurement on her pair of the quantum channel. If the measurement outcome is $B_1$ or $B_2$, the other particle of Alice's and Bob's quantum channel is, respectively, in maximally entangled state $|B_1\rangle_{b'c}$ or $|B_2\rangle_{b'c}$ where Alice's particle is denoted by $b'$. Otherwise, Alice's particle and Bob's quantum channel are not in a maximally entangled state

$$|B_3'\rangle_{b'c}=\frac{1}{\sqrt{N_\eta'}}(\cos^2\eta|\psi_+\rangle_{b'}|\psi_-\rangle_c+\sin^2\eta|\psi_-\rangle_{b'}|\psi_+\rangle_c), \tag{26}$$

$$|B_4'\rangle_{b'c}=\frac{1}{\sqrt{N_\eta'}}(\cos^2\eta|\psi_+\rangle_{b'}|\psi_-\rangle_c-\sin^2\eta|\psi_-\rangle_{b'}|\psi_+\rangle_c), \tag{27}$$

respectively, for measurement outcome of $B_3$ or $B_4$. $N_\eta'$ is a normalization factor. The probability $P_1$ and $P_2$ to obtain the maximally entangled state $|B_1\rangle_{b'c}$ and $|B_2\rangle_{b'c}$ are $P_1=P_2=\cos^2\eta\sin^2\eta$. In this way, no matter how small the initial entanglement is, it is possible to distill some maximally entangled coherent channels from partially entangled pure channels.

We now consider the concentration protocol when $\alpha$ is not large enough to neglect $\sin 2\theta$. In this case, only two Bell states $|B_2\rangle$ and $|B_4\rangle$ may be precisely measured. Extending the previous argument leading to Eq. (27), when the measurement outcome is $B_4$, the resulting state for particles $b'$ and $c$ is not maximally entangled. However, we may find that, for the measurement outcome of $B_2$, the resulting state is $|B_2\rangle_{b'c}$ even for the case of $\alpha$ small. The success probability $\mathcal{P}_2$ for this case is

$$\mathcal{P}_2(\theta,\eta)=\frac{\cos^4 2\theta\sin^2 2\eta}{4(1-\sin^2 2\theta\sin 2\eta)}, \tag{28}$$

where $\mathcal{P}_2\to 0$ for $\alpha\simeq 0$ and $\mathcal{P}_2\to\cos^2\eta\sin^2\eta$ for $\alpha\gg 1$.

## IV. DECAY OF THE ENTANGLED COHERENT CHANNEL: MEASURE OF ENTANGLEMENT

When the entangled coherent channel $|B_4\rangle$ is embedded in a vacuum environment, the channel decoheres and becomes a mixed state of its density operator $\rho_4(\tau)$, where $\tau$ stands for the decoherence time. To know the time dependence of $\rho_4(\tau)$, we have to solve the master equation [19]

$$\frac{\partial\rho}{\partial\tau}=\hat{J}\rho+\hat{L}\rho;\quad \hat{J}\rho=\gamma\sum_i a_i\rho a_i^\dagger,$$

$$\hat{L}\rho=-\sum_i\frac{\gamma}{2}(a_i^\dagger a\rho+\rho a_i^\dagger a_i), \tag{29}$$

where $\gamma$ is the energy decay rate. The formal solution of the master equation (29) may be written as

$$\rho(t)=\exp[(\hat{J}+\hat{L})\tau]\rho(0), \tag{30}$$

which leads to the solution for the initial single-mode dyadic $|\alpha\rangle\langle\beta|$

$$\exp[(\hat{J}+\hat{L})\tau]|\alpha\rangle\langle\beta|=\langle\beta|\alpha\rangle^{1-t^2}|\alpha t\rangle\langle\beta t|, \tag{31}$$

where $t=e^{-(1/2)\gamma\tau}$. For later use, we introduce a normalized interaction time $r$, which is related to $t$: $r=\sqrt{1-t^2}$.

To restrict our discussion in a $2\times 2$-dimensional Hilbert space even for the mixed case, the orthonormal basis vectors (3) and (4) are now $\tau$-dependent

$$|\Psi_+(\tau)\rangle=\frac{1}{\sqrt{N_\Theta}}(\cos\Theta|t\alpha\rangle-\sin\Theta|-t\alpha\rangle), \tag{32}$$

$$|\Psi_-(\tau)\rangle=\frac{1}{\sqrt{N_\Theta}}(-\sin\Theta|t\alpha\rangle+\cos\Theta|-t\alpha\rangle), \tag{33}$$

where $\sin 2\Theta=\exp(-2t^2\alpha^2)$. The unknown state to teleport and the Bell-state bases are newly defined according to the basis vectors Eqs. (32) and (33).

Any two-dimensional bipartite state may be written as

$$\rho=\frac{1}{4}\left(\mathbb{1}\otimes\mathbb{1}+\vec{v}\cdot\vec{\sigma}\otimes\mathbb{1}+\mathbb{1}\otimes\vec{s}\cdot\vec{\sigma}+\sum_{m,n=1}^{3}t_{nm}\sigma_n\otimes\sigma_m\right), \tag{34}$$

where coefficients $t_{nm}=\mathrm{Tr}(\rho\sigma_m\otimes\sigma_n)$ form a real matrix $T$. Vectors $\vec{v}$ and $\vec{s}$ are local parameters that determine the reduced density operator of each mode

$$\rho_b=\mathrm{Tr}_c\rho=\frac{1}{2}(\mathbb{1}+\vec{v}\cdot\vec{\sigma}), \tag{35}$$

$$\rho_c=\mathrm{Tr}_b\rho=\frac{1}{2}(\mathbb{1}+\vec{s}\cdot\vec{\sigma}), \tag{36}$$

while the matrix $T$ is responsible for correlation [23]

$$\mathcal{E}(a,b)=\mathrm{Tr}(\rho\vec{a}\cdot\vec{\sigma}\otimes\vec{b}\cdot\vec{\sigma})=(\vec{a},T\vec{b}). \tag{37}$$

With use of Eqs. (11) and (29), we find $\vec{v}$, $\vec{s}$, and $T$ for the mixed channel $\rho_4(\tau)$ as follows:

$$\vec{v}=\vec{s}=\left(\frac{B}{N_\theta},\ 0,\ 0\right), \tag{38}$$

$$T=\frac{1}{2N_\theta}\begin{pmatrix}A+D & 0 & 0\\ 0 & -A+D & 0\\ 0 & 0 & A-C\end{pmatrix}, \tag{39}$$

where $A$, $B$, $C$, and $D$ are defined as

$$A=(1-\Gamma)\exp(-4t^2\alpha^2),$$

$$B=(1-\Gamma)\exp(-2t^2\alpha^2),$$

$$C=2-(1+\Gamma)\exp(-4t^2\alpha^2),$$

$$D=-2\Gamma+(1+\Gamma)\exp(-4t^2\alpha^2),$$

$$\Gamma=\exp[-4(1-t^2)\alpha^2]. \tag{40}$$

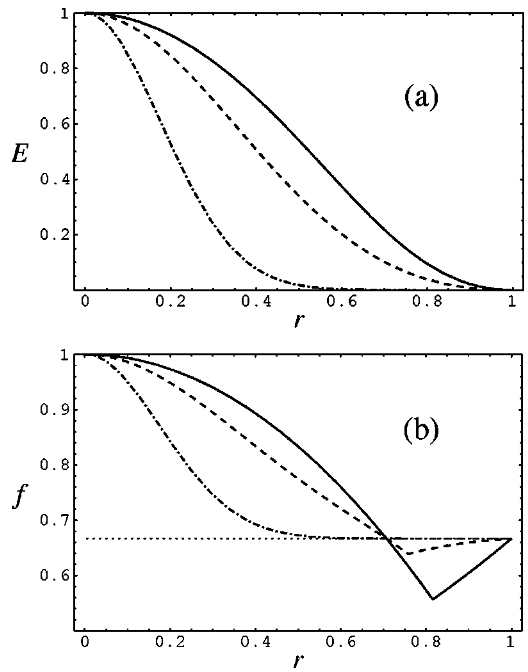


FIG. 2. (a) Entanglement $E$ for the mixed entangled coherent channel against the normalized decoherence time $r=\sqrt{1-e^{-\gamma\tau}}$. (b) Optimal fidelity $f$ of quantum teleportation with the mixed entangled coherent channel. The maximum fidelity 2/3 obtained by classical teleportation is plotted by a dotted line. We can clearly see that the mixed channel is not useful in quantum teleportation from $r=1/\sqrt{2}$ even though it is always entangled. $\alpha=0.1$ (solid line), $\alpha=1$ (long dashed), and $\alpha=2$ (dot dashed).

Note that $N_\theta$ is a time-independent normalization factor and $\rho_4(\tau\neq 0)$ may not be represented by a Bell-diagonal matrix.

The necessary and sufficient condition for separability of a two-dimensional bipartite system is the positivity of the partial transposition of its density matrix [12]. Consider a density matrix $\rho$ for a $2\times 2$ system and its partial transposition $\rho^{T_2}$. The density matrix $\rho$ is inseparable iff $\rho^{T_2}$ has any negative eigenvalue(s). We define the measure of entanglement $E$ for $\rho$ in terms of the negative eigenvalues of $\rho^{T_2}$ [11]. The measure of entanglement $E$ is then defined as

$$E=-2\sum_i \lambda_i^-, \tag{41}$$

where $\lambda_i^-$ are the negative eigenvalue(s) of $\rho^{T_2}$ and the factor two is introduced to have $0\leqslant E\leqslant 1$.

For $\rho_4(\tau)$, we find the time evolution of the measure of entanglement

$$E(\tau)=\frac{\sqrt{16B^2+(C-D)^2}-(2A+C+D)}{4N_\theta}. \tag{42}$$

Initially, the state $|B_4\rangle$ is maximally entangled, i.e., $E(\tau=0)=1$, regardless of $\alpha$. It is seen in Fig. 2(a) that the mixed state $\rho_4(\tau)$ is never separable at the interaction time $\tau<\infty$. It should be noted that the larger the initial amplitude $\alpha$, the more rapidly the entanglement is degraded. It is known that the speed of destruction of quantum interference depends on the distance between the coherent component states [20]. When the amplitudes of coherent component states are larger, the entanglement due to their quantum interference is more fragile.

### V. TELEPORTATION VIA A MIXED CHANNEL

The optimal fidelity of teleportation in any general scheme by means of trace-preserving local quantum operations and classical communication via a single channel may be obtained from the maximal singlet fraction of the channel [21]. The relation is

$$f(\rho)=\frac{F(\rho)N+1}{N+1}, \tag{43}$$

where $f(\rho)$ is the optimal fidelity for the given quantum channel $\rho$, $F(\rho)$ is the maximal singlet fraction of the channel, and $N$ is the dimension of the related Hilbert space $C^N\otimes C^N$. $F(\rho)$ is defined as $\max\langle\Phi|\rho|\Phi\rangle$ where the maximum is taken over all the $N\times N$ maximally entangled states.

Any $2\times 2$ channel becomes useless for quantum teleportation when the optimal fidelity $f(\rho)$ is less than the classical limit 2/3. In other words, when $F(\rho)\leqslant 1/2$, the channel is useless for quantum teleportation. To find the maximally entangled basis in which a given channel has the highest fraction of a maximally entangled state, it suffices to find rotations that diagonalize $T$ [22]. In the case of $\rho_4$, $T$ in Eq. (39) is always a diagonal matrix. This means that the Bell bases constructed from Eqs. (32) and (33) give the maximal singlet fraction at any decay time. The optimal fidelity $f(\rho_4)$ obtained by Eq. (43) and the definition of the maximal singlet fraction is

$$f(\rho_4)=\frac{1}{3}\max\left\{1+\frac{e^{4\alpha^2}-e^{4t^2\alpha^2}}{e^{4\alpha^2}-1},\frac{e^{4t^2\alpha^2}-e^{4r^2\alpha^2}+2e^{4\alpha^2}-2}{e^{4\alpha^2}-1}\right\}. \tag{44}$$

Because the initially defined Bell bases always give the maximal singlet fraction, the optimal fidelity is obtained by the standard teleportation scheme with Bell measurement and unitary operations. This means that the experimental proposal in Sec. III for pure channel may also be used for a mixed channel to obtain the optimal fidelity. The optimal fidelity for the standard teleportation scheme is

$$f_s(\rho_4)=\max\left[\frac{1}{2}\left(1-\frac{1}{3}\mathrm{Tr}(TO)\right)\right]=f(\rho_4), \tag{45}$$

where the maximum is taken over all possible rotations $O=O^+(3)$ [23]. As the interaction time varies, parameters $\vec{v}$, $\vec{s}$, and $T$ are changed. For the decoherence model we consider in this paper, $T$ alone affects the fidelity of teleportation.

Figure 2(b) shows the optimal fidelity at the normalized decay time $r$. The channel is always entangled as shown in Fig. 2(a). However, after the characteristic time $r_c=1/\sqrt{2}$ the

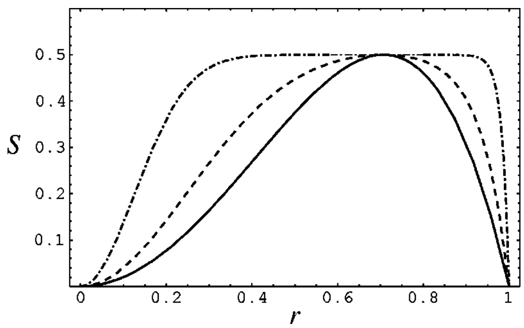


FIG. 3. Mixedness $S$ quantified by the linear entropy for the mixed entangled coherent state against the normalized decoherence time $r$. The mixedness becomes maximized at the characteristic time $r_c$ after which the channel is no longer useful for teleportation. $\alpha=0.1$ (solid line), $\alpha=1$ (long dashed), and $\alpha=2$ (dot dashed).

channel becomes useless for teleportation. It is worth noting that the characteristic time does not depend on the initial $\alpha$ value. This is confirmed by the fact that the only real solution of the equation $f(\rho_4)=2/3$ is $r=1/\sqrt{2}$ regardless of $\alpha$. Bennett *et al.* [24] have pointed out that some states with nonzero entanglement do not have the maximal singlet fractions higher than 1/2. The decohered entangled coherent channel $\rho_4(r\geqslant r_c)$ is an example of such a case.

Bose and Vedral [25] found that not only entanglement but also mixedness of quantum channels affect the fidelity of teleportation. We may conjecture that the higher entanglement and the lower mixedness (higher purity) result in the better fidelity. In this case, it is shown to be true only when the channel is useful for teleportation. The mixedness of a given state $\rho$ can be quantified by its linear entropy $S(\rho)=1-\mathrm{Tr}(\rho^2)$. For the decohered entangled coherent channel, the linear entropy is

$$S(\rho_4)=\frac{(e^{8r^2\alpha^2}-1)(e^{8t^2\alpha^2}-1)}{2(e^{4\alpha^2}-1)^2}, \tag{46}$$

which increases to the maximal value and then decreases to zero as shown in Fig. 3 because the channel interacts with the vacuum and the state for $\tau\rightarrow\infty$ approaches to the two-mode vacuum, which is a pure state. We found that mixedness becomes maximized at the characteristic time $r_c$. It is confirmed by solving the equation $\partial S(\rho_4)/\partial r=0$, which yields a unique real solution $r=1/\sqrt{2}\ =r_c$ again regardless of $\alpha$. It is easily checked that von-Neumann entropy as a measurement of mixedness gives exactly the same result.

Horodecki *et al.* [22] showed that any entangled $2\times2$ density matrix may be distilled to a singlet form by local filtering [17,26] and entanglement concentration protocol [16]. If sufficiently many entangled $2\times2$ channels are given, no matter how small the entanglement of the channels is, some maximally entangled channels may be obtained from the original pairs. Because the decohered channel $\rho_4$ is entangled at any decay time, the ensemble represented by $\rho_4(\tau)$ may be purified to obtain some maximally entangled channels. We have seen that the singlet fraction $F(\rho_4)$ becomes smaller than 1/2 after $r_c$, meanwhile the purification protocol in [16] may be applied when the singlet fraction of a given density matrix is larger than 1/2. Therefore, if the decay time is longer than $r_c$, a local filtering or a generalized measurement [22] should be first performed on $\rho_4$ for purification. It has been pointed out that the filtering process allows one to transfer the entanglement hidden in the relation between $\vec{v}$, $\vec{s}$, and $T$ (the entanglement added by change of the local states) to $T$ [22].

## VI. USEFULNESS FOR CONTINUOUS-VARIABLE TELEPORTATION

We have studied entangled coherent states in $2\times2$ Hilbert space. However, entangled coherent states are in fact continuous-variable states in infinite-dimensional Hilbert space. If $|B_2\rangle$ and $|B_4\rangle$ are considered in infinite-dimensional Hilbert space, they are not maximally entangled any more [27]. It is thus natural to ask such a question: how useful are the entangled coherent states for teleportation of continuous-variable states?

In the protocol proposed in [28] and demonstrated experimentally in [29] for continuous-variable teleportation, a two-mode squeezed state is used as the quantum channel and a joint homodyne measurement as Alice's measurement. An unknown quantum state in Eq. (13) may be teleported by a two-mode squeezed state, and the fidelity becomes unity for the limit of infinite squeezing.

Assume that a coherent state of an unknown amplitude is the state to teleport via an entangled coherent state, $|C_2\rangle$ in Eq. (2) with $\varphi'=0$. After a straightforward calculation, the fidelity is obtained [30]

$$f(\alpha)=\frac{1+\exp(-2\alpha_r^2)}{2[1+\exp(-4\alpha_r^2)]}. \tag{47}$$

Note that $f(\alpha)$ is independent from the amplitude of the unknown coherent state to teleport. It depends only on the real part of coherent amplitude $\alpha$ of the quantum channel. We find from Eq. (47) that the fidelity is always better than 1/2. The maximal value is about 0.6 when $\alpha_r\simeq\pm0.7$.

## VII. REMARKS

We have studied a mixed entangled coherent channel in $2\times2$ Hilbert space. We constructed orthogonal Bell bases with entangled coherent states to consider their entanglement and usefulness for teleportation in a dissipative environment. A pure entangled coherent channel is shown to teleport perfectly some quantum information. We investigated an experimental scheme for teleportation and entanglement concentration with a realizable Bell-measurement method.

It is found that a mixed entangled coherent state is always entangled regardless of the decay time. The larger initial amplitude $\alpha$, the more rapidly entanglement is degraded. This is in agreement with the fact that macroscopic quantum effects are not easily seen because it is more fragile.

Because a decohered entangled coherent channel is entangled at any decay time, its ensemble can be purified by an entanglement purification protocol [16] and used for reliable teleportation. On the other hand, it is shown that the optimal fidelity of teleportation attainable using a single pair is better than the classical limit 2/3 only until a certain characteristic time $r_c$, at which the mixedness of the channel becomes maximized. The maximal singlet fraction of the state is not more than 1/2 after $r_c$, even though it is still entangled.

Entanglement and mixedness [25] of quantum channels are important factors that affect teleportation. Until the characteristic time, both entanglement and purity decrease, which causes the decrease of teleportation fidelity. After the time $r_c$, the purity of the channel is recovered back even though entanglement decreases further. The experimental realization of purification for the mixed channels deserves further investigation.

## ACKNOWLEDGMENTS

This work has been supported by the U.K. Engineering and Physical Sciences Research Council (EPSRC) (GR/R 33304). J.L. thanks the Brain Korea 21 Project No. (D-1099) of the Korea Ministry of Education for financial support.


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