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Entanglement between motional states of a single trapped ion and light

F. L. Semião, A. Vidiella-Barranco, and J. A. Roversi
Instituto de Física "Gleb Wataghin," Universidade Estadual de Campinas, 13083-970 Campinas São Paulo Brazil
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We propose a generation method of Bell-type states involving light and the vibrational motion of a single trapped ion. The trap itself is supposed to be placed inside a high-Q cavity sustaining a single mode, quantized electromagnetic field. Entangled light-motional states may be readily generated if a conditional measurement of the ion's internal electronic state is made after an appropriate interaction time and a suitable preparation of the initial state. We show that all four Bell states may be generated using different motional sidebands (either blue or red), as well as adequate ionic relative phases.

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The investigation of trapped ions manipulated by laser beams [1] is of importance not only due to the fundamental physics involved, but also because of potential applications, such as precision spectroscopy [2] and quantum computation [3]. The laser fields couple the (quantized) internal degrees of freedom in the ion to the (quantized) vibrational motion of the ion's center of mass, but the fields themselves are usually treated as classical. The quantization of the field of course brings new possibilities. Within that realm, it has been already investigated the influence of the field statistics on the ion dynamics [4,5], as well as the transfer of coherence between the motional states and light [6]. There is much interest in the generation of nonproduct, entangled states, and trapped ions seem to constitute a suitable system for doing that [7]. Entangled states involving atoms rather than photons may also be used for testing Bell's inequality [8,9], as it has been already experimentally demonstrated [8]. In general, what has been achieved so far is either the entanglement between the internal degrees of freedom of a single ion (electronic states) with the vibrational motion states of the ion itself, or the entanglement between internal states of several ions [7,10]. Nevertheless, there are few discussions about possibilities of entanglement between the quantized field and the vibrational motion of the ion. This might be of special interest in quantum information; an entangled state of a subsystem that may *store quantum information* (vibrational motion) with a subsystem that can be used for the *propagation of quantum information* (light). As another example of entanglement between matter and light, we may refer to a recently reported scheme for entangling light with atoms in a Bose-Einstein condensate [11].

In this contribution we present a simple scheme through which there could be produced entanglement between the (center of mass) vibrational motion of a single trapped ion and the electromagnetic field. We show that it is possible to generate the whole Bell-state basis simply by choosing either the blue or the red sideband (with different relative phases between ionic states).

We consider a single trapped ion, within a Paul trap, which is by its turn placed inside a high-Q cavity [12], so that the cavity mode couples to the internal electronic states of the ion as well as to the vibrational degrees of freedom, as it has been already discussed in [5]. The Hamiltonian corresponding to such a system may be written as

$$\hat{H}=\hbar\nu\hat{a}^\dagger\hat{a}+\hbar\omega\hat{b}^\dagger\hat{b}+\hbar\frac{\omega_0}{2}\sigma_z+\hbar g(\sigma_+ + \sigma_-)\times(\hat{b}^\dagger+\hat{b})\sin\eta(\hat{a}^\dagger+\hat{a}). \tag{1}$$

Here $\hat{a}^\dagger$ ($\hat{a}$) denote the creation (annihilation) operators of the center-of-mass vibrational motion of the ion (frequency ν), $\hat{b}^\dagger$ ($\hat{b}$) are the creation (annihilation) operators of photons in the field mode (frequency ω), ω_0 is the atomic frequency, g is the ion-field coupling constant, and $\eta=2\pi a_0/\lambda$ is the Lamb-Dicke parameter, being a_0 the amplitude of the harmonic motion and λ the wavelength of light. In the Lamb-Dicke regime, i.e., if the ion is confined in a region much smaller than light's wavelength ($\eta\ll 1$), we may write $\sin\eta(\hat{a}^\dagger+\hat{a})\approx\eta(\hat{a}^\dagger+\hat{a})$. This is of course a convenient way of linearizing the Hamiltonian, although another approach, based on a unitary transformation of the Hamiltonian and which avoids the application of Lamb-Dicke approximation from the beginning, is also possible [13]. If we tune the light field to the first red sideband, i.e., $\delta=\omega_0-\omega=\nu$, we obtain, after discarding the rapidly oscillating terms, the following interaction picture Hamiltonian:

$$\hat{H}_I^r=\eta\hbar g(\sigma_-\hat{a}^\dagger\hat{b}^\dagger+\sigma_+\hat{a}\hat{b}). \tag{2}$$

Such a Hamiltonian describes the simultaneous process of creation (annihilation) of one quanta of vibrational motion, one quanta of the field, while the atom has its internal energy decreased (increased). The corresponding evolution operator $\hat{U}^r(t)=\exp(-i\hat{H}_I^r t/\hbar)$ will be, in the atomic basis,

$$\hat{U}^r(t)=\hat{C}_{n+1}|e\rangle\langle e|+\hat{C}_n|g\rangle\langle g|-i\hat{S}_{n+1}\hat{a}\hat{b}|e\rangle\langle g| -i\hat{a}^\dagger\hat{b}^\dagger\hat{S}_{n+1}|g\rangle\langle e|, \tag{3}$$

where

$$\hat{C}_{n+1}=\cos[\eta g\sqrt{(\hat{a}^\dagger\hat{a}+1)(\hat{b}^\dagger\hat{b}+1)}t], \tag{4}$$

$$\hat{C}_n=\cos[\eta g\sqrt{\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}t], \tag{5}$$

and

$$\hat{S}_{n+1}=\frac{\sin[\eta g\sqrt{(\hat{a}^{\dagger}\hat{a}+1)(\hat{b}^{\dagger}\hat{b}+1)}t]}{\sqrt{(\hat{a}^{\dagger}\hat{a}+1)(\hat{b}^{\dagger}\hat{b}+1)}}. \tag{6}$$

We may now investigate the time evolution of the state vector having the following initial condition for the ion-field state

$$|\Psi(0)\rangle=|n\rangle_f|m\rangle_v(\cos\theta|e\rangle+e^{i\phi}\sin\theta|g\rangle), \tag{7}$$

or the field prepared in a number state $|n\rangle_f$ containing n photons, the ion's center-of-mass motion prepared in a number state $|m\rangle_v$ containing m quanta, and the ion's internal levels prepared in a coherent superposition of two energy eigenstates $|\varphi\rangle=\cos\theta|e\rangle+e^{i\phi}\sin\theta|g\rangle$. This particular initial condition is crucial for the generation of entanglement between the states of ionic vibration and the electromagnetic field. Such an initial superposition state in Eq. (7) may be prepared through the convenient application of laser pulses. At a time t, the ion-field state vector will become

$$\begin{aligned}|\Psi(t)\rangle=&[\cos\theta\cos(\eta g\sqrt{(n+1)(m+1)}t)|n\rangle_f|m\rangle_v\\&-ie^{i\phi}\sin\theta\sin(\eta g\sqrt{nm}t)|n-1\rangle_f|m-1\rangle_v]|e\rangle\\&+[e^{i\phi}\sin\theta\cos(\eta g\sqrt{nm}t)|n\rangle_f|m\rangle_v\\&-i\cos\theta\sin(\eta g\sqrt{(n+1)(m+1)}t)\\&\times|n+1\rangle_f|m+1\rangle_v]|g\rangle.\end{aligned} \tag{8}$$

The resulting state above is an entangled state involving the ion's internal (electronic) degrees of freedom, the vibrational motion, and the cavity field. If one measures the internal state of the ion (either in $|g\rangle$ or $|e\rangle$), that action will collapse the state $|\Psi\rangle$ into entangled states of ionic vibrational motion and the cavity field. For instance, we may consider that the ion's motion is initially cooled down to the vacuum state $|0\rangle_v$ and the cavity field is also in its vacuum state $|0\rangle_f$. In this case, for interaction times $t_k=\pi(4k+1)/2\eta g$ ($k=0,1,2,\ldots$) and for equally weighted ionic states ($\theta=\pi/4$), if one measures (via fluorescence) the ion in its internal state $|g\rangle$, the resulting vibration-light-field state will become

$$|\psi\rangle=\frac{1}{\sqrt{2}}(e^{i\phi}|0\rangle_f|0\rangle_v-i|1\rangle_f|1\rangle_v), \tag{9}$$

which is a Bell-type state, or an entangled state having as subsystems both the ion and cavity field. Note that the ionic relative phase ϕ in the superposition of electronic states $|e\rangle$ and $|g\rangle$ is fully transferred to the resulting entangled state. Although there is no direct interaction between the motion of the trapped ion and the cavity field, the coupling of the ion's internal (electronic) levels to both vibration and light makes possible the generation of states of the type shown above in Eq. (9).

There is also the possibility of tuning the light field to the first blue sideband, or $\delta=-\nu$. The corresponding interaction Hamiltonian will then read

$$\hat{H}_I^b=\eta\hbar g(\sigma_-\hat{a}\hat{b}^{\dagger}+\sigma_+\hat{a}^{\dagger}\hat{b}). \tag{10}$$

In this case, while the ion has its internal-energy increased, a quanta of its vibrational motion is created and a photon is annihilated. We may follow a similar procedure as we have done for the first red sideband, and find out what type of entangled states may be generated under such circumstances. The corresponding evolution operator is

$$\begin{aligned}\hat{U}^b(t)=&\hat{C}'_{n+1}|e\rangle\langle e|+\hat{C}'_n|g\rangle\langle g|-i\hat{S}'_n\hat{b}\hat{a}^{\dagger}|e\rangle\langle g|\\&-i\hat{b}^{\dagger}\hat{a}\hat{S}'_n|g\rangle\langle e|,\end{aligned} \tag{11}$$

where

$$\hat{C}'_{n+1}=\cos[\eta g\sqrt{(\hat{b}^{\dagger}\hat{b}+1)\hat{a}^{\dagger}\hat{a}}t], \tag{12}$$

$$\hat{C}'_n=\cos[\eta g\sqrt{\hat{b}^{\dagger}\hat{b}(\hat{a}^{\dagger}\hat{a}+1)}t], \tag{13}$$

and

$$\hat{S}'_n=\frac{\sin[\eta g\sqrt{(\hat{b}^{\dagger}\hat{b}+1)\hat{a}^{\dagger}\hat{a}}t]}{\sqrt{(\hat{b}^{\dagger}\hat{b}+1)\hat{a}^{\dagger}\hat{a}}}. \tag{14}$$

We may prepare the initial state as the one in Eq. (7), but with $n=0$ (field in the vacuum state) and $m=1$ (ion vibrational motion in the first excited state). For interaction times t_k (the same as to the red sideband case), after having detected the ion in the internal state $|g\rangle$, the resulting state will be

$$|\psi'\rangle=\frac{1}{\sqrt{2}}(e^{i\phi}|0\rangle_f|1\rangle_v-i|1\rangle_f|0\rangle_v), \tag{15}$$

which is also a Bell-type state involving the quantized cavity field as well as the ion's vibrational motion. In fact, by taking $\phi\pm\pi/2$ in Eqs. (15) and (9), we are able to obtain the four states constituting the *Bell-state basis*

$$|\psi\rangle_{\pm}=\frac{1}{\sqrt{2}}(|0\rangle_f|0\rangle_v\pm|1\rangle_f|1\rangle_v), \tag{16}$$

and

$$|\psi'\rangle_{\pm}=\frac{1}{\sqrt{2}}(|0\rangle_f|1\rangle_v\pm|1\rangle_f|0\rangle_v). \tag{17}$$

More general states could also be generated, depending on the initial conditions. For instance, in the red sideband case ($\delta=\omega_0-\omega=\nu$), if the ion is initially prepared in a state having m excitations, $|m\rangle$, and the field in the vacuum state, the resulting entangled state will be (after measuring the ion in the internal state $|g\rangle$)

$$|\psi\rangle=\frac{1}{\sqrt{2}}(e^{i\phi}|0\rangle_f|m\rangle_v-i|1\rangle_f|m+1\rangle_v). \tag{18}$$

Note that entanglement here involves states belonging to different kinds of physical systems (although both subspaces have infinite dimension), but because of their nature (light and massive trapped ions) new possibilities for quantum information processing might arise. Needless to say that the generation of such entangled states is of importance for addressing fundamental issues in quantum theory as well. We have so far considered an ideal situation in which losses are not taken into account. Dissipation introduced by the finite-Q cavity will of course become an important origin for decoherence, together with losses related to the ion trap itself. Therefore the Bell state generation here proposed must be accomplished within the cavity decay time, which is long enough in high-Q cavities available nowadays (around 0.2 s) [14]. The loss of a photon will obviously destroy the Bell state generated in such a scheme. Nevertheless we shall remark that losses, which are normally regarded as responsible for decoherence effects, might not only induce nonclassical behavior [15] but also assist the generation of pure states [16]. In the method presented in Ref. [16], an atom-atom entangled state is generated through a (properly monitored) photon decay. In our case, although both subsystems constituting the Bell states are vulnerable to losses, it would be possible to preserve their integrity for relatively long times [14].

In summary, we have proposed a generation scheme of entangled states of two-coupled harmonic oscillators, the ionic vibrational motion, and a cavity field. We have shown that it is possible to generate Bell-type states having rather simple initial state preparation, e.g., the vacuum state for both cavity field and the ion motion.

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