Coupled cavities for enhancing the cross-phase-modulation in electromagnetically induced transparency

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We propose an optical double-cavity resonator whose response to a signal is similar to that of an electromagnetically induced transparency (EIT) medium. A combination of such a device with a four-level EIT medium can serve for achieving large cross-Kerr modulation of a probe field by a signal field. This would offer the possibility of building a quantum logic gate based on photonic qubits. We discuss the technical requirements that are necessary for realizing a probe-photon phase shift of π caused by a single-photon signal. The main difficulty is the requirement of an ultralow reflectivity beam splitter, and we must be able to operate a sufficiently dense cool EIT medium in a cavity.

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I. INTRODUCTION

Strong cross-Kerr phase modulation [1,2] based on electromagnetically induced transparency (EIT) [3] (for a review of EIT, see, e.g., Refs. [4,5]) has been of increasing interest for quantum information processing. If a single photon can change the phase of another photon, then one can use the effect for constructing a controlled-NOT quantum gate. To realize a maximally large cross-phase-modulation (XPM) on a single-photon level, both cavity [6] and free-medium [7,8] regimes have been considered.

The advantage of the cavity regime is that a relatively strong electromagnetic field can be built up inside a cavity, and that the photons can interact during a relatively long time. However, the combination of the cavity properties with the ultralow group velocity of light in the EIT medium leads to an extremely narrow line of the transmitted light [9], which introduces some limits of the applications that are possible in principle [10–12]. Another disadvantage results from the fact that, even for the best mirrors presently available absorption is of the same order of magnitude as transmission [13]. Thus, even in the case of exact resonance, only a fraction of about ≈25% of the incident light is transmitted, whereas a fraction of about ≈50% is absorbed by the mirrors (see Appendix A). Obviously, devices giving rise to such huge losses cannot serve for quantum information processing.

In the free-medium regime, a probe pulse, which is to be influenced by a signal pulse, moves in a free EIT medium, and has thus a very small group velocity [14]. A signal pulse whose midfrequency is sufficiently far from the medium resonance moves with a velocity close to c. The interaction time of the signal and probe is thus limited by the short overlap time of the two pulses [7]. To overcome this problem, it was suggested to use a second EIT medium for the signal, so that both signal and probe have the same (small) group velocities v_g [8]. Both pulses can then interact for a long time. The disadvantage of such a scheme is that most of the signal-photon energy is wasted in dressing the atoms of the (second) EIT medium [15], and only a small fraction $\sim v_g/c$ of it is available for shifting the phase of the probe.

In this paper, we suggest a regime in which the probe pulse moves in a free EIT medium, whereas the signal field is confined to a special double-cavity (Michelson-like) resonator whose linear response is analogous to that of an EIT medium. Thus a strong signal can interact with the probe for a long time. The cavity-cavity and input-output couplings are accomplished with weakly reflecting beam splitters, so that losses at the mirrors are avoided. Construction of such beam splitters may be a technical challenge, but could be achievable in principle.

The paper is organized as follows. In Sec. II the double-cavity system is studied. The combined action of the double-cavity system and the EIT medium are studied in Sec. III, and the single-photon phase shift is calculated. Finally, some concluding remarks are given in Sec. IV.

II. COUPLED CAVITIES

A. Linear response and "Rabi" splitting

Let us consider the two-cavity scheme shown in Fig. 1 and disregard the medium and the probe beam for the moment. The distances are denoted by L_1 to L_5, the lengths of the vertical and horizontal cavities being $L_V = L_1 + L_4 + L_5$ and $L_H = L_2 + L_3$, respectively. The vertical cavity is coupled to the outside field by a beam splitter BS_1, and the two cavities are coupled to each other by a beam splitter BS_2, with

$$\mathbf{T}_l = \begin{pmatrix} t_l & i r_l \\ i r_l & t_l \end{pmatrix} \tag{1}$$

($l = 1$, and 2) being the characteristic transformation matrices of the two beam splitters. Here $r_l^2 \equiv R_l$ and $t_l^2 \equiv T_l$, respectively, are the reflectivities and transmissivities satisfying the relations $R_l + T_l + A_l = 1$, where A_l are the absorption coefficients. In what follows we are interested in weakly reflecting beam splitters, such that $R_l \ll 1$ and $T_l \approx 1$ for $A_l \ll R_l$. The reflection and small absorption coefficients of the four mirrors M_m ($m = 1$, 2, 3, and 4) are R_{Mm} and A_{Mm}, respectively, with $R_{Mm} + A_{Mm} \approx R_{Mm} \approx 1$.

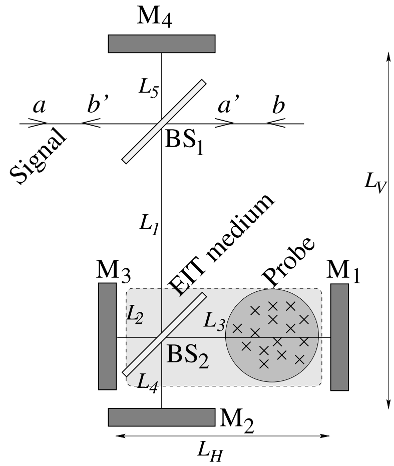


FIG. 1. Scheme of the double-cavity system. The horizontal cavity is filled with an EIT medium. The signal enters (a) and leaves (a') the system via the beam splitter BS_1. The probe propagates across the horizontal cavity in a direction perpendicular to the plane of the plot.

The input-output relations for the complex amplitudes a (a') and b (b') of the incoming (outgoing) waves of wave number $k=\omega/c$ are then found by successive application of the transformations realized by the beam splitters and the mirrors. The result can be given in the form of

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}, \tag{2}$$

where

$$G_{11}=t_1\frac{1+(1-A_1)\sqrt{1-A_{M4}}e^{i2kL_5}\mathcal{B}}{1+T_1\sqrt{1-A_{M4}}e^{i2kL_5}\mathcal{B}}, \tag{3}$$

$$G_{12}=-\frac{R_1\mathcal{B}}{1+T_1\sqrt{1-A_{M4}}e^{i2kL_5}\mathcal{B}}, \tag{4}$$

$$G_{21}=\frac{R_1\sqrt{1-A_{M4}}e^{i2kL_5}}{1+T_1\sqrt{1-A_{M4}}e^{i2kL_5}\mathcal{B}}, \tag{5}$$

$$G_{22}=G_{11}, \tag{6}$$

with

$$\mathcal{B}=\frac{\mathcal{B}_1+\mathcal{B}_2+\mathcal{B}_3}{\mathcal{B}_4}, \tag{7}$$

$$\mathcal{B}_1=(1-A_2)^2\sqrt{(1-A_{M1})(1-A_{M2})(1-A_{M3})}\times e^{i2k(L_1+L_4+L_H)}, \tag{8}$$

$$\mathcal{B}_2=-T_2\sqrt{1-A_{M2}}e^{i2k(L_1+L_4)}, \tag{9}$$

$$\mathcal{B}_3=R_2\sqrt{1-A_{M3}}e^{i2k(L_1+L_2)}, \tag{10}$$

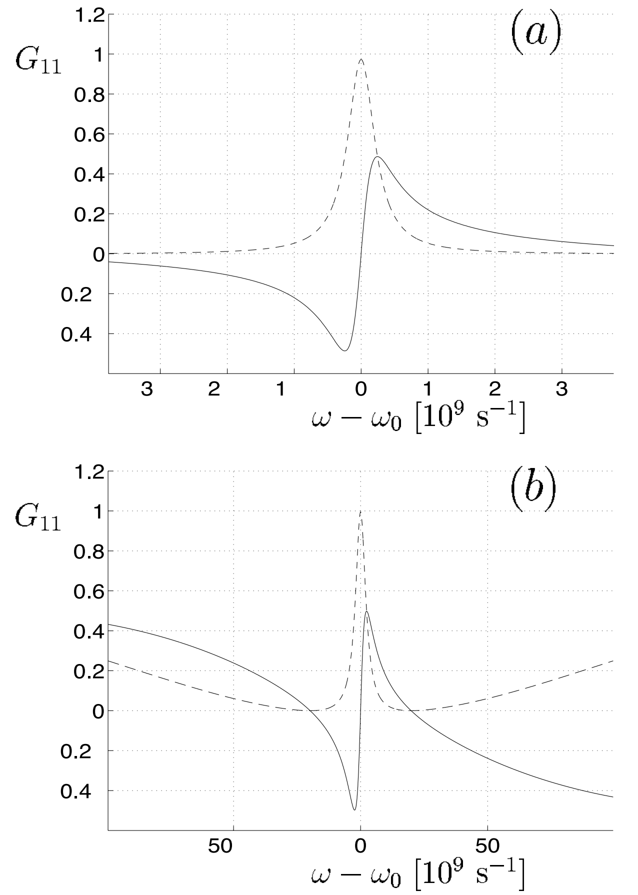


FIG. 2. Response function G_{11} for $L_V=120\lambda_0$, $L_H=30\lambda_0$ (with $\lambda_0=2\pi/k_0=795$ nm, $\omega_0=k_0c$), $A_{Mm}=10^{-6}$, $R_1=10\%$, and $R_2=10^{-6}$ (a) and $R_2=10^{-5}$ (b). Dashed line, $\mathrm{Re}\,G_{11}$; full line, $\mathrm{Im}\,G_{11}$.

$$\mathcal{B}_4=1-\sqrt{1-A_{M1}}[T_2\sqrt{1-A_{M3}}e^{i2kL_H}-R_2\sqrt{1-A_{M2}}e^{i2k(L_3+L_4)}]. \tag{11}$$

Here only relations for c numbers (a,a',b,b') are considered. Note that for obtaining the operator input-output relations, Eq. (2) must be complemented with an absorption matrix acting on some device variables [16].

An example of the response function of the transmitted signal G_{11} is shown in Fig. 2. This corresponds to the case where the two cavities have the same resonance wave number $k_0=n_V\pi/L_V=n_H\pi/L_H$, with n_V and n_H being integers. Were there only a vertical cavity ($R_2=0$), a signal of wave number $k=k_0$ would be reflected, and thus $|G_{11}|\approx 0$ would be observed. Due to the coupling introduced by the beam splitter BS_2 ($R_2\neq 0$), the resonance of the double-cavity system is split (quasi-Rabi splitting), so that now $|G_{11}|\approx 0$ for $k=k_0\pm\Delta k$, where

$$\Delta k\approx\sqrt{\frac{R_2}{L_HL_V}}. \tag{12}$$

In the middle of the interval between the two resonant wave-number values, the system is transparent, i.e., $G_{11}\approx 1$ for $k=k_0$.

Let us consider, for a moment, the isolated system consisting of the two cavities that are decoupled from the external world ($R_1=0$), but coupled to each other ($R_2\neq 0$). Solving (for nonabsorbing devices) the eigenmode equation, we find that the symmetric and antisymmetric combinations of the modes of the individual cavities form modes of the double-cavity system which just differ in the value of Δk given by Eq. (12). Hence an incoming wave of wave number k can be regarded as being coupled by the beam splitter BS_1 to these two modes. Whereas for $k=k_0\pm\Delta k$ one of the two modes can be excited, the couplings to them interfere destructively for $k=k_0$, so that the incoming field is virtually decoupled from the cavities and the system becomes transparent.

The behavior of a double-cavity resonator resembles the behavior of several other physical systems (for details, see Appendixes B–D). The system has many similarities to a three-level EIT medium (Appendix B) and to a one-dimensional atom in a cavity (Appendix C), and there is a close relationship to the so-called interaction-free-measurement and the quantum Zeno effect (Appendix D).

B. Time delay

From the input-output relation [Eq. (2)] it follows that outgoing and incoming pulses in the time domain are related to each other as

$$a_{out}(t)=\int G(\tau)a_{in}(t-\tau)d\tau, \tag{13}$$

where

$$G(\tau)=\frac{1}{2\pi}\int e^{-i\omega\tau}G_{11}(\omega)d\omega, \tag{14}$$

with $G_{11}(\omega)$ from Eq. (3). When absorption can be disregarded ($A_l, A_{Mm}=0$) and the reflection coefficients R_l are small, then $G_{11}(\omega)$ can be expanded around $\omega=\omega_0$ to obtain

$$\begin{aligned} G_{11}(\omega) &\approx (1+iL_D\delta k-L_D^2\delta k^2)\\ &\approx \exp(iL_D\delta k-\tfrac{1}{2}L_D^2\delta k^2) \end{aligned} \tag{15}$$

[$\delta k=(\omega-\omega_0)/c\ll 1/L_D$], where

$$L_D=\frac{R_1L_H}{2R_2}. \tag{16}$$

Let us consider an incoming pulse that has a midfrequency $\omega_0=k_0c$ and a spectral width smaller than c/L_D. In this case, the response function $G(\tau)$ in Eq. (14) can be obtained from Eq. (15), with the approximate expression of $G_{11}(\omega)$ given in Eq. (15),

$$G(\tau)\approx\frac{1}{\sqrt{2\pi}\tau_D}\exp\left[-\frac{(\tau-\tau_D)^2}{2\tau_D^2}\right], \tag{17}$$

where

$$\tau_D=\frac{L_D}{c}=\frac{R_1L_H}{2R_2c}. \tag{18}$$

Comparing with Eq. (13), we see that the outgoing pulse is delayed by τ_D and its envelope is broadened by the same amount τ_D (i.e., the envelope of the power intensity is broadened by $\tau_D/\sqrt{2}$).

C. Internal fields

The complex amplitudes (in the frequency domain) of the fields inside the double-cavity system are given in Appendix E. Expanding the amplitudes of the fields in the horizontal cavity, [Eqs. (E5), (E7), (E10), and (E11)], around $\omega=\omega_0$ we find, on neglecting again losses, that $-a_{2,M1}\approx a_{M1,2}\approx -a_{2,M3}\approx a_{M3,2}\equiv a_H(\omega)$, where

$$a_H(\omega)=\frac{1}{2}\sqrt{\frac{R_1}{R_2}}(1+i\,\delta kL_D-\delta k^2L_D^2)a(\omega) \tag{19}$$

($b=0$). We compare Eq. (19) with Eq. (15), and see that

$$a_H(\omega)\approx\frac{1}{2}\sqrt{\frac{R_1}{R_2}}G_{11}(\omega)a(\omega) \tag{20}$$

($\delta k\ll 1/L_D$). Thus the resonant field in the horizontal cavity has an intensity that is $R_1/(4R_2)$ times larger than that of the incoming field. In the time domain, it is obviously a pulse that is delayed and broadened by τ_D, i.e.,

$$a_H(t)\approx\frac{1}{2}\sqrt{\frac{R_1}{R_2}}\int G(\tau)a_{in}(t-\tau)d\tau. \tag{21}$$

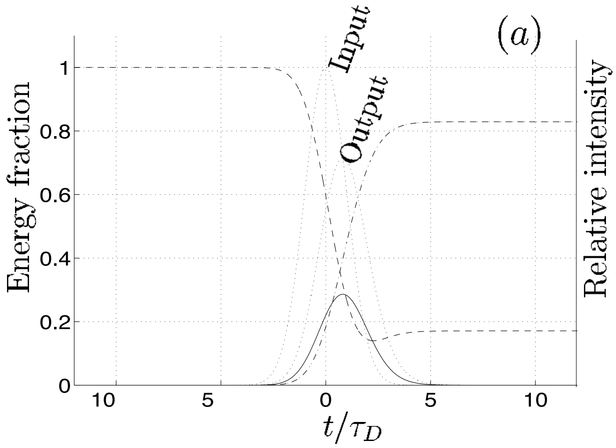
In the same approximation, from Eqs. (E1), (E3), (E4), (E8), and (E9), we find that the leading term of the field amplitudes in the vertical cavity is $a_V(\omega)=ia(\omega)\sqrt{R_1}/2$. That is, the intensity of the field in the vertical cavity is only a fraction of $R_1/4$ of the intensity of the incoming field. Since noticeable XPM in the probe will require a large field in the horizontal cavity, it is crucial that R_2 be as small as possible.

The temporal evolution of field energies on the left-hand side of the system (incoming and reflected fields), in the horizontal cavity, and on the right-hand side of the system (transmitted field), as well as the input and output field intensities, are illustrated in Fig. 3. One can see the importance of a proper choice of the signal pulse length. For short pulses (relative to τ_D), a strong electric intensity develops in the horizontal cavity, but the output pulse is deformed with a large fraction of the pulse being reflected. Long pulses keep their shape, but cannot produce very strong intensities.

D. Influence of losses

Let us comment on the conditions under which the material absorption of the mirrors and beam splitters may be disregarded. From an expansion of the expressions in Eqs. (3)–(6), it follows that when

$$A_1, A_{M2}, A_{M4}\ll R_1, \tag{22}$$



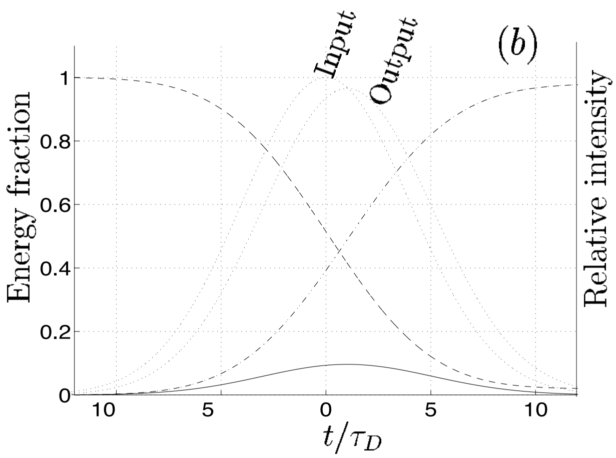


FIG. 3. Time evolution of the energy fraction in the space in front of the system (broken line), inside the horizontal cavity (full line), and in the space behind the system (dash-dotted line), and relative intensities entering and leaving the system at BS_1 (dotted lines). The input pulse is a Gaussian with half widths of τ_D (a) and $4\tau_d$ (b), respectively. The cavity parameters are $L_V=120\lambda$, $L_H=30\lambda$, $R_1=0.1$, and $R_2=10^{-6}$, and there is no absorption.

$$A_2, A_{M1}, A_{M3} \ll \frac{R_2}{R_1}, \tag{23}$$

then the absorption coefficients can be neglected. Since R_2 should be as small as possible, condition (23) is the most restrictive.

The probabilities of absorption of monochromatic photons are given by $P_a=1-|G_{11}|^2-|G_{12}|^2$ and $P_b=1-|G_{22}|^2-|G_{21}|^2$. In particular, for small absorption, we have $P_a\approx P_b\equiv P$, and, if k is close to k_0, expansion yields

$$P\approx A_1+A_{M4}\frac{R_1}{4}+(A_{M1}+A_{M3}+A_2)\frac{R_1}{4R_2}\left(1-\frac{R_1^2}{4}\right)+\delta k^2 L_D^2\left[\frac{2A_1+A_{M2}+A_{M4}}{R_1}-(A_{M1}+A_{M3}+A_2)\times\frac{R_1}{4R_2}(1+R_1)\right]. \tag{24}$$

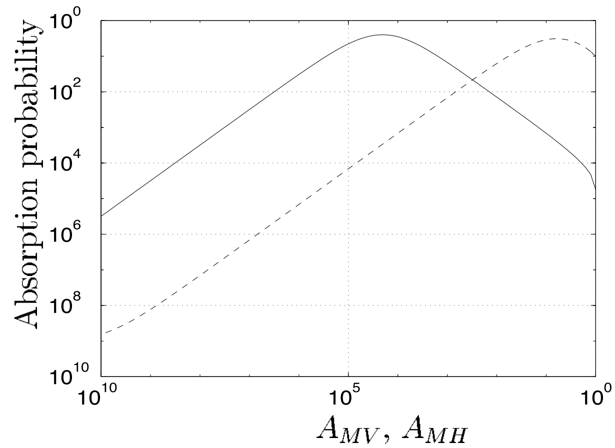


FIG. 4. Absorption probability of a photon in a Gaussian wave packet with a temporal half-width of τ_D in dependence on the mirror absorption coefficients. Full line: $A_{M2}=A_{M4}=0$, and $A_{M1}=A_{M3}=A_{MH}$; dashed line: $A_{M1}=A_{M3}=0$ and $A_{M2}=A_{M4}=A_{MV}$. The other cavity parameters are $L_H=30\lambda$, $L_V=120\lambda$, $R_1=10^{-1}$, $R_2=10^{-6}$, and $A_1=A_2=0$.

To obtain the absorption probability of a photon associated with a wave packet, one has to average over the corresponding k distribution.

Figure 4 illustrates the dependence of the absorption probability on the mirror absorption coefficients for a Gaussian wave packet of the temporal half-width τ_D. [Note that for such a wave packet the second-order expansion in δk, as given in Eq. (24), fails, because the k distribution of the wave packet is too broad.] It is seen that the absorption in the horizontal cavity (full line) plays a dominant role. The decrease of the absorption probability for large absorption coefficients (right to the knee of the full line) corresponds to the interaction-free measurement effect (Appendix D).

E. Low-reflectivity beam splitters

As already pointed out, the reflectivity of the second beam splitter, R_2, should be as small as possible. To achieve this goal, we first assume that it is made from a thin dielectric plate of thickness d ($d\ll\lambda$) and refractive index n (the refractive index of the medium be ≈ 1), so that

$$R_2=r_2^2\approx\left[\frac{n^2-1}{\sqrt{2}}kd\right]^2. \tag{25}$$

For $d\approx 10$ nm ($kd\approx 10^{-2}$), R_2 can thus be estimated to be $R_2\approx 10^{-4}$, which will be not small enough. Let us anticipate the progress of nanotechnology, assuming the possibility to build very thin and sparse dielectric nanostructures, so that $R_2\approx 10^{-6}$ becomes possible. This value would be realized by a dielectric plate of 1-nm thickness. Another possibility would be to construct a sparse networklike structure of 10-nm fibers, which would cover about 10% of the (otherwise empty) beam-splitter area.

One could also think about a cavity made of some solid material. Then, including a weakly reflecting beam splitter in

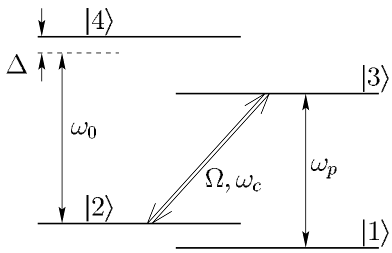


FIG. 5. Four-level scheme of the EIT medium.

this would not be difficult, because it could be realized by a thin layer of material with slightly different index of refraction. A disadvantage may be the somewhat increased absorption losses. Although EIT was successfully observed in doped solid materials [17], in such materials a smaller achievable XPM than in gases is expected, because of the lower dipole moments of the doping atoms.

III. CONDITIONAL PHASE SHIFT

A photonic qubit can be realized by a single photon which can travel along two alternate paths (e.g., two branches of an interferometer), the corresponding states being denoted by $|0\rangle$ and $|1\rangle$. Let us consider two qubits such that, in state $|11\rangle$ the paths of the two photons partly overlap in a Kerr nonlinear medium. Further, let the nonlinearity be so strong that the presence of one photon changes the refractive index appreciated by the other photon in order to introduce a π phase shift. State $|11\rangle$ thus transforms as $|11\rangle \rightarrow -|11\rangle$, while the other states remain unchanged: $|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow |01\rangle$, and $|10\rangle \rightarrow |10\rangle$. Such a transformation, accompanied by single-qubit transformations (which are trivial for the photonic qubit realization considered), can serve as a building block for quantum computation (see, e.g., Ref. [18]), or for achieving quantum teleportation [19].

A. EIT medium

To realize the desired large XPM, let us combine the action of the double-cavity system and that of a four-level EIT medium of the type studied in Ref. [1] (Fig. 5). We assume an atomic medium with a four-level structure as in Fig. 5, whose transition $|2\rangle \leftrightarrow |3\rangle$ is driven by an external coupling field of Rabi frequency Ω. The probe field (midfrequency ω_p) is resonant with the $|1\rangle \leftrightarrow |3\rangle$ transition, whereas the signal field (midfrequency $\omega_s = \omega_0$) is detuned by Δ from the $|2\rangle \leftrightarrow |4\rangle$ transition. Using fourth-order perturbation theory, one can find that the Kerr index of refraction "felt" by the probe is

$$n_K = \frac{N\mu_{13}^2\mu_{24}^2}{8\epsilon_0\hbar^3|\Omega|^2\Delta}|E_s|^2, \tag{26}$$

where N is the atomic density, μ_{13} and μ_{24} are the dipole matrix elements of transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$, respectively and E_s is the electric field strength of the signal. The group velocity v_g of the probe can be given by [14]

$$v_g = c\left[n_0 + \omega_p \frac{dn_0}{d\omega_p}\right]^{-1} \approx \frac{\epsilon_0\hbar\lambda|\Omega|^2}{4\pi N\mu_{13}^2}, \tag{27}$$

where n_0 is the signal-independent part of the refractive index.

During the propagation in the EIT medium, the signal and probe suffer from several kinds of losses. The most important losses result from the finite frequency window for the probe (frequency components outside a very narrow interval with respect to the $|1\rangle \leftrightarrow |3\rangle$ transition are partially absorbed), and two-photon losses due to the simultaneous absorption of probe and signal photons (exciting atomic state $|4\rangle$). The loss coefficient α_1 (inverse absorption length) for single-photon probe absorption can be given by [2,7]

$$\alpha_1 = \frac{32\pi^2 N\mu_{13}^2\gamma_3\delta^2}{\epsilon_0\hbar\lambda|\Omega|^4}, \tag{28}$$

where δ is the detuning of the probe from the $|1\rangle \leftrightarrow |3\rangle$ transition, and γ_3 is the decay rate of state $|3\rangle$. The loss coefficient α_2 for the (simultaneous) two-photon absorption can be written in the form of [2]

$$\alpha_2 = \frac{\pi^2 N\mu_{13}^2\mu_{24}^2\gamma_4}{2\epsilon_0\hbar^3\lambda|\Omega|^2\Delta^2}|E_s|^2, \tag{29}$$

where γ_4 is the atomic decay rate of state $|4\rangle$.

B. Conditional single-photon phase shift

Let us assume that the signal and the probe are sent, according to Fig. 1, to a double-cavity system complemented by the EIT medium. The conditional phase shift $\Delta\phi$ of the probe due to XPM is

$$\Delta\phi = \int n_K k_p dl, \tag{30}$$

where the integral runs over the propagation length of the probe. For a pulse propagating with the group velocity v_g in an otherwise homogeneous medium, we may write $dl = v_g dt$ and use Eqs. (26) and (27), so that

$$\Delta\phi = \frac{\mu_{24}^2}{16\hbar^2\Delta}\int |E_s(t)|^2 dt. \tag{31}$$

The integration runs over the time during which the probe pulse is inside the medium. Let the input signal be a single-photon Gaussian pulse of half-width τ_s, i.e., its electric field reads

$$E_s^{(\mathrm{in})} = E_0 \exp\left(-\frac{t^2}{4\tau_s^2} - i\omega_0 t\right), \tag{32}$$

where

$$E_0^2 = \sqrt{\frac{2}{\pi}}\frac{\hbar\omega_0}{c\epsilon_0 S\tau_s}, \tag{33}$$

with S being the cross-sectional area of the pulse. We apply transformation (21) to obtain the amplitude of the intracavity field propagating from BS_2 to the mirrors of the horizontal cavity in the form

$$E_{s,H}=\frac{E_0}{2}\sqrt{\frac{R_1}{R_2}}\left(1+\frac{\tau_D^2}{2\tau_s^2}\right)^{-1/2}\times\exp\left[-\frac{(t-\tau_D)^2}{4}\tau_s^2\left(1+\frac{\tau_D^2}{2\tau_s^2}\right)-i\omega_0 t\right]. \quad (34)$$

This field combines with the reflected fields to a standing wave whose squared amplitude averaged along L_H is $|E_s(t)|^2=2|E_{s,H}|^2$. When the probe group velocity is sufficiently low so that the time which the probe spends in the medium is longer than the time duration of the signal in the cavity,

$$L/v_g \gtrsim 4\sqrt{\tau_s^2+\tau_D^2/2}, \quad (35)$$

then the time integral in Eq. (31) can be approximated by integration in infinite limits as

$$\int |E_s(t)|^2 dt=\frac{R_1}{R_2}\frac{2\pi\hbar}{\epsilon_0\lambda S}\left(1+\frac{\tau_D^2}{2\tau_s^2}\right)^{-1/2}. \quad (36)$$

Thus, from Eqs. (36) and (31), the phase shift of the probe is estimated to be

$$\Delta\phi\approx\frac{\pi}{8}\frac{R_1}{R_2}\frac{\mu_{24}^2}{\epsilon_0\hbar\lambda S\Delta}\left(1+\frac{\tau_D^2}{2\tau_s^2}\right)^{-1/2}. \quad (37)$$

C. Conditions for the parameters

From Eq. (37) it is seen that the duration of the signal pulse, τ_s, should be as long as possible, but at least comparable with τ_D. On the other hand, τ_s should be limited by the transmission time of the probe according to condition (35). Otherwise, the full signal would not be used. Equation (37) further reveals that the phase shift does not explicitly depend on the parameters N, μ_{13}, and $|\Omega|$. However, these parameters must be chosen such that the group velocity [Eq. (27)] is low enough to satisfy condition (35). Assuming $L\approx L_H$ and

$$\left(1+\frac{\tau_D^2}{2\tau_s^2}\right)^{-1/2}\approx\frac{1}{2}, \quad (38)$$

from Eqs. (35) and (18) we obtain the following condition for the group velocity:

$$v_g\lesssim\frac{\sqrt{6}}{8}\frac{R_2}{R_1}c. \quad (39)$$

Recalling Eq. (27), we see that the Rabi frequency of the driven transition must satisfy the condition

$$|\Omega|^2\lesssim\frac{\sqrt{6}\pi}{2}\frac{R_2}{R_1}\frac{c\mu_{13}^2}{\epsilon_0\hbar\lambda}N. \quad (40)$$

Further, the parameters should also be chosen such that the losses are sufficiently small. The probability of two-photon absorption,

$$P_2\approx\int dl\,\alpha_2(|E_s|^2)=v_g\int dt\,\alpha_2(|E_s|^2), \quad (41)$$

can be calculated with the help of Eqs. (27), (29), and (36). The condition that $P_2\ll 1$ then reads

$$P_2\approx\frac{\pi^2}{4}\frac{\mu_{24}^2\gamma_4}{\epsilon_0\hbar S\lambda\Delta^2}\frac{R_1}{R_2}\left(1+\frac{\tau_D^2}{2\tau_s^2}\right)^{-1/2}\ll 1, \quad (42)$$

and the condition of the two-photon absorption probability being much smaller than the phase shift [Eq. (37)] is

$$\frac{P_2}{\Delta\phi}\approx\frac{2\pi\gamma_4}{\Delta}\ll 1, \quad (43)$$

which corresponds to the result in Ref. [1]. The condition of negligible single-photon absorption of the probe $\alpha_1 L\ll 1$ can be written, on recalling Eq. (28),

$$\frac{32\pi^2 NL\mu_{13}^2\gamma_3\delta^2}{\epsilon_0\hbar\lambda|\Omega|^4}\ll 1. \quad (44)$$

We have assumed that during the interaction with the signal, the whole probe pulse is inside the medium. Thus the time duration $\tau_p\approx\delta^{-1}$ of the probe must be shorter than its propagation time through the medium. From Eq. (27), it then follows that this assumption yields the condition that

$$\frac{1}{\delta}\ll\frac{L}{v_g}=\frac{4\pi NL\mu_{13}^2}{\epsilon_0\hbar\lambda|\Omega|^2}. \quad (45)$$

By combining conditions (44) and (45), we obtain a condition for the atomic density,

$$N\gg\frac{2\epsilon_0\hbar\lambda\gamma_3}{\mu_{13}^2 L}, \quad (46)$$

and a condition for the linewidth of the probe:

$$\delta\ll\frac{|\Omega|^2}{8\pi\gamma_3}. \quad (47)$$

From Eq. (37) it follows that the ratio R_1/R_2 should be chosen as large as possible. This would suggest using large R_1, but such a choice would mean large probability of reflection of the signal photon. In particular, expansion of G_{11} and G_{21} for the resonant signal yields $G_{11}\approx 1-R_1^2/8$ and $G_{21}\approx R_1/2$. We can see that using $R_1\approx 0.1$ is a reasonable choice.

D. Experimental parameters

In the experimental demonstration of ultraslow group velocity in Ref. [20] a gas of rubidium atoms was used. In this case, $\lambda = 795$ nm, $\mu_{13} \approx \mu_{24} \approx 10^{-29}$ C m, and $\gamma_3 \approx \gamma_4 \approx 10^6$ s^{-1}. Let the signal beam diameter be $\approx L_H \approx 10$ μm so that $S \approx 10^{-10}$ m^2. Condition (43) is satisfied, $P_2/\Delta\phi \approx 0.1$, if $\Delta = 10^8$ s^{-1}. From Eq. (37), it then follows that for

$$\frac{R_1}{R_2} \approx 10^5 \tag{48}$$

a phase shift of $\Delta\phi \approx \pi$ can be achieved.

Condition (46) requires that $N \gg 10^{12}$ cm^{-3}. Let us assume an atomic density of $N \approx 10^{14}$ cm^{-3}. Since the detuning Δ is very small, the gas has to be laser cooled to avoid the Doppler broadening. Condition (40) then implies that the Rabi frequency of the driven atomic transition should be $|\Omega| \approx 10^9$ s^{-1}. With these values, we find, from Eqs. (26) and (27) for the group velocity $v_g \approx 10^3$ ms^{-1}, the switching time being $L_H/v_g \approx 10$ ns. According to condition (45), the length of the probe pulse should chosen such that $\tau_p \gg 10^{-11}$ s. Choosing $\tau_p \approx 1$ ns (i.e., $\delta \approx 10^9$ s^{-1}), Eq. (28) yields $\alpha_1 \approx 10^4$ m^{-1}, so that the linear absorption is relatively small ($\alpha_1 L \approx 0.1$ and $L \approx L_H$).

Condition (48) is a strong requirement for building the subtle beam splitter needed; for $R_1 \approx 10^{-1}$ one needs $R_2 \approx 10^{-6}$. With respect to condition (23), we see that the absorption coefficients of the mirrors should be at least one order of magnitude smaller than the ratio R_2/R_1. Thus the quality of presently available mirrors (absorption $\approx 10^{-6}$ [13]) is nearly sufficient.

E. Comparison with free-medium scheme

Let us compare the performance of the coupled-cavity scheme with the free-medium scheme in Ref. [8]. Assuming a diameter of $l_D \approx 10$ μm of the copropagating signal and probe, and a pulse linewidth of $\Delta\omega \approx 1$ MHz, the signal field would produce a shift of the refraction index for the probe of the order $n_K \approx 10^{-4}$. Thus, in the free-medium scheme, a propagation length of $l_0 \approx 1$ cm would be necessary to achieve a phase shift of the order of π, which implies a switching time of $l_0/v_g \approx 1$ ms. This is about 10^5 times longer than the switching time in the coupled-cavity scheme. Note that the length l_0 is about 10^2 times longer than the Rayleigh distance $l_R \approx l_D^2/\lambda \approx 100\mu$ (see, e.g., Ref. [21]). Thus a sophisticated waveguide structure or a system of refocusings would be necessary which makes the free-medium approach technically demanding as well, so that other possibilities of confining the interacting beams have been of interest [22].

IV. CONCLUSION

A suitable combination of optical resonators with an EIT medium can increase the XPM effect to be used in quantum information processing. The cavity system emulates the EIT response for the signal: the *signal* is slowed down, and simultaneously its intensity is enhanced. The *probe* is not influenced by the cavity, but it propagates in the EIT medium whose properties are determined by the signal. By coupling the cavities with weakly reflecting beam splitters rather than with weakly transmitting mirrors, large losses can be avoided. This makes the system especially attractive for applications in quantum information processing.

To achieve a large ($\approx \pi$) phase shift by means of a single signal photon, several technical problems must be solved. As the most important, very low-reflectivity ($R \lesssim 10^{-6}$) beam splitters must be available and a suitable method for keeping the cooled EIT atoms in an optical cavity must be developed. The device then could serve as a relatively fast (≈ 10 ns switching time) quantum logic gate.

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APPENDIX A: LINEAR RESPONSE OF A CONVENTIONAL CAVITY

Consider a cavity constructed from two parallel mirrors separated by length L. Assume the transformation rule for the mirrors as

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} it & -r \\ -r & it \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}, \tag{A1}$$

where a and b are the input field amplitudes, a' and b' are the output field amplitudes, $r^2 \equiv R \approx 1$ is the mirror reflectivity, $t^2 \equiv T \ll 1$ is the transmissivity, and $R + T + A = 1$, with A being the absorption coefficient. When a unit input field of wave number k is fed into the cavity, then the amplitude of the output field that is generated by reflection at the entrance mirror is

$$q = r \frac{(1-A)e^{i2kL} - 1}{1 - Re^{i2kL}}, \tag{A2}$$

and the amplitude of the (transmitted) output field at the other mirror is

$$p = -\frac{Te^{ikL}}{1 - Re^{i2kL}}. \tag{A3}$$

Close to resonance, $k \approx k_0$ ($k_0 L = n\pi$), the transmission through the cavity is then

$$\mathcal{T} \equiv |p|^2 \approx \frac{T^2}{(T+A)^2} \frac{1}{1 + (k-k_0)^2/\Delta k^2}, \tag{A4}$$

and the reflection at the cavity reads

$$\mathcal{R}\equiv|q|^2\approx\frac{A^2R}{(T+A)^2}\frac{1}{1+(k-k_0)^2/\Delta k^2}+(1-A)\frac{(k-k_0)^2}{\Delta k^2}\frac{1}{1+(k-k_0)^2/\Delta k^2}, \quad \text{(A5)}$$

where the linewidth Δk is given by

$$\Delta k=\frac{T+A}{2\sqrt{RL}}. \quad \text{(A6)}$$

Thus, if $A\approx T$, we obtain $\mathcal{T}\approx 1/4$ and $\mathcal{R}\approx R/4$ in the resonance regime.

APPENDIX B: ANALOGY BETWEEN COUPLED RESONATORS AND EIT MEDIA

The action on light of the double-cavity system is analogous to that of a three-level EIT medium (see the level scheme in Fig. 5, without $|4\rangle$). The empty double-cavity system corresponds to atomic (ground) state $|1\rangle$, and light in the vertical cavity corresponds to excited atomic state $|3\rangle$. Accordingly, light in the horizontal cavity corresponds to the (auxiliary) atomic state $|2\rangle$. Coupling of atomic states $|3\rangle$ and $|2\rangle$, which gives rise to the dressed (Rabi-split) states, is analogous to the cavity coupling by the beam splitter BS_2.

From Fig. 2 it is seen that the linear response of the cavity system to the incoming light resembles the response of an EIT medium. Physically, the two systems are of course quite different. In particular, frequency components that are slightly off-resonant are absorbed by the EIT medium, but are reflected by the cavity system.

Recently large phase shifts in moving media with EIT-type dispersion were predicted [23]. A corresponding effect is observed in the double-cavity system when it is moved along the signal-propagation direction. The produced phase shift is $\Delta\phi_v=\omega_0\tau_D v/c$, where v is the velocity of the system. It can be large if τ_D [Eq. (18)] is sufficiently large, even when the motion of the system is relatively slow.

APPENDIX C: ANALOGY BETWEEN COUPLED RESONATORS AND ONE-DIMENSIONAL ATOMS

The double-cavity system is also analogous to a *one-dimensional atom* in the bad-cavity regime [24]. Here a two-level atom inside a cavity is resonantly coupled to the cavity field, such that the Rabi splitting is larger than the spontaneous decay rate of the atom, whereas the decay rate of the cavity field is larger than the Rabi splitting. In the double-cavity system, the horizontal cavity plays the role of the atom, and the vertical cavity plays the role of the cavity, provided that the condition $R_2\ll R_1$ is satisfied. The transmission-line splitting is then quite similar to the Rabi splitting mentioned above (compare Fig. 2 with Figs. 3 and 4 in Ref. [24]).

APPENDIX D: RELATION TO THE INTERACTION-FREE MEASUREMENT AND THE QUANTUM ZENO EFFECT

If one (or both) of the mirrors of the horizontal cavity is replaced by a completely absorbing object (i.e., $A_{M1}=1$ and/or $A_{M3}=1$), then in the resonance regime the double-cavity system changes from an almost perfectly transmitting device to an almost perfectly reflecting device. To be more specific, while for perfect mirrors of the horizontal cavity, in resonance the fraction of the reflected light is $\approx R_1^2/4\ll 1$ and the fraction of the transmitted light is $\approx 1-R_1^2/4$, for a fully absorbing horizontal cavity one obtains for the transmitted fraction $\approx 4T_1R_2^2/R_1^2\ll 1$ and the reflected fraction $\approx 1-4T_1R_2/R_1\approx 1$. Thus, almost without touching the absorber (and being lost), a photon can tell us whether the absorber is in the horizontal cavity or not. The probability of such a loss is $\approx 4T_1R_2(R_1-R_2)/R_1^2\approx 4T_1R_2/R_1$ and can be, in principle, made as small as we wish (see Fig. 4). This is effect is also called *interaction-free measurement* [25]. This was suggested for a different coupled cavity system (coupling by a highly reflecting, partially transmitting mirrors) in Ref. [26], and demonstrated experimentally (for coupling by polarization rotation) in Ref. [27].

One can also understand the phenomenon in terms of the *quantum Zeno effect*. Let us consider a two-state system where a photon can be either in the vertical or in the horizontal cavity, the coupling between the two states being realized by BS_2. The evolution starts with the photon in the vertical cavity, while the presence of a photon in the horizontal cavity is monitored by a detector. After a time interval Δt (much shorter than the period of oscillation between the two states), the two-level system evolves to a state where with probability $\propto\Delta t^2$ the photon is in the horizontal cavity. If the detector ''sees'' no photon (which happens in most cases), then the state is projected back into the original state. Thus the photon is prevented from entering the horizontal cavity by the presence of the detector (for two-mode photonic Zeno effects in other schemes, see Ref. [28]). Let us emphasize that the quantum nature of the effect is observed on a single-photon level. The effect of inhibiting waves from entering the absorbing part of the resonator can of course be explained classically.

APPENDIX E: INTRACAVITY FIELDS

The field propagating from BS_1 to BS_2 has an amplitude a_{12} given by

$$a_{12}=ir_1\frac{-t_1\sqrt{1-A_{M4}}e^{i2kL_5}a+b}{1+T_1\sqrt{1-A_{M4}}e^{i2kL_5}\mathcal{B}}, \quad \text{(E1)}$$

with a and b the input amplitudes in Eq. (2) and $\mathcal{B}$ given in Eq. (7). The field arriving at BS_1 from BS_2 has an amplitude a_{21} given by

$$a_{21}=\mathcal{B}a_{12}. \quad \text{(E2)}$$

The field propagating from BS_1 to M_4 has an amplitude $a_{1,M4}$ given by

$$a_{1,M4}=ir_1\frac{a+t_1\mathcal{B}b}{1+T_1\sqrt{1-A_{M4}}e^{i2kL_5}\mathcal{B}}, \tag{E3}$$

and the field arriving at BS_1 from M_4 has an amplitude $a_{M4,1}$ given by

$$a_{M4,1}=-\sqrt{1-A_{M4}}e^{i2kL_5}a_{1,M4}. \tag{E4}$$

The field propagating from BS_2 to M_1 has an amplitude $a_{2,M1}$ given by

$$a_{2,M1}=-ir_2t_2(\sqrt{1-A_{M3}}e^{i2kL_2} + \sqrt{1-A_{M2}}e^{i2kL_4})\mathcal{B}_5^{-1}e^{ikL_1}a_{12}, \tag{E5}$$

where

$$\mathcal{B}_5=1-\sqrt{1-A_{M1}}e^{i2kL_3}(T_2\sqrt{1-A_{M3}}e^{i2kL_2} - R_2\sqrt{1-A_{M2}}e^{i2kL_4}); \tag{E6}$$

and the field arriving at BS_2 from M_1 has an amplitude $a_{M1,2}$ given by

$$a_{M1,2}=-\sqrt{1-A_{M1}}e^{i2kL_3}a_{2,M1}. \tag{E7}$$

The field propagating from BS_2 to M_2 has an amplitude $a_{2,M2}$ given by

$$a_{2,M2}=t_2(1-\sqrt{(1-A_{M1})(1-A_{M3})} \times(1-A_2)e^{i2k(L_2+L_3)})\mathcal{B}_5^{-1}e^{ikL_1}a_{12}, \tag{E8}$$

and the field arriving at BS_2 from M_2 has an amplitude $a_{M2,2}$ given by

$$a_{M2,2}=-\sqrt{1-A_{M2}}e^{i2kL_4}a_{2,M2}. \tag{E9}$$

The field propagating from BS_2 to M_3 has an amplitude $a_{2,M3}$ given by

$$a_{2,M3}=-ir_2(1+\sqrt{(1-A_{M1})(1-A_{M2})} \times(1-A_2)e^{i2k(L_3+L_4)})\mathcal{B}_5^{-1}e^{ikL_1}a_{12}, \tag{E10}$$

and the field arriving at BS_2 from M_3 has an amplitude $a_{M3,2}$ given by

$$a_{M3,2}=-\sqrt{1-A_{M3}}e^{i2kL_2}a_{2,M3}. \tag{E11}$$

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