

Analysis of a two-atom double-slit experiment based on environment-induced measurements

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To investigate the effect of the environment on a quantum-mechanical system we consider two two-level atoms in a free radiation field in the presence of a screen. By assuming that the screen causes *continuous ideal measurements* on the free radiation field we derive a quantum jump description for the state of the atoms. Our results are consistent with the master equation for *dipole* interacting atoms, but give more insight in the time evolution of a *single* system. To illustrate this we derive a necessary and sufficient criterion for interference in a two-atom double-slit experiment and analyze bunching in the statistics of photons emitted in a certain direction.

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I. INTRODUCTION

In this paper we study the effect of the environment on a simple quantum-mechanical system. The experimental setup we consider as an example is shown schematically in Fig. 1. It consists of two two-level atoms continuously driven by a resonant laser field and stored at a fixed distance r from each other. The atoms are surrounded by a free radiation field and spontaneously emit photons. Each photon causes a “click” at a certain point on a screen. If enough photons are emitted, these clicks add up and form an interference pattern.

The Hamiltonian H of the quantum-mechanical system, which consists here of the two atoms, the laser, and the free radiation field, is well known [1]. However, solving the corresponding Schrödinger equation does not explain that the atoms spontaneously emit photons. On the other hand, a purely wave mechanical description of the emitted photons can predict the interference pattern [2,3] but does not allow us to determine higher-order time correlations in the photon statistics.

The aim of this paper is to show that the experiment pictured in Fig. 1 can be explained purely quantum mechanically from first principles with the help of the projection postulate for ideal measurements [4]. We show that the environment surrounding the system—the screen—has the same effect as continuous measurements on the free radiation field. That each photon causes a click on the screen at a point that depends only on the direction of its wave vector \mathbf{k} suggests that the screen measures whether a photon has been emitted or not. If so it determines its direction $\hat{\mathbf{k}} = \mathbf{k}/k$. As these measurements are caused by the interaction of the free radiation field with the screen, we call them *environment-induced measurements*.

Between consecutive measurements the state of the atoms and the field develops with the Hamiltonian H and all components of the quantum-mechanical system become entangled. A measurement on the free radiation field therefore

also has an effect on the atomic state. In case of a click on the screen the state of the atoms changes abruptly. It *jumps* into the reset state which can be obtained by applying the *reset operator* $R_{\hat{\mathbf{k}}}$ to the state $|\psi\rangle$ of the two atoms before the emission.

By deriving the reset operator $R_{\hat{\mathbf{k}}}$ [5] we specify the quantum jump approach for two *dipole* interacting atoms [6,7] which predicts the no photon time evolution with the help of the conditional Hamiltonian H_{cond} but does not distinguish between photon emissions in different directions $\hat{\mathbf{k}}$. To justify the assumptions and approximations on which our results are based we show that they are consistent with the master equation for two dipole interacting atoms [8–10]. Both approaches, the quantum jump approach and the master equation, are widely used in quantum optics and both have their respective merits.

A quantum jump description [11–14] is well suited for predicting all possible trajectories of a *single* system. Using this approach, it has been shown, for instance, that environment-induced measurements can assist in the realization of universal gates for quantum computing [15]. A possible application of the reset operator $R_{\hat{\mathbf{k}}}$ is given by a recently proposed scheme by Cabrillo *et al.* [16] for entangling distant atoms by interference. The master equation has considerable advantages in the description of an *ensemble* of systems and is well suited for determining stationary states.

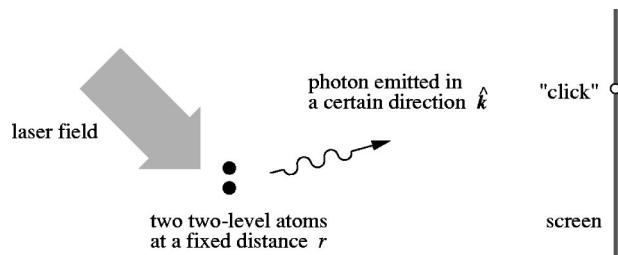


FIG. 1. Experimental setup. Two two-level atoms are placed at a fixed distance r from each other and are continuously driven by a resonant laser. This leads to spontaneous photon emissions. Each photon causes a click on a screen in a direction $\hat{\mathbf{k}}$ away from the atoms.

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The main reason to consider in this paper an experimental setup with *two* atoms is that this leads to spatially dependent effects which do not occur in single atom experiments. Verifying these effects experimentally shows that the quantum jump approach is not only an artifact of the master equations obtained from an unraveling of these equations [17] but a self-consistent approach. The aim of this paper is to show that the quantum jump approach can be applied to all experiments in which a single system spontaneously emits photons and is surrounded by “white” walls of a laboratory forming the screen.

The experimental setup shown in Fig. 1 has been discussed widely in the literature [3,18–22] and it has been realized as a quantum mechanical two-atom double-slit experiment by Eichmann *et al.* [23] in 1993. The slits of the classical version of this experiment are there replaced by two atoms which are likewise the sources of the light reaching the screen. In spite of its simplicity and the fact that this experiment is one of the basic experiments in quantum mechanics its discussion never came to an end. For other recent and related quantum-mechanical double-slit experiments see Refs. [24–28].

Here we show, in agreement with Refs. [29] and [30], that the reset operator $R_{\hat{\mathbf{k}}}$ allows us to determine *directly* the interference pattern of the experiment by Eichmann *et al.* [23]. To demonstrate the advantage of the quantum jump approach we derive a *necessary and sufficient* criterion for interference. In good agreement with Refs. [25] and [31–33], it is shown that interference arises from the fact that in quantum mechanics the wave functions, and not the probabilities, of different paths contributions have to be added to determine the probability for a certain event to happen. Other authors attributed interference in quantum mechanical double-slit experiments to the position-momentum uncertainty relation, Bohr’s complementarity principle, and to the absence of the *which way* information [26,34–43]. It is shown here for the experimental setup of Fig. 1 that the interference vanishes if and only if the *which way* information is, at least in principle, available in the experiment.

To give a further application of our results we analyze the effect of bunching in the statistics of photons emitted in a certain direction $\hat{\mathbf{k}}$. In agreement with Refs. [44] and [30] we predict arbitrary strong bunching even if the atoms are several wavelengths apart from each other. An intuitive explanation for this effect is given following the reasoning of Ref. [6].

This paper is organized as follows. In Sec. II we derive the reset operator $R_{\hat{\mathbf{k}}}$ which represents the main result of our paper. In Sec. III we give a short overview of the quantum jump approach and show its consistency with the master equation for two dipole interacting atoms [10]. In Sec. IV we discuss the experimental setup shown in Fig. 1 and derive a necessary and sufficient interference criterion. Afterwards we discuss spatially dependent bunching in the statistics of the photons emitted by the two atoms. Finally, our results are summarized in Sec. VI.

II. THE RESET OPERATOR

In this section we derive an analytic expression for the reset operator $R_{\hat{\mathbf{k}}}$ which can be used to determine the state of

the atoms after an emission in a certain direction $\hat{\mathbf{k}}$ from first principles. If the state of the atoms just before an emission is $|\psi\rangle$ it can, as we show below, immediately afterwards be written as

$$|\hat{\psi}_{\hat{\mathbf{k}}}\rangle \equiv R_{\hat{\mathbf{k}}}|\psi\rangle / \|\cdot\|, \quad (1)$$

which is a pure state. This equation defines the operator $R_{\hat{\mathbf{k}}}$ up to a proportionality factor. For practicality we choose this factor such that the probability density for a photon emission in the $\hat{\mathbf{k}}$ direction, $I_{\hat{\mathbf{k}}}(\psi)$, equals

$$I_{\hat{\mathbf{k}}}(\psi) \equiv \|R_{\hat{\mathbf{k}}}|\psi\rangle\|^2, \quad (2)$$

which is a density in time and solid angle.

To derive an analytic expression for the reset operator let us first write down the Hamiltonian of the quantum-mechanical system consisting of two two-level atoms and the free radiation field. In the following $|1\rangle_i$ and $|2\rangle_i$ denote the ground state and the excited state of atom i and $S_i^- = |1\rangle_i\langle 2|$ and $S_i^+ = |2\rangle_i\langle 1|$ are the corresponding lowering and raising operators. The energy separation between the levels is given by $\hbar\omega_0$. The annihilation operator for a single photon of the mode (\mathbf{k}, λ) of the free radiation field is denoted by $a_{\mathbf{k}\lambda}$ where \mathbf{k} is its wave vector, λ characterizes its polarization, and $\boldsymbol{\epsilon}_{\mathbf{k}\lambda}$ is the polarization vector. The coupling constant between the free radiation field and atom i is given by $g_{\mathbf{k}\lambda}^{(i)}$. For simplicity we assume that both atoms have the same transition dipole moment \mathbf{D}_{21} which gives $g_{\mathbf{k}\lambda}^{(1)} = g_{\mathbf{k}\lambda}^{(2)} = g_{\mathbf{k}\lambda}$ with

$$g_{\mathbf{k}\lambda} = ie \left(\frac{\omega_k}{2\epsilon_0 \hbar L^3} \right)^{1/2} \mathbf{D}_{21} \cdot \boldsymbol{\epsilon}_{\mathbf{k}\lambda}, \quad (3)$$

where $\omega_k = k/c$ and L^3 is the quantization volume. In addition, we assume that both atoms are irradiated by a laser field which has the (complex) Rabi frequency $\Omega^{(i)}$ with respect to atom i . If both atoms interact with the same laser the relative phase of the two Rabi frequencies depends on the direction of the incoming beam. Using this notation the interaction Hamiltonian H_I with respect to the interaction-free Hamiltonian is given by

$$H_I = \hbar \sum_{i=1,2} \sum_{\mathbf{k}, \lambda} e^{i(\omega_0 - \omega_k)t} e^{i\mathbf{k} \cdot \mathbf{r}_i} g_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda} S_i^+ + \text{H.c.} \\ + \frac{\hbar}{2} \sum_{i=1,2} \Omega^{(i)} S_i^+ + \text{H.c.} \quad (4)$$

In the experimental setup of Fig. 1, each emitted photon causes a click at a certain *point* on the screen. To describe this we assume that the presence of the screen leads to repeated measurements on the free radiation field as to whether a photon has been emitted or not. If so it determines its direction $\hat{\mathbf{k}}$. Here we do not discuss what exactly causes these environment-induced measurements but show later that the results derived from this assumption are consistent with the master equation for two dipole interacting atoms [10] and in good agreement with the experimental results of Ref. [23].

To determine the state of the atoms in the case of a click we make use of the projection postulate for ideal measurements [4].

Let us first consider a situation in which the screen is replaced by detectors which measure with each photon also its wave vector \mathbf{k} and polarization λ . As in Ref. [14], we assume that the atoms are initially in state $|\psi\rangle$ and the free radiation field is in the vacuum state $|0_{\text{ph}}\rangle$. After a time Δt , which should not be too long so that in Δt only the one-photon states become populated, the detector performs a measurement on the free radiation field. According to the projection postulate the unnormalized state of the atom-field system in the case of a click caused by a photon $|1_{\mathbf{k}\lambda}\rangle$ equals

$$|1_{\mathbf{k}\lambda}\rangle|\psi_{\mathbf{k}\lambda}\rangle \equiv |1_{\mathbf{k}\lambda}\rangle\langle 1_{\mathbf{k}\lambda}|U_I(\Delta t,0)|0_{\text{ph}}\rangle|\psi\rangle. \quad (5)$$

Here $U_I(\Delta t,0)$ is the time development operator with respect to the interaction Hamiltonian (4) which entangles the state of the atoms with the state of the free radiation field. The measurement of the free radiation field therefore also has an effect on the atomic state. It makes the atoms *jump* into the state $|\psi_{\mathbf{k}\lambda}\rangle$.

A comparison of both sides of Eq. (5) shows that the unnormalized state of the atoms after the click of the detector equals

$$|\psi_{\mathbf{k}\lambda}\rangle = \langle 1_{\mathbf{k}\lambda}|U_I(\Delta t,0)|0_{\text{ph}}\rangle|\psi\rangle. \quad (6)$$

From first-order perturbation theory and Eq. (4) we find

$$|\psi_{\mathbf{k}\lambda}\rangle = -ig_{\mathbf{k}\lambda}^* \int_0^{\Delta t} dt e^{-i(\omega_0 - \omega_k)t} \sum_{i=1,2} e^{-i\mathbf{k}\cdot\mathbf{r}_i} S_i^- |\psi\rangle. \quad (7)$$

According to the projection postulate [4], the squared norm of this vector equals the probability density for the emission of a photon $|1_{\mathbf{k}\lambda}\rangle$ during the time interval Δt . Assuming $\Delta t \gg 1/\omega_0$ we obtain in analogy to Ref. [14]

$$\begin{aligned} I_{\mathbf{k}\lambda}(\psi) &= \lim_{\Delta t \rightarrow 0} \frac{\| |\psi_{\mathbf{k}\lambda}\rangle \|^2}{\Delta t} \\ &= 2\pi |g_{\mathbf{k}\lambda}|^2 \delta(\omega_0 - \omega_k) \left\| \sum_{i=1,2} e^{-i\mathbf{k}\cdot\mathbf{r}_i} S_i^- |\psi\rangle \right\|^2. \end{aligned} \quad (8)$$

The proportionality of this equation to $\delta(\omega_0 - \omega_k)$ shows that all emitted photons have, within the approximations made, the wave number $k_0 = \omega_0 c$. The normalized state of the atoms after an emission therefore equals

$$|\hat{\psi}_{\hat{\mathbf{k}}}\rangle = \left(\sum_{i=1,2} e^{-ik_0 \hat{\mathbf{k}}\cdot\mathbf{r}_i} S_i^- |\psi\rangle \right) / \|\cdot\|, \quad (9)$$

which depends only on the direction $\hat{\mathbf{k}}$ of the emitted photon but not on k and λ .

Let us now consider again the situation where each emitted photon is detected by a click on the screen which determines only its direction $\hat{\mathbf{k}}$. To find the state of the atoms after an emission in this case we can proceed as above but have to replace the projector $|1_{\mathbf{k}\lambda}\rangle\langle 1_{\mathbf{k}\lambda}|$ in Eq. (5) by

$$P_{\hat{\mathbf{k}}} = \sum_{k,\lambda} |1_{k\hat{\mathbf{k}}\lambda}\rangle\langle 1_{k\hat{\mathbf{k}}\lambda}|. \quad (10)$$

This operator projects onto all one-photon states with a wave vector in the $\hat{\mathbf{k}}$ direction. By doing so we find that the reset state of the atom-field system equals

$$\sum_{k,\lambda} |1_{k\hat{\mathbf{k}}\lambda}\rangle|\psi_{k\hat{\mathbf{k}}\lambda}\rangle \equiv \sum_{k,\lambda} |1_{k\hat{\mathbf{k}}\lambda}\rangle\langle 1_{k\hat{\mathbf{k}}\lambda}|U_I(\Delta t,0)|0_{\text{ph}}\rangle|\psi\rangle. \quad (11)$$

As shown above, only terms with $k = k_0$ contribute with a nonvanishing amplitude to the right-hand side of this equation. From Eq. (9) one can then see that Eq. (11) is of the form

$$\sum_{k,\lambda} |1_{k\hat{\mathbf{k}}\lambda}\rangle|\psi_{k\hat{\mathbf{k}}\lambda}\rangle = \sum_{\lambda} c_{\lambda} |1_{k_0\hat{\mathbf{k}}\lambda}\rangle|\hat{\psi}_{\hat{\mathbf{k}}}\rangle, \quad (12)$$

where c_{λ} is a complex number. Normalizing this state we find that $|\hat{\psi}_{\hat{\mathbf{k}}}\rangle$ of Eq. (9) is indeed the reset state (1) of the atoms.

The probability density for a click on the screen in the direction $\hat{\mathbf{k}}$ away from the atoms can be obtained from the relation

$$I_{\hat{\mathbf{k}}}(\psi) = \sum_{\lambda} \left(\frac{L}{2\pi} \right)^3 \int_0^{\infty} dk k^2 I_{k\hat{\mathbf{k}}\lambda}(\psi). \quad (13)$$

Using Eq. (8) this leads to

$$I_{\hat{\mathbf{k}}}(\psi) = \frac{3A}{8\pi} (1 - |\hat{\mathbf{D}}_{21} \cdot \hat{\mathbf{k}}|^2) \left\| \sum_{i=1,2} e^{-ik_0 \hat{\mathbf{k}}\cdot\mathbf{r}_i} S_i^- |\psi\rangle \right\|^2, \quad (14)$$

where

$$A = \frac{e^2 \omega_0^3 |\mathbf{D}_{21}|^2}{3\pi \epsilon_0 \hbar c^3} \quad (15)$$

is the spontaneous decay rate of a single atom.

From Eqs. (9) and (14) we can now derive an expression for the reset operator $R_{\hat{\mathbf{k}}}$ of Eqs. (1) and (2) and find

$$R_{\hat{\mathbf{k}}} = R_{\hat{\mathbf{k}}}^{(1)} + R_{\hat{\mathbf{k}}}^{(2)} \quad (16)$$

with

$$R_{\hat{\mathbf{k}}}^{(i)} = \left[\frac{3A}{8\pi} (1 - |\hat{\mathbf{D}}_{21} \cdot \hat{\mathbf{k}}|^2) \right]^{1/2} e^{-ik_0 \hat{\mathbf{k}}\cdot\mathbf{r}_i} S_i^-. \quad (17)$$

In the same way as shown here for two atoms, one can derive the reset operator for the situation when only atom i is emitting photons while the other atom is far away and cannot emit a photon onto the same point on the screen. Proceeding as above we find that the reset operator in this case is given

by $R_{\mathbf{k}}^{(i)}$ of Eq. (17) alone. The reset operator for *both* atoms is the sum of the reset operators for each individual atom. This fact will play an important role in the discussion of a two-atom double-slit experiment in Sec. IV.

III. QUANTUM JUMP APPROACH VERSUS MASTER EQUATION

Before we apply our results to the experimental setup of Fig. 1 we shortly summarize the quantum jump approach [45] and show that the results obtained in Sec. II are consistent with the master equation for two dipole interacting atoms [10].

A. The quantum jump approach

The quantum jump approach [11–14] can be used to predict all possible trajectories of a single quantum-mechanical system which stochastically emits photons. At all times t the probability density for a photon emission is known. If this happens the state of the atoms changes abruptly. It jumps into another state which can be determined with the help of the reset operator. Between two photon emissions the system undergoes a continuous time evolution which can be described by the conditional Hamiltonian H_{cond} .

To derive H_{cond} for two dipole interacting atoms one can proceed as in Sec. II. Assuming again that the environment performs repeated measurements on the free radiation field one can determine the state of the system in the case of no photon emission by replacing the projector $|1_{\mathbf{k}\lambda}\rangle\langle 1_{\mathbf{k}\lambda}|$ in Eq. (5) by the projector onto the vacuum state $|0_{\text{ph}}\rangle\langle 0_{\text{ph}}|$. In this way one finds that the state of the atom-field system equals in the case of no photon emission after a time interval Δt

$$|0_{\text{ph}}\rangle U_{\text{cond}}(\Delta t, 0)|\psi\rangle \equiv |0_{\text{ph}}\rangle\langle 0_{\text{ph}}| U_{\text{I}}(\Delta t, 0)|0_{\text{ph}}\rangle|\psi\rangle. \quad (18)$$

Using second-order perturbation theory this leads, as in Ref. [6], to

$$H_{\text{cond}} = \frac{\hbar}{2i} \left[A \sum_{i=1,2} S_i^+ S_i^- + C \sum_{i \neq j} S_i^+ S_j^- \right] + \frac{\hbar}{2} \sum_{i=1,2} \Omega^{(i)} S_i^+ + \text{H.c.} \quad (19)$$

with the complex dipole interaction coupling constant

$$C = \frac{3A}{2} e^{ik_0 r} \left[\frac{1}{ik_0 r} (1 - |\hat{\mathbf{D}}_{21} \cdot \hat{\mathbf{r}}|^2) + \left(\frac{1}{(k_0 r)^2} - \frac{1}{i(k_0 r)^3} \right) (1 - 3|\hat{\mathbf{D}}_{21} \cdot \hat{\mathbf{r}}|^2) \right]. \quad (20)$$

As in Sec. II, we assume here that the dipole moment \mathbf{D}_{21} is the same for both atoms.

The probability for no photon emission in Δt can be obtained from Eq. (18) by taking the norm squared and equals

$$P_0(\Delta t, \psi) = \|U_{\text{cond}}(\Delta t, 0)|\psi\rangle\|^2. \quad (21)$$

B. Consistency with the master equation for two dipole interacting atoms

Another way to describe two atoms inside a free radiation field is to use the master equation. It provides linear differential equations which govern the time evolution of the density matrix ρ corresponding to an ensemble of single systems. It can be derived by averaging over all possible trajectories. By doing so we show here that our results are consistent with the master equation for two dipole interacting atoms.

Let us now consider an ensemble of systems with initial state ρ . After a time Δt this ensemble consists of many subensembles. The subensemble without photon emissions develops with the conditional Hamiltonian H_{cond} and can, at time Δt , be described by the density matrix

$$\rho_0(\Delta t) = U_{\text{cond}}(\Delta t, 0)\rho U_{\text{cond}}^\dagger(\Delta t, 0). \quad (22)$$

Equation (21) shows that the trace over this matrix is equal to the probability for no photon emission in $(0, \Delta t)$ and to the relative size of the subensemble without photon emissions. Using Eqs. (1) and (2) we see that the density matrix of the subensemble of systems with a photon emission in $\hat{\mathbf{k}}$ direction equals

$$\rho_{\hat{\mathbf{k}}}\Delta t = R_{\hat{\mathbf{k}}}\rho R_{\hat{\mathbf{k}}}^\dagger \Delta t \quad (23)$$

and the trace over this matrix gives the relative size of this subensemble.

If Δt is not too long so that the probability for more than one emission can again be neglected, the density matrix of the whole ensemble at Δt equals

$$\rho(\Delta t) = \rho_0(\Delta t) + \sum_{\hat{\mathbf{k}}} \rho_{\hat{\mathbf{k}}}\Delta t. \quad (24)$$

From Eqs. (17), (19), and (20) we find

$$\sum_{\hat{\mathbf{k}}} \rho_{\hat{\mathbf{k}}} = (A + \text{Re } C)R_+\rho R_+^\dagger + (A - \text{Re } C)R_-\rho R_-^\dagger \quad (25)$$

with

$$R_{\pm} = (S_1^- \pm S_2^-) / \sqrt{2}. \quad (26)$$

Considering Δt as a continuous parameter this leads to the differential equation

$$\dot{\rho} = -\frac{i}{\hbar} [H_{\text{cond}}\rho - \rho H_{\text{cond}}^\dagger] + (A + \text{Re } C)R_+\rho R_+^\dagger + (A - \text{Re } C)R_-\rho R_-^\dagger. \quad (27)$$

A comparison with Ref. [10] shows that this is the master equation for two dipole interacting atoms.

IV. ANALYSIS OF THE TWO-ATOM DOUBLE-SLIT EXPERIMENT

To demonstrate the usefulness of the reset operator $R_{\hat{\mathbf{k}}}$ we apply it in this section to the two-atom double-slit experiment shown in Fig. 1. A necessary and sufficient criterion for interference is derived. The interference pattern we predict has the same spatial dependence as the one observed experimentally by Eichmann *et al.* [23].

A. A necessary and sufficient criterion for interference

Before we discuss the two-atom double-slit experiment in which the atoms are continuously driven by a laser field, let us first consider a simplified version of the setup shown in Fig. 1. We assume that the atoms are repeatedly prepared in the same pure state $|\psi\rangle$. By observing the emitted photons one can measure the spatially dependent probability density $I_{\hat{\mathbf{k}}}(\psi)$.

To calculate $I_{\hat{\mathbf{k}}}(\psi)$ we determine first the unnormalized reset state $|\psi_{\hat{\mathbf{k}}}\rangle$ of the two atoms in the case of an emission in the $\hat{\mathbf{k}}$ direction. From Eqs. (1) and (16) we find that it is a superposition of two wave functions, each corresponding to a different situation, but both leading to a click at the same point on the screen,

$$|\psi_{\hat{\mathbf{k}}}\rangle = R_{\hat{\mathbf{k}}}^{(1)}|\psi\rangle + R_{\hat{\mathbf{k}}}^{(2)}|\psi\rangle. \quad (28)$$

The amplitude $R_{\hat{\mathbf{k}}}^{(i)}|\psi\rangle$ describes the state of the atoms after a photon emission by atom i alone. We denote the probability density for such an emission by $I_{\hat{\mathbf{k}}}^{(i)}(\psi)$. Analogously to Eq. (2) it equals

$$I_{\hat{\mathbf{k}}}^{(i)}(\psi) = \|R_{\hat{\mathbf{k}}}^{(i)}|\psi\rangle\|^2. \quad (29)$$

The probability density $I_{\hat{\mathbf{k}}}(\psi)$ can be obtained by taking the squared norm of the reset state $|\psi_{\hat{\mathbf{k}}}\rangle$ and we find

$$I_{\hat{\mathbf{k}}}(\psi) = I_{\hat{\mathbf{k}}}^{(1)}(\psi) + I_{\hat{\mathbf{k}}}^{(2)}(\psi) + 2 \operatorname{Re}\langle\psi|R_{\hat{\mathbf{k}}}^{(2)\dagger}R_{\hat{\mathbf{k}}}^{(1)}|\psi\rangle. \quad (30)$$

This differs by the last term from the sum of the probability densities for an emission either by atom 1 or atom 2 and describes the *interference* in the light emitted by the two atoms quantitatively. Interference results from the joint coupling of both atoms to the same free radiation field. There is only no interference if the last term in Eq. (30) vanishes for all directions $\hat{\mathbf{k}}$, i.e.,

$$\operatorname{Re}\langle\psi|R_{\hat{\mathbf{k}}}^{(2)\dagger}R_{\hat{\mathbf{k}}}^{(1)}|\psi\rangle = 0 \quad \text{for all } \hat{\mathbf{k}}. \quad (31)$$

This condition is equivalent to the reset states $R_{\hat{\mathbf{k}}}^{(1)}|\psi\rangle$ and $R_{\hat{\mathbf{k}}}^{(2)}|\psi\rangle$ being orthogonal to each other and we find using Eq. (17) that

$$\langle\psi|S_2^+S_1^-|\psi\rangle \neq 0 \quad (32)$$

is a necessary and sufficient criterion for interference. Whether this criterion is fulfilled or not depends only on the initial state $|\psi\rangle$ of the atoms.

Summarizing this, we have shown that interference in the two-atom double-slit experiment can be attributed to the fact that the amplitudes of the wave function corresponding to a click at the same point on the screen have to be added to determine the probability for this to happen. This is opposed to classical probability theory where the probabilities of all contributing paths have to be added, and which would not yield the last term in Eq. (30). Attributing interference to the superposition of wave functions is one of the basic concepts in quantum mechanics [32,33]. However, the quantum jump approach allowed us to calculate the amplitudes of the wave function for the concrete experimental setup shown in Fig. 1 explicitly and to identify each amplitude with a certain path.

B. The which way information

Other authors showed that interference in quantum-mechanical double-slit experiments vanishes in the presence of the which way information (see, for instance, Scully and Drühl [34]). Englert [39] derived an inequality which relates the fringe visibility to the which way knowledge available in the experiment. In the following, we show that this is in good agreement with the criterion given in Eq. (32).

To do so we first point out that a which way interpretation automatically implies the assumption that each photon is emitted either by atom 1 or by atom 2. Assuming this, the quantum jump approach predicts that the reset state of the atoms for a certain emission equals $R_{\hat{\mathbf{k}}}^{(i)}|\psi\rangle$ with the corresponding probability density $\|R_{\hat{\mathbf{k}}}^{(i)}|\psi\rangle\|^2$ where i equals 1 or 2. This is in contradiction with Eq. (30) which shows that the probability density for an emission in the $\hat{\mathbf{k}}$ direction equals $\|R_{\hat{\mathbf{k}}}|\psi\rangle\|^2$ and not $\|R_{\hat{\mathbf{k}}}^{(1)}|\psi\rangle\|^2 + \|R_{\hat{\mathbf{k}}}^{(2)}|\psi\rangle\|^2$.

Nevertheless, there is one situation in which one cannot distinguish whether both atoms are cooperatively emitting or whether one can assign each photon to one of the two atoms. This is the case if

$$R_{\hat{\mathbf{k}}}^{(1)}|\psi\rangle \perp R_{\hat{\mathbf{k}}}^{(2)}|\psi\rangle \quad \text{for all } \hat{\mathbf{k}}. \quad (33)$$

Then one can find out which atom emitted the photon by measuring whether the atoms are either in the state $R_{\hat{\mathbf{k}}}^{(1)}|\psi\rangle$ or in $R_{\hat{\mathbf{k}}}^{(2)}|\psi\rangle$. Equation (17) shows that Eqs. (31) and (33) are equivalent. This means that the interference vanishes if and only if the which way information is available in the experiment.

C. Interference from two continuously driven atoms

In the previous two subsections we assumed that the state of the atoms by the time of an emission is always $|\psi\rangle$. This is not the case for the experimental setup of Fig. 1 in which the atoms are continuously driven by a laser field. To apply our results to this situation we have to describe the atoms at the time of an emission by the steady-state matrix ρ^{ss} . From

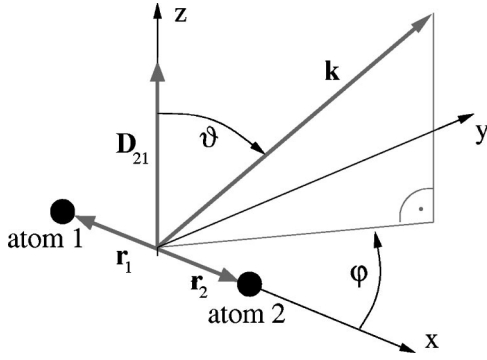


FIG. 2. Coordinate system with the spatial angles ϑ and φ characterizing the direction of the wave vector \mathbf{k} . We assume that the atomic dipole moment \mathbf{D}_{21} is perpendicular to the line connecting both atoms.

Eq. (2) we find that the probability density for an emission in the $\hat{\mathbf{k}}$ direction equals

$$I_{\hat{\mathbf{k}}}(\rho^{ss}) = \text{Tr}(R_{\hat{\mathbf{k}}}\rho^{ss}R_{\hat{\mathbf{k}}}^\dagger). \quad (34)$$

In analogy to Eq. (32) a necessary and sufficient criterion for interference is now given by the condition

$$\text{Tr}(S_2^+ S_1^- \rho^{ss}) = \text{Tr}(S_1^- \rho^{ss} S_2^+) \neq 0. \quad (35)$$

Using Eqs. (16) and (17) we obtain

$$I_{\hat{\mathbf{k}}}(\rho^{ss}) = \frac{3A}{8\pi} (1 - |\hat{\mathbf{D}}_{21} \cdot \hat{\mathbf{k}}|^2) [\text{Tr}(S_1^- \rho^{ss} S_1^+) + \text{Tr}(S_2^- \rho^{ss} S_2^+) + 2 \text{Re} \text{Tr}(e^{-i\mathbf{k}_0 \cdot (\mathbf{r}_1 - \mathbf{r}_2)} S_1^- \rho^{ss} S_2^+)], \quad (36)$$

where the last term describes the interference effects.

To discuss a concrete example, it is convenient to introduce Dicke states,

$$\begin{aligned} |g\rangle &= |11\rangle, & |s\rangle &= (|12\rangle + |21\rangle)/\sqrt{2}, \\ |e\rangle &= |22\rangle, & |a\rangle &= (|12\rangle - |21\rangle)/\sqrt{2}, \end{aligned} \quad (37)$$

and to use the spatial angles ϑ and φ as defined in Fig. 2. In the following we choose the dipole moments \mathbf{D}_{21} to be perpendicular to the line connecting both atoms. Using this notation we find from Eq. (36) in good agreement with Ref. [21]

$$I_{\hat{\mathbf{k}}}(\rho^{ss}) = \frac{3A}{8\pi} \sin^2 \vartheta [2\rho_{ee} + \rho_{ss} + \rho_{aa} + (\rho_{ss} - \rho_{aa}) \cos(k_0 r \sin \vartheta \cos \varphi) + 2 \text{Im} \rho_{sa} \sin(k_0 r \sin \vartheta \cos \varphi)], \quad (38)$$

where $\rho_{xy} \equiv \langle x | \rho^{ss} | y \rangle$ are the matrix elements of the steady-state density matrix ρ^{ss} . The last two terms in Eq. (38) describe the interference and result from the last term in Eq. (36).

In the classical double-slit experiment, interference vanishes if the waves emanating from both slits have a random

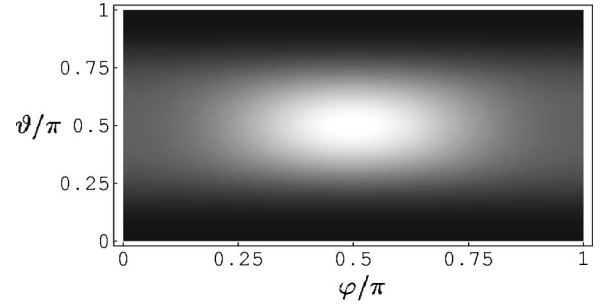


FIG. 3. Density plot of the emission rate $I_{\hat{\mathbf{k}}}(\rho^{ss})$ for two continuously driven two-level atoms, $r = \lambda_0/\pi$ and $\Omega = 0.3A$. White areas correspond to spatial angles with maximal intensity.

phase relation. The same is true for the phase difference of the Rabi frequencies driving both atoms. It enters Eq. (38) through the steady-state matrix ρ^{ss} . As an example, we assume in the following that both atoms see the same (real) Rabi frequency

$$\Omega^{(1)} = \Omega^{(2)} = \Omega. \quad (39)$$

From Eq. (27) and the condition $\dot{\rho}^{ss} = 0$ we find

$$\begin{aligned} \rho_{gg} &= \frac{(A^2 + \Omega^2)^2 + A^2(2A + \text{Re } C)\text{Re } C + A^2(\text{Im } C)^2}{N}, \\ \rho_{ss} &= \frac{\Omega^2(2A^2 + \Omega^2)}{N}, & \rho_{ee} = \rho_{aa} &= \frac{\Omega^4}{N}, & \text{Im } \rho_{sa} &= 0 \end{aligned} \quad (40)$$

with

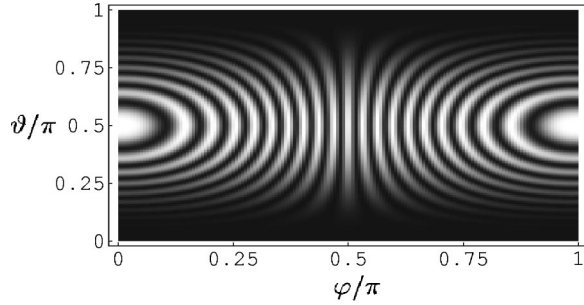
$$N = (A^2 + 2\Omega^2)^2 + A^2(2A + \text{Re } C)\text{Re } C + A^2(\text{Im } C)^2. \quad (41)$$

As it can be seen from these equations, the dipole interaction between the atoms has only a small influence on the depth but does not affect the form of the interference pattern. For $r > 2\lambda_0$ one can neglect all terms proportional to the dipole coupling constant C . This leads to [46]

$$I_{\hat{\mathbf{k}}}(\rho^{ss}) = \frac{3}{4\pi} \frac{A\Omega^2}{(A^2 + 2\Omega^2)^2} \sin^2 \vartheta [A^2 + 2\Omega^2 + A^2 \cos(k_0 r \sin \vartheta \cos \varphi)] \quad (42)$$

which is in good agreement with experimental results by Eichmann *et al.* [23].

To illustrate this we show in Figs. 3 and 4 density plots of the emission rate $I_{\hat{\mathbf{k}}}(\rho^{ss})$ for different atomic distances r . White areas correspond to spatial angles with maximal intensity. The interference effects of the photons emitted by the two atoms are more distinct in Fig. 4 which shows stronger oscillations of the intensity with the polar angle φ . These become more frequent the larger the distance between the atoms.

FIG. 4. As in Fig. 3 but with $r = 10\lambda_0$.

Finally we note that every change of the stationary state ρ^{ss} in Eq. (36) can affect the depth of the interference pattern. This has been discussed in Refs. [47] and [48] where an additional coupling of the two atoms via the mode of an optical cavity has been assumed. Another situation, in which the density matrix ρ^{ss} is different from Eq. (40) is when the atomic state is continuously monitored. This can be done with the help of an additional rapidly decaying level and a second laser field [34,49] or by using two four-level atoms and detecting the polarization of the emitted photons [23,31]. Alternatively, it has been proposed to use two microwave cavities as which way detectors [35]. As a consequence of the knowledge of the which way information in these setups the interference vanishes. This is in good agreement with our discussion in the previous subsection.

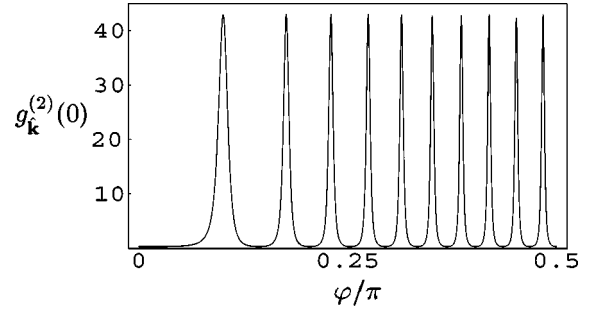
V. BUNCHING EFFECTS IN THE PHOTON STATISTICS OF TWO DISTANT ATOMS

As another application of the quantum jump approach we investigate in this section the second-order correlations in the photon statistics of two continuously driven two-level atoms. The experimental setup we consider is again the same as in Fig. 1 but in the following we replace the screen by a single photon detector which registers only photons emitted in a certain direction $\hat{\mathbf{k}}$. In this section we predict strong spatially dependent *bunching* — the effect that a photon emission in the $\hat{\mathbf{k}}$ direction increases the probability density for yet another emission in the same direction [1]. Our results are in good agreement with the results of Ref. [30]. An intuitive explanation for bunching, following the reasoning of Ref. [6], is given.

To obtain a simple mathematical description of bunching we define, analogously to Eq. (4) of Ref. [6], the second-order correlation function by

$$g_{\hat{\mathbf{k}}}^{(2)}(0) \equiv \frac{I_{\hat{\mathbf{k}}}(R_{\hat{\mathbf{k}}}\rho^{ss}R_{\hat{\mathbf{k}}}^\dagger/\text{Tr}(\cdot))}{I_{\hat{\mathbf{k}}}(\rho^{ss})}. \quad (43)$$

The denominator of this function is the steady-state photon emission rate in the $\hat{\mathbf{k}}$ direction while the numerator equals the probability density for an emission in the same direction *immediately* after an emission. Therefore the photons emitted in the $\hat{\mathbf{k}}$ direction are bunched if $g_{\hat{\mathbf{k}}}^{(2)}(0) > 1$ and antibunched if $g_{\hat{\mathbf{k}}}^{(2)}(0) \leq 1$.

FIG. 5. The second-order photon correlation function $g_{\hat{\mathbf{k}}}^{(2)}(0)$ as a function of φ for $\Omega = 0.3A$, $r = 10\lambda_0$, and $\vartheta = \pi/2$.

A. The photon correlation function for two continuously driven atoms

With present ion trapping technology atomic distances larger than a few wavelengths are easier to prepare. We consider therefore in the following the case $r > 2\lambda_0$ and neglect again the dipole interaction between the atoms. Assuming, as in Eq. (39), that the Rabi frequency of the driving laser field is the same for both atoms we find from Eqs. (16), (17), (40), and (41)

$$g_{\hat{\mathbf{k}}}^{(2)}(0) = \left[1 - \frac{\cos(k_0 r \sin \vartheta \cos \varphi)}{1 + 2 \left(\frac{\Omega}{A} \right)^2 + \cos(k_0 r \sin \vartheta \cos \varphi)} \right]^2. \quad (44)$$

As can be seen from this result, bunching occurs for all directions $\hat{\mathbf{k}}$ with $\cos(k_0 r \sin \vartheta \cos \varphi) < 0$ and does not depend on the concrete choice of the Rabi frequency Ω . This is different from the statistics of photons emitted into *all* spatial directions where bunching can only occur for distances with $r < 2\lambda_0$ [6].

Figure 5 shows as an example the second-order correlation function $g_{\hat{\mathbf{k}}}^{(2)}(0)$ for different spatial angles φ , $\vartheta = \pi/2$, $r = 10\lambda_0$, and $\Omega = 0.3A$. For these parameters $g_{\hat{\mathbf{k}}}^{(2)}(0)$ can adopt values larger than 40 which corresponds to very strong bunching. For weaker driving, $\Omega/A \rightarrow 0$, the correlation function can even become infinitely large. This seems unphysical but corresponds to angles for which the photon intensity (42) vanishes for $\Omega/A \rightarrow 0$.

B. An intuitive explanation of strong bunching

The quantum jump approach allows us not only to calculate easily photon correlation functions but also to obtain a good intuitive understanding of this phenomenon. To do so we proceed as proposed in Ref. [6] and investigate how the state of the atoms changes during a photon emission in a direction with bunching. According to Eq. (44) we get maximal bunching if

$$\cos(k_0 r \sin \vartheta \cos \varphi) = -1. \quad (45)$$

For this direction the corresponding reset operator (16) can be written as

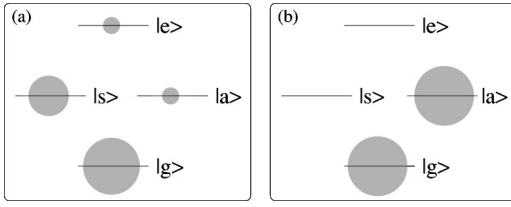


FIG. 6. The population of the Dicke states $|g\rangle$, $|s\rangle$, $|a\rangle$, and $|e\rangle$ for the steady state ρ^{ss} (a) and for the normalized state immediately after the emission of a photon into a direction with maximal bunching (b) pictured by circles. The area of each circle is proportional to the population of the corresponding level.

$$R_{\hat{\mathbf{k}}} = \alpha(|a\rangle\langle e| - |g\rangle\langle a|), \quad (46)$$

where α is a complex number. For the same direction the probability density for an emission (34) equals

$$I_{\hat{\mathbf{k}}}(\rho^{\text{ss}}) = |\alpha|^2(\rho_{ee} + \rho_{aa}) \quad (47)$$

and is proportional to the population in the states $|a\rangle$ and $|e\rangle$.

Figure 6(a) illustrates the population in the atomic levels for the steady state matrix ρ^{ss} and Fig. 6(b) for the state of the atoms immediately after an emission in the direction $\hat{\mathbf{k}}$ of Eq. (45). The area of each circle is proportional to the population of the corresponding level. In the steady state, there is nearly no population in the levels a and e and the probability density for an emission in the direction of Eq. (45) is therefore relatively low. It equals

$$I_{\hat{\mathbf{k}}}(\rho^{\text{ss}}) = \frac{2\Omega^4|\alpha|^2}{(A^2 + 2\Omega^2)^2}. \quad (48)$$

During an emission a redistribution of the population takes place according to the reset operator (46). The population of level a goes over to level g and the population of level e goes to level a while the population of the two other levels vanishes. Afterwards the reset state has to be normalized. A comparison of Figs. 6(a) and (b) shows that the emission of a photon causes in this way an increase of the population in the states $|a\rangle$ and $|e\rangle$ and therefore also an increase of the probability density for a further emission in the same direction, which is given by

$$I_{\hat{\mathbf{k}}}(R_{\hat{\mathbf{k}}}\rho^{\text{ss}}R_{\hat{\mathbf{k}}}^\dagger/\text{Tr}(\cdot)) = \frac{1}{2}|\alpha|^2, \quad (49)$$

which is larger than $I_{\hat{\mathbf{k}}}(\rho^{\text{ss}})$ of Eq. (48).

Summarizing this, we see that bunching results from the fact that the detection of a photon is always connected with a measurement on the atomic state. During this measurement the state of the atoms might change in such a way that the probability density for a further emission in the same direction is increased.

VI. CONCLUSIONS

As long as a quantum-mechanical system does not couple to its environment one can predict its time evolution by the Schrödinger equation. This is not possible for open systems like spontaneously emitting atoms. To describe them the quantum jump approach [45] has been derived from the assumption that the environment performs continuous measurements on the free radiation field as to whether a photon is emitted by the atoms or not. The time evolution of the atoms under the condition of no photon emission can be described by a Schrödinger equation based on the conditional Hamiltonian H_{cond} . In the case of a photon emission the state of the atoms changes abruptly.

In this paper we assumed that the environment of the atom-field system, here in form of a screen, detects each emitted photon and, if so, determines its direction $\hat{\mathbf{k}}$. This ansatz was motivated by the experimental setup of Fig. 1 in which each photon causes a click at a certain point on the screen. From this assumption of environment-induced measurements we derived in Sec. II the reset operator $R_{\hat{\mathbf{k}}}$. It can be used to determine the state of the atoms immediately after an emission in the $\hat{\mathbf{k}}$ direction. Initially in a pure state, the state of the atoms remains always pure. This extension of the quantum jump approach allows us now to predict all individual trajectories of a single atomic system. We think that all quantum optical experiments with “white” walls in the laboratory can be described by a quantum jump approach.

In Sec. III we showed that our results are consistent with the master equations for two dipole interacting atoms [10]. The dipole interaction results from the fact that both atoms interact with the same free radiation field and exchange virtual photons. This is described by the dipole coupling constant C in the conditional Hamiltonian H_{cond} . Also the reset operator $R_{\hat{\mathbf{k}}}$ leads to terms proportional C in the master equation.

The advantage of our generalization of the quantum jump approach [6,7] is that it can now be applied to further experiments such as the scheme by Cabrillo *et al.* [16] to entangle distant atoms by interference. In this paper we discussed in Sec. IV, as an example, the two-atom double-slit experiment shown in Fig. 1 and derived a necessary and sufficient interference criterion. Another application of the reset operator $R_{\hat{\mathbf{k}}}$ was given in Sec. V, where we predicted in agreement with Ref. [30] strong bunching for the photons emitted into certain directions $\hat{\mathbf{k}}$. An intuitive explanation for this effect was given.

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- [1] P. Meystre and M. Sargent, *Elements of Quantum Optics* (Springer, Berlin, 1990).
- [2] W. Heitler, *The Quantum Theory of Radiation* (Dover, New York, 1984).
- [3] L. Mandel, Phys. Rev. A **28**, 929 (1983).
- [4] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, NJ, 1955); G. Lüders, Ann. Phys. (Leipzig) **8**, 323 (1951).
- [5] The reset operator for one atom for a slightly different situation has been presented in Eq. (68) in K. Mølmer, Y. Castin, and J. Dalibard, J. Opt. Soc. Am. B **10**, 524 (1993), where it has been obtained from an unraveling of the master equation.
- [6] A. Beige and G. C. Hegerfeldt, Phys. Rev. A **58**, 4133 (1998).
- [7] A. Beige and G. C. Hegerfeldt, Phys. Rev. A **59**, 2385 (1999); S. Addicks, A. Beige, M. Dakna, and G. C. Hegerfeldt, e-print quant-ph/0002093.
- [8] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
- [9] R. H. Lehmer, Phys. Rev. A **2**, 883 (1970); **2**, 889 (1970).
- [10] G. S. Agarwal, *Quantum Optics*, Springer Tracts of Modern Physics Vol. 70 (Springer-Verlag, Berlin, 1974).
- [11] G. C. Hegerfeldt and T. S. Wilser, in *Classical and Quantum Systems*, Proceedings of the Second International Wigner Symposium, 1991, edited by H. D. Doebner, W. Scherer, and F. Schroeck (World Scientific, Singapore, 1992), p. 104.
- [12] J. Dalibard, Y. Castin, and K. Mølmer, Phys. Rev. Lett. **68**, 580 (1992).
- [13] H. Carmichael, *An Open Systems Approach to Quantum Optics*, Lecture Notes in Physics Vol. 18 (Springer, Berlin, 1993).
- [14] G. C. Hegerfeldt, Phys. Rev. A **47**, 449 (1993).
- [15] A. Beige, D. Braun, B. Tregenna, and P. L. Knight, Phys. Rev. Lett. **85**, 1762 (2000).
- [16] C. Cabrillo, J. I. Cirac, P. Garcia-Fernandez, and P. Zoller, Phys. Rev. A **59**, 1025 (1999).
- [17] H. J. Carmichael and K. Kim, Opt. Commun. **179**, 417 (2000).
- [18] U. Fano, Am. J. Phys. **29**, 539 (1961).
- [19] T. Richter, J. Mod. Opt. **38**, 47 (1991); Opt. Commun. **80**, 285 (1991).
- [20] H. Huang, G. S. Agarwal, and M. O. Scully, Opt. Commun. **127**, 243 (1996).
- [21] T. G. Rudolph, Z. Ficek, and B. J. Dalton, Phys. Rev. A **52**, 636 (1995); T. Rudolph and Z. Ficek, *ibid.* **58**, 748 (1998).
- [22] R. G. Brewer, Phys. Rev. A **52**, 2965 (1995); Phys. Rev. Lett. **77**, 5153 (1996).
- [23] U. Eichmann, J. C. Berquist, J. J. Bollinger, J. M. Gilligan, W. M. Itano, D. J. Wineland, and M. G. Raizen, Phys. Rev. Lett. **70**, 2359 (1993).
- [24] P. Grangier, A. Aspect, and J. Vigue, Phys. Rev. Lett. **54**, 418 (1985).
- [25] P. G. Kwiat, A. M. Steinberg, and R. Y. Chiao, Phys. Rev. A **45**, 7729 (1992).
- [26] S. Dürr, T. Nonn, and G. Rempe, Nature (London) **395**, 33 (1998); S. Dürr and G. Rempe, Opt. Commun. **179**, 323 (2000).
- [27] P. D. D. Schwindt, P. G. Kwiat, and B.-G. Englert, Phys. Rev. A **60**, 4285 (1999).
- [28] Y.-H. Kim, R. Yu, S. P. Kulik, Y. Shih, and M. O. Scully, Phys. Rev. Lett. **84**, 1 (2000).
- [29] T. Wong, S. M. Tan, M. J. Collett, and D. F. Walls, Phys. Rev. A **55**, 1288 (1997).
- [30] C. Skornia, J. von Zanthier, G. S. Agarwal, E. Werner, and H. Walther, e-print quant-ph/0103031.
- [31] W. M. Itano, J. C. Bergquist, J. J. Bollinger, D. J. Wineland, U. Eichmann, and M. G. Raizen, Phys. Rev. A **57**, 4176 (1998).
- [32] P. A. M. Dirac, *The Principles of Quantum Mechanics*, 3rd ed. (Clarendon Press, Oxford, 1947).
- [33] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison Wesley, Reading, MA, 1965), Vol. 3.
- [34] M. O. Scully and K. Drühl, Phys. Rev. A **25**, 2208 (1982).
- [35] M. O. Scully, B.-G. Englert, and H. Walther, Nature (London) **351**, 111 (1991).
- [36] P. Storey, S. Tan, and M. Collett, Nature (London) **367**, 626 (1994); P. Storey, S. Tan, M. Collett, and D. F. Walls, *ibid.* **375**, 368 (1995).
- [37] B.-G. Englert, M. O. Scully, and H. Walther, Nature (London) **375**, 367 (1995).
- [38] H. Wiseman and F. Harrison, Nature (London) **377**, 584 (1995); H. M. Wiseman, F. E. Harrison, M. J. Collett, S. M. Tan, D. F. Walls, and R. B. Killip, Phys. Rev. A **56**, 55 (1997).
- [39] B.-G. Englert, Phys. Rev. Lett. **77**, 2154 (1996).
- [40] P. L. Knight, Nature (London) **395**, 12 (1998).
- [41] Y. Abranyos, M. Jakob, and J. Bergou, Phys. Rev. A **60**, R2618 (1999); **61**, 013804 (2000).
- [42] I. Kim and G. Mahler, Phys. Lett. A **269**, 287 (2000).
- [43] H. T. Dung and K. Ujihara, Phys. Rev. Lett. **84**, 254 (2000).
- [44] M. Wiegand, J. Phys. B **16**, 1133 (1983).
- [45] For a recent review, see M. B. Plenio and P. L. Knight, Rev. Mod. Phys. **70**, 101 (1998).
- [46] This result agrees with the result of Ref. [30] up to a factor proportional $\sin^2\vartheta$ but predicts the same dependence of the interference pattern on φ .
- [47] P. Kochan, H. J. Carmichael, P. R. Morrow, and M. G. Raizen, Phys. Rev. Lett. **75**, 45 (1995).
- [48] G. Yeoman, Phys. Rev. A **58**, 764 (1998).
- [49] R. L. Ingraham, Phys. Rev. A **50**, 4502 (1994).