Nonclassical two-photon interferometry and lithography with high-gain parametric amplifiers

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Optical parametric amplification is a process that leads to the generation of quantum states of light. In the limit of low single-pass gain, this process is often referred to as parametric down conversion, and produces entangled two-photon states. Such states have played a key role in recent studies of quantum optical effects such as quantum teleportation. As the gain of the parametric amplification process is increased, the generated light field still possesses strong quantum correlations, but not of the sort associated with two-photon states. Here we present an analysis of the output state of a parametric amplifier as a function of the single-pass gain, and we find certain signatures of quantum light (such as the vanishing of the coincidence rate in a Hong-Ou-Mandel interferometer) disappear in the high-gain limit, whereas others (such as the existence of two-photon interference fringes) remain. Consequently, the intense light field generated by a high-gain parametric amplifier can be utilized in such applications of quantum optics.

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I. INTRODUCTION

Parametric down conversion and parametric amplification are two related processes that are used to produce light fields possessing strong quantum features. The term parametric down conversion is often reserved for the low-gain limit of parametric amplification. In this limit, the generated light field is typically quite weak and consists of correlated and entangled photon pairs. However, certain applications require the use of more intense light fields, of the sort that can be produced by parametric amplification. The output of a parametric amplifier consists of highly correlated pairs of modes, in which each mode contains more than one photon. Although strong quantum correlations are present in the output in both limits, the detailed nature of the quantum correlations can be quite different. In the present paper we present an analysis of the quantum-statistical properties of the output of a parametric amplifier for arbitrary values of the single-pass gain. We find that certain signatures of quantum states of light are badly degraded through use of large values of the gain whereas other signatures remain. These results should prove useful in the design of laboratory techniques for the production of quantum states of light for various applications such as quantum information and quantum imaging.

Parametric down conversion is a versatile source of two-photon entangled states. Photon pairs generated by this process show entanglement with respect to a variety of different physical attributes such as time of arrival [1] and state of polarization [2] and have been analyzed under a variety of experimental situations [3–5]. Photon pairs are especially useful in the context of interferometric studies [6], where they can be used to demonstrate a variety of nonclassical features.

The analysis of entangled states has recently been extended to include transverse spatial effects [7,8] and in particular the impact of entangled sources on the properties of interferometric patterns [9]. An interesting question is the extent to which increased gain and the additional multiphoton states that result impact on entanglement-related interferometric effects.

II. THE HONG-OU-MANDEL INTERFEROMETER

The behavior of entangled photon pairs has been extensively analyzed in the context of interferometry. A well-known example is the Hong-Ou-Mandel interferometer [10], where entangled light beams are directed into the two input ports of a 50/50 beam splitter and photon counting detectors are placed at each output (see part (a) of Fig. 1). When photon pairs from a spontaneous parametric down converter are used as input beams, the rate of coincidence counts is found to vanish for equal optical path lengths of the two input beams. If a path length difference is introduced between the two input beams, the coincidence count rate becomes nonzero and increases toward an asymptotic value as the path length difference is increased still further. The *absence* of coincidence counts is a sensitive signature of the two-photon, entangled-state nature of the light field entering a Hong-Ou-Mandel interferometer.

Let us next analyze this situation more explicitly, paying careful attention to how the coincidence count rate depends on the strength (i.e., the single-pass gain) of the parametric interaction. We consider two light fields $\hat{a}_1$ and $\hat{b}_1$ that are generated by parametric amplification. Under general circumstances, these field operators can be related to those of the input light fields $\hat{a}_0$ and $\hat{b}_0$ by means of the relations

$$\hat{a}_1 = U\hat{a}_0 + V\hat{b}_0^\dagger, \tag{1}$$

and

$$\hat{b}_1 = U\hat{b}_0 + V\hat{a}_0^\dagger. \tag{2}$$

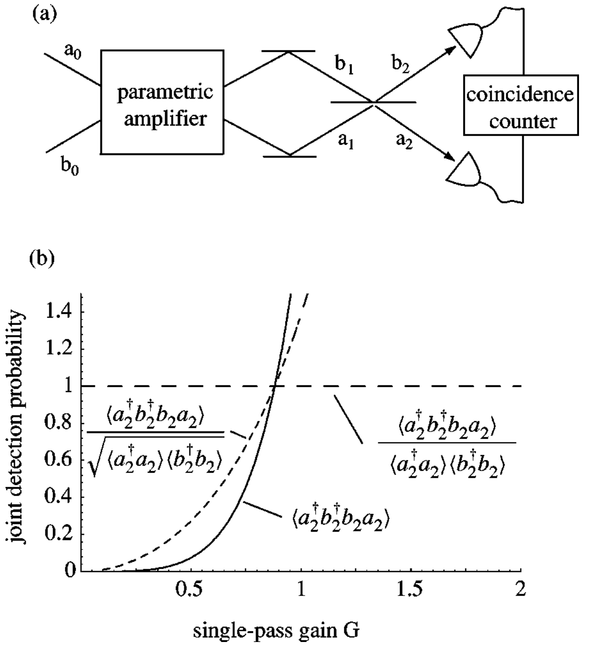

FIG. 1. (a) Hong-Ou-Mandel interferometer in which the light source is a parametric amplifier. (b) Dependence of the coincidence count rate on the single-pass gain of the parametric amplifier. For illustrative purpose, the coincidence rate is also shown normalized by the single-channel count rate in two different ways.

The input light fields are often but not always assumed to be in the vacuum state. The coefficients U and V describe the strength of the nonlinear coupling. For parametric amplification these coefficients are given by coefficients of the form

$$U = \cosh G, \tag{3}$$

$$V = -i \exp(i\varphi) \sinh G, \tag{4}$$

where G represents the single-pass gain of the process and φ is a phase shift describing the interaction. The gain factor G may be written as $G = g|E_p|L$, where L is the interaction path length, $|E_p|$ is the pump laser amplitude, and g is a gain coefficient proportional to the second-order susceptibility $\chi^{(2)}$ [11].

We assume that these two generated beams are directed into the two input ports of the 50/50 beam splitter shown in part (a) of Fig. 1, where they are combined. We describe the beam splitter by means of the standard transfer relations

$$\hat{a}_2 = \frac{1}{\sqrt{2}}[-\hat{a}_1 + i\hat{b}_1], \tag{5}$$

$$\hat{b}_2 = \frac{1}{\sqrt{2}}[i\hat{a}_1 - \hat{b}_1]. \tag{6}$$

The fields leaving the beam splitter can then be expressed as

$$\hat{a}_2 = \frac{-1}{\sqrt{2}}[(U\hat{a}_0 + V\hat{b}_0^\dagger) - i(U\hat{b}_0 + V\hat{a}_0^\dagger)] \tag{7}$$

and

$$\hat{b}_2 = \frac{-1}{\sqrt{2}}[-i(U\hat{a}_0 + V\hat{b}_0^\dagger) + (U\hat{b}_0 + V\hat{a}_0^\dagger)]. \tag{8}$$

The individual count rates for channels $\hat{a}_2$ and $\hat{b}_2$ are given by $\langle \hat{a}_2^\dagger \hat{a}_2\rangle$ and $\langle \hat{b}_2^\dagger \hat{b}_2\rangle$ and the coincidence count rate is given by $\langle \hat{a}_2^\dagger \hat{b}_2^\dagger \hat{b}_2 \hat{a}_2\rangle$. For a vacuum state input to the material, the indvidual count rates are hence given by

$$\langle 0,0|\hat{a}_2^\dagger \hat{a}_2|0,0\rangle = \langle 0,0|\hat{b}_2^\dagger \hat{b}_2|0,0\rangle = |V|^2, \tag{9}$$

and the coincidence count rate is given by

$$\langle 0,0|\hat{a}_2^\dagger \hat{b}_2^\dagger \hat{b}_2 \hat{a}_2|0,0\rangle = |V|^4, \tag{10}$$

which clearly does not in general vanish. This result shows that the disappearance of the coincidence count rate is not a generic property of the light generated by a parametric amplifier but occurs only in the limiting case of a vanishingly small V. The coefficients U and V for parametric amplification are given as functions of the single-pass gain G by Eqs. (3) and (4). From these results, the coincidence count rate can be plotted as a function of G as shown in part (b) Fig. 1. Because the coincidence count rate increases rapidly with the single-pass gain G, in this figure we also, for clarity, show this rate normalized by the product of the product of the two single-channel count rates and normalized by the square root of this product. We can see that the coincidence rate becomes appreciable even for values of G of the order of unity, where the mean output photon number is also of the order of unity. Thus, only in the limit of extremely weak fields does the joint detection probablilty vanish.

The appearance of joint detection events with increasing single-pass gain can be understood on the basis of the following argument. For a vacuum-state input, the output state leaving the parametric amplifier can be written as the sum of states of the form $|n,n\rangle$ [12]. As the gain is increased, the relative contribution of states with large n also increases. It is the presence of these states that leads to the appearance of coincidence counts, which never occur for a $|1,1\rangle$ state. It is straightforward to show that a state $|\psi\rangle_1 = |n,n\rangle$ injected into the two input ports of the beam splitter of Fig. 1(a) described by Eqs. (5) and (6) produces a coincidence count rate at the output of $\langle \hat{a}_2^\dagger \hat{b}_2^\dagger \hat{b}_2 \hat{a}_2\rangle = \frac{1}{2}n(n-1)$. This quantity vanishes only for $n=1$, that is, for the $|1,1\rangle$ input state. As the nonlinear interaction strength is increased and more photons are produced, states with $n>1$ will make an appreciable contribution to the output beam and an increase in coincidence counts is an expected consequence.

III. FOURTH-ORDER COINCIDENCE STATISTICS USING A MACH-ZEHNDER INTERFEROMETER

The Mach-Zehnder arrangement illustrated in Fig. 2 has been shown to exhibit strong, nonclassical fourth-order interference effects when a biphoton input is used [13]. It is a member of the more general class of two-beam interferom-

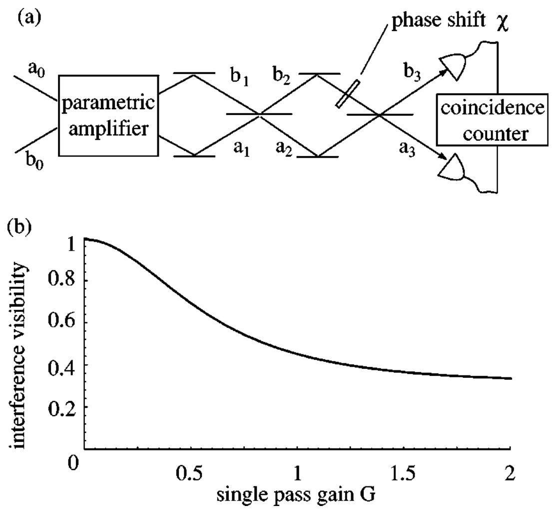

FIG. 2. (a) Mach-Zehnder interferometer for studying fourth-order interference. χ is the phase difference between the two arms of the interferometer. (b) Variation of the fourth-order coincidence visibility with single-pass gain.

eters that have been used to explore the entangled-state properties of the spontaneous parametric down converter [6]. In particular, the joint detection probablility $\langle \hat{a}_3^\dagger \hat{b}_3^\dagger \hat{b}_3 \hat{a}_3 \rangle$ for the situation shown in Fig. 2 is found to oscillate harmonically as the classical phase shift χ introduced in one arm of the interferometer is varied. In practice, this phase shift can be varied by translating vertically the rightmost beam splitter shown in the figure. With a $|1,1\rangle$ input, a "visibility" of unity of the resulting interference pattern is obtained. It is well known that a visibility of greater than 50% is a signature of nonclassical light. As the gain of the parametric amplification process producing the photon pairs is increased, this visibility decreases. We can analyze the effect of increasing the gain by explicitly including the process of parametric amplification in the calculation of the joint detection probability. By performing a calculation analogous to that given above, we find that the operators at the exit ports of the interferometer are given by

$$\hat{a}_3 = \frac{1}{2}[(1-e^{i\chi})(U\hat{a}_0 + V\hat{b}_0^\dagger) - i(1+e^{i\chi})(U\hat{b}_0 + V\hat{a}_0^\dagger)] \tag{11}$$

$$\hat{b}_3 = \frac{1}{2}[-i(1+e^{i\chi})(U\hat{a}_0 + V\hat{b}_0^\dagger) - (1-e^{i\chi})(U\hat{b}_0 + V\hat{a}_0^\dagger)]. \tag{12}$$

For a vacuum state input, the joint detection probability is then readily found to be given by

$$\langle \hat{a}_3^\dagger \hat{b}_3^\dagger \hat{b}_3 \hat{a}_3 \rangle = |V|^2[|V|^2 + \tfrac{1}{2}|U|^2(1+\cos 2\chi)]. \tag{13}$$

Since for the process of parametric amplification it is necessarily true that $|U|^2 - |V|^2 = 1$, the joint detection probability can be expressed as

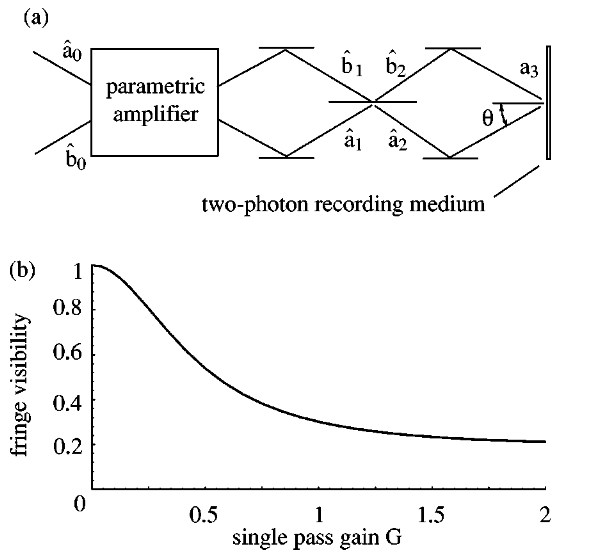

FIG. 3. (a) Setup for the recording of sub-Rayleigh fringes using entangled states of light. (b) Variation of the two-photon fringe visibility with single-pass gain.

$$\langle \hat{a}_3^\dagger \hat{b}_3^\dagger \hat{b}_3 \hat{a}_3 \rangle = |V|^2[\tfrac{1}{2}(1+\cos 2\chi) + |V|^2(\tfrac{3}{2} + \tfrac{1}{2}\cos 2\chi)]. \tag{14}$$

The visibility is thus given by $(1+|V|^2)/(1+3|V|^2)$. For values of the single pass gain G comparable to or greater than unity, the visibility drops below 50%, which as mentioned above is the limiting value below which nonclassical features are not displayed. In the limit of large gain, the visibility drops to 1/3.

IV. QUANTUM LITHOGRAPHY

Let us next analyze a different experimental situation, which is shown schematically in part (a) of Fig. 3. Here, as above, the two light beams leaving the parametric amplifier are combined at a 50/50 beam splitter. However, in this case the two beams leaving the beam splitter are allowed to interfere and the resulting interference pattern is recorded on a two-photon absorbing medium. Boto *et al.* [9] have recently analyzed this situation in the low-gain limit and show that the resulting interference pattern will have half the period of the classical (one-photon) interference pattern involving two light waves. They propose that this interaction, which they name quantum lithography, can be usefully exploited to write feaures that are finer than those allowed by the Rayleigh criterion. In their initial proposal, they suggested using photon pairs from a parametric down converter as the light source. Such photon pairs possess the proper quantum entanglement to produce sub-Rayleigh features, but such a light source may be too weak to efficiently excite the two-photon absorption process. Here we analyze the nature of the resulting interference pattern when the parametric down converter of Boto *et al.* is replaced by a high-gain parametric amplifier, which can produce much more intense output beams. A brief account of the results of this calculation have been published recently [14].

Through use of Eqs. (7) and (8), we find that the field at the recording plane can be written

$$\hat{a}_3=\frac{1}{\sqrt{2}}[(-e^{i\chi}+i)(U\hat{a}_0+V\hat{b}_0^\dagger)+(ie^{i\chi}-1)(U\hat{b}_0+V\hat{a}_0^\dagger)], \tag{15}$$

where χ, the phase shift between the two arms, arises from the difference in phase accumulated by the two output modes at various points along the detection plane. Because we have assumed that only two modes, each with wave vector of magnitude k, are populated, we can write this phase shift as $\chi=2kx\sin\theta$ where θ is the angle of incidence of each of the beams onto the recording plane and x is the transverse coordinate in this plane. We next calculate the two-photon absorption rate at the image plane, again assuming a vacuum-state input to the interaction region. We obtain

$$\begin{aligned}\langle 0,0|\hat{a}_3^\dagger\hat{a}_3^\dagger\hat{a}_3\hat{a}_3|0,0\rangle &= \frac{1}{4}|i-e^{i\chi}|^4|V|^2\left[4\left|\frac{ie^{i\chi}-1}{i-e^{i\chi}}\right|^2(|U|^2\right.\\ &\quad\left.+|V|^2)+2|V|^2\left(1+\left|\frac{ie^{i\chi}-1}{i-e^{i\chi}}\right|^4\right)\right]\\ &=4|V|^2[|U|^2\cos^2\chi+2|V|^2]. \end{aligned}\tag{16}$$

Noting that $\cos^2\chi$ can be expressed as $\frac{1}{2}(1+\cos 2\chi)$, we see that the two-photon excitation pattern has a period half that of the corresponding classical (one-photon) interference pattern. This peculiar property of the interference pattern is a consequence of the exotic properties of entangled light beams. In the case considered by Boto *et al.* [9], in which the output of the parametric amplifier is in the ideal $|1,1\rangle$ state, this pattern has a visibility of unity. In general, the visibility of this pattern is dependent on the values of the coefficients U and V.

We next calculate the fringe visibility of the two-photon interference pattern. To do so, we first calculate the extrema of the two-photon absorption pattern. We find that the maxima occur at $\chi=m\pi$ with value $\langle 0,0|\hat{a}_3^\dagger\hat{a}_3^\dagger\hat{a}_3\hat{a}_3|0,0\rangle=4|V|^2[|U|^2+2|V|^2]$ and minima at $\chi=(2m+1)\pi/2$ with value $\langle 0,0|\hat{a}_3^\dagger\hat{a}_3^\dagger\hat{a}_3\hat{a}_3|0,0\rangle=8|V|^4$. Consequently, the visibility at the image plane of the two-photon interference pattern becomes

$$C_{\text{vis}}=\frac{|U|^2}{|U|^2+4|V|^2}. \tag{17}$$

With U and V given by Eqs. (3) and (4), the two-photon interference visibility can be determined as a function of the single-pass gain G. This dependence is shown in part (b) of Fig. 3.

We see that for small values of G, the visibility is equal to unity, but that as G increases, the visibility approaches the value 1/5. The visibility drops to the value 0.5 when $|V|^2=1/3$, under such conditions the mean photon number in each beam is also equal to 1/3. This result shows that the fringe visibility begins to drop as soon as the number of photons per mode becomes of the order of unity. Nonetheless, the fringe visibility never drops to a value smaller than 1/5. Although a visibility closer to 3/5 is often necessary for production standards, we feel that this level will be sufficient to demonstrate this method in the laboratory. In such a proof-of-principle laboratory demonstration, it might prove preferable to make use of a two-photon detector array instead of a traditional lithographic plate. In this context it is reassuring to note that two-photon absorption is routinely used in the construction of autocorelators for the characterization of ultrashort laser pulses.

It can also be noted that the correlated beams from either a parametric down converter or a high-gain parametric amplifier have twice the wavelength of the pump light, so by using the pump beam and standard lithography one could achieve the same resolution as that produced by the arrangement discussed in this section. However, use of parametric amplification to generate the correlated beams is only for proof of principle, and for a practical system one would replace the amplifier with a degenerate process such as four-wave mixing to achieve true resolution doubling.

The key result, that the fringe visibility decreases from unity to the limiting value 1/5 with increasing gain, can be understood on the basis of the following simple argument. With increasing gain G, the number n of photons emitted into each of the modes a_1 and b_1 can exceed unity. In this case, there will be a contribution to the two-photon interference pattern from photons coming both from b_1 or both from b_2. This contribution is of the form $I_1=n(n-1)K(1.5-0.5\cos 2\chi)$, where K is a constant and is to be added to the contribution from photons coming one each from b_1 and b_2, which is of the form $I_2=n^2K(1+\cos 2\chi)$. For $n=1$, the first contribution vanishes and we are left with only the contribution considered by Boto *et al.*, which corresponds to a fringe visibility of unity. But for n large, the interference pattern is of the form $I=n^2K(2.5+0.5\cos 2\chi)$, which corresponds to a fringe visibility of 1/5.

V. DISCUSSION AND SUMMARY

In summary, we have examined the process of parametric amplification for arbitrary single-pass gain as a source of entangled light beams. We find that the output beams possess strong quantum correlations in both the low- and high-gain limits. However, only in the limit of low gain does the output consist of a stream of photon pairs. We have examined the use of such light beams in the context of three laboratory configurations that display quantum effects of light. In the first, the Hong-Ou-Mandel interferometer, we find that the laboratory signature of quantum aspects to the input beam (that is, the vanishing of the coincidence count rate) is quickly lost as the single-pass gain becomes of the order of unity or greater. In the second example, we find that the visibility of the joint detection probability of the output of a Mach-Zehnder interferometer decreases to an asymptotic value of 1/3 as the gain of the parametric source used to provide an input to the interferometer is increased. For the third example, a recently proposed quantum lithography setup that can write fringes finer than those allowed by the Rayleigh criterion, we find that the fringe visibility is degraded through the use of high single-pass gain but is always at least 20%. We conclude that high-gain parametric amplifiers possess great promise as intense sources of entangled photons for certain applications in quantum optics.

ACKNOWLEDGMENTS

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