

Quantum phase gate with a selective interaction

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(Received 9 February 2001; published 12 July 2001)

We present a proposal for implementing quantum phase gates using selective interactions. We analyze selectivity and the possibility to implement these gates in two particular systems, namely, trapped ions and cavity QED.

DOI: 10.1103/PhysRevA.64.024304

PACS number(s): 03.67.Lx, 42.50.Vk, 32.80.Qk

Quantum computers would perform certain tasks, such as factoring a number and searching a data in an array, faster than a classical computer [1,2]. The core of quantum computing are the quantum logic gates. In fact, it is known that any quantum computation can be reduced to a sequence of universal two-qubit logic gates and one-qubit local operations [3]. Since the original formulation of quantum computation, a number of experimental systems have been proposed as candidates for the practical implementation of these quantum gates. We may mention trapped ions [4], cavity QED [5,6], liquid-state nuclear magnetic resonance (NMR) [7], quantum dots [8], optical lattices [9], among others. These systems have shown to be good enough not only for testing quantum logic gates but also for some basic quantum operations with a few qubits. For example, the realization of quantum logic gates in trapped ions [10], cavity QED [11,12], and NMR [13] have already been possible. The aim of the mentioned experiments was essentially the practical realization in a bipartite system of two equivalent kinds of universal two-qubit logic gates: a quantum controlled NOT (CNOT) gate and a quantum phase gate (QPG) [14]. The CNOT gate and the QPG differ from each other only by local operations (single qubit rotations). In a given bipartite system, consisting of the so-called control and target qubits, a CNOT operation changes the target-qubit state only if the control-qubit state is in a specific state. Explicitly, a CNOT operation acting on the initial arbitrary state

$$|\Psi\rangle = \alpha|\downarrow, \downarrow\rangle + \beta|\uparrow, \downarrow\rangle + \gamma|\downarrow, \uparrow\rangle + \delta|\uparrow, \uparrow\rangle \quad (1)$$

produces

$$|\Psi'\rangle = \alpha|\downarrow, \downarrow\rangle + \beta|\uparrow, \uparrow\rangle + \gamma|\downarrow, \uparrow\rangle + \delta|\uparrow, \downarrow\rangle, \quad (2)$$

where the first label in the kets refers to the control qubit and the second one refers to the target qubit. On the other hand, a QPG acting on the same initial state produces

$$|\Psi''\rangle = -\alpha|\downarrow, \downarrow\rangle + \beta|\uparrow, \downarrow\rangle + \gamma|\downarrow, \uparrow\rangle + \delta|\uparrow, \uparrow\rangle. \quad (3)$$

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It can be easily seen that a QPG can act as a CNOT gate if we rotate the target bit before and after operating the gate according to the following steps: (i) A single qubit rotation in the target qubit

$$|\downarrow\rangle \rightarrow |-\rangle = |\downarrow\rangle - |\uparrow\rangle/\sqrt{2}, \quad (4)$$

$$|\uparrow\rangle \rightarrow |+\rangle = |\downarrow\rangle + |\uparrow\rangle/\sqrt{2},$$

also known as Hadamard gate. (ii) A QPG; and (iii) another Hadamard rotation in the target qubit. Because of their fundamental interest, in quantum-logic tests as well as in the search of scalable quantum computing, it is always useful to find ways of implementing CNOT and QPG gates in the laboratory. This fact motivated a number of proposals for realizing such gates in different quantum systems. The main problem these systems face, when scalability is the goal, is decoherence. For the quantum computing schemes to work it is essential to keep coherence of the qubits themselves and among them. However, when the dimension of the system and the number of operations increase, decoherence effects can become more and more important. Therefore, simplifying the operations on the qubits [15] as well as making faster logic gates [16] are the main purpose of many of these works.

In this paper, we will be concerned with turning the computational process simpler: we propose the use of a selective interaction that would realize a quantum phase gate with a single pulse, i.e., without changing experimental parameters during the process. We will discuss our method in a trapped ions system and in the domain of CQED.

First, we will consider an array of N ions of mass m in a Paul trap. The ions will be treated as two-level systems that interact with each other through a coulombian force, so that collective vibrational modes can be conveniently introduced [17]. The manipulation of both electronic states and vibrational collective modes can be done by means of laser beams tuned to appropriate frequencies.

Our task is then to produce an interaction that performs the transformation described in Eq. (3). This interaction must act effectively on two chosen ions (j and k) in the array, ion j being the control qubit and ion k the target qubit. To achieve this, we address ion j with a Raman laser pair [18] described by the electric field $\vec{E}_1 = \vec{E}_0 e^{i(qz - \omega_1 t)}$ and ion k

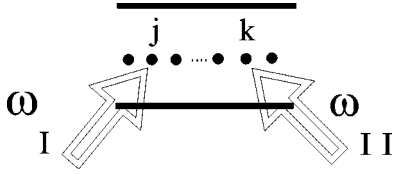


FIG. 1. Two Raman beams of frequencies ω_I and ω_{II} interacting dispersively with ions j and k , respectively.

with another Raman laser pair described by the electric field $\vec{E}_{II} = \vec{E}_{oII} e^{i(qz - \omega_{II}t)}$ (see Fig. 1). The Raman systems have different effective frequencies, ω_I and ω_{II} , that are quasis resonant to an electronic transition associated with the angular frequency ω_o . The Hamiltonian corresponding to this situation is

$$\hat{H} = \hat{H}_o + \hat{H}_{int}, \quad (5)$$

with

$$\hat{H}_o = \hbar \omega_o (\hat{S}_{+j} \hat{S}_{-j} + \hat{S}_{+k} \hat{S}_{-k}) + \hbar \nu \hat{a}^\dagger \hat{a} + \sum_n \hbar \nu_n \hat{b}_n^\dagger \hat{b}_n \quad (6)$$

and

$$\begin{aligned} \hat{H}_{int} = & \hbar \Omega \{ (\hat{S}_{+j} + \hat{S}_{-j}) \{ \exp[i(q\hat{z}_j - \omega_I t)] + \exp[-i(q\hat{z}_j \\ & - \omega_{II} t)] \} + (\hat{S}_{+k} + \hat{S}_{-k}) \{ \exp[i(q\hat{z}_k - \omega_{II} t)] \\ & + \exp[-i(q\hat{z}_k - \omega_I t)] \} \}. \end{aligned} \quad (7)$$

Here, $\hat{S}_{+i} = |\uparrow_i\rangle\langle\downarrow_i|$, $\hat{S}_{-i} = |\downarrow_i\rangle\langle\uparrow_i|$, and the state $|\uparrow_i\rangle$ ($|\downarrow_i\rangle$) corresponds to the i th ion in the excited (fundamental) state. \hat{a}^\dagger (\hat{a}) is the creation (annihilation) operator associated with the harmonic oscillation with frequency ν of the c.m. mode. \hat{b}_n^\dagger (\hat{b}_n) is the creation (annihilation) operator associated with the harmonic oscillation with frequency ν_n of the other collective modes. All frequencies ν_n are bigger than ν [17]. \hat{z}_j and \hat{z}_k are the operators corresponding to the positions of the ions j and k , respectively, and can be rewritten as linear combinations of the operators corresponding to the collective coordinates. \hat{H}_o is the free Hamiltonian that corresponds to the internal energy of the two ions plus the energy of the c.m. mode and of the other collective modes. \hat{H}_{int} is the interaction Hamiltonian describing the position-dependent dipolar interaction of ions j and k with the two Raman beams.

We choose the frequencies ω_I and ω_{II} to be quasis resonant to the first upper c.m. sideband and to the carrier, respectively. Specifically, $\omega_I = \omega_o + \nu - \delta$ and $\omega_{II} = \omega_o + \delta$, where δ is the detuning of each Raman beam with respect to the mentioned resonant vibronic transitions, such that $\omega_I + \omega_{II} = 2\omega_o + \nu$. Similarly to what was done in [19,20], the Hamiltonian in Eq. (7) can be expanded in terms of creation and annihilation operators of the normal modes and rewritten in the interaction picture. Then, following the standard procedure described in [21], we can make the rotating-wave approximation (RWA) and discard the terms that oscillate with

higher frequencies in the dispersive limit $\Omega \ll \delta \ll \nu$. We do not claim the Lamb-Dicke regime. In this way, the following effective time-independent Hamiltonian

$$\begin{aligned} H_{eff} = & \hbar \Omega_o \hat{G}_o^2 \left\{ i \eta \hat{S}_{+j} \hat{S}_{+k} e^{2i\phi} [\hat{a}^\dagger \hat{F}_o - \hat{F}_o \hat{a}^\dagger] \hat{F}_1 \right. \\ & + \frac{1}{2} \hat{S}_{+j} \hat{S}_{-j} [\eta^2 \hat{a}^\dagger \hat{F}_1^2 \hat{a}] - \frac{1}{2} \hat{S}_{-j} \hat{S}_{+j} [\eta^2 \hat{F}_1 \hat{a} \hat{a}^\dagger \hat{F}_1] \\ & \left. - \frac{1}{2} \hat{S}_{+k} \hat{S}_{-k} [\hat{F}_o^2] + \frac{1}{2} \hat{S}_{-k} \hat{S}_{+k} [\hat{F}_o^2] + \text{H.c.} \right\} \end{aligned} \quad (8)$$

can be derived as was done in [19,20]. Here, $\Omega_o = \Omega^2 / \delta$ and $\eta = q \sqrt{\hbar / 2Nm\nu}$ is the Lamb-Dicke parameter associated with the c.m. motion. The functions

$$\hat{F}_k = \sum_n f_k(n) |n\rangle\langle n|, \quad (9)$$

with

$$f_k(n) = e^{-\eta^2/2} \frac{n!}{(n+k)!} L_n^k(\eta^2), \quad (10)$$

where $L_n^k(\eta^2)$ are the generalized Laguerre polynomials, can always be written in terms of the c.m. mode number operator \hat{n} . \hat{G}_o is a similar nonlinear function involving the number operators related to all other normal modes. The exact form of function \hat{G}_o is irrelevant in our case, since we will suppose in this paper that all collective modes are cooled down to the ground state and that only the c.m. mode will be excited. In this case \hat{G}_o contributes to the effective Rabi frequency with only a constant numerical factor of the order of one. The first term of H_{eff} and its Hermitian conjugate describe the common excitation of both electronic states and the c.m. mode, similar to a nonlinear anti-Jaynes-Cummings interaction. The other terms are motional-dependent self-energy terms. The main difference of the Hamiltonian of Eq. (8), when compared to the one described in Ref. [20], is that now the subspace $\{|\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle\}$ remains untouched, a natural consequence of the ionic individual addressing demanded in the present scheme.

Selectivity arises from the \hat{n} dependence of the self-energy corrections in Eq. (8). The stark shift of the electronic states of the ions depends explicitly on the number of phonons of the c.m. mode through \hat{F}_o^2 . We can adjust the laser-beam frequencies to compensate this shift for one particular subspace transition tuning it to resonance. The new frequencies depend strongly on the selected vibronic subspace we want to excite. This will yield another selective interaction, different from the one discussed in [20], that only performs resonant transitions inside the closed subspace $\{|\downarrow\downarrow\rangle|n\rangle, |\uparrow\uparrow\rangle|n+1\rangle\}$ without producing transitions inside the subspace $\{|\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle\}$. For the sake of simplicity, we only label the c.m. mode excitations, since the other vibronic states are cooled down to the vacuum state and in this way are not affected by the interaction Hamiltonian.

We are now ready to show how a QPG can be implemented using the interaction described in the Hamiltonian of Eq. (8). We study the effect of the associated evolution operator \hat{U} over some relevant states of the Hilbert space

$$\begin{aligned}\hat{U}|\downarrow\downarrow,0\rangle &= \cos(\Omega_{eff}t)|\downarrow\downarrow,0\rangle + \sin(\Omega_{eff}t)|\uparrow\uparrow,1\rangle, \\ U|\uparrow\downarrow,0\rangle &= |\uparrow\downarrow,0\rangle, \\ U|\downarrow\uparrow,0\rangle &= |\downarrow\uparrow,0\rangle, \\ \hat{U}|\uparrow\uparrow,0\rangle &= |\uparrow\uparrow,0\rangle,\end{aligned}\quad (11)$$

with $\Omega_{eff} = i\eta\Omega_o g_o^2(0)f_1(0)[f_o(0) - f_o(1)]$. The QPG is implemented by letting this evolution operator act during a time interval equivalent to a 2π pulse over the initial state

$$|\Phi\rangle = [\alpha|\downarrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle + \gamma|\uparrow\downarrow\rangle + \delta|\uparrow\uparrow\rangle]|0\rangle, \quad (12)$$

obtaining

$$|\Phi''\rangle = [-\alpha|\downarrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle + \gamma|\uparrow\downarrow\rangle + \delta|\uparrow\uparrow\rangle]|0\rangle, \quad (13)$$

as desired. The CNOT gate could also be implemented by means of additional local operations following the recipe given at the beginning of this work.

The main feature of this method is that we were able to implement a universal quantum logic gate with a single collective Rabi flip. The interaction needs to be turned on just once with fixed parameters. In fact, although two ions must be addressed individually at the same time, one needs just one laser-beam split in two separated pairs. These facts may be considered as advantages when comparing our proposal to other ones requiring several consecutive and differently adjusted interactions [4,15,16]. Although we are dealing with a dispersive interaction, instead of a resonant one, which results in a slower process, we are less exposed to errors arising from the sequential switching of lasers. It is also worth noticing that the experimental tools to implement many of these schemes, including the one proposed here, are available. An experiment by Sackett *et al.* produced four ions in a linear array, cooled down to their collective ground state [22], and individual ionic addressing has been achieved by Nägerl *et al.* [23].

In the domain of CQED, we can find another possibility of making use of a selective interaction for implementing a QPG. CQED has already been recognized as a system where quantum logical operations can be implemented [11,12]. There are a number of proposals where logical gates are performed either in the electronic states of the atoms crossing the cavity or in a combined atom-field system [5]. We will be dealing here with a system where logical operations are performed in quantized modes of the cavity field. The two qubits are encoded in two nondegenerated modes of the electromagnetic field inside a high- Q cavity that can have either one or zero photons. In this case, atoms will serve only as catalyzers of the logical operation. Modes of the electromagnetic field in a cavity have been proposed as a possible environment where quantum logical operations can be done [6]. Nevertheless, the setup we will discuss here is different

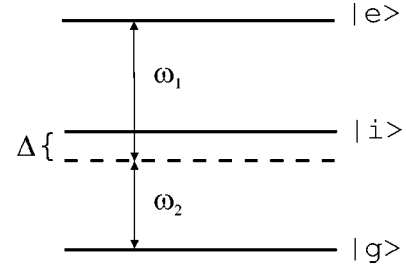


FIG. 2. Three-level atom interacting dispersively with two cavity modes of different frequencies ω_1 and ω_2 .

in some fundamental aspects. While in [6] each qubit corresponds to one mode inside a different cavity, in our scheme there are two nondegenerated modes inside a single cavity. Our scheme presents an additional advantage, it is not necessary to make the atom cross the cavity more than once. We will show that, again, a QPG can be implemented with only a single interaction pulse of a three-level atom crossing the cavity.

The experimental setup considered is easily identified as the one corresponding to a nondegenerated two-photon micromaser [24]. The cavity is crossed by a three-level atom that interacts with the cavity field during a time interval much smaller than the atomic and field decay time. Here, an effective Hamiltonian corresponding to a two-photon transition can be derived in the limit where transitions from the lower level $|g\rangle$ and upper level $|e\rangle$ to an intermediate level $|i\rangle$ are not resonant, the detuning being such that $\Delta = \omega_1 - (E_e - E_i)/\hbar \gg (\Omega_{ei}^2 + \Omega_{ig}^2)/\Delta$ (see Fig. 2). In this formula, E_e and E_i are the energies of the levels $|e\rangle$ and $|i\rangle$, respectively, and Ω_{ei} (Ω_{ig}) is the Rabi frequency corresponding to the transition $|e\rangle \rightarrow |i\rangle$ ($|i\rangle \rightarrow |g\rangle$). All these calculations are found in [24].

Explicitly, the effective two-photon interaction Hamiltonian reads, not writing the self-energy terms,

$$\hat{H}_{int} = \hbar\Omega(|e\rangle\langle g|\hat{a}_1\hat{a}_2 + \text{H.c.}), \quad (14)$$

where $\Omega = \Omega_{ei}\Omega_{ig}/\Delta$ is the effective Rabi frequency, Ω_{ei}, Ω_{ig} taken as real. Supposing that each cavity mode has either one or zero photons and that the decoupled atom is in the ground state, the most general initial pure state for the combined atom-field system is

$$|\psi\rangle = [\alpha|0,0\rangle + \beta|1,0\rangle + \gamma|0,1\rangle + \delta|1,1\rangle]|g\rangle. \quad (15)$$

It is clear that states in the subspace $\{|0,1\rangle, |1,0\rangle\}|g\rangle$ and the ground state $|0,0\rangle|g\rangle$ will not evolve under the interaction described in Eq. (14). Only $|1,1\rangle|g\rangle$ will suffer Rabi oscillations with the effective frequency Ω

$$|1,1\rangle|g\rangle \rightarrow \cos(\Omega t)|1,1\rangle|g\rangle + \sin(\Omega t)|0,0\rangle|e\rangle. \quad (16)$$

By selecting the atom velocity, in such a way that the interaction time corresponds to a π pulse, the final state

$$|\psi''\rangle = [\alpha|0,0\rangle + \beta|1,0\rangle + \gamma|0,1\rangle - \delta|1,1\rangle]|g\rangle \quad (17)$$

is produced after the atom leaves the cavity. This corresponds to the action of a QPG on the state of Eq. (15). This result shows an alternative way of implementing universal quantum gates in the modes of the electromagnetic field in a high- Q cavity with a rather known scheme.

Both schemes presented here show an interesting feature about quantum gates. Although they are proposed as logic gates for the electronic levels of the trapped ions and the electromagnetic modes of a cavity, they can be thought of, as well, as three-qubit quantum gates where the auxiliary CM vibronic mode and the atomic electronic states, respectively, are now the target qubits. In this sense, these schemes perform a Deutsch gate (controlled-controlled rotation) [25] where the two qubits of the ionic electronic levels and of the electromagnetic modes of the cavity plays the role of the control qubits. This is a characteristic of logic gates implemented by quantum systems where, since the operations are unitary, the process can be regarded as two-way logic gates.

To summarize, we have shown that a selective interaction can be useful for implementing quantum logic schemes in trapped ions and in the domain of CQED. Specifically, we made a proposal for implementing a QPG with a single pulse of a selective interaction in these two systems. In both cases, we find that our scheme reduces significantly the number of steps required for the gate operation, which may be attractive when thinking of scalable quantum-logical processes.

E.S. would like to thank N. Zagury and R. L. de Matos Filho for useful comments on the ionic quantum gates. The authors also acknowledge the support of Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ), Fundação Universitária José Bonifácio (FUJB), and Programa de Apoio a Núcleos de Excelência (PRONEX).

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