# Generalized measurements on atoms in microtraps

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Neutral atoms can be trapped in microscopic potentials resulting from electric and magnetic fields. These microtraps might be used to build simple quantum devices, and ultimately be combined to networks for performing quantum computation. Considering quantum information applications, it is of interest to be able to perform generalized measurements on qubits. A scheme to accomplish this when the qubits are carried by neutral atoms in microtraps is suggested.

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## I. INTRODUCTION

There has recently been growing experimental interest in constructing microscopic traps, where neutral atoms are trapped by means of electric and magnetic forces; see, for example, [1–4]. The microtraps may be integrated on a surface to form "atom chips" [3], an atom-optics analog of semiconductor heterostructures. Simple devices such as beam splitters have already been realized [5,6], and one can envisage building networks leading to more complicated structures for quantum information applications [7]. Cold collisions [8–10] and induced dipole interactions [11] have been proposed as means to achieve entanglement and quantum logic gates for atoms trapped in optical lattices or in magnetic microtraps. The advantage with neutral atoms is that they couple weakly to the environment, which implies less decoherence. In previous work, we have investigated possible uses of microtrap networks, in particular, mechanisms to achieve conditional logic [12–14].

Any quantum communication protocol or quantum computational task involves a final measurement. An ideal von Neumann measurement [15] is a projection onto the orthogonal eigenstates of some observable, and is not sufficient for optimally adressing all physical measurement situations. Examples include the simultaneous measurement of noncommuting observables [16,17] and distinguishing between nonorthogonal states [18–21]. For example, given a quantum system prepared in one of two nonorthogonal states and asked in which one it is, it is possible to perform a more general kind of measurement with three outcomes. Two of them will correspond to each one of the states, telling us with certainty that the system was in this state. In addition, there will always be a finite probability for the third outcome, which provides no information at all. This procedure is an example of a probability operator measure (POM) strategy, also referred to as a positive operator-valued measure (POVM) strategy [22,23]. An important difference between von Neumann measurements and generalized measurements is that the number of outcomes need not be the same as the number of available preparations, or the same as the dimensionality of the Hilbert space of the system to be measured.

In principle, a generalized measurement can always be realized by introducing auxiliary degrees of freedom, and then performing a von Neumann measurement on the combined system. How to realize this in practice is another question. Actual experiments implementing generalized measurement protocols have so far only been performed on photons [24–27]. An experimental realization when the qubit states are different internal states of ions or atoms is suggested in [28]. This paper considers generalized measurements on atomic particles trapped in microtraps. A brief introduction to generalized measurements is offered in the next section. Section II outlines the measurement procedure on atoms in microtraps. An explicit example, distinguishing between three linearly dependent states, is given in Sec. III, followed by conclusions.

## II. GENERALIZED MEASUREMENTS

A POM measurement can be carried out via a coupling to an auxiliary system, followed by a von Neumann measurement on the combined system [23]. Let $\rho_i$ be the density matrix of the initial system prepared in state $i$, and $\rho^{aux}$ that of the auxiliary system, prepared in a known state. The different outcomes, labeled by $j$, of the measurement on the combined system correspond to orthogonal projectors $\hat{\Pi}_j^{comb}$. These operators act in the combined Hilbert space. The probability to obtain result $j$ after preparation $i$ is

$$P_j = \mathrm{Tr}[\hat{\Pi}_j^{comb}(\rho_i \otimes \rho^{aux})] = \sum_{mr,ns} (\hat{\Pi}_j^{comb})_{mr,ns}(\rho_i)_{nm}(\rho^{aux})_{sr}. \quad (1)$$

If we now define

$$(\hat{\Pi}_j)_{mn} = \sum_{rs} (\hat{\Pi}_j^{comb})_{mr,ns}(\rho^{aux})_{sr}, \quad (2)$$

then the probability $P_j$ to obtain the result labeled by $j$ is given by

$$P_j = \mathrm{Tr}(\hat{\Pi}_j \hat{\rho}_i), \quad (3)$$

where each possible outcome $j$ of the generalized measurement is associated with a Hermitian operator $\hat{\Pi}_j$ acting on

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the original system, referred to as the elements of the POM. For a von Neumann measurement, these operators are the projectors onto the orthonormal eigenstates of the observable to be measured, and there is no need for an auxiliary system. In general, $\hat{\Pi}_j$ are neither orthogonal nor normalized, but the following conditions will always hold: All the eigenvalues of $\hat{\Pi}_j$ are positive or zero, and $\hat{\Pi}_j$ form a decomposition of the identity operator, that is, $\Sigma_j \hat{\Pi}_j = \hat{\mathbf{1}}$.

### A. Implementation via auxiliary states

Let us assume that we can extend the Hilbert space of the original system by adding auxiliary basis states. Given the projectors $\hat{\Pi}_j$, it is always possible to find a set of orthogonal projectors in the extended space, such that $\hat{\Pi}_j$ results from projecting these operators back onto the original Hilbert space [23]. In order to have $N$ different outcomes, we need to map the original system, with $M \leq N$ dimensions, onto $N$ orthogonal states of an extended system.

It can be proven that for most relevant measurement tasks there always exists an optimal POM consisting of rank-1 matrices $\hat{\Pi}_j = |\Psi_j\rangle\langle\Psi_j|$ [29]. Hence, we can consider the orthonormal extended states

$$|\Psi_j'\rangle = |\Psi_j\rangle + |\phi_j\rangle, \tag{4}$$

where $|\Psi_j\rangle$ are linear combinations of the $M$ original basis states, and $|\phi_j\rangle$ are linear combinations of the $N-M$ auxiliary basis states. These states $|\Psi_j'\rangle$ are always possible to find and span the whole extended $N$-dimensional Hilbert space. The generalized measurement can be implemented as a projection onto the states $|\Psi_j'\rangle$ in the extended Hilbert space [23]. Sometimes the original quantum system has states available for this extension, otherwise, it is necessary to introduce an ancillary system.

We can construct the unitary operator

$$\hat{U} = \sum_{j=1}^{N} |j\rangle\langle\Psi_j'|, \tag{5}$$

which, applied to an initial state $|\phi\rangle$ in the Hilbert space of the original system, yields

$$\hat{U}|\phi\rangle = \sum_{j=1}^{N} |j\rangle\langle\Psi_j'|\phi\rangle = \sum_{j=1}^{N} |j\rangle\langle\Psi_j|\phi\rangle. \tag{6}$$

This means that the probability to find the system in state $|j\rangle$ is exactly $P_j = \mathrm{Tr}(\hat{\Pi}_j \hat{\rho})$, with $\hat{\Pi}_j$ given by $|\Psi_j\rangle\langle\Psi_j|$. In order to effect the desired POM measurement we only need to apply this unitary transformation, coupling original and auxiliary degrees of freedom, and then to measure the final population in the basis states.

## III. REALIZATION WITH ATOMS IN A MICROTRAP NETWORK

A network of microtraps could be used to build quantum devices and possibly for performing quantum computational tasks. One possibility is to let each atom represent a qubit, with the ground states of the different channels being the qubit basis states, so that each qubit occupies, for example, two neighboring channels. The atoms then travel through the network, interacting with the device potential and each other. As an alternative to moving through a network of channels, the atoms could be stored in time-dependent microtraps.

To perform a generalized measurement on an atomic qubit in a microtrap network, the auxiliary states would be extra channels. The atom whose state is to be measured is incident in a superposition of the ground states of channels 1 to $M$ and the auxiliary input ports $M+1$ to $N$ are unused. The qubit basis modes and the auxiliary modes are then coupled, effecting the unitary transform $\hat{U}$ in Eq. (5), and the readout is done by detecting at which of the $N$ output ports the atom emerges.

For measurements on a single qubit, only single-qubit operations are needed. Phase shifts on individual qubits may be achieved with appropriate differences in path length or shifts of the trapping potential. When atoms are trapped with magnetic fields arising from current-carrying structures, for example, along wires, the trapping potential is typically varying a few milliKelvin on the length scale of a few micrometers [7]. Thus, with additional current-carrying structures, it would be possible to shift the potential with a $\Delta E$ on the milliKelvin scale [30]. Since $\Delta\phi = \Delta E t/\hbar$, a phase shift of $\pi$ would require the atom to experience this energy shift for about $10^{-8}$ s. Again, the distance where the potential is varying is of the order of some micrometers, hence, this implies that the velocity of an atom passing through the region where the phase shift occurs should be less than $10^2$ m/s. In experiments, the longitudinal velocity of the cold atoms is much less, implying that much smaller additional energy shifts would already be sufficient for the desired phase shifts.

Two channels belonging to the same qubit may also be coupled by bringing them close together, so that a particle could tunnel from one channel to the other [12]. Using the WKB approximation, we find that particle in a one-dimensional double-well potential $U(x)$ will tunnel across the barrier with a rate

$$T \approx \exp\left\{ -\int \sqrt{\frac{2m}{\hbar^2}[U(x)-E]}dx \right\}, \tag{7}$$

where the integral should be evaluated over the potential barrier separating the two wells. The coupling can be understood in terms of the eigenfunctions of the double-well potential. The ground state is symmetric, $\psi_S$, with energy $E_S$, and the first excited state is antisymmetric, $\psi_A$, with energy $E_A$. We define the tunneling frequency $2\Omega$ according to

$$E_A = \overline{E} + \hbar\Omega, \qquad E_S = \overline{E} - \hbar\Omega. \tag{8}$$

The time evolution of any initial state can easily be found using the eigenstates. We are interested in where the particle is localized; thus we form the states

$$|\phi_L\rangle = \frac{1}{\sqrt{2}}(|\psi_S\rangle + |\psi_A\rangle),$$

$$|\phi_R\rangle = \frac{1}{\sqrt{2}}(|\psi_S\rangle - |\psi_A\rangle), \tag{9}$$

where the subscripts $L$ ($R$) denote left (right) localization. The time-evolution operator is then given by

$$\hat{U}(t) = \cos\Omega t(|\phi_L\rangle\langle\phi_L| + |\phi_R\rangle\langle\phi_R|) + i\sin\Omega t(|\phi_L\rangle\langle\phi_R| + |\phi_R\rangle\langle\phi_L|). \tag{10}$$

This is equivalent to an optical-fibre coupler or a beam splitter. Together with phase shifts on single channels, it is enough for realizing any unitary single-qubit operation. By varying the distance between the channels, thus altering the tunneling frequency $\Omega$, and by changing the tunneling time, making the interaction region longer or shorter, the transmission and reflection coefficients can be tuned. In reality, $\Omega$ is of course not constant when the channels approach; the role of $\Omega t$ is played by $\int\Omega dt$. This changes nothing in the qualitative description, and for simplicity, we picture the case with a constant $\Omega$.

This kind of beam-splitter configuration, with magnetic fields guiding an atomic cloud, has recently been realized experimentally [6]. Also, somewhat related, for cesium atoms, oscillations between the two localized states in a double-well potential in a far off-resonant optical lattice have been observed [31].

To go one step further and perform joint generalized measurements on many qubits, one needs to be able to implement a universal two-qubit gate, such as the quantum controlled-NOT gate. Cold collisions [8–10] and induced dipole interactions [11] have been proposed as means to achieve entanglement and conditional quantum logic for atoms trapped in optical lattices and magnetic microtraps. Together with single-qubit operations, a universal two-qubit gate is enough for implementing any unitary transform $\hat{U}$, coupling qubit states and auxiliary states. The measurement result is then obtained by detecting in which output states the atoms emerge. On photons, only single-qubit generalized measurements have been performed. In contrast to this, atoms do interact, making a joint generalized measurement on many qubits possible.

Questions that have to be solved experimentally are how to achieve single-mode propagation of single atoms, and how to control the trapping potentials very accurately. Steps towards loading of Bose-condensed atoms into microtraps are taken [32]. The atom cloud to be guided on an atom chip, typically containing $10^6$ atoms, is usually trapped in a magneto-optical trap (MOT) and subsequently loaded into the guiding structures. A MOT may, however, also be used to trap small numbers of atoms [33–35]. These very few atoms in a MOT may be loaded into an optical dipole trap with 100% efficiency [36]. At the output ports, the single atoms might be detected by ionization.

## IV. DISTINGUISHING THREE LINEARLY DEPENDENT STATES

In the following, we will consider an explicit example of a POM strategy and how it could be realized on atoms in a network of microtraps. Assume that a qubit is prepared in one of the three nonorthogonal symmetric states

$$|\phi_1\rangle = -\frac{1}{2}(|1\rangle + \sqrt{3}|2\rangle),$$

$$|\phi_2\rangle = -\frac{1}{2}(|1\rangle - \sqrt{3}|2\rangle), \tag{11}$$

$$|\phi_3\rangle = |1\rangle$$

referred to as the trine states [37–39], and that we are interested to know in which one. The states $|1\rangle$ and $|2\rangle$ are orthonormal basis states.

Due to the finite overlap between the trine states, it is not possible to perfectly distinguish them without errors. The optimal measurement requires three possible outcomes, and is not a straightforward von Neumann projection. If the states $|\phi_j\rangle$ are equiprobable, it can be shown that the POM strategy maximizing the probability to obtain a correct result has the elements [22]

$$\hat{\Pi}_j = |\Psi_j\rangle\langle\Psi_j| = \tfrac{2}{3}|\phi_j\rangle\langle\phi_j|. \tag{12}$$

The probability to obtain the correct result is 2/3, and the probability to obtain either one of the erroneous results is 1/6.

We will now explicitly show how it is possible to perform this measurement by enlarging the Hilbert space of the system. Let us refer to the states $|\Psi_j\rangle$ as

$$|\Psi_j\rangle = \sqrt{\frac{2}{3}}|\phi_j\rangle = a_{j,1}|1\rangle + a_{j,2}|2\rangle \tag{13}$$

and denote an auxiliary state with $|3\rangle$. It is now possible to "extend" the states $|\Psi_j\rangle$,

$$|\Psi'_j\rangle = a_{j,1}|1\rangle + a_{j,2}|2\rangle + b_{j,3}|3\rangle = |\Psi_j\rangle + \sqrt{\frac{1}{3}}|3\rangle, \tag{14}$$

so that the states $|\Psi'_j\rangle$, living in the Hilbert space of the combined system, are orthonormal. Consider the unitary transformation

$$\hat{U} = |1\rangle\langle\Psi'_1| + |2\rangle\langle\Psi'_2| + |3\rangle\langle\Psi'_3|. \tag{15}$$

Applying $\hat{U}$ to an initial state $|\phi\rangle$ in the $\{|1\rangle,|2\rangle\}$ subspace we get

$$\hat{U}|\phi\rangle = |1\rangle\langle\Psi_1|\phi\rangle + |2\rangle\langle\Psi_2|\phi\rangle + |3\rangle\langle\Psi_3|\phi\rangle, \tag{16}$$

and the probabilities to find the system in the states $|1\rangle$, $|2\rangle$, and $|3\rangle$ now are exactly $P_j = \mathrm{Tr}(\hat{\Pi}_j\hat{\rho})$, with $\hat{\rho} = |\phi\rangle\langle\phi|$. This means that the POM strategy can be implemented using the unitary transform $\hat{U}$ followed by a von Neumann projection on the states of the composite three-dimensional system.

The unitary operator $\hat{U}$ can be realized via a series of two-by-two beam splitters and phase shifts. Here we will decompose $\hat{U}$ following Reck *et al.* [40]. Let us consider the matrix

$$U=\begin{bmatrix} a^*_{1,1} & a^*_{1,2} & b^*_{1,3} \\ a^*_{2,1} & a^*_{2,2} & b^*_{2,3} \\ a^*_{3,1} & a^*_{3,2} & b^*_{3,3} \end{bmatrix}=\begin{bmatrix} -1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \\ -1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ \sqrt{2/3} & 0 & 1/\sqrt{3} \end{bmatrix}, \tag{17}$$

with the coefficients of the states $|\Psi'_j\rangle$ as elements. A general $N\times N$ matrix will require $N(N-1)/2$ two-by-two transformations, but because of the zero element in position (3,2), we will get around with only two subtransformations. With

$$R^\dagger_{13}=\begin{bmatrix} 1/\sqrt{3} & 0 & -\sqrt{2/3} \\ 0 & 1 & 0 \\ -\sqrt{2/3} & 0 & -1/\sqrt{3} \end{bmatrix}, \tag{18}$$

which couples the first and third components, we obtain

$$UR^\dagger_{13}=\begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{19}$$

The effective dimension of the matrix $U$ is reduced by 1, so that $UR^\dagger_{13}$ performs a transformation on the two-dimensional subspace formed by the first two components only. With

$$R^\dagger_{12}=\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{20}$$

we obtain

$$UR^\dagger_{13}R^\dagger_{12}=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{21}$$

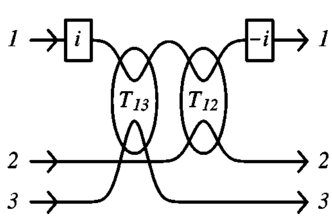


FIG. 1. The generalized meaurement, which distinguishes optimally between the trine states, can be implemented by coupling the channels two at a time. The state to be measured is incident in channels 1 and 2; the auxiliary channel 3 is needed in order to have three outcomes corresponding to outputs 1, 2, and 3. The square boxes indicate phase shifts of $\pm i$, and $T_{13}$ and $T_{12}$ are beam-splitter transformations as explained in Sec. III.

and the whole transformation $U$ is decomposed as

$$U=[R^\dagger_{13}R^\dagger_{12}(-I)]^{-1}=-IR_{12}R_{13}. \tag{22}$$

We have to effect the transforms $R_{13}$ and $R_{12}$ with phase shifts and beam splitters as in Eq. (10). $R_{13}$ is obtained by letting channels 1 and 3 approach each other, with $\cos\Omega t$ equal to $-1/\sqrt{3}$ and $\sin\Omega t$ equal to $\sqrt{2/3}$, and performing a phase shift of $i$ on both input and output port number 1:

$$R_{13}=\begin{bmatrix} 1/\sqrt{3} & 0 & -\sqrt{2/3} \\ 0 & 1 & 0 \\ -\sqrt{2/3} & 0 & -1/\sqrt{3} \end{bmatrix}=P_1(i)T_{13}P_1(i) \tag{23}$$

$$=\begin{bmatrix} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1/\sqrt{3} & 0 & i\sqrt{2/3} \\ 0 & 1 & 0 \\ i\sqrt{2/3} & 0 & -1/\sqrt{3} \end{bmatrix}\times\begin{bmatrix} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{24}$$

$R_{12}$ is effected by coupling channels 1 and 2 and letting $-\cos\Omega t=\sin\Omega t=1/\sqrt{2}$, and again adding phase shifts of $-i$ on port one before and after the coupling,

$$R_{12}=\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}=P_1(-i)T_{12}P_1(-i) \tag{25}$$

$$=\begin{bmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1/\sqrt{2} & i/\sqrt{2} & 0 \\ i/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\times\begin{bmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{26}$$

As $U=-IR_{12}R_{13}$, the middle phase shifts of $-i$ and $i$ cancel. The whole transformation $U$ is illustrated in Fig. 1, where the overall phase shift resulting in the factor $-1$ has been dropped since it is not needed for the realization of the generalized measurement.

The actual realization might not allow the channels to cross as in the figure; this is the case for present realization of atom chips. The two channels that are to be coupled can still effectively be brought next to each other by "swapping" neighboring channels using beam-splitter transformations as in Eq. (10) with $\Omega t=\pm\pi$. Effectively, this amounts to reordering the channels. If the experiment allows for layered channels, one or more bus channels in a second layer could be used.

## V. CONCLUSIONS

We have described a scheme for realizing generalized measurements on atomic qubits in microtrap networks, when the basis states are the ground states of different channels. Many experimental challenges remain to be solved, including the preparation and detection of atoms, and the precise control of the network potential. Even though the experimental challenges are major, progress in the field is rapid. In the future, it may well be possible to use neutral-atom microtraps for quantum-information applications, including the generalized measurements outlined in this paper.

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[1] N.H. Dekker, C.S. Lee, V. Lorent, J.H. Thywissen, S.P. Smith, M. Drndic, R.M. Westerwelt, and M. Prentiss, Phys. Rev. Lett. **84**, 1124 (2000).
[2] P. Rosenbusch, B.V. Hall, I.G. Hughes, C.V. Saba, and E.A. Hinds, Phys. Rev. A **61**, 031404(R) (2000).
[3] R. Folman, P. Krüger, D. Cassettari, B. Hessmo, T. Maier, and J. Schmiedmayer, Phys. Rev. Lett. **84**, 4749 (2000).
[4] W. Hänsel, J. Reichel, P. Hommelhoff, and T.W. Hänsch, Phys. Rev. Lett. **86**, 608 (2001).
[5] D. Cassettari, B. Hessmo, R. Folman, T. Maier, and J. Schmiedmayer, Phys. Rev. Lett. **85**, 5483 (2000).
[6] D. Müller, E.A. Cornell, M. Prevedelli, P.D.D. Schwindt, A. Zozulya, and D.Z. Anderson, Opt. Lett. **25**, 1382 (2000).
[7] J. Schmiedmayer, Eur. Phys. J. D **4**, 57 (1998).
[8] D. Jaksch, H.-J. Briegel, J.I. Cirac, C.W. Gardiner, and P. Zoller, Phys. Rev. Lett. **82**, 1975 (1999).
[9] H.-J. Briegel, T. Calarco, D. Jaksch, J.I. Cirac, and P. Zoller, J. Mod. Opt. **47**, 415 (2000).
[10] T. Calarco, H.-J. Briegel, D. Jaksch, J.I. Cirac, and P. Zoller, J. Mod. Opt. **47**, 2137 (2000).
[11] G.K. Brennen, C.M. Caves, P.S. Jessen, and I.H. Deutsch, Phys. Rev. Lett. **82**, 1060 (1999).
[12] E. Andersson, M.T. Fontenelle, and S. Stenholm, Phys. Rev. A **59**, 3841 (1999).
[13] E. Andersson and S.M. Barnett, Phys. Rev. A **62**, 044301 (2000).
[14] E. Andersson and S. Stenholm, J. Mod. Opt. **48**, 965 (2001).
[15] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, NJ, 1955).
[16] E. Arthurs and J.L. Kelly, Bell Syst. Tech. J. **44**, 725 (1965).
[17] S. Stenholm, Ann. Phys. (N.Y.) **218**, 233 (1992).
[18] I.D. Ivanovic, Phys. Lett. A **123**, 257 (1987).
[19] D. Dieks, Phys. Lett. A **126**, 303 (1988).
[20] A. Peres, Phys. Lett. A **128**, 19 (1988).
[21] A. Chefles and S.M. Barnett, J. Mod. Opt. **45**, 1295 (1998).
[22] C.W. Helstrom, *Quantum Detection and Estimation Theory* (Academic, New York, 1976).
[23] A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic, Dordrecht, MA, 1993).
[24] B. Huttner, A. Müller, J.D. Gautier, H. Zbinden, and N. Gisin, Phys. Rev. A **54**, 3783 (1996).
[25] S.M. Barnett and E. Riis, J. Mod. Opt. **44**, 1061 (1997).
[26] R.B.M. Clarke, A. Chefles, S.M. Barnett, and E. Riis, Phys. Rev. A **63**, 040305(R) (2001).
[27] R.B.M. Clarke, V. Kendon, A. Chefles, S.M. Barnett, E. Riis, and M. Sasaki, Phys. Rev. A **64**, 012303 (2001).
[28] S. Franke-Arnold, E. Andersson, S. M. Barnett and S. Stenholm, Phys. Rev. A **63**, 040305(R) (2001).
[29] E.B. Davies, IEEE Trans. Inf. Theory **IT-24**, 596 (1978).
[30] The trapping fields in a quadrupole guide are orthogonal to the guiding wire. To prevent spin flips at $B=0$, one may add an additional field $B_{ip}$ parallel to the wire, obtaining a Ioffe-Pritchard guide. For example, increasing $B_{ip}$ locally with another current-carrying wire orthogonal to the guiding wire would shift the guide potential. Increasing or decreasing this additional current would tune the phase shift without changing the hardware on the atom chip.
[31] D.L. Haycock, P.M. Alsing, I.H. Deutsch, J. Grondalski, and P.S. Jessen, Phys. Rev. Lett. **85**, 3365 (2000).
[32] M. Hammes, D. Rychtarik, V. Druzhinina, U. Moslener, I. Manek-Hönninger, and R. Grimm, e-print physics/0005035.
[33] Z. Hu and H.J. Kimble, Opt. Lett. **19**, 1888 (1994).
[34] F. Ruschewitz, D. Betterman, J.L. Peng, and W. Ertmer, Europhys. Lett. **34**, 651 (1996).
[35] D. Haubrich, H. Schadwinkel, F. Strauch, B. Ueberholz, R. Wynands, and D. Meschede, Europhys. Lett. **34**, 663 (1996).
[36] D. Frese, B. Ueberholz, S. Kuhr, W. Alt, D. Schrader, V. Gomer, and D. Meschede, Phys. Rev. Lett. **85**, 3777 (2000).
[37] A.S. Holevo, Problemy Peredachi Informatsii **9**, 31 (1973) [Probl. Inf. Transm. **9**, 110 (1973)].
[38] A. Peres and W. Wootters, Phys. Rev. Lett. **66**, 1119 (1992).
[39] P. Hausladen and W.K. Wootters, J. Mod. Opt. **41**, 2385 (1994).
[40] M. Reck *et al.*, Phys. Rev. Lett. **73**, 58 (1994).