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Heliumlike geonium atom

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We study the quantum dynamics of two interacting electrons confined in a Penning trap. We determine the characteristic frequencies of this system and we propose a way to perform measurement on spin and cyclotron degrees of freedom as well as a way of preparing the geonium atom in any spin state.

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I. INTRODUCTION

It is well known that a single electron stored in a Penning trap (PT) [1] permits accurate measurements and provides a simple system to investigate the fundamental laws of nature [2]. This system has been called a hydrogenlike geonium atom since it resembles a hydrogen atom for which the binding for the electron was replaced by external trapping fields [3]. Recently, the geonium system has been also studied to implement some interesting quantum optics situations, such as quantum nondemolition measurements [4], generation and characterization of nonclassical states [5], and implementation of quantum logic operations [6].

In a previous work [6], we have shown how to use the spin and motional degrees of freedom of the confined electron in order to store and to manipulate two qubits through a controlled-NOT gate (the qubit being the basic unit of information in a quantum computer [7]). It would be, however, interesting to replace the qubit associated with the motional degree of freedom by a qubit associated with a second spin. There are two immediate advantages of real spin over pseudospin: First, the qubit represented by a real spin 1/2 is always a well-defined qubit in the sense that the two-dimensional Hilbert space is the entire space avalaible; thus there are no extra dimensions into which the qubit state could "leak." Second, since the spin motion in a PT decays via magnetic dipole radiation, which is exceedingly small, we can guarantee the phase coherence of the qubits during the computation time. The use of another spin as a qubit can be naturally achieved by introducing a second electron in the trap. This system consisting of two electrons confined in a PT is an analog of a helium atom; thus we name it a *heliumlike geonium atom*.

In this work we will study the energy spectrum of the heliumlike atom and we will propose possible ways to measure and to manipulate the quantum state of the system.

It is well known [3] that the problem of finding the energy spectrum of a single electron in a PT is analytically solvable. Its motion can be decomposed into three independent harmonic oscillators: axial, cyclotron, and magnetron, each of them characterized by a well-defined frequency of oscillation. However, this analytical simplicity is destroyed when a second electron enters in the PT. Actually, the motion of two trapped electons is a three-body problem (the two electrons plus the confinement fields) [8] and, in general, cannot be solved exactly. Thus, in order to determine the energy spectrum of the heliumlike atom we will use perturbative techniques. To the second order of approximation, the motion of the heliumlike geonium atom can be decomposed into six noninteracting harmonic oscillators, each of them characterized by its own frequency.

We shall also show that the use of an additional field, known as a "magnetic bottle" [3], allows one to get quantum information about the heliumlike geonium atom. Namely, we shall relate the axial frequency shifts induced by the "magnetic bottle" field with the axial spin projection as well as with the cyclotron quantum numbers. We also show how to use this measurement technique and a small driving oscillatory field to prepare our geonium atom in any desired spin state.

This paper is organized as follows: In Sec. II we present the model and we separate the Hamiltonian in the center-of-mass (c.m.) and the relative contributions. In Sec. III each of these two parts is studied and their characteristic frequencies are determined. Sec. IV presents a way of getting information from the system by using a "magnetic bottle" field. Finally, Sec. V concludes.

II. MODEL

We are considering a system of two electrons of mass m_e and charge e inside a Penning trap. The confinement in this trap is realized by a uniform magnetic field **B** along the positive z axis and a static quadrupole potential

$$V=V_0\frac{x^2+y^2-2z^2}{4d^2}, \qquad (1)$$

where d characterizes the dimension of the trap and V_0 is the potential applied to the trap electrodes [3]. By ignoring for the moment the Coulomb repulsion between the two electrons, we can write the Hamiltonian $h^{(i)}$ for each trapped electron ($i=1,2$) as the counterpart of the classical one, with the addition of the spin term, that is

$$h^{(i)}=\frac{1}{2m_e}\left[\vec{\wp}^{(i)}-\frac{e}{c}\vec{A}^{(i)}\right]^2+eV^{(i)}-\frac{g}{2}\frac{e}{m_e c}\vec{S}^{(i)}\cdot\vec{B}, \qquad (2)$$

where g is the electron's g factor, c the speed of light, and $\vec{A}^{(i)}=\frac{1}{2}\vec{B}\wedge\vec{x}^{(i)}$ the vector potential. The position and conjugate momentum operators are $\vec{x}^{(i)}\equiv(x^{(i)},y^{(i)},z^{(i)})$ and $\vec{\wp}^{(i)}$

$\equiv(\wp_x^{(i)},wp_y^{(i)},\wp_z^{(i)})$, respectively. The spin operator $\vec{S}^{(i)}$ is related to the Pauli matrices through $\vec{S}^{(i)}\equiv\frac{1}{2}\hbar(\sigma_x^{(i)},\sigma_y^{(i)},\sigma_z^{(i)})$.

To obtain the total Hamiltonian for the two trapped electrons, we must sum the individual contributions (2) with the repulsive Coulomb energy, that is,

$$H=\sum_{i=1}^{2} h^{(i)}+\frac{e^2}{|\vec{x}^{(1)}-\vec{x}^{(2)}|}. \tag{3}$$

We can write this Hamiltonian in a more convenient form by introducing the c.m. coordinates $\vec{R}\equiv(X,Y,Z)=\frac{1}{2}(\vec{x}^{(1)}+\vec{x}^{(2)})$, $\vec{P}\equiv(P_X,P_Y,P_Z)=(\vec{\wp}^{(1)}+\vec{\wp}^{(2)})$, and the total mass $M=2m_e$, as well as the relative coordinates $\vec{r}\equiv(\xi,\upsilon,\zeta)=(\vec{x}^{(1)}-\vec{x}^{(2)})$, $\vec{p}\equiv(p_\xi,p_\upsilon,p_\zeta)=\frac{1}{2}(\vec{\wp}^{(1)}-\vec{\wp}^{(2)})$ and the reduced mass $\mu=m_e/2$. With these new variables Eq. (3) transforms into

$$H=H_{\text{c.m.}}+H_r+H_S, \tag{4}$$

where

$$H_{\text{c.m.}}=\frac{\vec{P}^2}{2M}+\frac{M}{8}(\omega_c^2-2\omega_z^2)\vec{\Gamma}^2+\frac{\omega_z}{2}\vec{e}_z\cdot(\vec{\Gamma}\times\vec{P})+\frac{M}{2}\omega_z^2Z^2, \tag{5}$$

$$H_r=\frac{\vec{p}^2}{2\mu}+\frac{\mu}{8}(\omega_c^2-2\omega_z^2)\vec{\rho}^2+\frac{\omega_z}{2}\vec{e}_z\cdot(\vec{\rho}\times\vec{p})+\frac{\mu}{2}\omega_z^2\zeta^2+\frac{e^2}{|\vec{r}|}. \tag{6}$$

Here we introduced the cyclotron and axial frequencies defined by $\omega_c=|eB|/m_e$ and by $\omega_z=[eV_0/md^2]^{1/2}$, respectively:

$$H_S=\omega_s S_z. \tag{7}$$

We have used the notation $\vec{\Gamma}\equiv(X,Y)$, $\vec{\rho}\equiv(\xi,\upsilon)$, and we have indicated by $\vec{e}_z$ the unit vector along the z axis. The first two terms in Eq. (4) commute with each other; the last gives the spin contribution and it is also called the Zeeman energy. The spin precession angular frequency is defined by $\omega_s=g|eB|/2m_e$.

III. ORBITAL MOTION

Comparing the c.m. Hamiltonian (5) with that of a single electron in the PT [3] we conclude immediately that the dynamics is the same in both cases. That is, the c.m. motion is the result of three independent harmonic oscillators: two (cyclotron and magnetron) oscillating in the x-y plane with frequencies $\omega_\pm$ and the third (axial) oscillating along the z-axis with frequency ω_z. The frequencies $\omega_\pm$ are defined by

$$\omega_\pm=\frac{1}{2}(\omega_c\pm\sqrt{\omega_c^2-2\omega_z^2}). \tag{8}$$

Let us stress that these frequencies coincide with those obtained for a single trapped electron and they are well separated in the energy scale, i.e., $\omega_+\gg\omega_z\gg\omega_-$.

Let us now define the ladder operators

$$A_z=\sqrt{\frac{M\omega_z}{2\hbar}}Z+i\sqrt{\frac{1}{2\hbar M\omega_z}}P_Z, \tag{9}$$

$$\begin{aligned}A_\pm=\sqrt{\frac{M}{2\hbar(\omega_+-\omega_-)}}\Bigg\{&\left[\frac{P_X}{M}-\left(\frac{\omega_c}{2}-\omega_\mp\right)Y\right]\\&\mp i\left[\frac{P_Y}{M}+\left(\frac{\omega_c}{2}-\omega_\mp\right)X\right]\Bigg\},\end{aligned} \tag{10}$$

obeying the commutation relations $[A_i,A_j^\dagger]=\delta_{ij}$, with $(i,j=+,-,z)$. Then, with the aid of these operators we may reduce Eq. (5) to the form

$$\begin{aligned}H_{\text{c.m.}}=\hbar\omega_Z\left(A_Z^\dagger A_Z+\frac{1}{2}\right)+\hbar\omega_+\left(A_+^\dagger A_++\frac{1}{2}\right)\\-\hbar\omega_-\left(A_-^\dagger A_-+\frac{1}{2}\right),\end{aligned} \tag{11}$$

where the three harmonic oscillators appears as Z axial, $+$ cyclotron, and $-$ magnetron. The negative energy of the latter denotes its unstable motion which, however, takes place on a long time scale, its frequency being the smallest.

If we neglect the Coulomb repulsion between electrons in the relative Hamiltonian (6), we end up with a Hamiltonian equivalent to Eq. (11). It means that the motion of the reduced particle of mass μ would be the result of three harmonic oscillators with frequencies equal to those of c.m. oscillators. However, as we are going to show, the Coulomb interaction will remove this degeneracy in the energy levels.

The presence of the Coulomb energy in the relative Hamiltonian introduces a nonlinear coupling between the three harmonic oscillators. In order to overcome the mathematical difficulty associated with the Coulomb interaction, we develop the relative Hamiltonian in power series around its stationary points,

$$\vec{\rho}_s=\vec{0},\quad z_s=\left[\frac{e^2}{\mu\omega_z^2}\right]^{1/3},\quad \vec{p}_s=\vec{0}, \tag{12}$$

given by the ratio between the Coulomb and axial energy.

The development of Hamiltonian H_r to the second order yields

$$\begin{aligned}H_r=H_0+\frac{\vec{p}^2}{2\mu}+\frac{\mu}{8}(\omega_c^2-6\omega_z^2)\vec{\rho}^2+\frac{\omega_z}{2}\vec{e}_z\cdot(\vec{\rho}\times\vec{p})\\+\frac{3}{2}\mu\omega_z^2\zeta^2,\end{aligned} \tag{13}$$

where $\vec{\rho}$, ζ, and $\vec{p}$ are the deviations of the relative positions and momenta with respect to the stationary points (12). The zero-order approximation is given by the constant energy $H_0=\frac{3}{2}\mu\omega_z^2z_s^2$. The second-order approximation of the series expansion will be a good one if the amplitude of the axial motion, ζ, is much smaller than the equilibrium distance z_s between the electrons:

$$\zeta \approx \sqrt{\hbar/\mu\omega_z} \ll z_s . \tag{14}$$

For a typical value $\omega_z = 6.64$ MHz of the axial frequency in a PT [3], the condition (14) is widely obeyed.

A simple comparison of Eq. (13) with Eq. (5) shows that the relative motion exhibits new frequencies $\omega'_+, \omega'_-, \omega'_z$ given by

$$\omega'_+ = \frac{1}{2}(\omega_c + \sqrt{\omega_c^2 - 6\omega_z^2}) < \omega_+ , \tag{15}$$

$$\omega'_- = \frac{1}{2}(\omega_c - \sqrt{\omega_c^2 - 6\omega_z^2}) > \omega_- , \tag{16}$$

$$\omega'_z = \sqrt{3}\omega_z > \omega_z . \tag{17}$$

Thus, the effect of Coulomb repulsion to the second order of approximation consists in removing the degeneracy in the energy levels of the six oscillators by introducing new frequencies. It should be noticed that this same correcting factor of $\sqrt{3}$ of the relative axial frequency was already noticed for two ions in a Paul trap [9]. The anharmonic corrections to the relative motion only appear at the third order of approximation.

The ladder operators for the relative motion, denoted by $a_\pm, a_\pm^\dagger, a_z, a_z^\dagger$, are defined analogously to those of the c.m. motion, i.e.,

$$a_z = \sqrt{\frac{\mu\omega'_z}{2\hbar}}\zeta + i\sqrt{\frac{1}{2\hbar\mu\omega'_z}}p_\zeta . \tag{18}$$

$$a_\pm = \sqrt{\frac{\mu}{2\hbar(\omega'_+ - \omega'_-)}}\left\{\left[\frac{p_\xi}{\mu} - \left(\frac{\omega_c}{2} - \omega'_\mp\right)\upsilon\right] \mp i\left[\frac{p_\upsilon}{\mu} + \left(\frac{\omega_c}{2} - \omega'_\mp\right)\xi\right]\right\}. \tag{19}$$

They lead to the following form for the relative Hamiltonian:

$$H_r = \hbar\omega'_z\left(a_z^\dagger a_z + \frac{1}{2}\right) + \hbar\omega'_+\left(a_+^\dagger a_+ + \frac{1}{2}\right) - \hbar\omega'_-\left(a_-^\dagger a_- + \frac{1}{2}\right), \tag{20}$$

which is an analog of Eq. (11).

Summarizing, we can describe the motion of the two confined electrons as the result of six independent harmonic oscillators: three associated with the c.m. motion, with frequencies $\omega_+, \omega_-, \omega_z$ and the other three associated with the relative motion, with frequencies $\omega'_+, \omega'_-, \omega'_z$. The spin Zeeman energies $E_S = M(\hbar\omega_s)/2$ $(M = -1,0,+1)$ are the eigenvalues of the spin Hamiltonian (7). For the same orbital quantum numbers, the triplet states are equally spaced and their splitting energy is $\hbar\omega_s$.

Now, the electrons, being fermions, must satisfy the Pauli principle. It is straightforward to show that the c.m. eigenfunctions are always symmetric under permutation of two electrons, while the symmetry of the relative motion is determined by means of $(-1)^{n+k+l}$, where n, k, and l are the quantum numbers of the cyclotron, axial, and magnetron relative oscillators. Then, in order to satisfy the Pauli exclusion principle we can conclude that a spin-singlet state must have $n+k+l$ even, while a spin-triplet state must have $n+k+l$ odd. Changing by 1 any of the quantum numbers n,k,l makes the trapped electrons oscillate between the singlet and triplet states. The use of additional standing waves in the trap configuration [5] may induce jumps in these quantum numbers.

IV. INFORMATION MEASUREMENTS

We recall that in the geonium system the measurements are performed by using the axial motion as a meter to investigate the other degrees of freedom, due to the nonexistence of good detectors in the microwave range [3]. In fact, the axial oscillating charge particle induces alternating image charges on the electrodes, which in turn cause an oscillating current to flow through the external circuit where the measurement is performed. The axial frequencies ω_z and ω'_z of the heliumlike geonium atom are in the radio-frequency range and can easily be measured through the external circuit. Small frequency shifts $(\Delta\omega/\omega < 10^{-8})$ have been routinely observed [3]. Then, using Eq. (17) we obtain $\omega'_z/\omega_z \approx \sqrt{3} \gg 10^{-8}$, showing that the relative and c.m. axial motions can be well resolved through an axial measurement and therefore it will be also possible to manipulate the c.m. degrees of freedom independently of the relative ones by applying, for instance, driving fields as described in Ref. [5].

We are going to show how to get some information about the quantum states by measuring the axial frequency shift induced by an additional "magnetic bottle" field [3], given by

$$\Delta\vec{B} = B_2\left[\left(z^2 - \frac{x^2+y^2}{2}\right)\vec{e}_z - z\vec{\rho}\right]. \tag{21}$$

The extra energy ΔH associated with this field is

$$\Delta H = -\frac{e}{m_e c}[\vec{\wp}^{(1)}\cdot\Delta\vec{A}^{(1)} + \vec{\wp}^{(2)}\cdot\Delta\vec{A}^{(2)}] - \frac{ge}{2m_e c}B_2[\vec{S}^{(1)}\cdot\Delta\vec{B}^{(1)} + \vec{S}^{(2)}\cdot\Delta\vec{B}^{(2)}], \tag{22}$$

where $\Delta\vec{A}^{(i)}$ is the vector potential associated with the inhomogeneous field of Eq. (21):

$$\Delta\vec{A}^{(i)} = \frac{1}{2}B_2(z^{(i)2} - \vec{\rho}^{(i)2}/4)\vec{e}_z \times \vec{\rho}^{(i)}. \tag{23}$$

We evaluate the contribution of ΔH to the energy levels by computing the matrix elements

$$\Delta E(N,K,L;n,k,l;M) =\langle m;n,k,l;N,K,L;|\Delta H|N,K,L;n,k,l;M\rangle, \tag{24}$$

where n,k,l (N,K,L) are the quantum numbers of the cyclotron, axial, and magnetron oscillators of the relative (c.m.) motion, and $M=-1,0,1$ is the quantum number associated with S_z. The shifts of the axial eigenfrequencies induced by the presence of the "magnetic bottle" are related to these matrix elements. The c.m. axial frequency shift

$$\Delta\omega_Z=\Delta\Omega_Z W \tag{25}$$

can be obtained by taking the difference between $\Delta E(N,K+1,L;n,k,l;M)$ and $\Delta E(N,K,L;n,k,l;M)$. Instead, the relative axial frequency shift

$$\Delta\omega_z'=\Delta\Omega_z' W \tag{26}$$

is obtained by taking the difference between $\Delta E(N,K,L;n,k+1,l;M)$ and $\Delta E(N,K,L;n,k,l;M)$. The factor W appearing in Eqs. (25) and (26) is given by

$$W=\left[\frac{g}{4}M+N+n+1+\frac{\omega_-}{\omega_+}\left(L+\frac{1}{2}\right)+\frac{\omega_-'}{\omega_+'}\left(l+\frac{1}{2}\right)\right], \tag{27}$$

while

$$\Delta\Omega_z=\frac{\hbar B_2\omega_z}{2m\omega_-B(\omega_+-\omega_-)} \tag{28}$$

and

$$\Delta\Omega_z'=\frac{\hbar B_2\omega_z}{2m\omega_-'B(\omega_+'-\omega_-')}. \tag{29}$$

In Eq. (27), the ratios ω_-/ω_+ and ω_-'/ω_+' that multiply L and l, respectively, are very small, and thus they can be ignored. Essentially, the relevant measurable quantity is

$$W\approx\left[\frac{g}{4}M+N+n+1\right]. \tag{30}$$

Comparing Eqs. (25) and (26) we conclude that they contain the same information about the quantum number *M*. Thus a measurement of any of the two axial frequency shifts is equally good to determine the spin projection along the *z* axis.

The *g* factor of the electron equals 2 to within about 1 part in 10^3. This and the above-mentioned fine resolution of the axial measurements guarantee the possiblity of distinguishing the three values of *M* appearing in the Eq. (28) or in Eq. (29).

After measuring the axial spin projection *M*, the geonium atom will be left in the spin state $|1,1\rangle$ if $M=1$ or in the state $|1,-1\rangle$ if $M=-1$. However, if the result of the measurement is $M=0$, the state can be either the singlet $|0,0\rangle$ or the triplet $|1,0\rangle$. In order to determine the symmetry of the spin function we have to do further measurements.

Before explaining how to do these measurements, let us study the effect of a small oscillatory magnetic driving field

$$\vec{b}(t)=b[\cos(\omega t)\vec{e}_x+\sin(\omega t)\vec{e}_y] \tag{31}$$

lying in the *xy plane*, on the spin state of the electrons. The total spin motion of the heliumlike atom is governed by the Hamiltonian

$$H_S=-g\frac{e\hbar}{2mc}\frac{1}{2}\vec{\sigma}\cdot[\vec{B}+\vec{b}(t)], \tag{32}$$

where

$$\vec{S}=\frac{\hbar}{2}\vec{\sigma}=\frac{\hbar}{2}(\vec{\sigma}_1+\vec{\sigma}_2)$$

is the total spin of the heliumlike atom. It is straighforward to show that the result of switching on the driving field $\vec{b}(t)$ on the spin state is the following: (a) If the electrons are in the initial singlet state $|0,0\rangle$, they will stay in it for any further time *t*. (b) If the initial state of the electrons is one of the three triplet states $|1,-1\rangle,|1,0\rangle$, or $|1,1\rangle$, then it will evolve to a superposition of the three. The transition probabilities between these three states are a function of time and depend on the Rabi frequency $\Omega_s=|e|b/2mc$ as well as in the detuning $\Delta\omega=\omega-\omega_s$. For instance, if the initial state is $|1,0\rangle$, the transition probabilities P_{01}, $P_{0,-1}$ to the states $|1,1\rangle$ and $|1,-1\rangle$ are given by

$$P_{01}=P_{0,-1}=\frac{\Omega_s^2[2\Delta\omega^2+\Omega_s^2+\Omega_s^2\cos(t\sqrt{\Omega_s^2+\Delta\omega^2})]\sin[\frac{1}{2}t\sqrt{\Omega_s^2+\Delta\omega^2}]^2}{(\Omega_s^2+\Delta\omega^2)^2} \tag{33}$$

and the probability of keeping in the same state $|1{,}0\rangle$ is

$$P_{00}=\frac{[\Delta\omega^2+\Omega_s^2\cos(t\sqrt{\Omega_s^2+\Delta\omega^2})]^2}{(\Omega_s^2+\Delta\omega^2)^2}. \tag{34}$$

The maximum values for the transition probabilities are attained in resonance ($\omega=\omega_s$). When the interaction time is $t_i=\pi/2\Omega_s$ we get $P_{00}=0$ and $P_{01}=P_{0,-1}=\frac{1}{2}$. Let us now assume that the axial frequency shift measurement left the electrons with axial spin projection $M=0$. Then we apply the driving field during a time $t_i=\pi/2\Omega_s$. If the system was initially in the singlet state $|0{,}0\rangle$, it will keep in this state. If the system was initially in the triplet state $|1{,}0\rangle$, then its state after a time t_i will be a superposition of the state $|1{,}1\rangle$ with the state $|1,-1\rangle$. Finally, we make a second measurement of the axial frequency shift (using the ''magnetic bottle'' technique) and, if we obtain again $M=0$, we know that the state is singlet; otherwise, the spin state is a triplet.

The technique that we have been describing can also be used to prepare the heliumlike geonium atom in any of its three triplet states, as well as in any hoped for superposition of them. Any final spin state can be reached by swithcing on the driving field during a convenient time interval t after the first measurement of M.

V. CONCLUSIONS

In this work we have studied the dynamics of two electrons in a Penning trap which we have named a *heliumlike geonium atom*.

We have shown that their motion can be reduced to six harmonic oscillators, each of them characterized by its own frequency. We have seen that the symmetry of the spatial wave function depends on the sum of the three relative motion quantum numbers.

Furthermore, we have shown how to extract quantum information about the spin and the total cyclotron number of the heliumlike geonium atom by using the ''magnetic bottle'' field and a small oscillatory driving field. These measurement techniques can also be used to prepare the electrons in any spin state.

The possibility of manipulating the quantum state of electrons can be used to store and process quantum information. In fact the present work suggests the possibility to implement quantum logic gates with the heliumlike geonium atom, using an extension of the arguments sketched in Ref. [6].

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