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Extremely slow propagation of a light pulse in an ultracold atomic vapor: A Raman scheme without electromagnetically induced transparency

M. G.Payne[1] and L. Deng[2]
[1]Department of Physics, Georgia Southern University, Statesboro, Georgia 30460
[2]Electron and Optical Physics Division, NIST, Gaithersburg, Maryland 20899
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We describe a Raman scheme where the group velocity of an optical pulse can be altered dramatically. With this none electromagnetically-induced-transparency scheme, we show that when on a two-photon resonance, a light pulse can propagate with extremely slow group velocity. Both pulse narrowing and broadening can occur depending upon the choice of two-photon detuning. When using a tuned far-off two-photon resonance, we show that the pulse propagates ‘‘superluminally’’ in the medium with pulse narrowing.

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The propagation of an optical pulse through dispersive media has been extensively studied [1–5]. A recent experiment [6] on the propagation of a light pulse in a Bose-Einstein condensed atomic vapor has demonstrated an order-of-magnitude reduction of the group velocity of nearly 7. The key element of the experiment is an on-one-photon-resonance electromagnetically induced transparency (EIT) [6,7] produced by a coupling laser that reduces the absorption of the probe laser to nearly zero. In this Rapid Communication, we describe a Raman scheme that is very different from the usual EIT scheme. The alternative scheme, which is not an EIT scheme, can, with appropriately selected parameters, achieve an order-of-magnitude reduction of the group velocity of an optical pulse of more than 9. Further, the alternative scheme predicts various pulse distortion regimes and, in particular, a regime where pulse narrowing will occur. When both one- and two-photon detunings are large compared with the linewidth of the driving field, by changing the single driving laser frequency to two closely spaced frequency components, the alternative scheme can easily account for the recent study [8] on the ‘‘superluminal’’ propagation of a light pulse. Our theory, however, shows that the superluminal propagation of the pulse is not due to the destructive interference [8]. Indeed, we show that it is the differential gain experienced by the different edges of the pulse that contributes to the ‘‘apparent pulse peak forward shifting’’ that gives the impression of superluminal propagation.

Consider a three-level atomic system that interacts with two laser fields (Fig. 1). We emphasize that our scheme is not an EIT scheme used in all slow group velocity experiments. The latter is inherently on-one-photon resonance and relies on a doublet created by the driving field to cancel the absorption to the probe field.

In our scheme, when the one-photon detuning δ_1 is large, but two-photon detuning δ_p is zero, the familiar two-photon Raman resonant process is established. In this Raman gain situation, as we will show later, a substantial group velocity reduction can be obtained. This is known as subluminal propagation, since the group velocity of the pulse is less than the speed of light in a vacuum. A recent experiment [9] falls in this category but no explanation of the pulse propagation is provided there. When both the one- and two-photon detunings are large, we will show that the pulse ‘‘appears’’ to travel superluminally, i.e., it appears to have exited the material before it would have if it had traveled through an equal distance of vacuum [4,5,8].

We begin our analysis by writing the wave function of the atomic system as

$$|\Psi(z,t)\rangle = a_0 e^{-i\omega_0 t}|0\rangle + a_1 e^{-i\omega_1 t}|1\rangle + a_2 e^{-i\omega_2 t}|2\rangle.$$

Applying the time-dependent Schrödinger equation, introducing one- and two-photon detunings $\delta_1 = \omega_c - \omega_{20}$ and $\delta_p = \omega_p - \omega_{21} - \delta_1$, we obtain three atomic equations of motion:

$$\dot{a}_0 = i\Omega_{02} e^{-ik_c z + i\delta_1 t} a_2,$$

$$\dot{a}_1 = i\Omega_{12}(t) e^{-ik_p z + i(\delta_1+\delta_p)t} a_2 - \frac{\gamma_1}{2} a_1,$$

$$\dot{a}_2 = i\Omega_{21}(t) e^{ik_p z - i(\delta_1+\delta_p)t} a_1 + i\Omega_{20} e^{ik_c z - i\delta_1 t} a_0 - \frac{\gamma_2}{2} a_2. \quad (1)$$

As usual, $\Omega_{ij} = D_{ij}E/(2\hbar)$ is one-half of the Rabi frequency for the transition $|j\rangle \to |i\rangle$. The requirement for the intensity of the driving field is such that the spontaneous Raman gain should be less than several *e* folds of the stimulated gain over the entire path. The probe and the driving field intensities

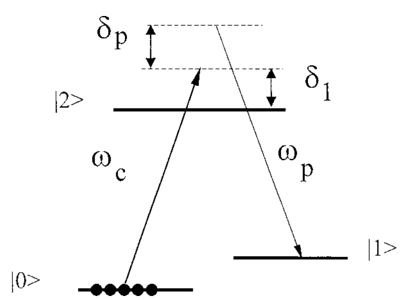

FIG. 1. Energy-level diagram for the far-off one-photon resonance Raman scheme showing relevant laser couplings. As discussed in the text, the subluminal scheme is obtained when $\delta_p \to 0$ and $\gamma_1 \tau_p \gg 1$, whereas the ‘‘superluminal’’ scheme is obtained when $\delta_p \tau \gg 1$.

should be such that the two-photon Rabi frequency times the probe pulse length is much less than 1, therefore, the depletion of the ground state can be neglected. To avoid significant group velocity mismatch and for mathematical simplicity, we will assume a cw driving field. In the case where $|\delta_1| \gg |\Omega_{20}|,|\Omega_{21}|$; $|\Omega_{20}| \gg |\Omega_{21}|$, and use $|a_0| \approx 1$, the third equation yields an adiabatic solution

$$a_2 \approx \frac{\Omega_{20}}{\delta_1 + i\gamma_2/2} e^{ik_c z - i\delta_1 t}. \quad (2)$$

In order to predict the propagation of the probe pulse, Eq. (1) must be solved together with Maxwell's equation for the probe field. For an unfocused beam within the slowly varying amplitude approximation, the wave equation for the positive frequency part of the field reads

$$\frac{\partial E_{p0}^{(+)}}{\partial z} + \frac{1}{c}\frac{\partial E_{p0}^{(+)}}{\partial t} = i\frac{4\pi\omega_p}{c} P_{\omega_p 0}^{(+)}, \quad (3)$$

where the polarization is given by

$$P_{\omega_p 0}^{(+)} = ND_{12} a_2 a_1^* e^{-ik_p z + i(\delta_1+\delta_p)t}$$
$$= -ND_{12} a_1^* e^{-i(k_p - k_c)z + i\delta_p t} \frac{\Omega_{20}}{\delta_1 + i\gamma_2/2}. \quad (4)$$

Differentiation Eq. (4) with respect to time yields

$$\frac{\partial}{\partial t} P_{\omega_p 0}^{(+)} = i(\delta_p + i\gamma_1/2) P_{w_p 0}^{(+)} - iND_{12}\Omega_{21}(t) \frac{|\Omega_{20}|^2}{\delta_1^2 + \gamma_2^2/4}. \quad (5)$$

Taking the Fourier transform of Eqs. (3) and (5), we obtain

$$\frac{\partial}{\partial z}\varepsilon_{p0}^{(+)} - i\frac{\omega}{c}\varepsilon_{p0}^{(+)} = i\kappa_{12}\varepsilon_{p0}^{(+)} W(\omega), \quad (6a)$$

where $\varepsilon_{p0}^{(+)}(z,\omega)$ is the Fourier transform of the probe field,

$$\kappa_{12} = \frac{2\pi N\omega_p |D_{12}|^2}{c} \frac{|\Omega_{20}|^2}{\delta_1^2 + \gamma_2^2/4} = \kappa_{12}^{(0)} \frac{|\Omega_{20}|^2}{\delta_1^2 + \gamma_2^2/4}, \quad (6b)$$

and

$$W(\omega) = \frac{1}{\omega + \delta_p + i\gamma_1/2}. \quad (6c)$$

The solution to the wave equation (6a) is given by

$$\varepsilon_{p0}^{(+)}(z,\omega) = \varepsilon_{p0}^{(+)}(0,\omega)\exp\left[i\frac{\omega}{c}z + i\kappa_{12} z W(\omega)\right]. \quad (7)$$

Upon carrying out the inverse transform, we immediately obtain

$$E_{p0}^{(+)}(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\omega\, \varepsilon_{p0}^{(+)}(0,\omega) \exp\left[-i\omega\left(t - \frac{z}{c} - \frac{\kappa_{12} z W(\omega)}{\omega}\right)\right]. \quad (8)$$

It is remarkable that Eq. (8) can predict many features that are of great interest to pulse propagation.

First, we consider the probe field in the limit of $\delta_p \tau_p \gg 1$ and $\gamma_1 \tau_p \gg 1$. Under these conditions, $W(\omega)$ can be expanded in a McLaurin series

$$W(\omega) = W^{(0)}(0) + W^{(1)}(0)\omega + \frac{1}{2}W^{(2)}(0)\omega^2 + O(\omega^3).$$

Keeping terms up to order ω^2, the probe field can be evaluated analytically and yields

$$E_{p0}^{(+)}(z,t) = \frac{E_{p0}^{(+)}(0,0) e^{i\kappa_{12} z W^{(0)}(0)}}{\sqrt{1 - i\dfrac{2\kappa_{12} z W^{(2)}(0)}{\tau_p^2}}} \times \exp\left[-\frac{[t - z/c - \kappa_{12} z W^{(1)}(0)]^2}{\tau_p^2(1 - 2i\kappa_{12} z W^{(2)}(0)/\tau_p^2)}\right]. \quad (9)$$

Several observations can be made quickly:

(i) The imaginary part of $W^{(0)}(0)$ gives rise to a gain, while as the real part of $W^{(0)}(0)$ will give a position-dependent oscillatory factor.

(ii) When $W^{(2)}(0)$ can be neglected, the group velocity is solely determined by $W^{(1)}(0)$. The sign of Re$[W^{(1)}(0)]$ determines the propagation is subluminal or superluminal, whereas the imaginary part of $W^{(1)}(0)$ will give rise to a position-dependent gain.

(iii) Both the real and imaginary parts of $W^{(2)}(0)$ contribute to the pulse width change as well as to the gain. In addition, $W^{(2)}(0)$ will contribute to the group velocity.

The explicit time-dependent factor of the probe field amplitude [Eq. (9)] can be expressed as

$$\propto \exp\left[-\frac{(t - z/c - c_1 + c_2 b_2/b_1)^2}{\tau_p^2(1 + b_2^2/b_1^2) b_1}\right], \quad (10)$$

where

$$c_1 = \kappa_{12} z\, \mathrm{Re}[W^{(1)}(0)], \quad c_2 = \kappa_{12} z\, \mathrm{Im}[W^{(1)}(0)], \quad (11a)$$

and

$$b_1 = 1 + 2\kappa_{12} z\, \mathrm{Im}[W^{(2)}(0)]/\tau_p^2,$$
$$b_2 = 2\kappa_{12} z\, \mathrm{Re}[W^{(2)}(0)]/\tau_p^2. \quad (11b)$$

Therefore, the group velocity can be expressed as

$$\frac{1}{V_g}=\frac{1}{c}+\kappa_{12}\,\mathrm{Re}[W^{(1)}(0)]-\kappa_{12}\,\mathrm{Im}[W^{(1)}(0)]\times\left(\frac{2\kappa_{12}Z\,\mathrm{Re}[W^{(2)}(0)]/\tau_p^2}{1+2\kappa_{12}z\,\mathrm{Im}[W^{(2)}(0)]/\tau_p^2}\right), \tag{12}$$

whereas the relative width of the pulse is given by

$$(\text{relative pulse width})=(1+2\kappa_{12}z\,\mathrm{Im}[W^{(2)}(0)]/\tau_p^2)\times\left[1+\left(\frac{2\kappa_{12}z\,\mathrm{Re}[W^{(2)}(0)]/\tau_p^2}{1+2\kappa_{12}z\,\mathrm{Im}[W^{(2)}(0)]/\tau_p^2}\right)^2\right]. \tag{13}$$

We now distinguish two regions of interest for further discussion.

(1) Propagation without pulse shape change. If $|2\kappa_{12}zW^{(2)}(0)/\tau_p^2|\ll 1$, we obtain from Eq. (9)

$$E_{p0}^{(+)}(z,t)=E_{p0}^{(+)}(0,0)e^{i\kappa_{12}zW^{(0)}(0)}\exp\left[-\frac{(t-z/V_g)^2}{\tau_p^2}\right], \tag{14}$$

where the group velocity is given by $1/V_g=1/c+\kappa_{12}\,\mathrm{Re}[W^{(1)}(0)]$, as can be seen from Eq. (12). In this limit, the probe pulse shape remains unchanged and there are two possibilities:

Subluminal propagation. Consider the case where the two-photon detuning is zero. In this case, the expansion parameter would be $\gamma_1\tau_p\gg 1$. Since in most cases state $|1\rangle$ is metastable, and the dephasing rate is very low (a few kHz), this condition requires a long pulse length. Let $\delta_p\to 0$, we immediately find from Eqs. (6b), (6c), and (12) that

$$V_g=\frac{c}{1+\dfrac{c\kappa_{12}}{(\gamma_1/2)^2}}\approx\left(\frac{\gamma_1\delta_1}{2|\Omega_{20}|^2}\right)^2 V_g^{(\mathrm{EIT})},\quad V_g^{(\mathrm{EIT})}=\frac{|\Omega_{20}|^2}{\kappa_{12}^{(0)}}. \tag{15}$$

This key result shows that (1) propagation is subluminal, and (2) with appropriately chosen parameters, the reduction of the group velocity can be significantly larger than can be achieved with the usual on-one-photon-resonance EIT scheme under the same conditions. Notice that in the usual on-one-photon-resonance EIT scheme $|\Omega_{20}|$ must be relatively large compared to linewidths of the laser and upper excited state in order to create a transparency gap that can significantly reduce the absorption of the probe pulse. Yet, large $|\Omega_{20}|$ is detrimental to the reduction of the group velocity of the pulse in the usual EIT scheme. Consider a case where the lifetimes of the two excited states and the one-photon detuning are taken as $\gamma_1\approx 5$ kHz, $\gamma_2\approx 10$ MHz, and $\delta_1\approx 1$ GHz, respectively. In this case, with a same coupling laser Rabi frequency $|\Omega_{20}|\approx 5$ MHz, the Raman scheme could easily give a factor of 100, a further reduction for the group velocity achievable with the usual on-one-photon-resonance EIT scheme under the same condition [10]. In addition, from Eq. (6c) we see that when $\delta_p\to 0$ and $\gamma_1\tau_p\gg 1$, $W^{(0)}(0)=$imaginary and negative, therefore, a gain is established in Eq. (14). This is very different from the usual on-one-photon-resonance EIT scheme where $W^{(0)}(0)=0$, indicating zero absorption in the center of the transparency gap. Therefore, the significant probe field loss associated with the EIT scheme is greatly reduced with the Raman scheme [11].

Superluminal propagation. For $\delta_p\gg\gamma_1$, the probe pulse group velocity is given by

$$V_g=\frac{c}{1-\dfrac{c\kappa_{12}}{(\delta_p^2+\gamma_1^2/4)^2}(\delta_p^2-\gamma_1^2/4)}\approx\frac{c}{1-\dfrac{c\kappa_{12}}{|\delta_p|^2}}. \tag{16}$$

Notice that this group velocity index can be very large yet negative, this is termed as superluminal propagation [3–5,8]. The interpretation of this result will be discussed later.

(2) Propagation with pulse shape change. In the case where $2\kappa_{12}zW^{(2)}(0)/\tau_p^2\ll 1$ is not satisfied, we must carefully evaluate the second derivative of the dispersion relation. Although both the real and imaginary parts of $W^{(2)}(0)$ contribute to the variation of the pulse width, a crude estimate of the pulse width variation can be made by examining the imaginary part of $W^{(2)}(0)$. We will see that the real part of $W^{(2)}(0)$ will always contribute to the pulse broadening and the neglect of the real part will shift the onset of the group velocity for superluminal propagation. Later, we will include both the real and imaginary parts and evaluate the pulse width numerically.

From Eq. (13), we notice that the pulse narrowing is dictated by the sign of

$$\mathrm{Im}(W^{(2)}(0))=-\frac{2}{(\delta_p^2+\gamma_1^2/4)^3}\left(3\delta_p^2\frac{\gamma_1}{2}-\frac{\gamma_1^3}{8}\right). \tag{17a}$$

It is immediately obvious that if $\delta_p>\gamma_1/(2\sqrt{3})$, the pulse width will be narrowed. Otherwise, the pulse width will be broadened. On the other hand, from Eq. (12) we see that the sign of

$$\mathrm{Re}(W^{(1)}(0))=-\frac{1}{(\delta_p^2+\gamma_1^2/4)^2}\left(\delta_p^2-\frac{\gamma_1^2}{4}\right), \tag{17b}$$

determines the characteristics of the group velocity. Therefore, when $\delta_p>\gamma_1/2$ the propagation is superluminal, whereas when $\delta_p<\gamma_1/2$, the propagation is subluminal. Combining Eqs. (17a) and (17b), we immediately conclude that for the two-photon detuning $\gamma_1/(2\sqrt{3})<\delta_p<\gamma_1/2$, we will have subluminal propagation with pulse narrowing. It is, therefore, possible to have either pulse broadening or narrowing for extreme slow propagation. In Fig. 2 we plot a typical result where we have chosen parameters to give $2\kappa_{12}z/\tau_p^2\approx 9.4\times 10^8$ for $|\Omega_{20}|^2/\delta_1^2=10^{-6}$. It is seen that the pulse width changes from broadening to narrowing, whereas the propagation characteristics change from subluminal to superluminal. In the case of $\delta_p>\gamma_1/2$, the propagation is superluminal and pulse narrowing will always occur as long

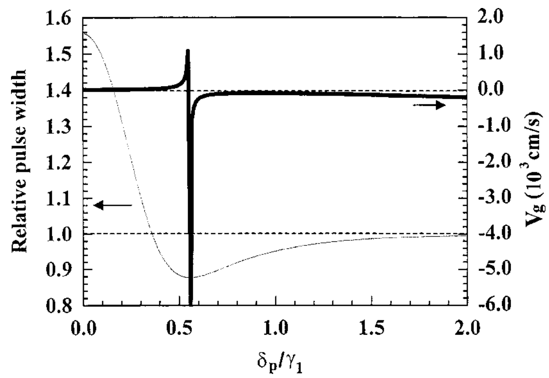

FIG. 2. A plot of relative pulse width and group velocity as a function of δ_p/γ_1. Notice the region where pulse may travel at lower group velocity with width deduction.

as the condition $2\kappa_{12}zW^{(2)}(0)/\tau_p^2 \ll 1$ is not satisfied. Notice, however, that $W^{(2)}(0) \propto 1/\delta_p^2$, therefore, as δ_p increases, $W^{(2)}(0)$ decreases rapidly and the condition $2\kappa_{12}zW^{(2)}(0)/\tau_p^2 \ll 1$ quickly sets in, and the pulse shape will remain the same as that of the input pulse.

The above treatment can be easily extended to the case where the pump laser has two closely spaced frequency components. An experiment [8] with this configuration has been reported recently and it has shown that the pulse travels superluminally. In this case, a straightforward calculation shows that the group velocity is given by

$$V_g = \frac{c}{1 - \frac{4\kappa_{12}c}{\Delta^2}\left[\left|\frac{\Omega_{20}^{(L1)}}{\delta_1}\right|^2 + \left|\frac{\Omega_{20}^{(L2)}}{\delta_1+\Delta}\right|^2\right]}.$$

It is trivial to reproduce the similar lead-time reported in Ref. [8] using the above expression and the data given in Ref. [8]. It should, however, be made clear that the superluminal propagation in this case is not caused by the destructive interference of the two frequency components. The destructive interference will mainly contribute to the reduction of absorption that is dependent upon $W^{(0)}(0)$. The group velocity, regardless whether it is superluminal or subluminal, is predominately due to $W^{(1)}(0)$ with some contribution from $W^{(2)}(0)$, but not $W^{(0)}(0)$. The apparent superluminal propagation of light pulse in the present case is due to the differential gain to the front and rear edges of the pulse. Such a different gain, as can be easily deduced from Eq. (10), leads to an apparent "shift" of the "peak," thereby giving an apparent superluminal effect.

In summary, we have presented a far-off one-photon resonance Raman scheme that can achieve an order-of-magnitude reduction on the group velocity of more than 9 without using the EIT effect. We have shown that the alternative scheme is superior to the usual on-one-photon-resonance EIT scheme in two aspects: (1) under the same operation conditions described here [10] the reduction of the group velocity is better than that of an EIT scheme, and (2) a gain rather than a significant loss of the probe field. In addition, our theory has revealed various regimes of subluminal propagation where both pulse broadening and pulse narrowing can occur, regimes that have never been studied so far. An added advantage of the theory is that it can also correctly predict superluminal propagation when the two-photon detuning is large.

Note added. Recently, the authors have learned of related work by Sprangle *et al.* [12].

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