Nonadiabatic approach to quantum optical information storage

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We show that there is no need for adiabatic passage in the storage and retrieval of information in the optically thick vapor of Lambda-type atoms. This information can be mapped into and retrieved out of long-lived atomic coherence with nearly perfect efficiency by strong writing and reading pulses with steep rising and falling edges. We elucidate similarities and differences between the ''adiabatic'' and ''instant'' light storage techniques, and conclude that for any switching time, an almost perfect information storage is possible if the group velocity of the signal pulse is much less than the speed of light in the vacuum c and the bandwidth of the signal pulse is much less then the width of the two-photon resonance. The maximum loss of the information appears in the case of instantaneous switching of the writing and reading fields compared with adiabatic switching, and is determined by the ratio of the initial group velocity of the signal pulse in the medium and speed of light in the vacuum c, which can be very small. Quantum restrictions to the storage efficiency are also discussed.

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I. INTRODUCTION

A possibility of classical storage of optical information by employing atomic coherence was recognized early on in stimulated photon-echo experiments in two-level systems [1] and multilevel media [2–4]. Storage of optical information here means that one seeks to ''store'' and ''retrieve'' an optical pulse on demand without distortion of the pulse shape.

Usually, the photon-echo technique does not provide complete quantum information about a signal pulse, i.e., the retrieved light pulse has a different shape compared with the initial shape. However, two- and three- excitation-pulse photon echoes can be used [5], but the decay time of the storage of information is determined by the transverse relaxation time, which is quite short.

To increase the storage time, photon echo based on Raman transitions has been realized [6,7]. By using stimulated photon echo via spectrally ordered long-lived Zeeman coherences [2] not only has the storage time been elegantly increased, but also a pulse-shape storage has been achieved.

Quite recently, it has been predicted that coherent atomic media are capable of demonstrating nonlinear optical effects at the few-photon level [8,10,11] and this gives an opportunity to store and retrieve quantum states of light [12,13]. Proof-of-principle experiments confirm this theoretical prediction [14–16] in the classical light limit.

The basic idea of light storage via atomic coherence can be understood in terms of light interaction with Λ-type atoms. Strong ''writing'' and weak ''signal'' light pulses propagate in a gas of three-level Λ type atoms and excite a spatial profile of a long-lived coherence between ground states $|c\rangle$ and $|b\rangle$ of the atom (Fig. 1). This coherence profile stores information about these pulses after they have left or have been absorbed by the medium. Subsequently, sending a strong ''reading'' pulse into the medium results in its Raman scattering off the atomic coherence and generation of a ''retrieved'' pulse. Ideally, the retrieved pulse can be identical to the signal pulse, i.e., it possesses: (i) the same carrying frequency, (ii) the same profile and quantum statistics, and (iii) propagates in the same direction as the signal pulse [12]. That is why the term ''light storage'' instead of the term ''information storage'' sometimes is used in the literature.

Concerning the issue of how we should think about ''stored light,'' we point to a similar situation in quantum teleportation. There, we are sending information from one point in space to another and claim that by using a light beam we may, in fact, teleport an atom having one state vector to an atom some place else in space having the identical state vector. One could argue that we are not teleporting atoms, we are simply teleporting information. However, all atoms are the same and so, as one argues in the teleportation game, once we have prepared any atom in a state identical to any other atom, we have indeed, teleported the atom. Now the same sort of logic could be applied to the ''stopped light'' experiments. If the ''readout'' in experiments [15,14] is, in principle, identical to the incident light and the quantum state of the light can be reproduced precisely in the spirit of the teleportation, then we might say that the light was ''stored'' in the medium and released after that.

However, generally speaking, the properties of the retrieved pulse depend on the reading pulse. For example, if the reading field pulse is centered about a frequency other than that of the the writing field, and propagates in the opposite direction to the writing field, then the retrieved pulse propagates in the same direction as the reading field and has indeed a different frequency from the incident writing field [16]. This is impossible if the light is really stored in the medium. Therefore, the exchange of the terms ''stored information'' and ''stored light'' is not always valid.

As was pointed out in Refs. [12], the necessary condition for the retrieved pulse to resemble the signal pulse is the adiabaticity. The switching time for the reading and writing

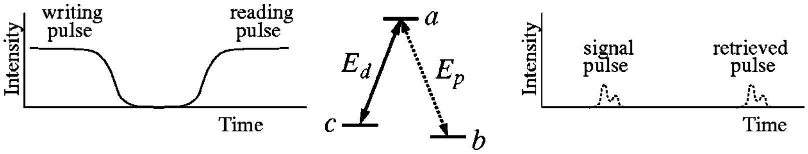

FIG. 1. Energy-level scheme and diagram of optical information storage experiment.

pulses should be long enough to avoid population of the excited atomic state $|a\rangle$ and subsequent spontaneous emission, that destroy the coherence and erode the stored signal. However, it has been noted in Ref. [15] that the adiabaticity condition can be relaxed. The importance of the question concerning the role of the adiabaticity in the process of quantum information storage was also recognized in [17].

In this paper, we show by numerical simulations that signal pulse information can be stored and retrieved even if the writing and reading pulses have rectangular shapes, i.e., very small rising and falling times. We also present simple analytical calculations that confirm our numerical simulations. The ''instantaneous'' switching is possible because strong fields are applied to the transition $|c\rangle \rightarrow |a\rangle$ with a nearly empty ground state so the atomic medium is not able to slow down the switching time. It is worth noting here that the switching time is less than the time of the light travel through the distance equal to the length of the sample L used for the information storage.

Our analytical calculations prove that storage of quantum information due to instantaneous switching of the writing and reading fields has features both similar to and essentially different from the properties of the storage based on the adiabatic passage. The common feature of both techniques is the coherence between atomic levels created by the electromagnetic fields. It is this coherence that is responsible for the signal pulse ''storage'' and restoration.

The difference lies in the energy exchange mechanisms between the fields and the medium. The case of adiabatic passage is more flexible. The complete pulse restoration appears for any field strength. It works for very strong writing and reading fields, analyzed in [12], such that the so-called ''dark polariton'' transforms to the signal field almost completely. For such strong fields, the signal pulse initially has group velocity v_g comparable with the speed of light in the vacuum c. However, as we show, even for weaker writing fields, adiabatic passage allows lossless storage of the signal light if the signal pulse is long enough and does not experience absorption in the coherent medium.

On the other hand, both the initial amplitude of the writing field and the group velocity should be small in the nonadiabatic case to store information effectively. The abrupt switching of the driving field leads to the absorption of the ''free field'' part of the ''dark polariton.'' Only the ''bound part,'' i.e., low-frequency atomic coherence, survives. The ratio between ''free'' and ''bound'' parts of the polariton is equal to v_g/c. Therefore, the smaller the initial group velocity of the signal pulse, the better the writing-reading quality.

It is instructive to mention here that the v_g/c ratio between ''free'' and ''bound'' parts of the polariton is of a general nature. It is true for any kind of slow propagation of a light pulse in Λ-type media, in two-level media, in periodic heterostructures, etc. [18]. The signal electromagnetic energy is always depleted once it is inside the medium. However, for a two-level medium, the energy is transferred to the atomic degrees of freedom [19] on the entrance to the medium, while in a Λ medium, the signal pulse is transferring a lion's share of its energy to the writing field. This part of the signal energy leaves the medium with the speed of light in the vacuum c. To restore the signal, the coupling field scatters on the atomic coherence on the exit of the medium. The delay between these stimulated absorption and scattering is determined by the group velocity.

As a result of our study of the adiabatic and nonadiabatic information storage techniques, we conclude that for *any* switching time, almost perfect information storage is possible if (i) the group velocity of the signal pulse is much less than c, (ii) the bandwidth of the signal pulse is much less than the width of the two-photon resonance, i.e., the signal pulse enters the medium without reflection and absorption. The maximum absorption appears in the case of instantaneous switching of the writing and reading fields and is determined by the ratio v_g/c.

We show that the quantum efficiency of information storage is almost perfect, i.e., any nonclassical state of light can be stored and retrieved without destroying it. It is this aspect of the process that allows one to say that system can ''store'' light.

Finally, we numerically simulate the possibility of ''time reversing'' the stored light, wherein the reading field applied to the original signal transition generates a temporally inverted pulse compared with the signal pulse [16].

II. BASIC EQUATIONS

Let us consider the interaction of two copropagating plane electromagnetic waves $E_p(z,t)\exp[-i(\nu_p t - k_p z)]$ and $E_d(z,t)\exp[-i(\nu_d t - k_d z)]$ with a gas of three-level Λ-type atoms shown in Fig. 1, where $E_p(z,t)$ and $E_d(z,t)$ are the slowly varying envelops, ν_p and ν_d are the carrier frequencies, and k_p and k_d are the wave numbers of the fields. Field E_p interacts with the transition $|a\rangle \rightarrow |b\rangle$, while field E_d interacts with the transition $|a\rangle \rightarrow |c\rangle$. We consider the case of the resonant tuning, i.e., the carrier frequencies of the waves coincide with the frequencies of the corresponding atomic transitions $\nu_p = \omega_{ab}$ and $\nu_d = \omega_{ac}$.

To describe the propagation of the light pulses through the atomic vapor, we use Maxwell-Bloch equations in the slowly varying amplitude and phase approximation. In this approximation, the Maxwell equations can be written as

$$\frac{\partial \alpha}{\partial z} + \frac{\partial \alpha}{\partial ct} = i\eta\rho_{ab}, \tag{1}$$

$$\frac{\partial \Omega}{\partial z} + \frac{\partial \Omega}{\partial ct} = i\eta\rho_{ac}, \tag{2}$$

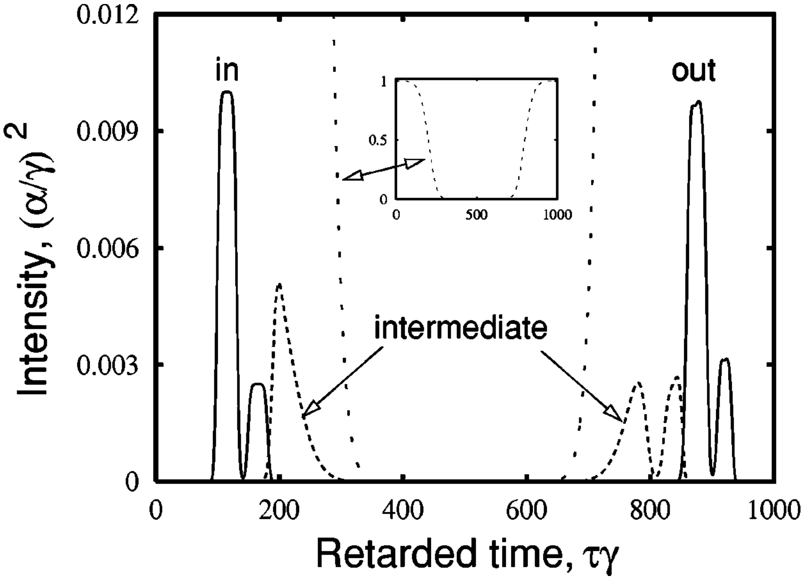

FIG. 2. Adiabatic "storage of light." Intensity of the signal field vs retarded time. The probe pulse denoted as "in" enters the medium. Switching off the writing field (shown by the dotted line) leads to the signal absorption. Subsequent switching on of the reading field leads to signal restoration. The retrieved pulse is denoted as "out." Intermediate profiles of signal and retrieved pulses (signal pulse not yet entirely absorbed and retrieved pulse not yet entirely emitted) are plotted as a dashed line. The full-scale drive field is shown in the inset. Retarded time and Rabi frequencies are presented in units of γ. Switching time is about $200/\gamma$.

where $\alpha=\wp_{ab}E_p/\hbar$ and $\Omega=\wp_{ac}E_d/\hbar$ are the Rabi frequency of the probe and drive fields, $\wp_{ab}$ and $\wp_{ac}$ are the dipole moments, ρ_{ab} and ρ_{ac} are the matrix elements of the corresponding atomic transitions, and c is the vacuum speed of the light. The coupling constant $\eta=3\lambda^2 N\gamma/8\pi$ depends on the density of the atomic vapor N and the wavelength of the atomic transitions $\lambda_{ac}\simeq\lambda_{ab}=\lambda$.

The Bloch equations for the matrix elements of the populations and polarizations of the atoms are

$$\dot{\rho}_{cc}=2\gamma_{ac}\rho_{aa}+i\Omega(\rho_{ac}-\rho_{ca}), \tag{3}$$

$$\dot{\rho}_{bb}=2\gamma_{ab}\rho_{aa}+i\alpha(\rho_{ab}-\rho_{ba}), \tag{4}$$

$$\dot{\rho}_{ab}=-(\gamma_{ab}+\gamma_{ac})\rho_{ab}-i\alpha(\rho_{aa}-\rho_{bb})+i\Omega\rho_{cb}, \tag{5}$$

$$\dot{\rho}_{ca}=-(\gamma_{ab}+\gamma_{ac})\rho_{ca}+i\Omega^*(\rho_{aa}-\rho_{cc})-i\alpha^*\rho_{cb}, \tag{6}$$

$$\dot{\rho}_{cb}=-\gamma_{cb}\rho_{cb}-i\alpha\rho_{ca}+i\Omega\rho_{ab}. \tag{7}$$

These equations are to be supplemented by the population conservation law

$$\rho_{aa}+\rho_{bb}+\rho_{cc}=1. \tag{8}$$

The solution of Eqs. (1)–(8) gives the complete evolution of the atom-field system at any moment of time. Because the general analytical study of the set is difficult, we first solve the problem numerically and make simple analytical calculations in order to better understand the physics. In the following, we assume that the coherence decay rate $\gamma_{bc}=0$, while $\gamma_{ac}\simeq\gamma_{ab}=\gamma/2$.

III. NUMERICAL SIMULATIONS

We solve the propagation problem numerically for a homogeneously broadened gas of Λ atoms. The results for

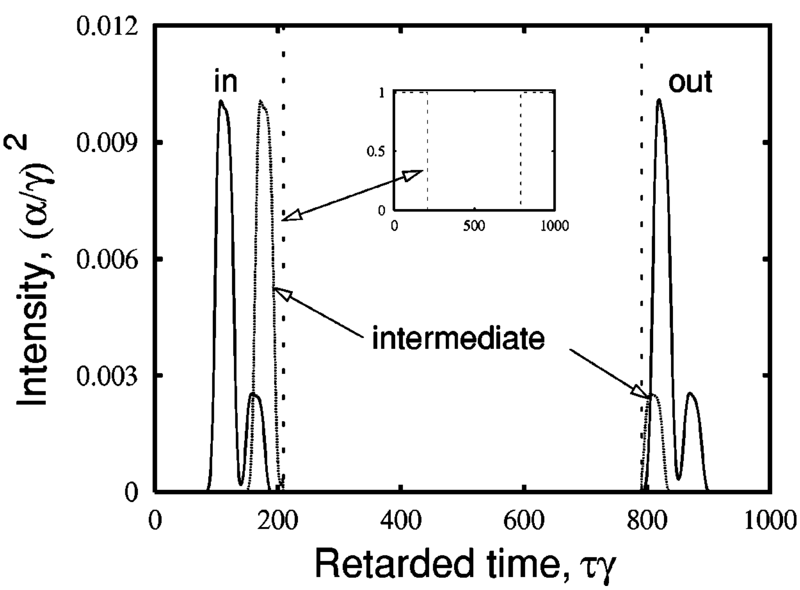

FIG. 3. "Storage of light" without adiabatic passage. All is the same as in Fig. 1 except the writing field is switching off and the reading field is switching on immediately. Numerically, the switching time was determined by the step of the grid $\delta t\simeq 0.1/\gamma$. There is no adiabaticity in this case.

adiabatic and instantaneous switching times are shown in Figs. 2 and 3, respectively.

In the numerical simulations, we first consider the case of slow switching times for the writing and reading pulses interacting with $|a\rangle \rightarrow |c\rangle$ atomic transitions so that the adiabaticity condition is fulfilled, i.e., the switching time T is much longer than $\sqrt{\eta\gamma L}/|\Omega|^2$ [9,10], $\gamma/|\Omega|^2$, and $1/\gamma$ [12], where Ω is the initial Rabi frequency of the writing pulse, and γ is the decay rate of both allowed atomic transitions.

The signal pulse enters the medium that has interacted with the driving field for a long time so that all atomic population is optically pumped into the state $|b\rangle$. When the entire signal pulse enters into the medium, we switch off the writing field, and this leads to the absorption of the signal. The energy associated with the signal pulse leaves the medium with the writing field. Really, our calculations show that in the adiabatic approximation the population of the excited state $|a\rangle$ is almost zero at any arbitrary moment of time. Therefore, the energy of internal degrees of freedom of the medium is vanishingly small. The absorption of the probe pulse appears via adiabatic passage $|b\rangle \rightarrow |a\rangle \rightarrow |c\rangle$ during which the writing field is amplified and the signal field is deamplified. Because the speed of the writing field in the medium is nearly equal to the speed of light in the vacuum, the energy transferred from the signal pulse to the writing field due to this stimulated quasi-Raman process, leaves the medium almost immediately.

After some delay, we launch the reading pulse probing the $|c\rangle \rightarrow |a\rangle$ transition and look for the pulse emitted on the frequency of the $|b\rangle \rightarrow |a\rangle$ transition. The shape of this retrieved pulse coincides with the shape of the signal pulse. Hence, our exact numerical simulations confirm the approximate calculations presented in [12].

We should mention here that in our numerical simulations, the pulse is always in the medium. We assume that any long probe pulse is not absorbed or distorted passing a coherently driven atomic medium because of electromagnetically induced transparency [21], so instead of solving a boundary-value problem, we solve an initial problem.

Let us consider now short switching times T, much less than $\sqrt{\eta\gamma L}/|\Omega|^2$, $\gamma/|\Omega|^2$, and $1/\gamma$, such that there is no adiabatic following of atomic coherence to the driving field. However, as can be easily seen from Fig. 3, the shape of the retrieved pulse is the same as the shape of the signal pulse. Therefore, nonadiabatic writing and reading processes leads to the same results as the adiabatic ones. To understand this result, we now turn to simple analytical calculations.

To compare how behavior of the atomic coherence ρ_{bc} depends on the behavior of the signal/retrieved light, we present Fig. 4. It is easy to see that the temporal shape of the coherence resembles the shape of the signal pulse when writing/reading fields are present. At the moment of signal pulse absorption (switching off the writing field) the coherence "freezes" on its value that it had had just a moment before the absorption process started. The spatial shape of the stored coherence is similar to the shape of the signal pulse [shown by dots in Fig. 4(b)].

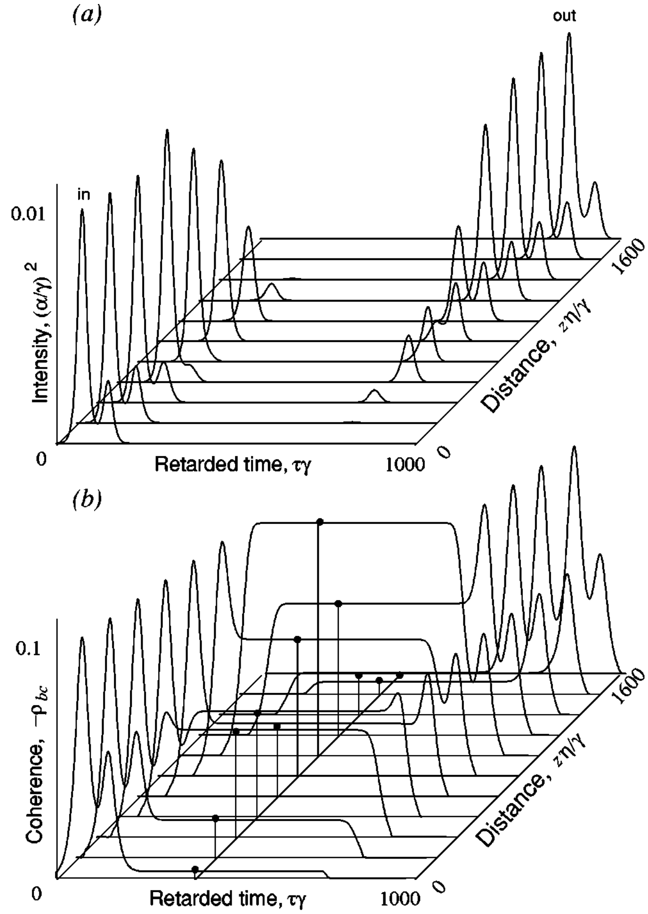

FIG. 4. (a) Normalized intensity of the signal field vs normalized retarded time and vs normalized propagation distance. The probe pulse denoted as "in" enters the medium. Switching off the writing field (not shown here, see Fig. 1 inset) leads to the signal absorption. Subsequent switching on of the reading field leads to signal restoration. The retrieved pulse is denoted as "out." (b) Atomic coherence $-\rho_{bc}$ vs normalized retarded time and vs normalized propagation distance. Because writing/reading field peak intensity is $(\Omega/\gamma)^2 = 1$ here, it is easy to see that $-\rho_{bc} = \alpha/\Omega$. The dependence of the spatial coherence profile for a fixed retarded time where the electromagnetic field is equal to zero is shown by dots.

IV. ADIABATIC FIELD SWITCHING

To consider the case of adiabatic field switching, we assume that the signal field is much weaker than the drive field at any point of time and at any point of space. This assumption follows from the exact numeric simulation discussed above. In this case, almost all atomic population is in the state $|b\rangle$ and the writing and reading fields propagate in the medium as in the vacuum, i.e.,

$$\frac{\partial\Omega}{\partial z} + \frac{\partial\Omega}{\partial ct} = 0. \tag{9}$$

Because of adiabaticity, the atoms stay in the "dark state" with respect to the fields and, therefore, the atomic coherence is

$$\rho_{cb} = -\frac{\alpha}{\Omega}. \tag{10}$$

Following [12], we construct a function

$$\Psi = \rho_{cb} - \frac{\Omega^*}{\eta c}\alpha \tag{11}$$

that obeys the wave equation

$$\frac{\partial \Psi}{\partial z} + \frac{|\Omega|^2 + \eta c}{|\Omega|^2 c}\frac{\partial \Psi}{\partial t} = 0, \tag{12}$$

which introduces the group velocity $v_g = c|\Omega|^2/(|\Omega|^2 + \eta c)$.

Equation (11) represents a ‘‘dark polariton.’’ The first term in the right-hand side (rhs) of the equation is the ‘‘bound’’ part of the polariton, while the second term is the ‘‘free’’ part of the polariton. The polariton is a normal mode of the propagation problem. Once created, it propagates without absorption and distortion.

It is easy to see from Eq. (11) that if the maximum amplitude of the writing and reading pulses is large enough $|\Omega|^2 \gg \eta c$, the process of the storage of the information in the medium is almost perfect. Really, the signal pulse enters the medium, adiabatically slows down with decreasing writing field, and disappears, transferring all its properties to the medium. The reading field leads to the adiabatic transfer of the medium properties back to the retrieved pulse, which is the same as the signal pulse.

However, when the signal pulse is long enough so that its spectral width is narrower than the width of the resonance of the electromagnetically induced transparency, there is no need for very strong writing-reading fields and the inequality $|\Omega|^2 \gg \eta c$ is redundant. In other words, when each spectral harmonic of the probe pulse is in the transparency window, the probe pulse propagates without absorption. This propagation occurs for moderate drive intensities, much less than the intensities determined by inequality $|\Omega|^2 \gg \eta c$. The same logic is applicable for the writing-reading process. The signal pulse enters the medium, transferring its energy to the driving field without absorption or reflection, and the restored pulse leaves the medium without absorption too, that makes lossless information storage possible.

V. NONADIABATIC FIELD SWITCHING

To study the light ‘‘storing’’ process using abrupt switching of the writing and reading pulses, we start with the simplified coupled Maxwell and density-matrix Eqs. (1)–(8) for the probe pulse propagation that can be rewritten as

$$\frac{\partial \alpha}{\partial z} + \frac{\partial \alpha}{\partial ct} = -\frac{\eta}{\gamma}[\alpha + \Omega\rho_{cb}], \tag{13}$$

$$\frac{\partial \rho_{cb}}{\partial t} = -\frac{\Omega}{\gamma}[\alpha + \Omega\rho_{cb}], \tag{14}$$

where we assume that $\rho_{bb} \simeq 1$, while $\rho_{cc} = \rho_{aa} = 0$, and

$$\rho_{ab} = \frac{i\alpha}{\gamma} + \frac{i\Omega}{\gamma}\rho_{cb}. \tag{15}$$

For simplicity, we also assume here that writing and reading pulses are applied to the same transition and that both these pulses are produced by the same laser with the same Rabi frequency Ω.

The set of equations (13), (14) is linear in the probe field. Therefore, we can use Fourier decomposition to solve the problem

$$\alpha(z,t) = \int_{-\infty}^{\infty} \alpha_k e^{ikz} dk, \tag{16}$$

$$\rho_{cb}(z,t) = \int_{-\infty}^{\infty} \rho_{k,cb} e^{ikz} dk. \tag{17}$$

The k modes of the probe field α_k and coherence $\rho_{k,cb}$ obey the ordinary differential equations

$$ik\alpha_k + \frac{\partial \alpha_k}{\partial ct} = -\frac{\eta}{\gamma}[\alpha_k + \Omega\rho_{k,cb}], \tag{18}$$

$$\frac{\partial \rho_{k,cb}}{\partial t} = -\frac{\Omega}{\gamma}[\alpha_k + \Omega\rho_{k,cb}]. \tag{19}$$

If the signal pulse duration is much longer than $\gamma/\Omega^2(t=0)$, i.e., the spectrum of the signal fits the electromagnetically induced transparency (EIT) window, the relation between $\rho_{k,cb}$ and α_k is

$$\rho_{k,cb} = -\frac{\alpha_k}{\Omega}. \tag{20}$$

Switching the driving field off, we write a probe pulse onto medium $\rho_{k,cb}$. The coherence *preserves* the value that it has before the switching, i.e., $\rho^0_{k,cb} = -\alpha^0_k/\Omega$, where α^0_k describes the signal pulse at the moment of switching off the writing field.

Now let us find out the condition under which we can restore the probe field. To do so we solve the initial value problem Eqs. (18), (19), wherein the initial value of the coherence is left after switching off the write pulse and there is no initial probe field α^0_k.

The general solution of Eqs. (18), (19) can be written as

$$(\alpha_k, \rho_{k,cb}) = C_1 \vec{e}_1 e^{\lambda_1 t} + C_2 \vec{e}_2 e^{\lambda_2 t}, \tag{21}$$

where $\vec{e}_1 = (\beta_1, 1)$, $\vec{e}_2 = (\beta_2, 1)$, $\beta_1 = -(\lambda_{1k}\gamma + \Omega^2)/\Omega$, $\beta_2 = -(\lambda_{2k}\gamma + \Omega^2)/\Omega$, and $\lambda_{1k,2k}$ are the eigenvalues

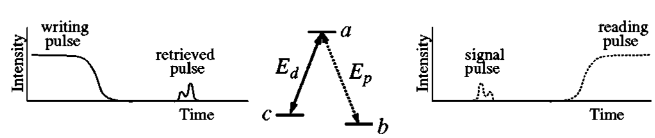

FIG. 5. Energy-level scheme and diagram of the ‘‘phase conjugated’’ optical information storage experiment.

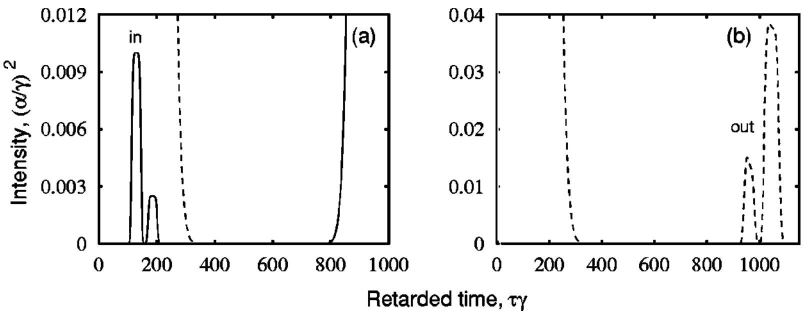

FIG. 6. "Time reversing" of the signal pulse. Intensity of the fields vs retarded time. (a) The signal pulse denoted as "in" enters the medium. Following switching off of the writing field, shown by the dotted line, leads to the signal absorption. (b) Subsequent switching on of the reading field leads to probe restoration in the channel of the writing field. Restored pulse, denoted as "out," is time reversed and amplified. The full scale writing and reading pulses are as in Fig. 2.

$$\lambda_{1k,2k} = -\frac{\Gamma_k + \frac{\Omega^2}{\gamma}}{2} \pm \sqrt{\left(\frac{\Gamma_k - \frac{\Omega^2}{\gamma}}{2}\right)^2 + \frac{\eta c \Omega^2}{\gamma^2}}, \quad (22)$$

$\Gamma_k = ikc + \eta c/\gamma$, C_1 and C_2 are some dimensionless constants.

The solution of Eqs. (18), (19) with the initial condition $\rho^0_{k,cb} = -\alpha^0_k/\Omega$, thus, has the form

$$\rho_{cb} = -\rho^0_{k,cb}\left(\frac{\beta_2}{\beta_1-\beta_2}e^{\lambda_{1k}t} + \frac{\beta_1}{\beta_2-\beta_1}e^{\lambda_{2k}t}\right), \quad (23)$$

$$\alpha_k = \frac{\beta_1\beta_2}{\beta_2-\beta_1}\rho^0_{k,cb}(e^{\lambda_{1k}t} - e^{\lambda_{2k}t}). \quad (24)$$

For small values of k, the eigenvalues can be simplified

$$\lambda_{1k} = -ikc\frac{\Omega^2}{\eta c + \Omega^2}, \quad \lambda_{2k} = -ikc\frac{\eta c}{\eta c + \Omega^2} - \frac{\eta c + \Omega^2}{\gamma}. \quad (25)$$

It is easy to see that $\lambda_{1k} = -i\omega_{1k}$ describes the slow wave and $\lambda_{2k} = -i\omega_{2k}$ describes the fast wave: $\omega_{1k} = kv_g$ and $\omega_{2k} = kc - i(\eta c + \Omega^2)/\gamma$, respectively, where $v_g = c\Omega^2/(\eta c + \Omega^2) \ll c$.

The maximum value of wave-number $k = \delta k$ that should be taken into account is determined by the smallest length in the system. Because the length of the pulse in the medium is shorter than the length of the medium, otherwise storage of the pulse is impossible, we have $\delta k \approx (v_g\tau)^{-1}$, where τ is duration of the pulse.

The fast wave, which has large absorption, almost does not introduce any input to the restored pulse, the expression for the Fourier amplitude of which is

$$\alpha_k = -\rho^0_{k,cb}\frac{(\Omega^2 - i\gamma k v_g)(i\gamma kc + \eta c)}{\Omega(i\gamma kc + \eta c + \Omega^2)}e^{\lambda_{1k}t}. \quad (26)$$

It is clear that, in general, the restored field is distorted compared with the signal pulse. However, if we meet simultaneously two conditions, namely $\Omega^2 \gg \gamma k v_g$ and $\eta c \gg \gamma kc$ (the last condition has the simple physical meaning that the length of the pulse should be much larger than the linear absorption length $L_p \gg L_{abs}$) there is no distortion, and the signal pulse can be restored,

$$\alpha_k e^{ikz} = -\frac{\Omega\,\eta c}{\eta c + \Omega^2}\rho^0_{k,cb}e^{ik(z-v_g t)}, \quad (27)$$

and propagate without absorption,

$$\alpha(z,t) \simeq \alpha^0(z - v_g t). \quad (28)$$

VI. TIME-REVERSING LIGHT

Let us turn now to the possibility of "time-reversing light" pointed out in [16]. There are two ways to do it. Leaving the writing procedure the same as above, we either send the reading pulse in the backward direction to the initial direction of the signal and writing pulses, or apply the reading pulse to the transition $|b\rangle \to |a\rangle$ instead of the transition $|c\rangle \to |a\rangle$ (Fig. 5). In the first case, the retrieved pulse goes out of the medium reversed in time, i.e., the head follows the tail. The result of the numerical simulations here looks the same as the result shown in Figs. 2 and 3. In the second, case the new-born field appears on the $|c\rangle \to |a\rangle$ transition and the shape of the restored pulse is time reversed compared to the initial signal pulse and, moreover, the retrieved pulse is phase conjugated to the signal pulse (see Figs. 6 and 7).

The difference between the signal and retrieved pulses for the phase conjugated time reversal appears because the process of reading here is different from the reading in the case of light restoration, which was discussed before. To produce the "time-reversed" pulse, we apply the reading field to the transition populated by the writing field. Hence, the "reading" field populates the level $|c\rangle$ and is absorbed in the medium. This mostly incoherent optical pumping process is accompanied by the scattering on the atomic coherence generated in the writing procedure. Because the coherent scattering is $|\Omega/\gamma|^2 \gg 1$ times faster than the incoherent optical pumping, the field restored by the coherent scattering interacts with an essentially empty transition and, therefore, leaves the medium almost instantly, while the reading field continues to populate level $|c\rangle$.

The time-reversing problem without phase conjugation can be easily solved analytically. The writing procedure cre-

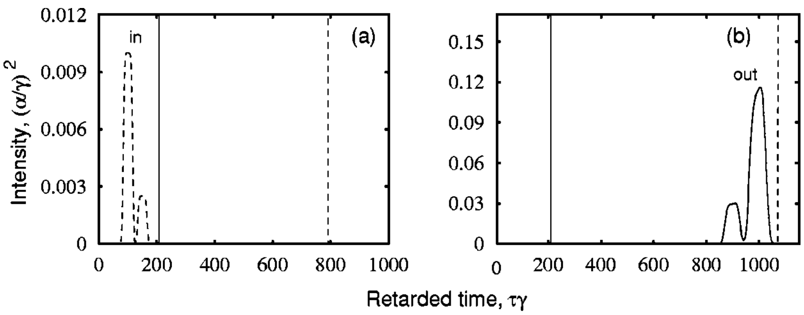

FIG. 7. ''Time reversing'' without adiabaticity. The full-scale writing and reading pulses are as in Fig. 3.

ates the coherence $\rho_{k,cb}=-\alpha_k^0/\Omega$. The retrieved pulse, propagating in the opposite direction to the writing field, and the atomic coherence obey the equations

$$\frac{\partial\alpha}{\partial z}-\frac{\partial\alpha}{\partial ct}=\frac{\eta}{\gamma}[\alpha+\Omega\rho_{cb}], \tag{29}$$

$$\frac{\partial\rho_{cb}}{\partial t}=-\frac{\Omega}{\gamma}[\alpha+\Omega\rho_{cb}], \tag{30}$$

which are similar to Eqs. (13), (14). Presenting the probe field and the coherence in the form

$$\alpha(z,t)=\int_{-\infty}^{\infty}\tilde{\alpha}_k e^{-ikz}dk, \quad \rho_{cb}(z,t)=\int_{-\infty}^{\infty}\tilde{\rho}_{k,cb}e^{-ikz}dk, \tag{31}$$

and substituting them into Eqs. (29), (30), we derive a set of equations one-to-one resembling Eqs. (18), (19) with change $\alpha\rightarrow\tilde{\alpha}$ and $\rho_{cb}\rightarrow\tilde{\rho}_{cb}$. Keeping in mind that the initial conditions here look like $\tilde{\rho}_{k,cb}=\rho_{-k,cb}=-\alpha_{-k}^0/\Omega$, we proceed with the solution and finally get

$$\tilde{\alpha}_k e^{-ikz}=-\frac{\Omega\eta c}{\eta c+\Omega^2}\tilde{\rho}_{k,cb}^0 e^{-ik(z+v_gt)}=\alpha_{-k}^0 e^{-ik(z+v_gt)}, \tag{32}$$

or

$$\alpha(z,t)\simeq\alpha^0(z+v_gt). \tag{33}$$

Equation (33) means that the envelope of the retrieved pulse is the same as the envelope of the signal pulse, but the retrieved pulse propagates in the direction opposite to the signal.

VII. QUANTUM INFORMATION STORAGE

We have been shown that the abrupt switching off and on of the writing-reading laser gives us also the way to store and to restore the signal pulse. Because the initial set of equations for the signal field is *linear* and there is almost no absorption in the medium, the above classical calculations remain true in the quantum case. We simply have to change the mean value of the field by the field operator and, therefore, it is possible to store and retrieve any quantum state of light without destroying it.

Let us calculate the quantum fluctuations for the problem. We enumerate all atoms in the cell and write quantum Langevin equations for the operators $\hat{\sigma}_{kl}^j=|k\rangle\langle l|^j$ of the jth atom,

$$\dot{\hat{\sigma}}_{cc}^j=\gamma\hat{\sigma}_{aa}^j+i\Omega(\hat{\sigma}_{ac}^j-\hat{\sigma}_{ca}^j)+\hat{F}_{cc}^j, \tag{34}$$

$$\dot{\hat{\sigma}}_{bb}^j=\gamma\hat{\sigma}_{aa}^j+i\hat{\alpha}(\hat{\sigma}_{ab}^j-\hat{\sigma}_{ba}^j)+\hat{F}_{bb}^j, \tag{35}$$

$$\dot{\hat{\sigma}}_{ba}^j=-\gamma\hat{\sigma}_{ba}^j-i\hat{\alpha}(\hat{\sigma}_{aa}^j-\hat{\sigma}_{bb}^j)+i\Omega\hat{\sigma}_{bc}^j+\hat{F}_{ba}^j, \tag{36}$$

$$\dot{\hat{\sigma}}_{ac}^j=-\gamma\hat{\sigma}_{ac}^j+i\Omega^*(\hat{\sigma}_{aa}^j-\hat{\sigma}_{cc}^j)-i\hat{\alpha}^\dagger\hat{\sigma}_{bc}^j+\hat{F}_{ac}^j, \tag{37}$$

$$\dot{\hat{\sigma}}_{bc}^j=-i\hat{\alpha}\hat{\sigma}_{ac}^j+i\Omega^*\hat{\sigma}_{ba}^j+\hat{F}_{bc}^j. \tag{38}$$

These equations are to be supplied by the population conservation law

$$\hat{\sigma}_{aa}^j+\hat{\sigma}_{bb}^j+\hat{\sigma}_{cc}^j=\mathbf{1}. \tag{39}$$

Here, $\hat{F}_{mn}^j$ are the quantum Langevin forces.

For the sake of simplicity, we assume that the classical reading and writing fields are much stronger than the quantum probe field $|\Omega|\gg|\langle\hat{\alpha}\rangle|$. Then, almost all atomic population is in the state $|b\rangle$. We also assume that the quantum signal still is quasiclassical, i.e., its expectation value exceeds its fluctuations. We restrict ourselves to the case of Gaussian statistics for the probe field, to be able to describe the system using statistical moments up to the second order (Fokker-Planck approximation).

We get from the set of equations (34)–(38) for the atomic coherence

$$\dot{\hat{\sigma}}_{bc}^j+\frac{|\Omega|^2+|\langle\hat{\alpha}\rangle|^2}{\gamma}\hat{\sigma}_{bc}^j=-\frac{\hat{\alpha}\Omega^*}{\gamma}+\hat{\mathcal{F}}_{bc}^j, \tag{40}$$

where

$$\hat{\mathcal{F}}_{bc}^j=-\frac{i}{\gamma}(\langle\hat{\alpha}\rangle\hat{F}_{ac}^j-\Omega^*\hat{F}_{ba}^j), \tag{41}$$

and

$$\langle \hat{F}^j_{ca}\hat{F}^j_{ac}\rangle = 2\gamma \frac{|\langle\hat{\alpha}\rangle|^2}{|\Omega|^2}\delta(t-t'),$$

$$\langle \hat{F}^j_{ba}\hat{F}^j_{ab}\rangle = 2\gamma\left(1-\frac{|\langle\hat{\alpha}\rangle|^2}{|\Omega|^2}\right)\delta(t-t'),$$

$$\langle \hat{F}^j_{ba}\hat{F}^j_{ac}\rangle = -2\gamma\frac{\langle\hat{\alpha}\rangle}{\Omega}\delta(t-t'),$$

$$\langle \hat{F}^j_{ca}\hat{F}^j_{ab}\rangle = -2\gamma\frac{\langle\hat{\alpha}^\dagger\rangle}{\Omega^*}\delta(t-t').$$

The other moments are negligibly small. The above expressions immediately give

$$\langle \hat{\mathcal{F}}^j_{bc}\hat{\mathcal{F}}^j_{cb}\rangle = \frac{2}{\gamma}(|\Omega|^2-|\langle\hat{\alpha}\rangle|^2)\delta(t-t'), \tag{42}$$

$$\langle \hat{\mathcal{F}}^j_{cb}\hat{\mathcal{F}}^j_{bc}\rangle = 0, \tag{43}$$

$$\langle \hat{\mathcal{F}}^j_{bc}\hat{\mathcal{F}}^j_{bc}\rangle = \langle \hat{\mathcal{F}}^j_{cb}\hat{\mathcal{F}}^j_{cb}\rangle^* = -\frac{2\langle\hat{\alpha}\rangle^2\Omega^*}{\gamma\Omega}\delta(t-t'). \tag{44}$$

To describe the interaction of the fields with the atoms, we introduce collective atomic operators

$$\hat{\sigma}_{ik}(z,t) = \frac{1}{N\mathcal{A}\Delta z}\sum_j \hat{\sigma}^j_{ik}, \tag{45}$$

where $\mathcal{A}$ is the laser beams cross-section area, Δz is a quantization length that exceeds the optical wavelength, but is small compared with the length scale determined by the fields change due to the absorption and dispersion of the medium. Collective fluctuation forces can be derived as of [20]. Neglecting terms of the order of an higher than $|\hat{\alpha}/\Omega|^2$, we rewrite Eq. (40) as

$$\dot{\hat{\sigma}}_{bc} + \frac{|\Omega|^2}{\gamma}\hat{\sigma}_{bc} = -\frac{\hat{\alpha}\Omega^*}{\gamma} + \hat{\mathcal{F}}_{bc}, \tag{46}$$

where

$$\langle \hat{\mathcal{F}}_{bc}\hat{\mathcal{F}}_{cb}\rangle = \frac{2|\Omega|^2}{\gamma N\mathcal{A}}\delta(t-t')\delta(z-z'). \tag{47}$$

On the other hand, for the probe field we have

$$\frac{\partial\hat{\alpha}}{\partial z} + \frac{\partial\hat{\alpha}}{\partial ct} = -\frac{\eta}{\gamma}[\hat{\alpha} + \Omega\hat{\sigma}_{bc} - i\hat{\mathcal{F}}_{ba}], \tag{48}$$

where

$$\hat{\mathcal{F}}_{ba} = -i\frac{\gamma}{\Omega^*}\hat{\mathcal{F}}_{bc}. \tag{49}$$

The set of Eqs. (46), (48), similar to Eqs. (13), (14), is linear in relation to the probe field. Therefore, we can use Fourier decomposition to solve the problem

$$\hat{\alpha}(z,t) = \int_{-\infty}^{\infty}\hat{\alpha}_k e^{ikz}dk,$$

$$\hat{\sigma}_{bc}(z,t) = \int_{-\infty}^{\infty}\hat{\sigma}_{k,bc} e^{ikz}dk,$$

$$\hat{\mathcal{F}}_{bc}(z,t) = \int_{-\infty}^{\infty}\hat{\mathcal{F}}_{k,bc} e^{ikz}dk,$$

where we assume $\Omega = \Omega^*$, and

$$\langle \hat{\mathcal{F}}_{k,bc}\hat{\mathcal{F}}_{k',cb}\rangle = \frac{4\pi|\Omega|^2}{\gamma N\mathcal{A}}\delta(t-t')\delta(k-k').$$

The k modes of the probe field $\hat{\alpha}_k$ and coherence $\sigma_{k,bc}$ obey the ordinary differential equations

$$ik\hat{\alpha}_k + \frac{\partial\hat{\alpha}_k}{\partial ct} = \frac{\eta}{\Omega}\frac{\partial\hat{\sigma}_{k,bc}}{\partial t}, \tag{50}$$

$$\frac{\partial\hat{\sigma}_{k,bc}}{\partial t} = -\frac{\Omega}{\gamma}[\hat{\alpha}_k + \Omega\hat{\sigma}_{k,cb}] + \hat{\mathcal{F}}_{k,bc}, \tag{51}$$

which have the general solution

$$\begin{aligned}\hat{\sigma}_{k,bc} = {} & \frac{\hat{\sigma}^0_{k,bc}\beta_2 - \hat{\alpha}^0_k}{\beta_2-\beta_1}e^{\lambda_{1k}t} - \frac{\hat{\sigma}^0_{k,bc}\beta_1 - \hat{\alpha}^0_k}{\beta_2-\beta_1}e^{\lambda_{2k}t} \\ & - \int_0^t\left[\frac{\beta_2 - \eta c/\Omega}{\beta_1-\beta_2}e^{\lambda_{1k}(t-t')}\right. \\ & \left. - \frac{\beta_1 - \eta c/\Omega}{\beta_1-\beta_2}e^{\lambda_{2k}(t-t')}\right]\hat{\mathcal{F}}_{k,bc}dt',\end{aligned}$$

$$\begin{aligned}\hat{\alpha}_k = {} & \beta_1\frac{\hat{\sigma}^0_{k,bc}\beta_2 - \hat{\alpha}^0_k}{\beta_2-\beta_1}e^{\lambda_{1k}t} - \beta_2\frac{\hat{\sigma}^0_{k,bc}\beta_1 - \hat{\alpha}^0_k}{\beta_2-\beta_1}e^{\lambda_{2k}t} \\ & - \int_0^t\left[\frac{(\beta_2 - \eta c/\Omega)\beta_1}{\beta_1-\beta_2}e^{\lambda_{1k}(t-t')}\right. \\ & \left. - \frac{(\beta_1 - \eta c/\Omega)\beta_2}{\beta_1-\beta_2}e^{\lambda_{2k}(t-t')}\right]\hat{\mathcal{F}}_{k,bc}dt',\end{aligned}$$

where $\hat{\sigma}^0_{k,bc}$ and $\hat{\alpha}^0_k$ are the initial values of the atomic and field operators.

There are two kinds of propagation problems: the initial-value problem and the boundary-value problem. Physically, the light "storage" should be described by the solution of both of them. In the first step, the signal pulse enters the medium. This is the boundary-value problem. In the second step, we switch off the writing field and, after some delay, switch on the reading field (actual light "storage" and "retrieving"). This is the initial-value problem. Finally, in the third step, the restored pulse leaves the medium. This is again the boundary-value problem.

The boundary problem was solved in [21]. In particular, it was shown there that a long probe pulse is not absorbed entering and leaving a coherently driven atomic medium. This is the essence of the phenomenon of electromagnetically induced transparency.

Our goal here is to show that manipulations by the writing and reading fields do not destroy the matter-field state, which automatically means that we are able to retrieve the signal pulse from the medium without distortion. We have solved the initial-value problem, not the boundary-value problem. We assume that the pulses are already in the medium. The initial conditions here does not describe the fields on the entrance on the medium, but rather, the values of the atomic and field operators at an arbitrary chosen moment of time when we start manipulating of the writing field. Therefore, $\hat{\alpha}_k^0$ and $\hat{\sigma}_{k,bc}^0$ are not independent, but depend on the boundary conditions.

The solution of the initial-value problem can be represented in the form of two normal modes

$$\hat{\Psi}_k \equiv \hat{\sigma}_{k,bc} - \frac{\hat{\alpha}_k}{\beta_2} = \hat{\Psi}_k^0 e^{\lambda_{1k}t} + \int_0^t dt' \hat{\mathcal{F}}_{k,bc}\left(1 - \frac{\eta c}{\beta_2 \Omega}\right) e^{\lambda_{1k}(t-t')}, \quad (52)$$

$$\hat{\Phi}_k \equiv \hat{\sigma}_{k,bc} - \frac{\hat{\alpha}_k}{\beta_1} = \hat{\Phi}_k^0 e^{\lambda_{2k}t} + \int_0^t dt' \hat{\mathcal{F}}_{k,bc}\left(1 - \frac{\eta c}{\beta_1 \Omega}\right) e^{\lambda_{2k}(t-t')}. \quad (53)$$

Let us note that $\hat{\Psi}$ is a slow propagating polaritonic mode without decay; $\hat{\Phi}$ is a fast propagating and fast decaying mode. Neglecting fast decaying terms $\sim e^{\lambda_{2k}t}$ in Eq. (53), we obtain [cf. (10)]

$$\hat{\sigma}_{k,bc} = \frac{\hat{\alpha}_k}{\beta_1} + \int_0^t \left(1 - \frac{\eta c}{\Omega \beta_1}\right) e^{\lambda_{2k}(t-t')} \hat{\mathcal{F}}_{k,bc} dt'. \quad (54)$$

To study light propagation in the medium, it is instructive to introduce the polaritonic mode as in Eq. (11),

$$\hat{\Psi} = \int_{-\delta k/2}^{\delta k/2} \hat{\Psi}_k e^{ikz} dk = \int_{-\delta k/2}^{\delta k/2} \left[\hat{\sigma}_{k,bc} - \frac{\hat{\alpha}_k}{\beta_2}\right] e^{ikz} dk. \quad (55)$$

This polaritonic mode obeys a wave equation

$$\frac{\partial \hat{\Psi}}{\partial z} + \frac{1}{v_{gr}} \frac{\partial \hat{\Psi}}{\partial t} = \int_{-\delta k/2}^{\delta k/2} \frac{ik\gamma}{\Omega^2} \hat{\mathcal{F}}_{k,bc} e^{ikz} dk \quad (56)$$

that can be easily solved by substitution $(z,t) \rightarrow (\zeta,t)$, where

$$\zeta = z - \int_0^t v_{gr}(t') dt'.$$

The solution of Eq. (56) is

$$\hat{\Psi}(z,t) = \hat{\Psi}\left(z - \int_0^t v_{gr}(t') dt', t=0\right) + \int_0^t \int_{-\delta k/2}^{\delta k/2} \frac{ikc\gamma}{\Omega^2 + \eta c} \hat{\mathcal{F}}_{k,bc} \times \exp\left[ik\left(z - \int_0^t v_{gr}(t') dt'\right)\right] dk dt''. \quad (57)$$

We change infinite boundaries of the integration over wave number k by finite values because Eq. (60) is derived in approximation of small k (long pulses). Generally, $\delta k \approx (v_{gr}\tau)^{-1}$, where τ is the pulse duration. The noise term in Eq. (57) scales as $\gamma/(\Omega^2\tau)$; that means that the fluctuation forces are negligibly small for long signal and retrieved pulses. Therefore, we get an unchanging state of the polaritonic quantum state in the medium.

The above result directly follows from Eq. (57), if we note that the lhs of the equation does not contain any decoherence terms. Such terms have been neglected due to their vanishing values under electromagnetically induced transparency conditions. Therefore, according to the fluctuation-dissipation theorem, there should not be any fluctuations larger than the negligible dissipation. This point of view is completely confirmed by our calculations.

With the above technique in hand, we are able to study a possibility of writing and retrieving quantum information. Let us do it first for the adiabatic case. Using Eq. (54) we rewrite Eq. (52) as

$$\hat{\Psi}_k \equiv \hat{\sigma}_{k,bc} - \frac{\hat{\alpha}_k}{\beta_2} \simeq -\frac{\hat{\alpha}_k}{\Omega} \frac{\eta c + \Omega^2}{\eta c} + \int_0^t \frac{\eta c + \Omega^2}{\Omega^2} e^{\lambda_{2k}(t-t')} \hat{\mathcal{F}}_{k,bc} dt'. \quad (58)$$

In the simplest case, when the initial and final Rabi frequencies of the writing and reading pulses are large $\Omega^2(t=0) = \Omega^2(t=T) \gg \eta c$ and $\Omega(t)$ is a slow function of time compared with $\exp(\lambda_{2k}t)$, we get $\hat{\alpha}_k(t=0) \simeq \hat{\alpha}_k(t=T)$, which means that the signal pulse can be stored and retrieved without distortion. This confirms the result obtained in [12].

It is very difficult to satisfy the condition $\Omega^2 \gg \eta c$ in a real experiment. Usually, the opposite condition $\eta c \gg \Omega^2$ is valid, which means that the group velocity of the signal pulse in the medium is always less than the speed of light in the vacuum $c \gg v_g$. Under this condition, the basic part of the ‘‘dark polariton’’ coincides with the atomic degree of freedom, i.e.,

$$\hat{\Psi}_k \equiv \hat{\sigma}_{k,bc} - \frac{\hat{\alpha}_k}{\beta_2} \simeq \hat{\sigma}_{k,bc}. \quad (59)$$

Hence, the atomic coherence is not destroyed by the adiabatic change of the writing and reading fields. On the other hand, an abrupt switching of the fields does not destroy the coherence, too. Therefore, both adiabatic and nonadiabatic switching of the writing and reading fields allows us to store quantum information if $\eta c \gg \Omega^2$.

We have considered a one-dimensional problem completely neglecting the transverse spatial structure of the electromagnetic fields. The transverse spatial structure would invalidate our claim that complete quantum optical information can be stored at least because the transverse effects might destroy the writing/reading procedure via interference of images with different $\vec{k}$ of the writing/reading fields. The problem is to be solved in the future to understand real abilities of the coherent information storage technique.

Finally, to compare the light "storage" technique with studies presented in [13] it is useful to rewrite equations (46)–(49) as

$$\frac{d}{dt}\hat{Q} = -i\kappa_1 \hat{E}_p E_d^* - \Gamma \hat{Q} + \hat{F}, \tag{60}$$

$$\frac{\partial \hat{E}_p}{\partial z} + \frac{\partial \hat{E}_p}{\partial ct} = i\frac{\kappa_2 \gamma}{|\Omega|^2} E_d \frac{d}{dt}\hat{Q}, \tag{61}$$

where $\hat{Q} = i\hat{\sigma}_{bc}$, $\hat{E}_p = \hbar\hat{\alpha}/\wp$, $E_d = \hbar\Omega/\wp$, $\kappa_1 = \wp^2/\gamma\hbar^2$, $\kappa_2 = \eta/\gamma$, $\Gamma = |\Omega|^2/\gamma$, and $\hat{F} = i\hat{\mathcal{F}}_{bc}$. The set of equations (60), (61) is very similar to the set of equations derived in [13] for the Raman-type scheme with the difference being that the rhs of Eq. (61) depends on $\dot{\hat{Q}}$, not $\hat{Q}$, as is in [13]. If we would assume that in the rhs of Eq. (61) $\dot{\hat{Q}} \approx -|\Omega|^2\hat{Q}/\gamma$ and exchange $\Delta \leftrightarrow \gamma$ (Δ is one-photon detuning in the Raman configuration), we could have one-to-one correspondence with the Raman scheme; however, this assumption is not correct because for the EIT regime $|\Omega|^2\hat{Q}/\gamma \gg \dot{\hat{Q}}$. Therefore, information storage via EIT is quite different from the Raman technique.

VIII. CONCLUSION

In conclusion, we propose an approach of storing coherent information in a vapor of Λ-type atoms. Our proposal is based on the instantaneous switching off and on of the writing and reading fields. This modification is complimentary to the light "storage" based on adiabatic passage technique. Our method allows us to reach the same results as it is possible to do with adiabatic passage and significantly broadens application ranges of the method.

The slowly varying amplitude approximation of the Maxwell-Bloch equations that describe the information storage process does not include actual optical frequencies but only one- and two-photon detunings of the fields from the relevant atomic transitions. As a result, there is no difference as to which one-photon transition the reading pulse is applied. The scattering of the reading pulse by the atomic coherence excited by the writing pulses is independent of the frequency of the writing pulse. This phenomenon could possibly be used as an effective multichannel optical switch, or image storage system.

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