Quantum interference between decay channels of a three-level atom in a multilayer dielectric medium

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Spontaneous emission of an atom with two close upper levels and one lower level in a multilayer dielectric medium, and quantum interference between transitions from the upper levels to the lower one, are considered. Since the medium is inhomogeneous along the direction normal to the interfaces of the dielectric layers, the vacuum has an anisotropic spatial structure, and quantum intereference between the two decay channels can appear even if the two dipole matrix elements are orthogonal to each other. If the atom is placed in a three-layer dielectric plate cavity, a strong quantum interference similar to that for an atom with two nearly parallel dipole moments in free space arises due to the influence of anisotropic vacuum. Putting the atom in a three-layer dielectric waveguide and a one-dimensional photonic crystal, there appears a strong quantum interference similar to that for an atom with nearly antiparallel dipole moments.

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I. INTRODUCTION

Since many interesting phenomena are relevant to spontaneous emission of an excited atomic system, for a long time it has been desirable to find an approach to control spontaneous emission. According to Fermi's golden rule [1], the rate of spontaneous emission of an atom with one excited level and one ground level is proportional to the squared atomic dipole matrix element between the two levels and the density of radiation modes at the atomic transition frequency. Therefore, there may be two ways to modify spontaneous emission. By placing an excited atom in a confined environment such as a photonic crystal and a resonance cavity [2–6], one can change or tailor the density of electromagnetic modes in the neighborhood of the transition frequency to control spontaneous emission. On the other hand, one may change the atomic dipole matrix element by adopting multiple-close-upper-level configurations of an atom. Zhu and Scully [7] suggested the cancellation of spontaneous emission of a multi-excited-level atom via quantum interference between different decay channels. This spontaneously generated quantum interference can lead to the appearance of ultranarrow spectral lines [9–13], gain without inversion [14], atomic population trapping in excited levels [8], and phase-dependent line shapes [11,12,15,16]. However, it should be noted that the existence of this quantum interference requires that the atomic dipole moments be nonorthogonal when the atom is placed in free space. Unfortunately, it is difficult to find such an atomic system which satisfies this stringent condition. In a very recent paper [17], Agarwal studied the spontaneous emission of a V-type three-level atom in an anisotropic vacuum, and found that quantum interference between two decay channels can appear even if the dipole matrix elements are orthogonal. In this way, instead of trying to find atomic systems with nonorthogonal dipole moments, we may now find the existence of quantum interference in a variety of new classes of systems.

In this paper, we consider a three-level atom with two close upper states and a single lower level embedded in a multilayered dielectric medium. Since there may exist many interfaces between the layers, the medium is not uniform along the direction normal to the interfaces. We investigate how the spatial anisotropy of the medium induces quantum interference between two transitions from the upper states to the lower level. This paper is organized as follows. In Sec. II the model is presented, and the rate of spontaneous emission and the interference coefficients between the two decay pathways are written as scalars formed from the Green function of the field in the medium and the electric dipole matrix elements. In Sec. III, the influence of the thickness of the layer on which the atom is placed on the quantum interference effect is investigated. The conclusions are summarized in Sec. IV.

II. MODEL AND FORMULA

Consider a V-configuration three-level atom, illustrated in Fig. 1, whose two upper levels are two Zeeman sublevels $|1\rangle=|j=1,m=1\rangle$ with energy $\hbar\omega_1$, and $|2\rangle=|j=1, m=-1\rangle$ with an energy $\hbar\omega_2$, and one ground state $|3\rangle=|j=0,m=0\rangle$ with an energy equal to zero. We choose the y direction as the quantization axis by applying a static magnetic field along the y direction. The atomic transitions from

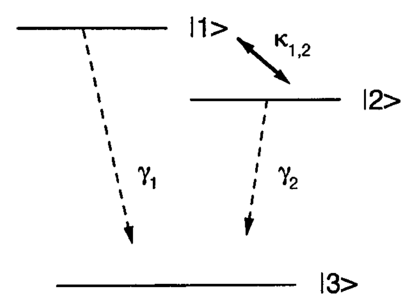

FIG. 1. Level diagram of the V-type three-level atom.

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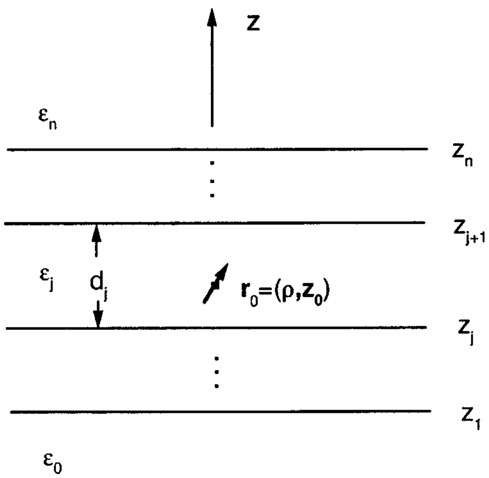

FIG. 2. A dielectric multilayer stack with an atom embedded in the jth layer.

$|1\rangle(|2\rangle)\leftrightarrow|3\rangle$ are linked by the same vacuum bath. The transition $|j=1,m=0\rangle\rightarrow|j=0,m=0\rangle$ is dropped because it does not involve quantum interference, as mentioned by Agarwal [17]. In the present study, the atom is placed in a multilayer dielectric medium whose permittivity function $\varepsilon(\mathbf{r})$ is position dependent, as shown in Fig. 2. We assume the direction normal to interfaces between layers as the z axis. With this choice, the atomic dipole moment is on the x-z plane. The atomic dipole moment operator is represented by $\mathbf{d}=d(A_{13}\boldsymbol{\varepsilon}_-+A_{23}\boldsymbol{\varepsilon}_+)+\text{H.c.}$, where $\boldsymbol{\varepsilon}_\pm=(1/\sqrt{2})(\mathbf{e}_z\pm i\mathbf{e}_x)$, the atomic operators $A_{\alpha\beta}=|\alpha\rangle\langle\beta|(\alpha,\beta=1,2,3)$, and d is chosen to be real.

Following the lines of the extensive discussions by Knoll *et al.* [18] and Glauber and Lwenstein [19] and using the rotating-wave approximation, we can easily derive out the equations of motion for the expectation values of the atomic operators governing the spontaneous emission for the V-type three-level atom as

$$\frac{d}{dt}\langle A_{nn}\rangle=-2\gamma_n\langle A_{nn}\rangle-\kappa_m\langle A_{nm}\rangle-\kappa_m\langle A_{mn}\rangle, \tag{1}$$

$$\frac{d}{dt}\langle A_{n3}\rangle=-(\gamma_n-i\omega_n)\langle A_{13}\rangle-\kappa_m\langle A_{m3}\rangle \quad (n\neq m,n,m=1,2), \tag{2}$$

$$\frac{d}{dt}\langle A_{12}\rangle=-[\gamma_1+\gamma_2+i(\omega_2-\omega_1)]\langle A_{12}\rangle-\kappa_1\langle A_{11}\rangle-\kappa_2\langle A_{22}\rangle, \tag{3}$$

where the coefficients γ_n and κ_n (n=1,2) are defined as

$$\gamma_n=d^2\omega_n^2\boldsymbol{\varepsilon}_-\,\text{Im}\,\overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r}_0;\omega_n)\cdot\boldsymbol{\varepsilon}_+, \tag{4}$$

$$\kappa_1=d^2\omega_1\omega_2\boldsymbol{\varepsilon}_+\,\text{Im}\,\overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r}_0;\omega_1)\cdot\boldsymbol{\varepsilon}_+, \tag{5}$$

$$\kappa_2=d^2\omega_1\omega_2\boldsymbol{\varepsilon}_-\,\text{Im}\,\overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r}_0;\omega_2)\cdot\boldsymbol{\varepsilon}_-. \tag{6}$$

The terms involving κ_1 and κ_2 in Eqs. (1)–(5) are responsible for quantum interferences between the two decay channels $|1\rangle\rightarrow|3\rangle$ and $|2\rangle\rightarrow|3\rangle$. Evidently, the spontaneous emission rates γ_1 and γ_2 and the coefficients κ_1 and κ_2 reflecting the strength of quantum interference depend on the imaginary part of the Green's tensor $\overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r}_0,\omega)$, which is the solution of the equation

$$\nabla\times\nabla\times\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)-\frac{\varepsilon(\mathbf{r})\omega^2}{c^2}\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)=\mu_0\overleftrightarrow{I}\delta(\mathbf{r}-\mathbf{r}'), \tag{7}$$

where $\overleftrightarrow{I}=\mathbf{e}_x\mathbf{e}_x+\mathbf{e}_y\mathbf{e}_y+\mathbf{e}_z\mathbf{e}_z$ is the unit dyadic [20].

Let us now consider the spontaneously generated quantum interference between the two decay channels when the atom embedded in the jth layer of the multilayer dielectric planar cavity shown in Fig. 2. Studying the spontaneous emission of atoms or molecules in such a dielectric planar cavity is a very important subject in semiconductor lasers [21] and photonic crystals [22]. The permittivity function $\varepsilon(\mathbf{r})=\varepsilon(z)$ of the dielectric multilayer is defined in a stepwise fashion, as depicted in Fig. 2. For the case under consideration, the Green's tensor $\overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r}_0,\omega)$, which is the solution of Eq. (7), is expressed as

$$\overleftrightarrow{G}_j(\mathbf{r}_0,\mathbf{r}_0;\omega)=\mathbf{e}_x\mathbf{e}_xG_{jxx}(z_0;\omega)+\mathbf{e}_y\mathbf{e}_yG_{jyy}(z_0;\omega)+\mathbf{e}_z\mathbf{e}_zG_{jzz}(z_0;\omega), \tag{8}$$

where

$$G_{jxx}(z_0;\omega)=G_{jyy}(z_0;\omega)=\frac{i\mu_0}{8\pi\tilde{k}_j^2}\int_0^\infty\frac{dkk}{\beta_j}\left[\frac{\beta_j^2}{D_{pj}}(1-r_{j-}^pe^{2i\beta_jz_0})(1-r_{j+}^p\times e^{2i\beta_j(d_j-z_0)})+\frac{\tilde{k}_j^2}{D_{sj}}(1+r_{j+}^se^{2i\beta_j(d_j-z_0)})\times(1+r_{j-}^se^{2i\beta_jz_0})\right], \tag{9}$$

$$G_{jzz}(z_0;\omega)=\frac{i\mu_0}{8\pi\tilde{k}_j^2}\int_0^\infty\frac{dkk}{\beta_j}\frac{2k^2}{D_{pj}}(1+r_{j-}^pe^{2i\beta_jz_0})\times(1+r_{j+}^pe^{2i\beta_j(d_j-z_0)}), \tag{10}$$

where $\tilde{k}_j=\sqrt{\varepsilon_j}(\omega/c)$, the parameter k is the magnitude of the vector $\mathbf{k}=(k_x,k_y)$, the conserved component of the wave vector is parallel to interfaces of the layers, and $\beta_j=\sqrt{\tilde{k}_j^2-k^2}$ is the magnitude of the z component of the wave vector in the jth layer. The symbol p stands for the electric field of the TM electromagnetic wave, whose polarization is in the plane formed by the two vectors $\mathbf{k}$ and $\mathbf{e}_z$, and s for the electric field of the TE electromagnetic wave, whose polarization along the $\mathbf{k}\times\mathbf{e}_z$ direction. These two kinds of electric fields are herafter referred to as p- and s- polarized waves or modes, respectively, as in Ref. [23]. In addition,

$$D_{qj}=1-r^q_{j-}r^q_{j+}\exp(2i\beta_j d_j), \tag{11}$$

with $r^q_{j\pm}=r^q_{j\not{n}(0)}$ being the reflection coefficient of the upper (lower) stack of layers bounding the jth layer. If there is no absorption in the multilayer medium, these coefficients obey the usual Fresnel recurrence relation

$$r^q_{ijk}=\frac{1}{1-r^q_{ji}r^q_{jk}\exp(2i\beta_j d_j)}[r^q_{ij}+r^q_{jk}\exp(2i\beta_j d_j)]. \tag{12}$$

For a single interface i-j, the coefficients reduce to

$$r^p_{ij}=\frac{\beta_i\varepsilon_j-\beta_j\varepsilon_i}{\beta_i\varepsilon_j+\beta_j\varepsilon_i}, \quad r^s_{ij}=\frac{\beta_i-\beta_j}{\beta_i+\beta_j}. \tag{13}$$

From Eqs. (8)–(10), we can see that the inhomogeneity of the medium along the z axis leads to a Green's tensor of the electromagnetic field exhibiting an asymmetric spatial property, i.e., the component along the z axis is different from those in the x-y plane. Substituting Eqs. (8)–(10) into Eqs. (4)–(6), we have

$$\gamma_n=\gamma_{nx}+\gamma_{nz}, \quad \kappa_n=\frac{\omega_1\omega_2}{\omega_n^2}(\gamma_{nz}-\gamma_{nx}) \quad (n=1,2), \tag{14}$$

where $\gamma_{nx}=d^2\omega_n^2\,\mathrm{Im}[G_{jxx}(z_0;\omega_n)]/2$ and $\gamma_{nz}=d^2\omega_n^2\,\mathrm{Im}[G_{jzz}(z_0;\omega_n)]/2$ represent the spontaneous decay rates of the x and z components of the atomic dipole moment, respectively. We also define the relative strength of quantum interference $p=\kappa_n/\sqrt{\gamma_1\gamma_2}$, which measures the effective parallelism of the atomic dipole matrix elements. Actually, in free space [7], if the two atomic dipole moments are perpendicular to each other, then the relative strength $p=0$; if the two atomic dipole moments are parallel, then $p=1$; when the two atomic dipole moments are antiparallel, $p=-1$. Evidently, when $\gamma_{nz}=\gamma_{nx}$, (that is, when the dielectric medium is homogeneous), $\kappa_1=\kappa_2=0$, and consequently $p=0$. However, if the dielectric structure is inhomogeneous, $\gamma_{nz}\neq\gamma_{nx}$, so κ_1 and κ_2 are not equal to zero. From Eq. (14), we also see that the larger the difference between γ_{nz} and γ_{nx}, the larger κ_n is. In conclusion, the anisotropy of the vacuum led by the inhomogeneity of the medium can induce quantum interference or the correlation between two transitions: $|1\rangle\rightarrow|3\rangle$ and $|2\rangle\rightarrow|3\rangle$.

III. RESULTS AND DISCUSSIONS

A. Atom placed between two parallel dielectric plates

First, we examine a three-layer dielectric structure, in which the permittivity of the layer cladding the atom is ε_3; the permittivity of the upper layer, which extends to infinity in the positive z direction, is ε_2; and the permittivity of the lower layer, which extends to infinity in the negative z direction, is ε_1. We assume that $\varepsilon_3<\varepsilon_1\leqslant\varepsilon_2$, and that the thickness of the layer with ε_3 is d.

For the case under consideration, there exist two kinds of electromagnetic waves in the medium. One is the propagating wave [β_j is real, i.e., $k<\sqrt{\varepsilon_3}(\omega/c)$], in which the atom emits energy, and the other one is the evanescent wave (β_j is imaginary). There is no contribution of the evanescent waves with $k>\sqrt{\varepsilon_2}(\omega/c)$ on the atomic spontaneous emission, because these waves cannot propagate out of the atom; only those with $\sqrt{\varepsilon_3}(\omega/c)<k<\sqrt{\varepsilon_2}(\omega/c)$, which may propagate in both the upper and lower layers or only in the upper layer, affect on atomic spontaneous emission. For simplicity, we assume that the two upper levels of the atom are nearly degenerate, so that $\omega_1\approx\omega_2=\omega$. The integrals in Eqs. (9) and (10) cannot be evaluated analytically any further for general cases. For the symmetric case, where $\varepsilon_1=\varepsilon_2=\varepsilon$ and $d\rightarrow 0$ (that is, as the distance between the two dielectric plates becomes very small), we can simplify them further and obtain

$$\gamma_{1z}=\gamma_{2z}=\gamma_z=\gamma_0\frac{\varepsilon^{5/2}}{2\varepsilon_3^2}, \quad \gamma_{1x}=\gamma_{2x}=\gamma_x=\varepsilon^{1/2}\gamma_0/2, \tag{15}$$

$$\gamma_1=\gamma_2=\gamma_0\frac{\sqrt{\varepsilon}(\varepsilon^2+\varepsilon_3^2)}{2\varepsilon_3^2}=\gamma, \tag{16}$$

$$\kappa_1=\kappa_2=\gamma_0\frac{\sqrt{\varepsilon}(\varepsilon^2-\varepsilon_3^2)}{2\varepsilon_3^2}=\kappa, \tag{17}$$

$$p=\kappa/\sqrt{\gamma_1\gamma_2}=\kappa/\gamma=\frac{\varepsilon^2-\varepsilon_3^2}{\varepsilon^2+\varepsilon_3^2}, \tag{18}$$

where γ_0 represents the spontaneous decay rate of the atom in free space. It is evident that the γ's and κ's depend on the permittivity ε of the upper and lower dielectrics. The decay rate of the atomic dipole component (γ_z) in the z direction (proportional to $\varepsilon^{5/2}$) is much larger than that of the atomic dipole component (γ_x) in the x direction (proportional to $\varepsilon^{1/2}$). This is because the medium is inhomogeneous in the z direction, and the electromagnetic wave emitted by the atom in the thin layer with ε_3 is reflected by the ε_1-ε_3 and ε_2-ε_3 interfaces. If the layer with ε_3 is very thin then, for the p-polarized waves in this layer, constructive interference occurs between the polarization component in the z direction of the reflected waves and that of the incident ones (the atom emits initially); however, for the polarization component in the x-y plane, destructive interference occurs. The s-polarized wave only contains the component in x-y plane, and destructive interference exits between the reflected waves and the incident waves. This leads to the mode density of electromagnetic waves with different polarization directions displaying inhomogeneity due to the anisotropic spatial distribution of the dielectric medium. Since the mode density of the electromagnetic waves whose polarization along the z direction is much higher than that in the x-y plane, the spontaneous decay of the atomic dipole in the z direction is much faster than that in the x direction, and the strengths κ_1 and κ_2 reflecting the quantum interference are different from zero. If the upper and lower layers are replaced by two ideal conductor plates, the emission from the component of the dipole moment parallel to the interfaces of the layers can be totally inhibited; however, that normal to the interfaces can be greatly greatly enhanced, when the separation between the plates becomes smaller than half of the wavelength $\lambda_0 = 2\pi/\omega/c$ of the atomic transition in free space [17,24]. In this case, the strongest quantum interference effect, i.e., $p=1$, arises [17]. For the present dielectric structure, however, γ_x is not equal to zero, and is moreover increased when ε becomes large and d approaches zero. Thus, we see that the inhomogeneous configuration of the dielectric medium in the z direction results in an anisotropic property of the vacuum. This results in a quantum interference strength κ different from zero. Equation (18) also indicates that with increasing ε the relative strength p of the quantum interference increases. Moreover, if $\varepsilon \gg \varepsilon_3$ and $d \to 0$, then $p \to 1$, which means that the strongest quantum interference between two decay channels $|1\rangle \to |3\rangle$ and $|2\rangle \to |3\rangle$ arises. This realization is very similar to the case where a V-type three-level atom has two parallel dipole matrix elements in free space [7]. This is because, with increasing ε, the evanescent modes involved in the spontaneous emission increase: the mode density of the electromagnetic field polarized along the z axis increases more evidently than that of the field polarized in the x-y plane.

In Fig. 3, we plot γ, κ and p versus the scaled thickness $d'(d'=2d/\lambda_0)$ of the middle layer. It is obvious that when d is smaller than the wavelength λ_0, the relative strength p of the quantum interference can reach a value larger than 0.9. Thus some new effects such as spectral narrowing [9–13] and gain without inversion [14], which are revealed in the system of a V-type three-level atom with two nearly parallel dipole elements in free space, will occur here because the anisotropy of the vacuum leads to an atom with two orthogonal dipole moments exhibiting a strong quantum interference. As we see, with increasing d', p decreases. This is because, with increasing d', for all modes with wave vectors $k<\sqrt{\varepsilon_2}(\omega/c)$, (which involve spontaneous emission), the destructive and constructive interferences occur simultaneously between the reflected and incident waves for TE and TM waves. The average effect leads to a reduction of the difference of in the mode density of the field polarized between the z axis and the x-y plane. Therefore, the anisotropy of the vacuum that the atom ''feels'' becomes much weaker than that when d is very small. From these figures, we also observe that when d' is small, γ, κ, and p are irrelevant to the position of the atom. When d' becomes large, all these quantities are strongly dependent on the position in which the atom is embedded. The physical reason for this phenomenon is as follows. In the medium ε_3, due to reflection through the ε_1-ε_3 and ε_2-ε_3 interfaces, there exist incident

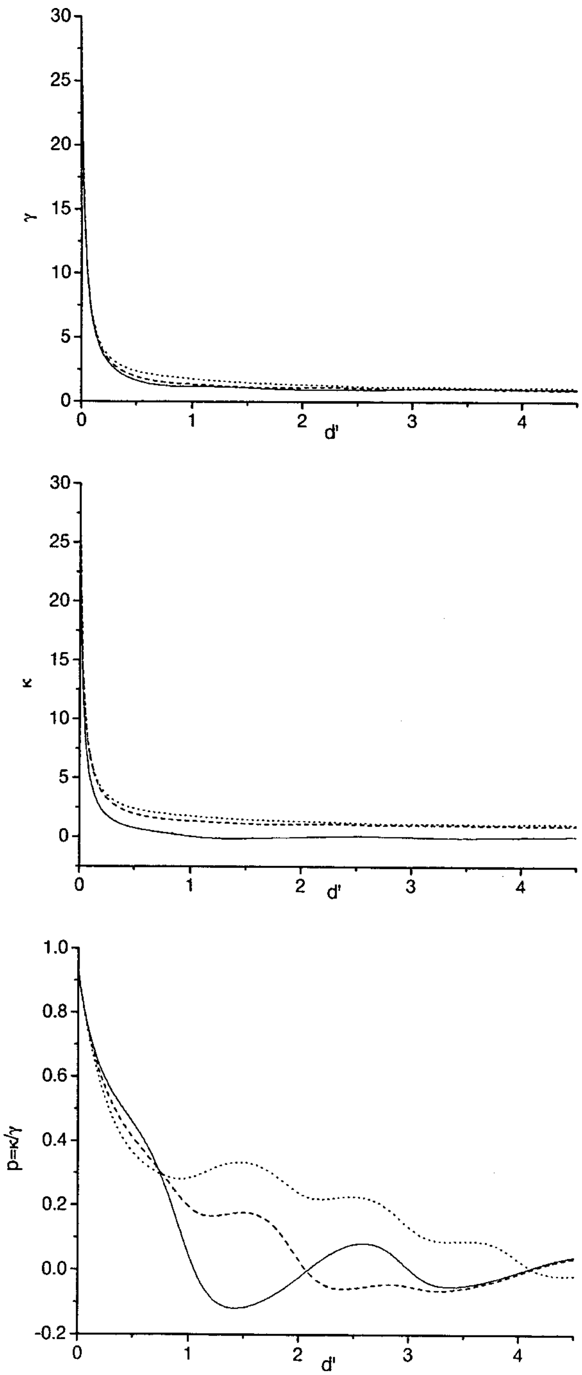

FIG. 3. The variation of the spontanoeus emission rate γ, the quantum interference strength κ, and the relative strength p of quantum interference with the scaled width d'. The permittivities are chosen as $\varepsilon_3=1.0$ and $\varepsilon_1=\varepsilon_2=5.0$. Here the solid line corresponds to $z_0'=d'/2$, the dashed line to $z_0'=d'/4$, and the dotted line to $z_0'=d'/8$. γ_0 has been set to 1.

waves and reflected waves. The superposition of the reflected and incident waves results in the mode density being position dependent. Thus the spontaneous emission rate γ, the quantum interference strength κ, and the relative strength p of quantum interference are strongly dependent on the atomic position.

B. Atom placed in a three-layer dielectric waveguide

If the permittivity coefficients of the three dielectric layers obey $\varepsilon_3 > \varepsilon_1 \geqslant \varepsilon_2$, the three-layer structure forms a dielectric waveguide. The electromagnetic modes in this dielectric structure can be classified into radiation modes, whose wave vectors satisfy $0 < k < \sqrt{\varepsilon_2}(\omega/c)$; substrate modes, with $\sqrt{\varepsilon_2}(\omega/c) < k < \sqrt{\varepsilon_1}(\omega/c)$; guided modes, with $\sqrt{\varepsilon_1}(\omega/c) < k < \sqrt{\varepsilon_3}(\omega/c)$; and evanescent modes, with $k > \sqrt{\varepsilon_3}(\omega/c)$. It is easily seen that there is no contribution of the evanescent modes in the waveguide to the spontaneous decay. The contributions of the radiation and the substrate modes to the spontaneous decay rates can be directly obtained by integrating Eqs. (9) and (10) numerically. Because the wave vectors of the guided modes are limited in the region $\sqrt{\varepsilon_1}(\omega/c) < k < \sqrt{\varepsilon_3}(\omega/c)$, the reflection coefficients of these modes are complex quantities with modulus 1, which can be expressed as

$$r_{\pm}^{q} = \exp(-2i\phi_{\pm}^{q})(q = p,s). \tag{19}$$

The wave vectors of the guided modes obey the resonance condition $|D_{qj}^{(1)}|^2 = 0$, i.e.,

$$\beta(k_q^{(m)})d - \phi_{-}^{q}(k_q^{(m)}) - \phi_{+}^{q}(k_q^{(m)}) = m\pi$$

$$(q = p,s; m = 0,1,2, \ldots, m_{\max}^{q}), \tag{20}$$

where $k_q^{(m)}$ is the wave number of the mth q-polarized guided mode in the x-y plane; $m_{\max}^{q}$ is the maximum integer for m to be chosen, which depends on the thickness of the middle layer and the mode polarization. For the s-polarization modes [25],

$$m_{\max}^{s} = \left[\sqrt{\varepsilon_3 - \varepsilon_2}\,\frac{2d}{\lambda_0} - \frac{1}{\pi}\arctan\left(\sqrt{\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_3 - \varepsilon_1}} \right) \right],$$

and, for the p-polarization modes,

$$m_{\max}^{p} = \left[\sqrt{\varepsilon_3 - \varepsilon_2}\,\frac{2d}{\lambda_0} - \frac{1}{\pi}\arctan\left(\frac{\varepsilon_3}{\varepsilon_2} \sqrt{\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_3 - \varepsilon_1}} \right) \right],$$

where $[\xi]$ denotes the largest integer that is smaller than or equal to ξ. These conditions show that for a certain thickness of the layer with permittivity ε_3, a finite number of guided modes exist. These field modes involve atomic spontaneous emission. The contributions of the p-polarized guided modes and the s-polarized guided modes on the spontaneous decay rate are expressed as

$$\gamma_{gz}^{p} = \frac{3\gamma_0}{4\varepsilon_3} \sum_m \frac{\pi x_p^{(m)}[1 + \cos(\pi\beta_3^{p'} d' - \phi_{+}^{p} - \phi_{-}^{p})\cos[\pi\beta_3^{p'}(d' - 2z_0') - \phi_{+}^{p} + \phi_{-}^{p}]}{\pi d' + \dfrac{\varepsilon_1\varepsilon_3}{[(\varepsilon_1 + \varepsilon_3)x_p^{(m)} - \varepsilon_1\varepsilon_3]\sqrt{x_p^{(m)} - \varepsilon_1}} + \dfrac{\varepsilon_2\varepsilon_3}{[(\varepsilon_2 + \varepsilon_3)x_p^{(m)} - \varepsilon_2\varepsilon_3]\sqrt{x_p^{(m)} - \varepsilon_2}}}, \tag{21}$$

$$\gamma_{gx}^{p} = \frac{3\gamma_0}{8\varepsilon_3} \sum_m \frac{\pi \beta_3^{p'2}[1 - \cos(\pi\beta_3^{p'} d' - \phi_{+}^{p} - \phi_{-}^{p})\cos[\pi\beta_3^{p'}(d' - 2z_0') - \phi_{+}^{p} + \phi_{-}^{p}]}{\pi d' + \dfrac{\varepsilon_1\varepsilon_3}{[(\varepsilon_1 + \varepsilon_3)x_p^{(m)} - \varepsilon_1\varepsilon_3]\sqrt{x_p^{(m)} - \varepsilon_1}} + \dfrac{\varepsilon_2\varepsilon_3}{[(\varepsilon_2 + \varepsilon_3)x_p^{(m)} - \varepsilon_2\varepsilon_3]\sqrt{x_p^{(m)} - \varepsilon_2}}}, \tag{22}$$

$$\gamma_{gx}^{s} = \frac{3\gamma_0}{8} \sum_m \frac{\pi[1 + \cos(\pi\beta_3^{s'} d' - \phi_{+}^{s} - \phi_{-}^{s})\cos[\pi\beta_3^{s'}(d' - 2z_0') - \phi_{+}^{s} + \phi_{-}^{s}]}{\pi d' + \dfrac{1}{\sqrt{x_s^{(m)} - \varepsilon_1}} + \dfrac{1}{\sqrt{x_s^{(m)} - \varepsilon_2}}}, \tag{23}$$

where $x_q^{(m)} = (k_q^{(m)})^2/\omega^2/c^2$, $z_0' = 2z_0/\lambda_0$, and $\beta_3^{q'} = \sqrt{\varepsilon_3 - x_q^{(m)}}$. The total decay rate is given by $\gamma = \gamma_z^p + \gamma_x^p + \gamma_x^s$. For the nearly degenerate case, the quantum interference strength is given by $\kappa = \gamma_z^p - \gamma_x^p - \gamma_x^s$, and the effective parallelism of the two dipole moments is given by $p = \kappa/\gamma$. Figure 4 shows γ, κ, and p versus the thickness of the layer with different relative positions of the atom. We see that when d' is small, p can take a value close to -1. This means equivalently, that the anisotropy of the vacuum induces quantum interference between the two decay channels $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$, similar to that in the system of the V-type three-level atom with two dipole moments nearly antiparallel to each other in free space [7]. From these figures, we also observe that, when d' is small, γ, κ, and p are irrelevant to the position of the atom. When d' becomes large, these quantities are strongly dependent on the position of the atom, and exhibit a quasiperiodic oscillation with d'.

These phenomena can be understood as follows. When d' is very small, only the lowest guided modes ($m = 0$) are supported by the dielectric waveguide [25]. Equation (20) shows that for certain d', the wave vector $k_s^{(0)}$ for the guide mode with s polarization is larger than $k_p^{(0)}$ for the guide mode with p polarization, although both of these approach $\sqrt{\varepsilon_2}(\omega/c)$. These guided modes mainly affect on the spontaneous emission rates γ_{gz}^{p} and γ_{gx}^{s}, and γ_{gx}^{p} is negligible (in fact, when $\varepsilon_1 = \varepsilon_2$ and $z_0' = d'/2$, the atom is in the node of

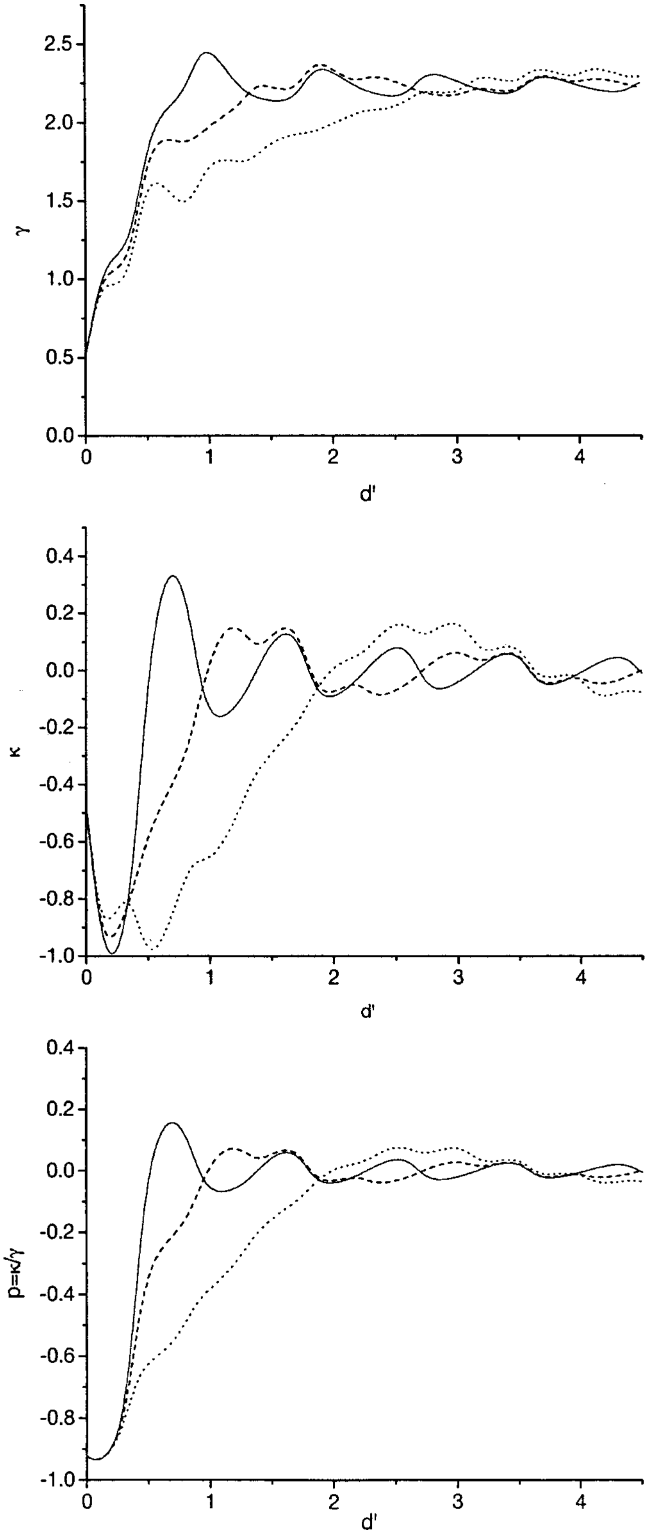

FIG. 4. Same as Fig. 3, except that the permittivities are chosen as $\varepsilon_1=\varepsilon_2=1.0$ and $\varepsilon_3=5.0$.

the guided modes and γ_{gx}^{p} is exactly equal to zero). Because of $k_s^{(0)}>k_p^{(0)}$, the mode density of the guide mode with p polarization is much smaller than that of guide mode with s polarization: $\gamma_{g//}>\gamma_{g\perp}$. Also, when $\varepsilon_1=\varepsilon_2=\varepsilon$ and $z_0'=d'/2\rightarrow 0$, the contribution of the radiation modes to the spontaneous emission rates also obey Eq. (15) except that $\varepsilon_3>\varepsilon$, that is, $\gamma_{rx}>\gamma_{rz}$. Therefore, different from the situation discussed in Sec. III A, here in the dielectric waveguide $p<0$. Because the mode density in the dielectric waveguide is generally position dependent, the spontaneous emission rate γ, the interference strength κ, and the relative strength p of the interference depend on the position of the atomic location. However, when d' is very small the mode density in the thin waveguide is nearly position independent, so γ, κ, and p are insensitive to d'. The oscillation behavior of the curves results from the fact that with an increase in the of the thickness of the layer, a new guide mode appears when m increases to $m+1$ in Eq. (20). For the symmetric case of $\varepsilon_1=\varepsilon_2$ and z_0', the atom is located in the nodes of the guide modes with even m ($m=0,2,4,\ldots$) and the antinodes of the guide modes with odd m ($m=1,3,\ldots$), so only the guided modes with even m affect the spontaneous emission. However, if the atom is located outside the center of the middle layer, all the guided modes are involved in the spontaneous emission. So as shown in Fig. 4, the periods of γ, κ, and p for $z_0'=d'/2$ are nearly as twice as those for $z_0'\neq d'/2$.

C. Atom placed in a one-dimensional quasiperiodic dielectric structure

Recently, one-, two-, and three-dimensional periodic structures [22,26–28] of optical materials have attracted a great deal of attention in the optics community, because these structures may exhibit the existence of photonic stop bands (or band gaps). The simplest case is one-dimensional periodic or quasiperiodic multilayer stacks, termed one-dimensional photonic crystal. Dowling and co-workers [22] studied the spontaneous emission of a two-level atom in a one-dimensional photonic crystal. They found that the spontaneous emission can be enhanced or inhibited. However, their model is essentially scalar, because only the field modes polarized parallel to the atomic dipole moment were taken into account. In this subsection, taking all the TE and TM polarized modes into account, we investigate the spontaneous emission of a V-type three-level atom and the quantum interference between the two decay channels $|1\rangle\rightarrow|3\rangle$ and $|2\rangle\rightarrow|3\rangle$.

We assume that the atom is embedded in the middle dielectric layer with permittivity ε_3 and thickness d. This layer is coated by two upper and lower distributed Bragg reflecting mirrors, each of them composed of N-period dielectric stacks, in which every period consists of two dielectric layers with permittivities ε_1 and ε_2 ($\varepsilon_3>\varepsilon_2>\varepsilon_1$) and widths d_1 and d_2. The multilayer medium is placed in air, so the permittivity of the top and bottom layers is modeled as $\varepsilon_0=1$. This structure forms a one-dimensional photonic crystal with a defect [28]. Similar to the three-layer dielectric waveguide discussed in Sec. III B, there are five kinds of field modes in this structure: one is a propagating mode with wave vector $k<\sqrt{\varepsilon_0}(\omega/c)$ which can be regarded as a running wave in all directions; the second is a guided mode with $\sqrt{\varepsilon_0}(\omega/c)<k<\sqrt{\varepsilon_1}(\omega/c)$ which forms an evanescent wave in the ε_0-ε_1 interface in the z direction; the third is a guided mode, but its wave vector obeys $\sqrt{\varepsilon_1}(\omega/c)<k<\sqrt{\varepsilon_2}(\omega/c)$, and it can be treated as an evanescent wave in the layers with ε_0 and ε_1 in the z direction; the fourth is also a guided mode with $\sqrt{\varepsilon_2}(\omega/c)<k<\sqrt{\varepsilon_3}(\omega/c)$, which forms an evanescent wave in the layers with ε_0, ε_1, and ε_2 along the z direction; the fifth is an evanescent mode with $k>\sqrt{\varepsilon_3}(\omega/c)$, which does not affect the spontaneous emission. Evidently, for all the guided modes, the reflection coefficients $r^q_{j\pm}$ $(q=p,s)$ in Eq. (11) are unit complex, which can be written as $r^q_{j\pm}=\exp(-2\phi^q_j)$. By use of Eqs. (9) and (10), the contribution of the guided modes to the spontaneous decay rates can be expressed as

$$\gamma^p_{g\perp}=\frac{3\gamma_0}{4\varepsilon_3}\sum_m \frac{\pi x_p^{(m)}[1+\cos(\pi\beta_3^{p'}d'-2\phi_j^p)\cos[\pi\beta_3^{p'}(d'-2z_0')]}{\pi d'-2\left.\dfrac{d\phi_j^p}{d\beta_3^{p'}}\right|_{x_p^{(m)}}}, \tag{24}$$

$$\gamma^p_{g\parallel}=\frac{3\gamma_0}{8\varepsilon_3}\sum_m \frac{\pi\beta_3^{p'2}[1-\cos(\pi\beta_3^{p'}d'-2\phi_j^p)\cos[\pi\beta_3^{p'}(d'-2z_0')]}{\pi d'-2\left.\dfrac{d\phi_j^p}{d\beta 3^{p'}}\right|_{x_p^{(m)}}}, \tag{25}$$

$$\gamma^s_{g\parallel}=\frac{3\gamma_0}{8}\sum_m \frac{\pi[1+\cos(\pi\beta_3^{s'}d'-2\phi_j^s)\cos[\pi\beta_3^{s'}(d'-2z_0')]}{\pi d'-2\left.\dfrac{d\phi_j^s}{d\beta_3^{s'}}\right|_{x_s^{(m)}}}, \tag{26}$$

where $x_q^{(m)}=(k_q^{(m)})^2/\omega^2/c^2$ are roots of the equation

$$\pi\beta_3(x_q^{(m)})d'-2\phi_j^q(x_q^{(m)})=m\pi \quad (m=0,1,2,\ldots). \tag{27}$$

By use of the Fresnel recurrence relation [Eq. (12)], we can obtain the reflection coefficients $r^q_{j\pm}$, the phase ϕ^q_j, and their derivations exactly. Substituting all these parameters into Eqs. (24)–(26), we can obtain the contribution of all the guided modes to the spontaneous decay rate numerically. The contribution of the radiation modes to the spontaneous decay rate can be directly calculated from Eqs. (9)–(11), together with Eq. (12), by numerical integration. In Figs. 5 and 6, we show the decay rate γ_r due to the radiation modes with $k<\sqrt{\varepsilon_0}(\omega/c)$, the total decay rate γ arising from all the radiation modes, and the guided modes with $\sqrt{\varepsilon_0}(\omega/c)<k<\sqrt{\varepsilon_3}(\omega/c)$, the quantum interference strength κ and the relative strength p of quantum interference vary with d' for different values of the periodic number of the coated stacks and the atomic position. These figures illustrate that γ, κ, and p display behaviors similar to these presented in Sec. III B for the waveguide case.

From Figs. 5 and 6 we see that by increasing the number of periods of the upper and lower stacks, the atomic decay rate γ_r due to the radiation modes can be greatly decreased for certain ranges d'. If we fix the thickness d of the layer in which the atom is embedded, then the variation of d' can be treated as a variation of the frequency ω. Since the decay rate γ_r is proportional to the local density of state for the radiation modes, the existence of ranges of d' corresponding to very small γ_r reflects the fact that there exist incomplete photonic band gaps for the radiation modes. This is because the dielectric stack under consideration displays a periodic structure in the z direction; when the period N is large, the stack should exhibit the essential feature of the standard one-dimensional photonic crystal [22], i.e., the appearance of photonic band gaps. However, different from Ref. [22], here we take all the TE and TM radiation modes with arbitrarily propagating directions into account, so there only appear incomplete gaps. The reason for this is that the stack is periodic only in the z direction; only the radiation modes propagating along the z direction can be completely inhibited, but the other radiation modes whose propagating directions are out of the z direction cannot be completely inhibited. We also see from Fig. 6 that the width of the band gap for the radiation modes is strongly dependent on the difference between the permittivities ε_2 and ε_1; the larger the difference, the wider the band gaps. With increasing d', the band gap becomes narrow; in the high-frequency region the band gap nearly disappears. These features are coincident with those in Ref. [22]. Also we find that γ_r is strongly dependent on the position where the atom is embedded; that is, at different positions the photonic density of states for the radiation modes are different, and the band gaps when $z_0'=d'/2$ are much wider than those when $z_0'\neq d'/2$. That is to say, to control the influence of the radiation modes on the properties of spontaneous emission, embedding the atom at the middle of the layer with ε_3 is much more efficient than doing so at other positions in this layer.

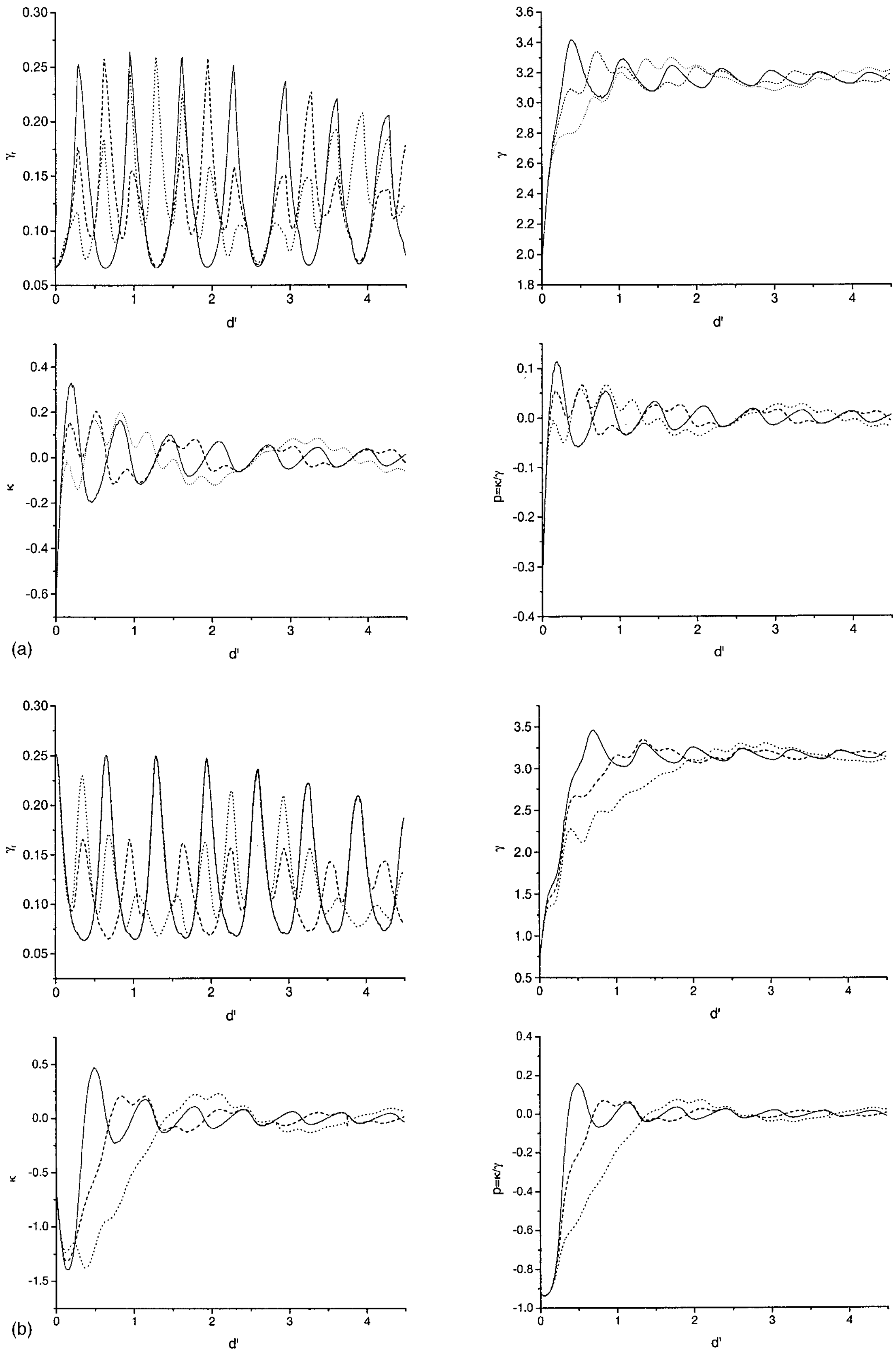

FIG. 5. (a) The variation of γ_r, γ, κ, and p with d'. Here the permittivity of the middle layer in which the atom embedded is $\varepsilon_3 = 10.0$, the periodic number of the upper and lower cladding dielectric layers is $N = 1$, and the permittivities of the two layers in each period are $\varepsilon_1 = 1.5$ and $\varepsilon_2 = 9.0$. The solid line corresponds to $z_0' = d'/2$, the dashed line to $z_0' = d'/4$, and the dotted line to $z_0' = d'/8$. (b) Same as (a), but $\varepsilon_2 = 2.0$.

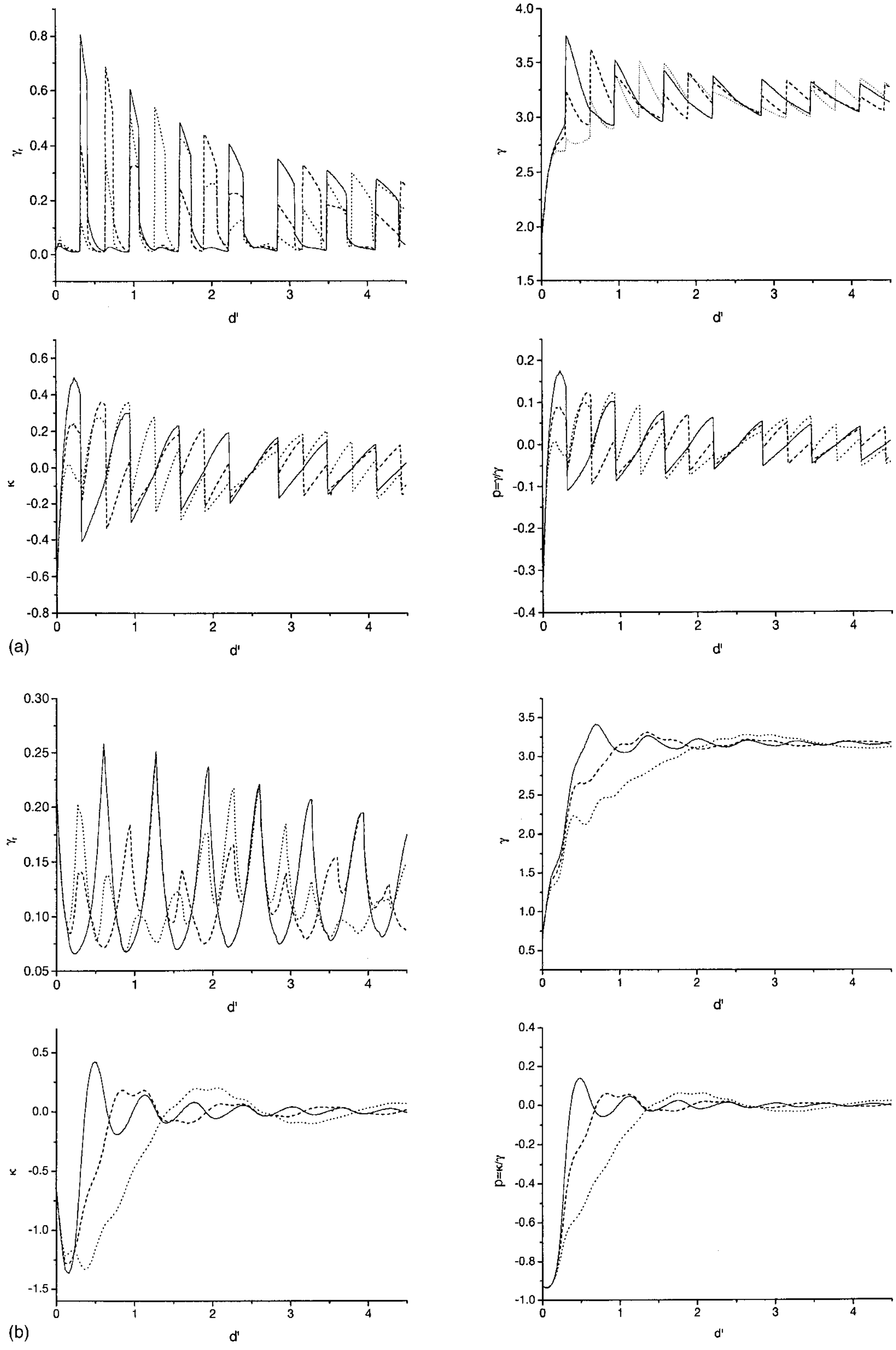

FIG. 6. (a) Same as Fig. 5(a), except that $N=5$. (b) Same as Fig. 5(c), except that $N=5$.

However, when we inspect the behavior of the quantum interference strength κ and the relative strength p of the quantum interference, the larger the difference between the permittivities ε_3 and ε_2, the stronger the quantum interference is. Therefore, if the atom is embedded in the present one-dimensional photonic crystal, then, in order to increase the relative strength of the quantum interference, the layer in which the atom is embedded should have a large permittivity

ε_3, and the difference between ε_3 and ε_2 should be large enough. At this moment, the band gaps for the radiation modes are very narrow.

IV. SUMMARY

We investigate spontaneous emission of an atom with two close upper levels and one single lower level, embedded in a multilayer dielectric medium, and quantum interference between two transitions from the upper levels to the common lower level. It is verified that in the anisotropic vacuum, quantum interference can exist even if the two dipole moments are orthogonal to each other. Choosing different arrangements and values of permittivity of the layers, we show two situations: a dielectric plate cavity and a dielectric waveguide. In the cavity case, we find when the separation between the two dielectric plates is small, the decay rate of the dipole moment component normal to interfaces of the layers is much larger than that parallel to the interfaces. This show that the vacuum is anisotropic. We show that the anisotropy of the vacuum can induce dipole matrix elements that are equivalently parallel to one another. In the dielectric waveguide case, we show that the anisotropy of the vacuum can induce dipole matrix elements that are, in effect, nearly antiparallel to one another. The decay rate and the quantum interference strength display an oscillation behavior with an increase of the thickness of the layer where the atom is embedded, because more guided wave modes are switched on. Increasing the number of the layers for appropriately large contrasts in the permittivity of the layers, the medium gradually becomes a one-dimensional photonic crystal. The enhancement and the inhibition of spontaneous emission of the atom into the radiation modes appear alternatively with variation of the thickness of the layer where the atom resides. This result clearly predicts the existence of photonic band gaps. In contrast to previous studies on one-dimensional photonic crystals, the present model is completely vectorial, and the photonic band gaps are incomplete.

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