Quantum teleportation of entangled coherent states

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We propose a simple scheme for the quantum teleportation of both bipartite and multipartite entangled coherent states with the success probability 1/2. The scheme is based only on linear optical devices such as beam splitters and phase shifters, and two-mode photon number measurements. The quantum channels described by the multipartite maximally entangled coherent states, are readily made by the beam splitters and phase shifters.

Quantum teleportation, first proposed by Bennett *et al.* [1], is a disembodied transport of quantum states between subsystems through a classical communication channel requiring a shared entangled state. Several experiments were implemented to demonstrate the teleportation [2]. Most of the studies were confined to the teleportation of single-body quantum states: quantum teleportation of two-level states [1], N-dimensional states [3], and continuous variables [4]. Recently Lee and Kim considered the teleportation of bipartite entangled states through noisy quantum channels [5]. Ikram *et al.* [6] and Shi *et al.* [7] proposed schemes for the quantum teleportation of a two-qubit entangled state. Teleportation of some pure entangled states of both discrete and continuous variables was considered by Gorbachev *et al.* [8]. A possibility to copy pure entangled states was studied by Koashi and Imoto [9].

In teleportation schemes we need certain types of maximally entangled states (MES's). We consider the following entangled coherent state (ECS) [10]:

$$|\alpha;\alpha\rangle_{12}^{\pm}=\frac{1}{\sqrt{2(1\pm e^{-4|\alpha|^2})}}(|\alpha\rangle_1|\alpha\rangle_2\pm|-\alpha\rangle_1|-\alpha\rangle_2),$$

$$(1)$$

where $|\alpha\rangle_i$ ($i=1$ and 2) is the coherent state of system i. It is interesting to see that the ECS $|\alpha;\alpha\rangle_{12}^{-}$ is a MES, irrespective of the parameter α [11]. The ECS $|\alpha;\alpha\rangle_{12}^{-}$ can be rewritten in the form

$$|\alpha;\alpha\rangle_{12}^{-}=\frac{1}{\sqrt{2}}(|\alpha\rangle_1^+|\alpha\rangle_2^-+|\alpha\rangle_1^-|\alpha\rangle_2^+)$$

$$(2)$$

in terms of the even and odd coherent states:

$$|\alpha\rangle_i^{\pm}=\frac{1}{\sqrt{2(1\pm e^{-2|\alpha|^2})}}(|\alpha\rangle_i\pm|-\alpha\rangle_i).$$

$$(3)$$

Equation (2) shows that the state $|\alpha;\alpha\rangle_{12}^{-}$ manifestly has one ebit of entanglement. In the limit $|\alpha|\rightarrow0$, the ECS reduces to the singlet-like state $|\Psi^+\rangle_{12}=(|0\rangle_1|1\rangle_2+|1\rangle_1|0\rangle_2)/\sqrt{2}$, where $|0\rangle_i$ and $|1\rangle_i$ are photon number states (Fock states).

van Enk and Hirota [11] discussed how to teleport a Schrödinger cat state of the form

$$|\alpha\rangle_{\text{cat}}=\mathcal{N}(\epsilon_+|\alpha\rangle+\epsilon_-|-\alpha\rangle),$$

$$\mathcal{N}=[|\epsilon_+|^2+|\epsilon_-|^2+2e^{-2|\alpha|^2}\text{Re}(\epsilon_+\epsilon_-^*)]^{-1/2},$$

$$(4)$$

through a quantum channel described by the MES $|\alpha;\alpha\rangle_{12}^{-}$, where $\epsilon_\pm$ are complex numbers. Inspired by their teleportation scheme, we consider the teleportation of the following ECS:

$$|\Phi\rangle_{12}=\mathcal{N}_\Phi(\epsilon_+|\alpha\rangle_1|\alpha\rangle_2+\epsilon_-|-\alpha\rangle_1|-\alpha\rangle_2),$$

$$\mathcal{N}_\Phi=[|\epsilon_+|^2+|\epsilon_-|^2+2e^{-4|\alpha|^2}\text{Re}(\epsilon_+\epsilon_-^*)]^{-1/2}.$$

$$(5)$$

In the teleportation of entangled states, particularly two-qubit pure states, one can use two EPR pairs: a four-qubit quantum channel, or a less expensive three-qubit GHZ state [8]. If we want to teleport the ECS $|\Phi\rangle_{12}$, we need at least a tripartite entangled state as the quantum channel. In a recent paper [12], we considered the following tripartite entangled states:

$$|\sqrt{2}\alpha;\alpha;\alpha\rangle_{345}^{\pm}=\frac{1}{\sqrt{2(1\pm e^{-8|\alpha|^2})}}(|\sqrt{2}\alpha\rangle_3|\alpha\rangle_4|\alpha\rangle_5$$

$$\pm|-\sqrt{2}\alpha\rangle_3|-\alpha\rangle_4|-\alpha\rangle_5).$$

$$(6)$$

The bipartite entanglement of the tripartite states can be characterized by one measure of entanglement, the concurrence [13]. The concurrence of the state $|\sqrt{2}\alpha;\alpha;\alpha\rangle_{345}^{\pm}$ between system i and systems j,k ($i\neq j\neq k\in\{3,4,5\}$) is denoted by $C_{i(jk)}^{\pm}$. The concurrences are obtained as [12]

$$C_{3(45)}^+=\tanh(4|\alpha|^2), \quad C_{3(45)}^-=1,$$

$$C_{4(35)}^{\pm}=C_{5(34)}^{\pm}=\frac{\sqrt{(1-e^{-4|\alpha|^2})(1-e^{-12|\alpha|^2})}}{1\pm e^{-8|\alpha|^2}}.$$

$$(7)$$

We see that system 3 with systems 4 and 5 is always maximally entangled in the state $|\sqrt{2}\alpha;\alpha;\alpha\rangle_{345}^{-}$. This tripartite state may be considered as a tripartite extension of the bipartite MES $|\alpha;\alpha\rangle_{12}^{-}$, and will act as a quantum channel in the following discussions. Now having the state $|\Phi\rangle_{12}$ to be teleported and the MES $|\sqrt{2}\alpha;\alpha;\alpha\rangle_{345}^{-}$ as a quantum channel, we begin to discuss our teleportation scheme.

We first briefly review the action of a beam splitter on coherent states. The lossless symmetric 50/50 beam splitter is described by $B_{12}=e^{i(\pi/4)(a_1^\dagger a_2+a_2^\dagger a_1)}$, which transforms the coherent states $|\alpha\rangle_1|\beta\rangle_2$ as

$$B_{12}|\alpha\rangle_1|\beta\rangle_2=|(\alpha+i\beta)/\sqrt{2}\rangle_1|(\beta+i\alpha)/\sqrt{2}\rangle_2. \quad (8)$$

Here a_i and $a_i^\dagger$ are the bosonic annihilation and creation operators of system i, respectively. By equipping the beam splitter by a pair of $-\pi/2$ phase shifters described by the unitary operator $P_2=e^{-i\pi a_2^\dagger a_2/2}$, we can have the operator $\mathcal{B}_{12}=P_2B_{12}P_2$ which transforms the state $|\alpha\rangle_1|\beta\rangle_2$ as

$$\mathcal{B}_{12}|\alpha\rangle_1|\beta\rangle_2=|(\alpha+\beta)/\sqrt{2}\rangle_1|(\alpha-\beta)/\sqrt{2}\rangle_2. \quad (9)$$

This transformation plays a key role in our teleportation scheme.

Now Alice wishes to teleport the ECS $|\Phi\rangle_{12}$ to a remote partner Bob by sharing the MES $|\sqrt{2}\alpha;\alpha;\alpha\rangle_{345}^-$. Systems 1, 2 and 3 are at Alice's side, and systems 4 and 5 are at Bob's side. The initial state of the whole system is then given by

$$|\Psi\rangle_{12345}=|\Phi\rangle_{12}|\sqrt{2}\alpha;\alpha;\alpha\rangle_{345}^-. \quad (10)$$

We first apply the transformation $\mathcal{B}_{21}=P_1B_{21}P_1$ to the initial state. From Eq. (9), the state after the transformation becomes a direct product of the vacuum state $|0\rangle_1$ with the unnormalized state

$$
\begin{aligned}
|\Psi'\rangle_{2345}=&\epsilon_+(|\sqrt{2}\alpha\rangle_2|\sqrt{2}\alpha\rangle_3|\alpha\rangle_4\otimes|\alpha\rangle_5-|\sqrt{2}\alpha\rangle_2| \\
&-\sqrt{2}\alpha\rangle_3|-\alpha\rangle_4\otimes|-\alpha\rangle_5) \\
&+\epsilon_-(|-\sqrt{2}\alpha\rangle_2|\sqrt{2}\alpha\rangle_3|\alpha\rangle_4\otimes|\alpha\rangle_5 \\
&-|-\sqrt{2}\alpha\rangle_2|-\sqrt{2}\alpha\rangle_3|-\alpha\rangle_4\otimes|-\alpha\rangle_5). \quad (11)
\end{aligned}
$$

Now system 1 is separated from the remain systems. Then, by applying the second transformation $\mathcal{B}_{23}$, we obtain

$$
\begin{aligned}
|\Psi''\rangle_{2345}=\mathcal{B}_{23}|\Psi'\rangle_{2345}=&\epsilon_+(|2\alpha\rangle_2|0\rangle_3|\alpha\rangle_4\otimes|\alpha\rangle_5 \\
&-|0\rangle_2|2\alpha\rangle_3|-\alpha\rangle_4\otimes|-\alpha\rangle_5)-\epsilon_-(|-2\alpha\rangle_2|0\rangle_3 \\
&\times|-\alpha\rangle_4\otimes|-\alpha\rangle_5-|0\rangle_2|-2\alpha\rangle_3|\alpha\rangle_4\otimes|\alpha\rangle_5). \\
&\quad (12)
\end{aligned}
$$

After these two transformations, Alice performs a two-mode number measurement on modes 2 and 3. The probability of finding n and m photons in modes 2 and 3 is given by

$$P(n,m)=|_2\langle n|_3\langle m|\Psi''\rangle_{2345}|^2. \quad (13)$$

The probability is zero if both n and m are nonzero, i.e., one of the two integers must be zero in order to have nonzero probability.

Let us suppose $n\neq0$ and $m=0$. In this case the state on Bob's side collapses into

$$|\Phi'\rangle_{45}=\epsilon_+|\alpha\rangle_4|\alpha\rangle_5-\epsilon_-(-1)^n|-\alpha\rangle_4|-\alpha\rangle_5, \quad (14)$$

For the cases $n=0$ and $m\neq0$, the state on Bob's side collapses into

$$|\Phi''\rangle_{45}=\epsilon_+|-\alpha\rangle_4|-\alpha\rangle_5-\epsilon_-(-1)^m|\alpha\rangle_4|\alpha\rangle_5. \quad (15)$$

Now Alice sends a classical information to Bob, and Bob makes a local transformation $(-1)^{a_4^\dagger a_4+a_5^\dagger a_5}$ on his state $|\Phi''\rangle_{45}$. The local transformation is a multiplication of two π phase shifters of modes 4 and 5, and the resultant state after the transformation is just the state $|\Phi'\rangle_{45}$.

We see that provided n is odd, the teleportation scheme works perfectly. However, for even n, the transformation for perfect teleportation is

$$|\alpha\rangle_4|\alpha\rangle_5\rightarrow|\alpha\rangle_4|\alpha\rangle_5, \quad (16a)$$

$$|-\alpha\rangle_4|-\alpha\rangle_5\rightarrow-|-\alpha\rangle_4|-\alpha\rangle_5, \quad (16b)$$

which is in general not a unitary transformation except the limit case $|\alpha|\rightarrow\infty$. From Eqs. (12) and (13), the probabilities $P(0,n)$ and $P(n,0)$ for odd n are obtained as

$$P(0,n)=P(n,0)=\frac{e^{-4|\alpha|^2}|2\alpha|^{2n}}{2n!(1-e^{-8|\alpha|^2})}. \quad (17)$$

Then the probability of success is given by

$$P_{odd}=2\sum_{\text{odd }n}P(n,0)=\frac{1}{2}. \quad (18)$$

As seen from Eq. (18), the success probability is independent on both α and $\epsilon_\pm$. One restriction to the ECS to be teleportated is that the mean photon number of the coherent state $|\alpha\rangle(|\alpha|^2)$ in system 1 (2) is the same as that in system 4 (5). In other words the state to be teleportated is strongly correlated in photon number to the quantum channel. Actually the scheme is used to teleport a qubit encoded in the ECS $|\Phi\rangle_{12}$. The teleportation scheme is not optimal; however, it indeed gives the nonzero probability 1/2 independent of α.

There is another problem left that if we can produce the tripartite maximally entangled coherent state which plays the role of the quantum channel. If we cannot, the scheme does not work. Fortunately we can create this MES in an easy way. We first prepare systems 3, 4, and 5 in the state $|2\alpha\rangle_3^-|0\rangle_4|0\rangle_5$. Then by applying the transformation $\mathcal{B}_{45}\mathcal{B}_{34}$, we obtain

$$
\begin{aligned}
\mathcal{B}_{45}\mathcal{B}_{34}|2\alpha\rangle_3^-|0\rangle_4|0\rangle_5=&\mathcal{B}_{45}(|\sqrt{2}\alpha\rangle_3|\sqrt{2}\alpha\rangle_4\otimes|0\rangle_5 \\
&-|-\sqrt{2}\alpha\rangle_3|-\sqrt{2}\alpha\rangle_4 \\
&\otimes|0\rangle_5)/\sqrt{2(1-e^{-8|\alpha|^2})} \\
=&|\sqrt{2}\alpha;\alpha;\alpha\rangle_{345}^-, \quad (19)
\end{aligned}
$$

which is just the tripartite MES in the teleportation scheme. In a short summary we can let the initial state of the whole composite system be $|\Phi\rangle_{12}|2\alpha\rangle_3^-|0\rangle_4|0\rangle_5$. We then apply a

transformation $\mathcal{B}_{23}\mathcal{B}_{21}\mathcal{B}_{45}\mathcal{B}_{34}$ to the initial state, and make the two-mode number measurement to implement the teleportation.

We would like to investigate further what the success probability is if we use a nonmaximally entangled state as the quantum channel in the teleportation scheme. We choose the state as $|\sqrt{2}\alpha;\alpha;\alpha\rangle^{+}_{345}$ (6). From Eq. (7), we see that the state $|\sqrt{2}\alpha;\alpha;\alpha\rangle^{+}_{345}$ is not maximally entangled except the limit case $|\alpha|\to\infty$. The entangled state $|\sqrt{2}\alpha;\alpha;\alpha\rangle^{+}_{345}$ can be generated similarly as the state $|\sqrt{2}\alpha;\alpha;\alpha\rangle^{-}_{345}$.

Following the same steps as before, after Alice measures n photons in mode 2 and zero photons in mode 3, Bob's state collapses into the state

$$|\tilde{\Phi}'\rangle_{45}=\epsilon_{+}|\alpha\rangle_{4}|\alpha\rangle_{5}+\epsilon_{-}(-1)^{n}|-\alpha\rangle_{4}|-\alpha\rangle_{5}. \quad (20)$$

The perfect teleportation is obtained for even n. The success probability is given by

$$P_{even}=\sum_{\text{even } n>0}[P(0,n)+P(n,0)]=\frac{(1-e^{-4|\alpha|^{2}})^{2}}{2(1+e^{-8|\alpha|^{2}})}, \quad (21)$$

which is independent on the parameters $\epsilon_{\pm}$. However, it depends on the parameter $|\alpha|$. In the limit $|\alpha|\to\infty$, the success probability becomes 1/2. In this limit case, the amount of entanglement in the state $|\sqrt{2}\alpha;\alpha;\alpha\rangle^{+}_{345}$ is one ebit and the probability can be 1/2. Except this case, the success probability is always less than 1/2 as the corresponding quantum channel is not a MES.

Our teleportation scheme can be generalized to the multipartite cases. For the sake of simplicity, we study only the tripartite case. We consider a tripartite ECS:

$$|\Phi\rangle_{123}=\epsilon_{+}|\sqrt{2}\alpha\rangle_{1}|\alpha\rangle_{2}|\alpha\rangle_{3}+\epsilon_{-}|-\sqrt{2}\alpha\rangle_{1}|-\alpha\rangle_{2}|-\alpha\rangle_{3}. \quad (22)$$

To teleport $|\Phi\rangle_{123}$, we may need the four-particle entangled state

$$|2\alpha;\sqrt{2}\alpha;\alpha;2\alpha\rangle^{-}_{4567}=|2\alpha\rangle_{4}|\sqrt{2}\alpha\rangle_{5}|\alpha\rangle_{6}|\alpha\rangle_{7}-|-2\alpha\rangle_{4}|-\sqrt{2}\alpha\rangle_{5}|-\alpha\rangle_{6}|-\alpha\rangle_{7}, \quad (23)$$

which is maximally entangled between the system 4 and systems 5, 6, and 7 [12]. First we disentangle system 3 from systems 1 and 2 by applying the operator $\mathcal{B}_{31}\mathcal{B}_{32}$ to the state $|\Phi\rangle_{123}$. We obtain

$$\mathcal{B}_{31}\mathcal{B}_{32}|\Phi\rangle_{123}=|0\rangle_{1}|0\rangle_{2}(\epsilon_{+}|2\alpha\rangle_{3}+\epsilon_{-}|-2\alpha\rangle_{3}). \quad (24)$$

Then we make the transformation $\mathcal{B}_{34}$, the two-mode number measurement on modes 3 and 4, and a classical communication from Alice to Bob to finish the teleportation process. The success probability is also 1/2. The quantum channel described by the four-particle entangled state can be obtained as

$$|2\alpha;\sqrt{2}\alpha;\alpha;2\alpha\rangle^{-}_{4567}=\mathcal{B}_{67}\mathcal{B}_{56}\mathcal{B}_{45}(|2\sqrt{2}\alpha\rangle_{4}$$
$$-|-2\sqrt{2}\alpha\rangle_{4})|0\rangle_{5}|0\rangle_{6}|0\rangle_{7}. \quad (25)$$

It is straightforward to generalize the teleportation scheme to teleport the multipartite (more than three) entangled ECS of the form $|\Phi\rangle_{123}$.

In the teleportation scheme described above the probability of success is 1/2, and independent of α. The it is interesting to consider the limit $|\alpha|\to 0$. The state $|\sqrt{2}\alpha;\alpha;\alpha\rangle^{-}_{345}$ can be rewritten as

$$|\sqrt{2}\alpha;\alpha;\alpha\rangle^{-}_{345}=\frac{1}{\sqrt{2}}(|\sqrt{2}\alpha\rangle^{-}_{3}|\alpha;\alpha\rangle^{+}_{45}+|\sqrt{2}\alpha\rangle^{+}_{3}|\alpha;\alpha\rangle^{-}_{45}), \quad (26)$$

which directly leads to

$$|\psi\rangle_{345}=\lim_{|\alpha|\to 0}|\sqrt{2}\alpha;\alpha;\alpha\rangle^{-}_{345}=\frac{1}{\sqrt{2}}(|1\rangle_{3}|00\rangle_{45}+|0\rangle_{3}|\Psi^{+}\rangle_{45}). \quad (27)$$

Here the state $|00\rangle_{45}\equiv|0\rangle_{4}|0\rangle_{5}$. We use the state $|\psi\rangle_{345}$ to teleport the state

$$|\phi\rangle_{12}=\frac{1}{\sqrt{|a|^{2}+|b|^{2}}}(a|00\rangle_{12}+b|\Psi^{+}\rangle_{12}). \quad (28)$$

Then the initial state of the whole system is given by

$$|\psi'\rangle_{12345}=|\phi\rangle_{12}|\psi\rangle_{345}. \quad (29)$$

It is straightforward to check that

$$\mathcal{B}_{12}|10\rangle_{12}=|\Psi^{+}\rangle_{12}, \quad \mathcal{B}_{12}|01\rangle_{12}=|\Psi^{-}\rangle_{12}, \quad (30a)$$

$$\mathcal{B}_{12}|\Psi^{+}\rangle_{12}=|10\rangle_{12}, \quad \mathcal{B}_{12}|\Psi^{-}\rangle_{12}=|01\rangle_{12}, \quad (30b)$$

where $|\Psi^{\pm}\rangle_{12}=(|10\rangle\pm|01\rangle)/\sqrt{2}$. We have used the identity $\mathcal{B}^{2}_{12}=1$. Then by applying the operator $\mathcal{B}_{13}\mathcal{B}_{12}$ to the initial state, we obtain

$$\mathcal{B}_{13}\mathcal{B}_{12}|\psi'\rangle_{12345}=\frac{1}{2\sqrt{|a|^{2}+|b|^{2}}}|0\rangle_{2}[|10\rangle_{13}(a|00\rangle_{45}$$
$$+b|\Psi^{+}\rangle_{45})+|01\rangle_{13}(-a|00\rangle_{45}$$
$$+b|\Psi^{+}\rangle_{45})+a|00\rangle_{13}|\Psi^{+}\rangle_{45}$$
$$+\frac{b}{\sqrt{2}}(|20\rangle_{13}-|02\rangle_{13})|00\rangle_{45}]. \quad (31)$$

Here we have used Eqs. (30a) and (30b). We see that system 2 decouples from the other systems. The teleportation scheme works perfectly if the resultant state of the two-mode number measurement is $|10\rangle_{13}$. If the resultant state is $|01\rangle_{13}$, then Alice needs to communicate classically with Bob, and Bob makes a local transformation $(-1)^{a^{\dagger}_{4}a_{4}+a^{\dagger}_{5}a_{5}}$

to finish the teleportation. Again the probability of success is 1/2. A setup similar to this teleportation scheme was proposed to perform optical state truncation [14], optical simulation of quantum logic [15], and quantum teleportation [16].

The measurement we have used in our teleportion scheme is a two-mode number measurement which must be sensitive enough to measure the number of photons and determine whether the number is even or odd. In practice this is difficult, especially for large numbers of photons, but in principle this can be done. Here we propose one method to distinguish even and odd n in the Fock state $|n\rangle$ by coupling the field to a two-level atom through dispersive interaction [17]. The Hamiltonian is given by $H = g a^{\dagger} a \sigma_x$, where g is the coupling constant and $\sigma_{\beta}(\beta = x, y, z)$ are the Pauli operators describing the peudospin of the two-level atom. This Hamiltonian can be realized in both cavity-QED and trapped-ion systems [17]. At time $t = \pi/(2g)$, the evolution operator $\exp(-iHt)$ becomes $U = \exp(-i\pi a^{\dagger} a \sigma_x/2)$. Initially let the atom be in the ground state and the field be in the Fock state $|n\rangle$. After applying the unitary operator U, we know that the photon number is even (odd) if the atom is found to be in the ground (excited) state. By this technique, we can distinguish even and odd photon numbers.

In conclusion we have proposed a simple scheme to teleport both the bipartite and multipartite ECS with the success probability 1/2, independent of α. As a crucial ingredient in the scheme, the quantum channel described by the multipartite ECS can be readily made only from linear optical devices such as beam splitters and phase shifters. Thus we provide a way to achieve all linear optical teleportation of quantum states [15,16]. The probability of success in our scheme is 1/2 due to the use of only linear operations and the absence of photon-photon interaction [18]. Both the measurement and preparation of the quantum channel can be implemented in the experiments by the present techniques. We expect that the present scheme can be used in the experiments to demonstrate the quantum teleportation of the entangled states.

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