Practical scheme for entanglement concentration

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We present a realistic purification scheme for pure nonmaximally entangled states. In the scheme, two distant parties Alice and Bob first start with two shared but less entangled photon pairs to produce a conditional four-photon Greenberger-Horne-Zeilinger state, and then perform a 45° polarization measurement onto one of the two photons at each location such that the remaining two photons are projected onto a maximally entangled state.

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Quantum entanglement has become an important resource for quantum computation [1,2], teleportation [3], dense coding [4], and cryptography [5]. In the past few years, a large number of experiments have shown that quantum computation and quantum communication are more efficient in many aspects than their classical counterparts. In these experiments, maximally entangled states are usually required. However, since there is decoherence during storage or transmission of particles over noisy channels, the quality of entanglement is easily degraded. There are two methods to overcome the effect of decoherence. One is the so-called quantum error-correction scheme [6], which makes quantum computation possible despite the effects of decoherence and imperfect apparatus. The alternative method is entanglement purification. From the quantum communication perspective, entanglement purification is more powerful than quantum error correction. In order to achieve quantum communication with high fidelity, entanglement purification is necessary to obtain maximally entangled states.

The basic idea of entanglement purification is to distill some pairs of particles in highly entangled states from less entangled states using local operations and classical communication. There have been several protocols [7–13] for purification of pure and mixed nonmaximally entangled states. In the Schmidt decomposition scheme [7], physical realization of local operations was achieved by collective measurements. But, in practice, it is very difficult to measure so many photons simultaneously. Another similar scheme called the Procrustean method [7], on the other hand, requires the states to be known in advance. Entanglement purification schemes [8–12] involving quantum logic gates are even more difficult to implement for mixed states. The difficulties associated with different schemes prevent experimental realization of purification. Recently, Bose *et al.* [13] suggested that one could investigate the purification of entangled states via entanglement swapping. By using Bell state measurements as local operations and the measurement results as classical communication, such a purification procedure could be easily realized by simple extension of an existing entanglement swapping experiment [14]. However, there one needs to know the coefficients in advance in order to reconstruct the same entangled states each time at Alice's or Bob's location.

In this paper, we present another protocol for entanglement concentration based on the principle of quantum erasure [15] and the Schmidt projection method. In our scheme, one can concentrate entanglement from arbitrary identical nonmaximally entangled pairs at distant locations. For distant nonmaximally entangled states we first erase the ''which-way'' information between the two nonmaximally entangled states by the process of quantum erasure such that we can produce a conditional four-particle maximally entangled state. Then, after performing simple Schmidt projection measurements [16] onto one of the two photons at each location, the remaining two photons are projected onto a maximally entangled state. Hence, we provide a realistic scheme for the original Schmidt decomposition idea [7]. On the other hand, we shall show that our scheme can also be used to concentrate entanglement from nonmaximally entangled multiphoton states, for example, to distill a Greenberger-Horne-Zeilinger (GHZ) state [17].

Figure 1 is a schematic drawing of our purification scheme. Consider two pairs of photons (1,2) and (3,4) in the

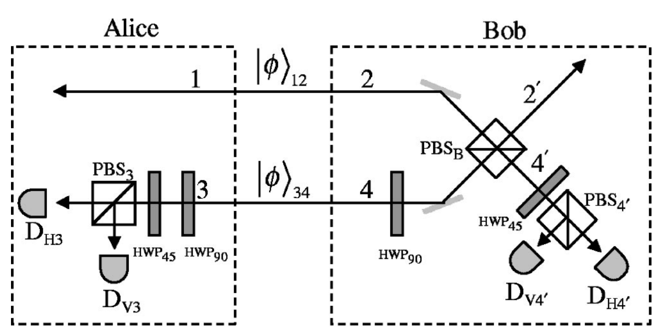

FIG. 1. A schematic drawing of our scheme for entanglement concentration. PBS_B, PBS_3, and $PBS_{4'}$ are three polarization beam splitters, which transmit the horizontal polarization component and reflect the vertical component. The half-wave plates HWP_{45} and HWP_{90} rotate the horizontal and vertical polarization by 45° and 90°, respectively; D_{H3}, D_{V3}, $D_{H4'}$, and $D_{V4'}$ are four single-photon detectors.

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following unknown polarization entangled states:

$$|\phi\rangle_{12}=\alpha|H_1\rangle|H_2\rangle+\beta|V_1\rangle|V_2\rangle, \tag{1}$$

$$|\phi\rangle_{34}=\alpha|H_3\rangle|H_4\rangle+\beta|V_3\rangle|V_4\rangle, \tag{2}$$

where $|\alpha|^2+|\beta|^2=1$, Alice holds photons 1 and 3, and Bob holds photons 2 and 4. Our nonmaximally entangled state is the same as the one described in Ref. [7].

Before proceeding to purify these states, the polarizations of photons 3 and 4 are rotated by 90° using two half-wave plates (HWP_{90} in Fig. 1). After passing through the two half-wave plates, the state of photons 3 and 4 becomes

$$|\phi\rangle'_{34}=\alpha|V_3\rangle|V_4\rangle+\beta|H_3\rangle|H_4\rangle. \tag{3}$$

Then we further forward photons 2 and 4 to a polarizing beam splitter PBS_B (see Fig. 1). Suppose that photons 2 and 4 arrive at PBS_B simultaneously so that they interfere at the PBS_B. Since the PBS transmits only the horizontal polarization component and reflects the vertical component, after photons 2 and 4 pass through the PBS_B the total state of photons 1, 2, 3, and 4 evolves into

$$\begin{aligned}|\Psi\rangle=&\alpha\beta|H_1\rangle|H_{2'}\rangle|H_3\rangle|H_{4'}\rangle+\alpha\beta|V_1\rangle|V_{2'}\rangle|V_3\rangle|V_{4'}\rangle\\&+\alpha^2|H_1\rangle|V_3\rangle|H_{4'}\rangle|V_{4'}\rangle+\beta^2|V_1\rangle|H_{2'}\rangle|V_{2'}\rangle|H_3\rangle.\end{aligned} \tag{4}$$

From the above equation, it is evident that Alice and Bob could observe a fourfold coincidence among modes 1, 2′, 3, and 4′ only for the terms $|H_1\rangle|H_{2'}\rangle|H_3\rangle|H_{4'}\rangle$ and $|V_1\rangle|V_{2'}\rangle|V_3\rangle|V_{4'}\rangle$. For the other two terms, there are always two particles in one of the two output modes of the PBS_B and no particle in the other mode. Therefore, by selecting only those events where there is exactly one photon at the output mode 4′, Alice and Bob can project the above state into a maximally entangled four-particle state

$$|\Psi\rangle_c=\frac{1}{\sqrt{2}}[|H_1\rangle|H_{2'}\rangle|H_3\rangle|H_{4'}\rangle+|V_1\rangle|V_{2'}\rangle|V_3\rangle|V_{4'}\rangle], \tag{5}$$

with a probability of $2|\alpha\beta|^2$.

Note that in the above description we have used the principle of quantum erasure so that after PBS_B some of the photons registered can no longer be identified as to which source they came from. The PBS_B plays the double role of both overlapping the two photons and erasing the ‘‘which-way’’ information. This principle, first proposed by Scully and Drühl [15] and realized by many other authors [18–22], has been used in several important experiments such as quantum teleportation [23], entanglement swapping [14], three-particle GHZ entanglement [24], and tests of the nonlocality of GHZ states [25].

To generate a maximally entangled two-photon state between Alice and Bob, they could further perform a 45° polarization measurement onto the photons 3 and 4′. As described in Fig. 1, Alice and Bob first rotate the polarizations of the photons 3 and 4′ by 45° with another two half-wave plate (HWP_{45} in Fig. 1). The unitary transformation of the photons 3 and 4′ through the half-wave plate is given by

$$|H_3\rangle\rightarrow\frac{1}{\sqrt{2}}(|H_3\rangle+|V_3\rangle), \tag{6}$$

$$|V_3\rangle\rightarrow\frac{1}{\sqrt{2}}(|H_3\rangle-|V_3\rangle), \tag{7}$$

$$|H_{4'}\rangle\rightarrow\frac{1}{\sqrt{2}}(|H_{4'}\rangle+|V_{4'}\rangle), \tag{8}$$

$$|V_{4'}\rangle\rightarrow\frac{1}{\sqrt{2}}(|H_{4'}\rangle-|V_{4'}\rangle). \tag{9}$$

After this operation, the state (5) will evolve into a coherent superposition of the following four combinations:

$$\frac{1}{2\sqrt{2}}|H_3\rangle|H_{4'}\rangle(|H_1\rangle|H_{2'}\rangle+|V_1\rangle|V_{2'}\rangle)+ \tag{10}$$

$$\frac{1}{2\sqrt{2}}|V_3\rangle|V_{4'}\rangle(|H_1\rangle|H_{2'}\rangle+|V_1\rangle|V_{2'}\rangle)+ \tag{11}$$

$$\frac{1}{2\sqrt{2}}|H_3\rangle|V_{4'}\rangle(|H_1\rangle|H_{2'}\rangle-|V_1\rangle|V_{2'}\rangle)+ \tag{12}$$

$$\frac{1}{2\sqrt{2}}|V_3\rangle|H_{4'}\rangle(|H_1\rangle|H_{2'}\rangle-|V_1\rangle|V_{2'}\rangle). \tag{13}$$

Now, Alice and Bob let the photons 3 and 4′ pass through the polarization beam splitters PBS_3 and $\mathrm{PBS}_{4'}$, respectively, and observe the coincidence between either detectors D_{H3} and $D_{H4'}$, or D_{V3} and $D_{V4'}$, or D_{H3} and $D_{V4'}$, or D_{V3} and $D_{H4'}$. Clearly, Alice and Bob will observe four possible coincidences, i.e., $|H_3\rangle|H_{4'}\rangle$, $|V_3\rangle|V_{4'}\rangle$, $|H_3\rangle|V_{4'}\rangle$, and $|V_3\rangle|H_{4'}\rangle$. Following Eq. (10), if both photons 3 and 4′ are observed to be in the same polarization state (either $|H_3\rangle|H_{4'}\rangle$ or $|V_3\rangle|V_{4'}\rangle$), then the remaining two photons 1 and 2′ are left in the state

$$|\phi^+\rangle_{12'}=\frac{1}{\sqrt{2}}(|H_1\rangle|H_{2'}\rangle+|V_1\rangle|V_{2'}\rangle). \tag{14}$$

Similarly, if photons 3 and 4′ are observed to be in different polarization states (either $|H_3\rangle|V_{4'}\rangle$ or $|V_3\rangle|H_{4'}\rangle$), then the remaining two photons 1 and 2′ are left in the state

$$|\phi^-\rangle_{12'}=\frac{1}{\sqrt{2}}(|H_1\rangle|H_{2'}\rangle-|V_1\rangle|V_{2'}\rangle). \tag{15}$$

In order to generate the same state $|\phi^+\rangle_{12'}$ at each successful run, either Alice or Bob could perform an additional local operation, i.e., a 180° phase shift (not shown in Fig. 1), to transform the state $|\phi^-\rangle_{12'}$ into $|\phi^+\rangle_{12'}$, on condition that the photons 3 and $4'$ are observed to be in different polarization states. After performing the polarization measurements and the conditional local operation, Alice and Bob can thus generate the maximally entangled state $|\phi^+\rangle_{12'}$ with a probability of $2|\alpha\beta|^2$, which is equal to the probability of obtaining the state $|\Psi\rangle_c$.

Here it is worthwhile to note that detection of fourfold coincidence is not necessary. In practice, with the help of a single-photon detector [26] it is sufficient to measure the photon number and polarizations in the 45° basis at the output modes 3 and $4'$. On condition that exactly one photon is detected at each of the two output modes 3 and $4'$, the remaining two photons 1 and $2'$ can be prepared in the state $|\phi^+\rangle_{12'}$ for further application.

Furthermore, one can easily verify that our scheme can also be used to concentrate entanglement from nonmaximally entangled multiparticle states. Let us, for example, consider Alice, Bob, and Cliff to share two pairs of nonmaximally entangled three-photon states $\alpha|H\rangle|H\rangle|H\rangle + \beta|V\rangle|V\rangle|V\rangle$ at three distant locations. Through a similar process of quantum erasure and a 45° polarization measurement one of the two photons at each of Alice, Bob, and Cliff will first share a conditional six-particle maximally entangled state, and then obtain a maximally entangled state, i.e., GHZ entanglement among three distant parties. The probability of obtaining the GHZ state is again $2|\alpha\beta|^2$.

Just as in many other schemes [7,13], while entanglement of some particles is concentrated by sacrificing the entanglement of other particles, our procedure is restricted to purification of two identical nonmaximally entangled states. Also, it should be noted that our scheme is not optimal since the whole amount of entanglement is decreased by a factor of 2 after finishing our purification procedure. However, our scheme is not involved in collective measurement like the Schmidt decomposition scheme [7] and does not require the states to be known in advance like the Procrustean method [7]. With the techniques developed in experiments on quantum teleportation [14,23] and multiphoton entanglement [24,25], our scheme is within the reach of current technology and thus is a feasible one for the original Schmidt decomposition scheme [7].

In summary, we present a practical scheme for purification of pure nonmaximally entangled states based on the principle of quantum erasure and the Schmidt projection measurement. Using this scheme we can obtain maximally entangled pairs, i.e., Bell states, by a simple 45° polarization measurement. Our scheme might be useful in future long-distance quantum communication.

Recently, the authors became aware of related work by Yamamoto *et al.*, who arrive at the same proposal [27]. Also, a more powerful purification scheme working for general mixed entangled states has been proposed recently by Pan *et al.* [28].

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