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Optomechanical scheme for the detection of weak impulsive forces

David Vitali,[1] Stefano Mancini,[1,2] and Paolo Tombesi[1]
[1]INFM, Dipartimento di Matematica e Fisica, Università di Camerino, I-62032 Camerino, Italy
[2]INFM, Dipartimento di Fisica, Università di Milano, Via Celoria 16, I-20133 Milano, Italy
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We show that a cooling scheme and an appropriate quantum nonstationary strategy can be used to improve the signal to noise ratio for the optomechanical detection of weak impulsive forces.

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A mechanical oscillator coupled to an optical mode by the radiation pressure provides a sensitive device able to detect very weak forces. Relevant examples are interferometers for the detection of gravitational waves [1] and atomic force microscopes [2]. Up to now, the major limitation to the implementation of sensitive optical measurements is given by thermal noise [3]. It has been proposed in Ref. [4] to reduce thermal noise by means of a feedback loop based on homodyning the light reflected by the oscillator, playing the role of a cavity mirror. The proposed scheme is a sort of continuous version of the stochastic cooling technique used in accelerators [5], because the homodyne measurement provides a continuous monitoring of the oscillator's position, and the feedback continuously ''kicks'' the mirror in order to put it in its equilibrium position. This proposal has been experimentally realized in Ref. [6], using the ''cold damping'' technique [7], which is physically analogous to that proposed in Ref. [4] and which amounts to applying a viscous feedback force to the oscillating mirror.

Both the ''stochastic cooling'' scheme of Ref. [4] and the cold damping scheme of Ref. [6] cool the mirror by overdamping it, thereby strongly decreasing its mechanical susceptibility at resonance. As a consequence, the oscillator does not resonantly respond to the thermal noise, yielding in this way an almost complete suppression of the resonance peak in the noise power spectrum, which is equivalent to cooling. However, the two feedback schemes cannot be directly applied to improve the detection of weak forces. In fact, the strong reduction of the mechanical susceptibility at resonance means that the mirror does not respond not only to the noise, but also to the signal, and we shall see that the signal to noise ratio (SNR) of the device in stationary conditions is actually never improved. Despite that, here we show how it is possible to design a *nonstationary* strategy able to significantly increase the SNR for the detection of *impulsive* classical forces acting on the oscillator. This may be of crucial importance in the field of metrology [8], as well as for the detection of gravitational waves [1]. We use a quantum treatment, allowing us to show why a classical approach provides an incomplete description of the optomechanical scheme.

Let us consider a simplified system with a single mechanical mode representing the movable mirror (with mass m and frequency ω_m) of a coherently driven optical cavity. The optomechanical coupling between the mirror and the cavity field is realized by the radiation pressure. In the adiabatic limit in which the mirror frequency is much smaller than the cavity free spectral range $c/2L$ (L is the cavity length) [9], one can focus on one cavity mode only (with annihilation operator b, frequency ω_c, and cavity decay rate γ_c) because photon scattering into other modes can be neglected. This adiabatic regime implies $\omega_m \ll \omega_c$, and therefore the generation of photons due to the Casimir effect, retardation and Doppler effects are completely negligible. The cavity mode is driven by a laser field with input power $\wp$ and frequency $\omega_0 \sim \omega_c$. The dynamics of the system can be described by the following set of coupled quantum Langevin equations (QLEs) (in the interaction picture with respect to $\hbar\omega_0 b^\dagger b$):

$$\dot{Q}(t) = \omega_m P(t), \tag{1a}$$

$$\dot{P}(t) = -\omega_m Q(t) - \gamma_m P(t) + G b^\dagger(t) b(t) + \mathcal{W}(t) + f(t), \tag{1b}$$

$$\dot{b}(t) = \left(-i\omega_c + i\omega_0 - \frac{\gamma_c}{2} \right) b(t) + 2iGQ(t)b(t) + E + \sqrt{\gamma_c}\, b_{in}(t), \tag{1c}$$

where Q and P are the dimensionless position and momentum operator of the movable mirror, γ_m is the mechanical damping rate, $G = (\omega_c/L)\sqrt{\hbar/2m\omega_m}$ is the coupling constant, $f(t)$ is the classical force to be detected, and $E = \sqrt{\wp\gamma_c/\hbar\omega_0}$. The noise terms in the QLEs are given by the usual input noise operator $b_{in}(t)$ [10], associated with the vacuum fluctuations of the continuum of electromagnetic modes outside the cavity, and by the random force $\mathcal{W}(t)$ describing the Brownian motion of the mirror caused by the coupling with other internal and external modes at the equilibrium temperature T. The optical input noise correlation function is $\langle b_{in}(t) b_{in}^\dagger(t') \rangle = \delta(t-t')$ [10], while that of the quantum Langevin force $\mathcal{W}(t)$ is given by [11,12] $\langle \mathcal{W}(t)\mathcal{W}(t') \rangle = (\gamma_m/2\pi\omega_m)[\mathcal{F}_r(t-t') - i\mathcal{F}_i(t-t')]$, where $\mathcal{F}_r(t) = \int_0^{\varpi} d\omega\, \omega \cos(\omega t) \coth(\hbar\omega/2k_B T)$, $\mathcal{F}_i(t) = \int_0^{\varpi} d\omega\, \omega \sin(\omega t)$ with ϖ the frequency cutoff of the reservoir spectrum. The QLEs (1), supplemented with the above correlation functions, provide an *exact* description of the system dynamics, valid at all temperatures [12].

In standard interferometric applications, the driving field is very intense. Under this condition the system is characterized by a semiclassical steady state with the internal cavity mode in a coherent state $|\beta\rangle$, and a new equilibrium position for the mirror, displaced by $G|\beta|^2/\omega_m$. Then the dynamics is well described by linearizing the QLEs (1) around the steady state, and if we rename with $Q(t)$ and $b(t)$ the operators describing the quantum fluctuations around the classical steady state, one gets

$$\dot{Q}(t)=\omega_m P(t), \tag{2a}$$

$$\dot{P}(t)=-\omega_m Q(t)-\gamma_m P(t)+G\beta[b(t)+b^\dagger(t)] +\mathcal{W}(t)+f(t), \tag{2b}$$

$$\dot{b}(t)=\left(-\frac{\gamma_c}{2}-i\Delta\right)b(t)+2iG\beta Q(t)+\sqrt{\gamma_c}b_{in}(t), \tag{2c}$$

where we have chosen the phase of the cavity mode field so that β is real and $\Delta=\omega_c-\omega_0-G^2\beta^2/\omega_m$ is the cavity mode detuning. We shall consider from now on $\Delta=0$, which can always be achieved by appropriately adjusting ω_0. In this case the dynamics becomes simpler, because only the phase quadrature $Y(t)=i[b^\dagger(t)-b(t)]/2$ is affected by the mirror position fluctuations $Q(t)$, while the amplitude field quadrature $X(t)=[b(t)+b^\dagger(t)]/2$ is not. The large cavity bandwidth limit $\gamma_c \gg G\beta$ is commonly considered, and in this case the cavity mode dynamics adiabatically follows that of the mirror position and it can be eliminated, i.e.,

$$Y(t)\simeq\frac{4G\beta}{\gamma_c}Q(t)+\frac{Y_{in}(t)}{\sqrt{\gamma_c}}, \tag{3}$$

where $Y_{in}(t)=i[b_{in}^\dagger(t)-b_{in}(t)]$. In this limit, the movable mirror can be used as a ponderomotive meter to detect a weak force $f(t)$ acting on it [13], which will be proportional to the displacement from the equilibrium position $Q(t)$. The measured quantity is the output homodyne photocurrent, $Y_{out}(t)=2\sqrt{\gamma_c}\eta Y(t)-\sqrt{\eta}Y_{in}^\eta(t)$ [10], where η is the detection efficiency, and $Y_{in}^\eta(t)$ is a generalized input noise, coinciding with the input noise $Y_{in}(t)$ in the case of perfect detection $\eta=1$, and taking into account the additional noise due to the inefficient detection in the general case $\eta<1$ [14]. This generalized input noise can be written as $Y_{in}^\eta(t)=i[b_\eta^\dagger(t)-b_\eta(t)]$, with $\langle b_\eta(t)b_\eta^\dagger(t')\rangle=\delta(t-t')$, and it is correlated with the input noise $b_{in}(t)$ according to $\langle b_{in}(t)b_\eta^\dagger(t')\rangle=\langle b_\eta(t)b_{in}^\dagger(t')\rangle=\sqrt{\eta}\delta(t-t')$ [14].

The output of the homodyne measurement may be used to devise a phase-sensitive feedback loop to control the dynamics of the mirror. The effect of the feedback loop has been described using quantum trajectory theory [15] and the master equation formalism in Ref. [4], and a classical description neglecting all quantum fluctuations in Ref. [6]. Here we shall use a more general description of feedback based on QLEs for Heisenberg operators, first developed in Ref. [16] and generalized to the nonideal detection case in Ref. [14]. In particular, we shall give a fully quantum description of the cold damping scheme [7]. The adoption of a quantum treatment is justified by the fact that, as we shall see, in the presence of feedback the radiation quantum noise has important effects, especially at low temperatures.

In the proposal of Ref. [4], feedback induces position shifts controlled by the output homodyne photocurrent $Y_{out}(t)$. This is described by an additional term in the QLE for a generic operator $\mathcal{O}(t)$ given by [14]

$$\dot{\mathcal{O}}_{fb}(t)=i\frac{\sqrt{\gamma_c}}{\eta}Y_{out}(t-\tau)[g_{sc}P(t),\mathcal{O}(t)], \tag{4}$$

where τ is the feedback loop delay time, and g_{sc} is the feedback gain. The feedback delay time is always much smaller than the typical time scale of the mirror dynamics and it can be neglected. After the adiabatic elimination of the cavity mode, the mirror QLEs become

$$\dot{Q}(t)=\omega_m P(t)+4G\beta g_{sc}Q(t)-\frac{g_{sc}}{2}\sqrt{\frac{\gamma_c}{\eta}}Y_{in}^\eta(t) +g_{sc}\sqrt{\gamma_c}Y_{in}(t), \tag{5a}$$

$$\dot{P}(t)=-\omega_m Q(t)-\gamma_m P(t)+\frac{2G\beta}{\sqrt{\gamma_c}}X_{in}(t)+\mathcal{W}(t)+f(t), \tag{5b}$$

where $X_{in}(t)=b_{in}^\dagger(t)+b_{in}(t)$.

In the cold damping scheme of Ref. [6], feedback is provided by the radiation pressure of another laser beam intensity-modulated by the time derivative of the output homodyne photocurrent, and therefore one has the additional term in the mirror QLEs:

$$\dot{\mathcal{O}}_{fb}(t)=\frac{i}{\eta\sqrt{\gamma_c}}\frac{dY_{out}(t-\tau)}{dt}[g_{cd}Q(t),\mathcal{O}(t)]. \tag{6}$$

Again, in the zero delay limit $\tau=0$, and adiabatically eliminating the cavity mode, one has

$$\dot{Q}(t)=\omega_m P(t), \tag{7a}$$

$$\dot{P}(t)=-\omega_m Q(t)-\gamma_m P(t)+\frac{2G\beta}{\sqrt{\gamma_c}}X_{in}(t)+\mathcal{W}(t)+f(t) -\frac{4G\beta g_{cd}}{\gamma_c}\dot{Q}(t)-\frac{g_{cd}}{\sqrt{\gamma_c}}\dot{Y}_{in}(t)+\frac{g_{cd}}{2\sqrt{\gamma_c\eta}}\dot{Y}_{in}^\eta(t). \tag{7b}$$

We have introduced the input noise $\dot{Y}_{in}(t)$, with the correlation function $\langle\dot{Y}_{in}(t)\dot{Y}_{in}(t')\rangle=-\ddot{\delta}(t-t')$, and the same holds for the associated generalized input noise $\dot{Y}_{in}^\eta(t)$.

The Eqs. (5) and (7) show that the two feedback schemes are not exactly equivalent. However, it is possible to see that they have very similar physical effects on the mirror dynamics, considering, for example, the Fourier transform of the mechanical susceptibility in the two cases, that is, $\chi_{sc}(\omega)=\omega_m/[\omega_m^2+g_1\gamma_m-\omega^2+i\omega(\gamma_m+g_1)]$ in the stochastic cooling feedback scheme of Ref. [4] ($g_1=-4G\beta g_{sc}$), and $\chi_{cd}(\omega)=\omega_m/[\omega_m^2-\omega^2+i\omega(\gamma_m+g_2)]$ in the cold damping feedback scheme of Ref. [6] ($g_2=4G\beta g_{cd}\omega_m/\gamma_c$). These expressions show that in both schemes the main effect of feedback is the modification of mechanical damping $\gamma_m \rightarrow \gamma_m + g_i$. In the stochastic cooling scheme, one also has a frequency renormalization $\omega_m^2 \rightarrow \omega_m^2 + g_1 \gamma_m$, which is, however, usually negligible since $\gamma_m \ll \omega_m$. If the gains g_i are appropriately chosen, one has a significant increase of damping and this increase is at the basis of the cooling mechanism proposed in Ref. [4] and realized in Ref. [6]. In fact, the mechanical susceptibility at resonance is inversely proportional to the damping coefficient and, in the presence of feedback, the oscillator becomes much less sensitive to the thermal noise, yielding a complete suppression of the resonance peak in the noise power spectrum.

The classical treatment of Ref. [6] (see also [17]) is equivalent to replace the QLEs (5) and (7) with classical stochastic equations in which the back action noise $2G\beta X_{in}(t)/\sqrt{\gamma_c}$ and the feedback-induced noise terms proportional to $Y_{in}(t)$, $Y_{in}^{\eta}(t)$, $\dot{Y}_{in}(t)$ and $\dot{Y}_{in}^{\eta}(t)$ are neglected. As a consequence, within the classical approach, the only effect of feedback is damping renormalization $\gamma_m \rightarrow \gamma_m + g_i$, while the increase of noise for increasing feedback gain, due to the presence of the feedback-induced terms, is completely missed. This gives the wrong impression that, at least in principle, unlimited cooling can be achieved for increasing feedback gain. With a quantum treatment, the tradeoff between damping renormalization and feedback-induced noise leads to the existence of an *optimal feedback gain*, corresponding to the best achievable cooling. This limit on thermal noise suppression would be present even in the case of an ideal feedback loop with no electronic noise. It is a manifestation of quantum effects, showing the conceptual difference with a purely classical description of cooling.

When the classical force we want to detect is characterized by a characteristic frequency, say, $f(t) = f_0 \exp[-(t - t_1)^2/2\sigma^2]\cos(\omega_f t)$, spectral measurements are commonly performed and the detected signal is

$$S(\omega) = \left| \int_{-\infty}^{+\infty} dt e^{-i\omega t} \langle Y_{out}(t) \rangle F_{T_m}(t) \right|, \tag{8}$$

where $F_{T_m}(t)$ is a "filter" function, approximately equal to one in the time interval $[0,T_m]$ in which the spectral measurement is performed, and equal to zero otherwise. Using Eq. (3) and the input-output relation, the signal can be rewritten as

$$S(\omega) = \frac{8G\beta\eta}{2\pi\sqrt{\gamma_c}} \left| \int_{-\infty}^{+\infty} d\omega' \chi(\omega') \tilde{f}(\omega') \tilde{F}_{T_m}(\omega - \omega') \right|, \tag{9}$$

where $\tilde{f}(\omega)$ and $\tilde{F}_{T_m}(\omega)$ are the Fourier transforms of the force and of the filter function, respectively, and $\chi(\omega)$ is equal to $\chi_{sc}(\omega)$ or $\chi_{cd}(\omega)$, according to the feedback scheme considered. The noise associated with the measurement of the signal of Eq. (8) is given by

$$N(\omega) = \left\{ \int_{-\infty}^{+\infty} dt F_{T_m}(t) \int_{-\infty}^{+\infty} dt' F_{T_m}(t') e^{-i\omega(t-t')} \times \langle Y_{out}(t) Y_{out}(t') \rangle_{f=0} \right\}^{1/2}, \tag{10}$$

where the subscript $f=0$ means evaluation in the absence of the external force. Using again Eq. (3) and the input noises correlation functions, the spectral noise can be rewritten as

$$N(\omega) = \left\{ \frac{(8G\beta\eta)^2}{\gamma_c} \int_{-\infty}^{+\infty} dt F_{T_m}(t) \int_{-\infty}^{+\infty} dt' F_{T_m}(t') \times e^{-i\omega(t-t')} C(t,t') + \eta \int_{-\infty}^{+\infty} dt F_{T_m}(t)^2 \right\}^{1/2}, \tag{11}$$

where $C(t,t') = \langle Q(t)Q(t') + Q(t')Q(t) \rangle/2$ is the symmetrized correlation function of the oscillator position. Spectral measurements are usually performed in the stationary case, that is, using a measurement time T_m much larger than the oscillator relaxation time, $T_m \gg 1/\gamma_m$. In this limit one has $F_{T_m}(t) \simeq 1 \ \forall \ t$, and the signal $S(\omega)$ simply becomes $S(\omega) = 8G\beta\eta|\chi(\omega)\tilde{f}(\omega)|/2\pi\sqrt{\gamma_c}$. The oscillator in this case is relaxed to equilibrium and $C(t,t')$ in Eq. (11) is replaced by the *stationary* correlation function $C(t-t')$. Defining the measurement time T_m so that $T_m = \int dt F_{T_m}(t)^2$, Eq. (11) assumes the usual form

$$N(\omega) = \left\{ \left[\frac{(8G\beta\eta)^2}{\gamma_c} N_Q(\omega) + \eta \right] T_m \right\}^{1/2}, \tag{12}$$

where $N_Q(\omega) = \int d\tau e^{-i\omega\tau} C(\tau)$. In this stationary case, the SNR can be calculated for both feedback schemes, and one gets

$$\frac{S(\omega)}{N(\omega)}^{st} = |\tilde{f}(\omega)| \left\{ T_m \left[\frac{\gamma_m \omega}{2\omega_m} \coth\left(\frac{\hbar\omega}{2kT} \right) + \frac{4G^2\beta^2}{\gamma_c} + \frac{\gamma_c}{64G^2\beta^2\eta} \left(\frac{g_2^2\omega^2}{\omega_m^2} + \frac{1}{|\chi_{cd}(\omega)|^2} \right) \right] \right\}^{-1/2} \tag{13}$$

for the cold damping scheme, and a similar expression for the stochastic cooling scheme [$g_2^2\omega^2$ is replaced by $g_1^2(\omega^2 + \gamma_m^2)$ and χ_{cd} by χ_{sc}]. In both cases, feedback does not improve this stationary SNR at any frequency, due to the g_i^2 term. This is not surprising because the effect of feedback is to decrease the mechanical susceptibility at resonance, so that the oscillator is less sensitive not only to the noise but also to the signal.

However, in the case of an *impulsive* force with a time duration $\sigma \ll 1/\gamma_m$, the force spectrum could still be well reproduced even if a much smaller time T_m, such that $\sigma \ll T_m \ll 1/\gamma_m$, is used. This corresponds to a nonstationary situation because the system is far from equilibrium during the whole measurement. In this case, the noise spectrum is very different from the stationary form of Eq. (12) and it is mostly determined by the initial state of the oscillator. It is therefore quite natural to devise a strategy in which the feedback cooling scheme is applied before the measurement, so that the state at the beginning of the measurement is just the cooled, equilibrium state in the presence of feedback, and turn off the feedback during the spectral measurement. In this way the noise remains small during the whole measurement because the heating time $1/\gamma_m$ is much larger than T_m, while at the same time the signal is not significantly suppressed because the mechanical susceptibility is just that in the absence of feedback. One expects that as long as the measurement time is sufficiently small, $T_m \ll 1/\gamma_m$, the SNR for the detection of the impulsive force [which has now to be evaluated using the most general expressions (9) and (11)] can be significantly increased by the above nonstationary strategy.

This scheme can be straightforwardly applied whenever the "arrival time" t_1 of the impulsive force is known: feedback has to be turned off just before the arrival of the force. However, the scheme can be easily adapted to the case of an impulsive force with an unknown arrival time, as it is the case of a gravitational wave passing through an interferometer. In this case, it is convenient to repeat the process many times, i.e., subject the oscillator to cooling-heating cycles. In fact, cyclic cooling has been proposed, in a qualitative way, to cool the violin modes of a gravitational waves interferometer in [17]. Feedback is turned off for a time T_m during which the spectral measurement is performed and the oscillator starts heating up. Then feedback is turned on and the oscillator is cooled, and then the process is iterated. Cyclic cooling is efficient if the cooling time T_{cool}, which is of the order of $(\gamma_m + g_i)^{-1}$, is much smaller than T_m. This is verified at sufficiently large gains and it has been experimentally proved in [17]. In the impulsive force limit $\sigma \ll T_m$, the performance of the scheme is well characterized by a time-averaged SNR, i.e.,

$$\langle S/N(\omega)\rangle \simeq \frac{1}{T_m + T_{cool}} \int_0^{T_m} dt_1 \frac{S(\omega, t_1)}{N(\omega)}. \tag{14}$$

This average SNR can be significantly improved by cyclic cooling, as it is shown in Fig. 1, where $\langle S/N(\omega)\rangle$ is plotted (full line) in the case of the impulsive Gaussian force chosen above: one has an improvement by a factor 10 at resonance with respect to the no feedback case (dashed line). Parameter values are those of Ref. [6], except that we have considered $T = 4$ K, and the optimal value of the feedback gain $g_2 \simeq 305\gamma_m$ maximizing the SNR. The dotted line refers to the classical $\langle S/N(\omega)\rangle$ calculated neglecting all quantum noises: the noise is underestimated, with a 15% error in the SNR at resonance. This shows that quantum noise has an appreciable effect already at liquid He temperatures and that a fully quantum treatment is needed for a faithful description of the physics. It is possible to see that at $T = 300$ K, at the corresponding optimal feedback gain, there is instead no appreciable difference between the classical and quantum predictions, and that $\langle S/N(\omega_m)\rangle$ becomes 16 times larger than that with no feedback.

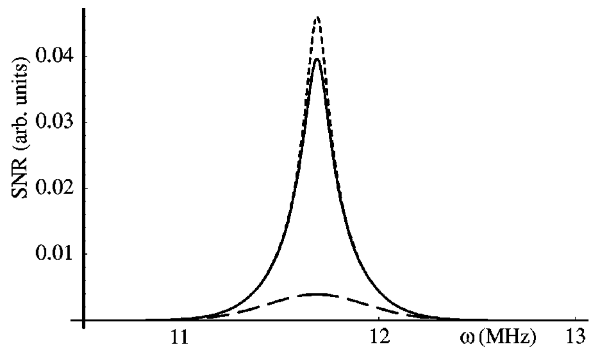

FIG. 1. The averaged spectral SNR of Eq. (14) in the presence of the cold damping feedback scheme with $g_2 = 82.4$ kHz in the full quantum treatment (full line), in the classical approximation (dotted line), and without feedback (dashed line). Other parameter values are: $\omega_f = \omega_m = 11.7$ MHz, $\gamma_m = 270$ Hz, $G^2\beta^2/\gamma_c = 10.3$ Hz, $T = 4$ K, $\eta = 0.99$, $\sigma = 3.7$ μsec, $T_m = 11$ μsec, $T_{cool} = 2$ μsec.

In conclusion, we have presented a completely quantum description of the cold damping feedback scheme [7], and we have shown how the cooling schemes of Refs. [4,6] may be used, within an appropriate nonstationary strategy, to improve the detection of weak impulsive forces.

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