Effects of Pauli blocking on semiconductor laser intensity and phase noise spectra

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In this paper we derive the intensity and phase noise spectra of a single-mode semiconductor laser on the basis of an operatorial Langevin set of equations that includes the dynamics of the microscopic variables, i.e., the carrier polarization and their distribution over the k states. In particular we take into account the fact that the carriers pumped into the active layer are subject to the blocking determined by Pauli exclusion principle. We demonstrate that, due to the fastness of the carrier scattering and polarization dephasing processes, the noise spectra can be determined on the basis of a macroscopic linearized set of three equations for the two quadrature components of the laser intensity and the total carrier number. A formal comparison with the paradigmatic results of [Yamamoto *et al.* Phys. Rev. A **34**, 4025 (1986)] allows to deduce that the only essential difference arises from Pauli-induced pump blocking, which has the effect of increasing the low-frequency branch of the intensity noise spectrum. We demonstrate that, even for very small amounts of pump blocking, the low-frequency intensity noise steeply rises with the stationary value of the carrier density in the active layer, which depends on a great number of parameters. This result can explain the erratic behavior of the experimental findings and their discrepancy with the standard theoretical predictions.

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I. INTRODUCTION

The paradigmatic analysis of the noise properties of semiconductor lasers is widely considered to be that of Yamamoto and coauthors [1]. For asymptotically high pump it predicts an intensity noise spectrum flat and equal to the standard quantum limit, the noise being contributed at low and high frequency by pump and zero-point field fluctuations, respectively. The phase noise spectrum is equal to the standard quantum limit at high frequency, but increases as the frequency diminishes because of unstationary phase diffusion, and is enhanced by nonlinear dispersion. The boundary between ''high'' and ''low''-frequency behavior is determined by the cavity bandwidth. These results allow to predict the possibility of generating intensity squeezed light if a sub-Poissonian pump is provided. Since ''high-impedance suppression'' of the pump noise takes place spontaneously in electrically driven semiconductor lasers at high pump level [2], Yamamoto's theory predicts, in the ideal case of unitary quantum efficiency, perfect squeezing at low frequency. The underlying reasoning is simple: since carriers are pumped regularly into the active layer and they all recombine to give a laser photon, if the observation time is long enough to let the photon leave the cavity, we should detect a regular beam of photons. In other words, electrical regularity ensures optical regularity under the conditions that (i) each pumped carrier is transformed into a photon and (ii) the frequency at which the output beam intensity noise is measured is small enough.

Despite the fact that squeezed light generation by semiconductor lasers has found many experimental confirmations [3–15], the levels of squeezing actually detected are generally smaller than those predicted by the paradigmatic analysis of [1]. Ensuring single-mode operation (be it a longitudinal, transverse, or polarization mode) has been generally recognized as an effective method to improve the experimental results, but also in this case, they may remain far from theoretical predictions and subject to a great variability as the specific device is changed.

A possible explanation of the erraticity in the semiconductor lasers intensity noise properties relies on the fact that, because of losses, the cavity spatial modes are not orthogonal to each other [16], and consequently the lasing mode will be ''contaminated'' by noise from subthreshold transverse modes [17]. A measure of the mode nonorthogonality is given by the increase of the Petermann excess noise factor K [18] from the value of 1, and in [19] it was shown that $K=1.5$ constitutes the boundary above which intensity squeezing becomes impossible. Since K ranges from approximately 1 for purely index-guided lasers to 15–25 for gain-guided lasers, the conclusion is that the field noise homodyned into the lasing mode from subthreshold transverse modes can seriously impede intensity squeezing [19].

The analysis presented in this paper is complementary to the one described in [16–19], since it is based on the assumption that the additional intensity noise does not derive from an optical coupling among the modes, but from the electronic processes that underlie the semiconductor laser operation. Coherently with this approach, in [20,21] a theoretical investigation has been started to determine the zero-frequency intensity noise of a single-mode semiconductor laser on the basis of a microscopic model that includes phenomena such as carrier scattering, spectral hole burning and pump blocking, which were not included in the original Yamamoto's theory. The conclusion was that scattering and spectral-hole burning do not contribute to zero-frequency intensity noise, and the only electronic phenomenon that may increase it is pump blocking. This paper is the natural sequel of [21], since it provides an expression of the intensity noise for all the frequencies, and determines also the phase noise spectrum.

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The calculation of the intensity noise spectrum confirms that the increase of the intensity noise by pump blocking is not a spurious effect that takes place only when the frequency is mathematically equal to zero, but extends to all the frequency range where squeezing is to be expected. A formal comparison with the expression of the intensity noise spectrum given in [1] allows to conclude that for asymptotically high pump the spectrum does not become flat, but at low frequency it exceeds the standard quantum limit by an amount that steeply increases with the carrier density in the active layer, and is exactly equal to zero only if the pump-blocking effect is completely absent. The phase noise spectrum does not differ substantially from the predictions of Yamamoto, and exhibits the same nonlinear dispersion contribution at low frequency. A parallelism between the intensity noise enhancement due to pump blocking and the phase noise enhancement due to nonlinear dispersion has been carried on in [22].

The material of the present paper is organized as follows: after this Introduction, Sec. II will present the microscopic equations that are the basis of our analysis, and deduce from them the exact set of linearized equations. In Sec. III we will show that, due to the fastness of polarization dephasing and carriers scattering, the exact linearized equations can be greatly simplified. Starting from the simplified equations, we will determine in Sec. IV the extracavity intensity and phase noise spectra. A formal comparison between our results and the standard results of [1] is carried out in Sec. V, and allows to conclude that the noise-increasing mechanism rests on pump blocking. The physical interpretation of this mechanism and a final comment on the results of the paper can be found in the concluding Sec. VI.

II. MICROSCOPIOC DYNAMICAL EQUATIONS

We start our analysis from the set of operatorial Langevin equations for the laser intracavity field $\hat{A}$, the microscopic material polarization $\hat{\sigma}_k$, the carrier distribution $\hat{n}_{\alpha k}$ ($\alpha = e,h$ for electrons and holes, respectively) and the total carrier number in each band $\hat{N}_\alpha \equiv \Sigma \hat{n}_{\alpha k}$, which have been derived in [21] (see also [23])

$$\frac{d\hat{A}}{dt} = -[\kappa_m + \kappa_l + i(\Omega - \nu)]\hat{A} - i\sum_k g_k^* \hat{\sigma}_k + \hat{F}_{\hat{A}}^m + \hat{F}_{\hat{A}}^l, \tag{1a}$$

$$\frac{d\hat{\sigma}_k}{dt} = -[\gamma + i(\omega_k - \nu)]\hat{\sigma}_k + i g_k \hat{A}(\hat{n}_{ek} + \hat{n}_{hk} - 1) + \hat{F}_{\hat{\sigma}_k}, \tag{1b}$$

$$\frac{d\hat{n}_{\alpha k}}{dt} = \gamma_p \frac{f_{pk}}{N_p}(1-\hat{n}_{ek})(1-\hat{n}_{hk}) - B_k \hat{n}_{ek}\hat{n}_{hk} - \gamma(\hat{n}_{\alpha k} - \hat{f}_{\alpha k}) + i(g_k^* \hat{A}^\dagger \hat{\sigma}_k - g_k \hat{\sigma}_k^\dagger \hat{A}) + \hat{F}_{\hat{n}_{\alpha k}}, \tag{1c}$$

$$\frac{d\hat{N}_\alpha}{dt} = \sum_k \gamma_p \frac{f_{pk}}{N_p}(1-\hat{n}_{ek})(1-\hat{n}_{hk}) - \sum_k B_k \hat{n}_{ek}\hat{n}_{hk} + i\sum_k (g_k^* \hat{A}^\dagger \hat{\sigma}_k - g_k \hat{\sigma}_k^\dagger \hat{A}) + \hat{F}_{\hat{N}_\alpha}. \tag{1d}$$

For generality we take into account the possibility of optical losses others than those due to the outcoupling mirror, so that the total field-decay rate is given by the sum

$$\kappa \equiv \kappa_m + \kappa_l, \tag{2}$$

where κ_m and κ_l are the field-decay rates associated with the mirror and the additional losses, respectively. Accordingly, the total field noise operator $\hat{F}_{\hat{A}}$ is defined as the sum of two uncorrelated noise operators $\hat{F}_{\hat{A}}^m$ and $\hat{F}_{\hat{A}}^l$

$$\hat{F}_{\hat{A}} \equiv \hat{F}_{\hat{A}}^m + \hat{F}_{\hat{A}}^l \tag{3}$$

associated with the two independent loss sources, respectively.

Let us briefly recall the meaning of the other symbols: Ω is the cold-cavity mode frequency, ν is the (unknown) laser field frequency, g_k is the light-matter coupling coefficient, γ is the polarization decay rate and the carrier scattering rate, ω_k is the frequency associated with the k-vector electron-hole pair recombination, B_k is the time rate associated with spontaneous emission, and $\hat{f}_{\alpha k}$ is the operator that describes the quasiequilibrium Fermi-Dirac distribution in each band. The noise operators $\hat{F}_{\hat{O}}$ associated with the operators of Eqs. (1) are assumed to be zero averaged and δ correlated in time.

The pump term, which is the first one on the right-hand side of Eq. (1d), deserves a more detailed comment: γ_p is the pump parameter, f_{pk} the spectral distribution of the pump carriers, and $N_p = \Sigma_k f_{pk}$ their total number. In our theoretical derivation we will always treat f_{pk} as an arbitrary function, so that all the formulas presented in the paper hold for any choice of the pumped carrier distribution. Only in the figures, where we must fix a particular distribution to exemplify the predictions of the theory, have we decided to follow the most common choice reported by the literature, i.e., that of a Fermi-Dirac distribution [23,24].

The explicit expressions of ω_k and g_k depend on the active medium. We consider a GaAs quantum well of thickness L_z and transverse dimensions L_x and L_y, and for simplicity we assume a single longitudinally bound state for both the electrons and the heavy holes, neglecting the confinement energies with respect to the band gap, and taking for electrons and holes the same average mass $m = 0.09 m_0$, where m_0 is the free electron mass. The conduction and valence bands E_k are separated by an energy gap $E_g = 1.424$ eV, and the energy associated with each electron-hole recombination event is

$$\hbar\omega_k = E_g + 2E_k = E_g + 2\frac{\hbar^2}{2m}k^2, \tag{4}$$

where $k=(k_x,k_y)$ is the transverse carrier wave vector. The light-matter coupling coefficient g_k has the dimensions of $(\text{time})^{-1}$ and is given by [25]

$$g_k=\frac{1}{\hbar}\left(\frac{\hbar\Omega c_f}{\epsilon_0 n^2 V}\right)^{1/2}\times\left\{\frac{E_g(E_g+\Delta)}{2\left(E_g+\frac{2}{3}\Delta\right)}\frac{\hbar^2}{2m_0}\left(\frac{m_0}{m_e}-1\right)\frac{e^2}{(\hbar\omega_k)^2}\right\}^{1/2}, \quad (5)$$

where $0<c_f\leqslant 1$ is the confinement factor, i.e. the fraction of the optical mode volume that overlaps with the active volume $V=L_xL_yL_z$. The fixed parameters are the GaAs refractive index $n=3.5$, the energy separation between the heavy hole and the split-off bands $\Delta=0.34$ eV, the electron mass in the material $m_e=0.065m_0$. The universal constants are

$$\hbar\approx(0.658\ \text{eV})(10^{-15}\ \text{s}),$$

$$\epsilon_0\approx(5.52\times10^{-3}\ \text{eV})(10^{-10}\ \text{m})^{-1}\ \text{V}^{-2},$$

$$\hbar^2/(2m_0)\approx(3.81\ \text{eV})(10^{-10}\ \text{m})^2,$$

and $e=1$ eV/V.

To determine the laser stationary state it is useful to introduce the intensity operator $\hat{I}$ and the average carrier number operator $\hat{N}$

$$\hat{I}=\hat{A}^\dagger\hat{A}, \quad (6a)$$

$$\hat{N}=(\hat{N}_e+\hat{N}_h)/2, \quad (6b)$$

and to define the expectation values

$$I=\langle\hat{I}\rangle, \quad (7a)$$

$$\sigma_k=\langle\hat{\sigma}_k\rangle, \quad (7b)$$

$$n_k=\langle\hat{n}_{ek}\rangle=\langle\hat{n}_{hk}\rangle, \quad (7c)$$

$$f_k=\langle\hat{f}_{ek}\rangle=\langle\hat{f}_{hk}\rangle, \quad (7d)$$

$$N=\langle\hat{N}\rangle. \quad (7e)$$

The stationary state is determined by the semiclassical counterpart of Eqs. (1): we have to take the expectation values of all the terms, factorize them, and set to zero all the time derivatives. First of all we set to zero the time derivative of the polarization σ_k, obtaining

$$\frac{d\langle\hat{A}\rangle}{dt}=\left[-\kappa+\sum_k\frac{\gamma}{\gamma^2+\Delta_k^2}|g_k|^2(2n_k-1)\right]\langle\hat{A}\rangle-i\left[(\Omega-\nu)+\sum_k\frac{\Delta_k}{\gamma^2+\Delta_k^2}|g_k|^2(2n_k-1)\right]\langle\hat{A}\rangle, \quad (8a)$$

$$\frac{dn_k}{dt}=\gamma_p\frac{f_{pk}}{N_p}(1-n_k)^2-B_kn_k^2-\gamma(n_k-f_k)-\frac{2\gamma}{\gamma^2+\Delta_k^2}I|g_k|^2(2n_k-1), \quad (8b)$$

$$\frac{dN}{dt}=\gamma_p\sum_k\frac{f_{pk}}{N_p}(1-n_k)^2-\sum_k B_kn_k^2-2\kappa I, \quad (8c)$$

where

$$\Delta_k\equiv\omega_k-\nu. \quad (9)$$

In [21] it has already been explained in detail how the nonlinear set of equation, Eqs. (8), can be solved via a numerical iterative procedure to give the exact stationary solution for the laser frequency ν, the carrier distribution n_k, and the intracavity photon number I. Once the exact solution is known, it can be verified that it can be approximated extremely well by the quasiequilibrium solution obtained assuming that in each band the carriers follow the Fermi-Dirac distribution f_k; this follows from the fact that in Eq. (8b) the scattering term proportional to γ is by far much larger than all the others [21]. The quasiequilibrium stationary solution is obtained by solving the nonlinear system given by the real and imaginary part of Eq. (8a)

$$\sum_k\frac{\gamma}{\gamma^2+\Delta_k^2}|g_k|^2(2f_k-1)=\kappa, \quad (10a)$$

$$\sum_k\frac{\Delta_k}{\gamma^2+\Delta_k^2}|g_k|^2(2f_k-1)=(\Omega-\nu). \quad (10b)$$

Let us now recall that f_k depends on N via the chemical potential $\mu(N)$

$$f_k=\frac{1}{1+e^{\beta[E_k-\mu(N)]}}, \quad (11)$$

where in the quantum well case it results [23]

$$\beta\mu=\ln\left[\exp\left(2\pi\beta\frac{\hbar^2}{2m}\frac{N}{L_xL_y}\right)-1\right], \quad (12)$$

with $\beta=1/K_BT$, $K_BT=0.025$ eV at room temperature. Consequently, our unknown variables in Eq. (10) are the superficial carrier density $N/(L_xL_y)$ and the laser frequency ν. We now define N_s and ν_s as the stationary values of the carrier number and of the laser frequency. Our numerical solutions show that, for any values of the laser parameters, ν_s is very close to Ω. On the other side, the dependence of $N_s/(L_xL_y)$ on κ, γ, L_z, $1/c_f$, and $\lambda_c=2\pi c/\Omega$ has been represented in Figs. 1(a)–1(e), respectively.

Once the stationary values N_s and ν_s are determined, we obtain the stationary value of the photon number in the cavity

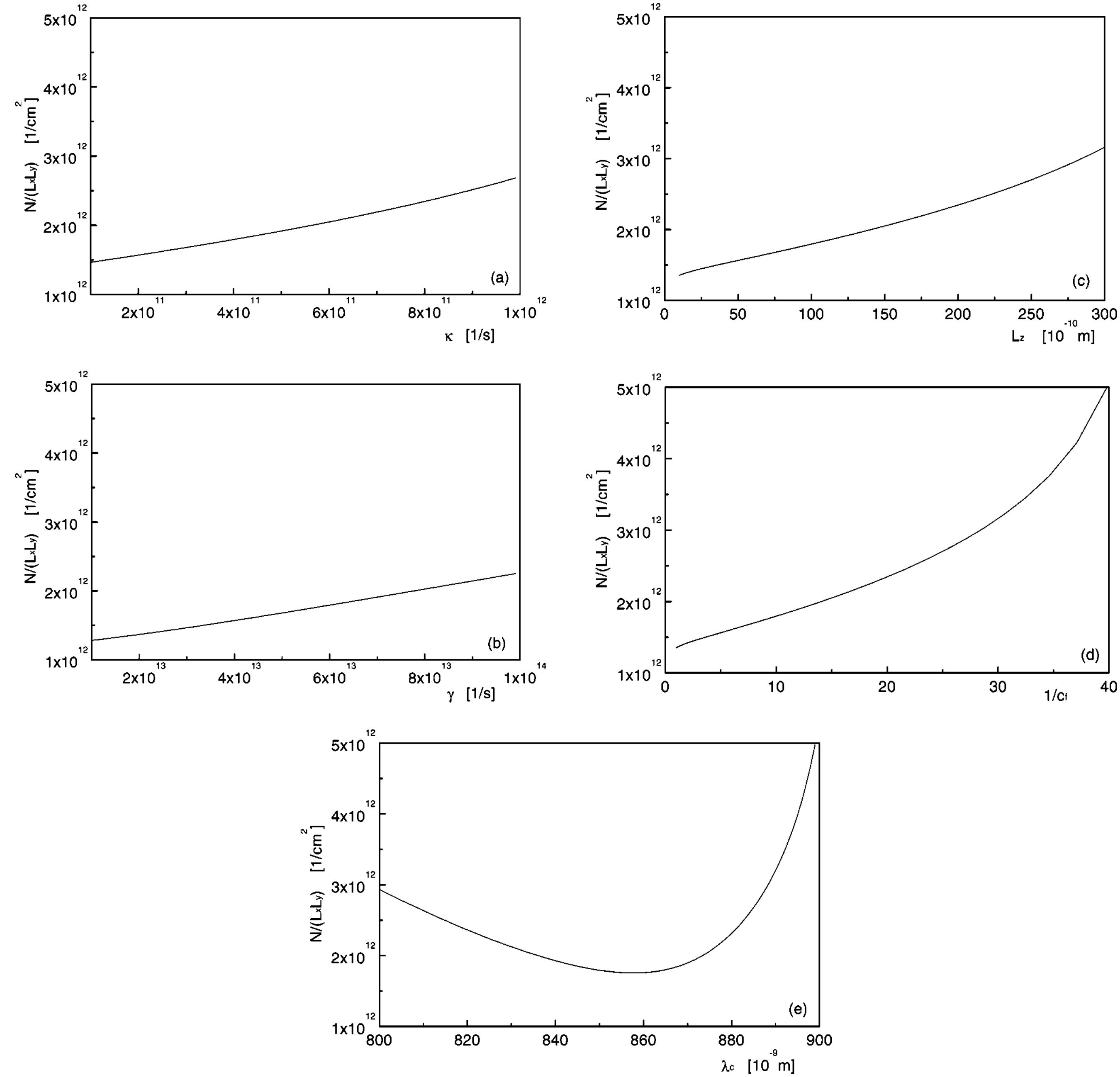

FIG. 1. Dependence of the stationary value of the superficial carrier density $N_s/(L_xL_y)$ on (a) the field decay rate κ, (b) the polarization decay rate γ, (c) the quantum well thickness L_z, (d) the reciprocal confinement factor $1/c_f$, and (e) the empty cavity eigenmode wavelength λ_c. The fixed parameters are $\kappa = 4\cdot 10^{11}\ \mathrm{s}^{-1}$, $\gamma = 6\times 10^{13}\ \mathrm{s}^{-1}$, $L_z = 100\times 10^{-10}$ m, $c_f = 0.1$, $\lambda_c = 850\times 10^{-9}$ m, and for this choice the carrier density results $N_s/(L_xL_y) \approx 1.8\times 10^{12}\ \mathrm{cm}^{-2}$.

$$I_s = \frac{\gamma_p \sum_k \frac{f_{pk}}{N_p}(1-f_k)^2 - \sum_k B_k f_k^2}{2\kappa} \tag{13}$$

and of the microscopic polarization

$$\sigma_k = \frac{i g_k \sqrt{I_s}(2f_k - 1)}{\gamma + i\Delta_k}. \tag{14}$$

To write Eq. (14) we have used the semiclassical relation

$$I_s = \langle \hat{A}_s^\dagger \hat{A}_s \rangle \approx \langle \hat{A}_s^\dagger \rangle \langle \hat{A}_s \rangle, \tag{15}$$

which hold under the condition $I_s \gg 1$, and for definiteness we have set to zero the arbitrary phase of $\langle \hat{A}_s^\dagger \rangle$ and $\langle \hat{A}_s \rangle$, so that it results

$$\langle \hat{A}_s \rangle \approx \langle \hat{A}_s^\dagger \rangle \approx \sqrt{I_s}. \tag{16}$$

Note that in Eq. (13) and Eq. (14) we should have written, in a more rigorous notation, $(f_k)_s$, $(\sigma_k)_s$, and $(\Delta_k)_s = \omega_k - \nu_s$, but for the sake of simplicity the subscript ''s'' has been dropped. From now on, we will always mean with f_k, σ_k, and Δ_k the values assumed by these variables in the stationary state.

Let us now define the quantities

$$\mathcal{G}\equiv\sum_k \frac{2\gamma}{\gamma^2+\Delta_k^2} I_s|g_k|^2(2f_k-1), \tag{17a}$$

$$\mathcal{H}\equiv\sum_k \frac{2\Delta_k}{\gamma^2+\Delta_k^2} I_s|g_k|^2(2f_k-1), \tag{17b}$$

$$\mathcal{P}\equiv\sum_k \gamma_p\frac{f_{pk}}{N_p}(1-f_k)^2-\sum_k B_k f_k^2\equiv\Lambda-\Lambda_{th}, \tag{17c}$$

which will turn out to be very useful in the following. As can be seen from Eqs. (8), $\mathcal{G}/2I_s$ and $\mathcal{H}/2I_s$ are the carrier-induced amplitude gain and the carrier-induced dispersion, respectively, or in other terms the imaginary and the real part of the active-material susceptivity [23], while $\mathcal{P}$ is the number of carriers that are actually transformed into stimulated photons per unit time, and it is given by the difference between the pump rate Λ and the pump threshold Λ_{th}. On the basis of Eqs. (8) it is easy to verify that in the stationary state it results

$$\frac{\mathcal{G}}{2I_s}=\kappa, \tag{18a}$$

$$\frac{\mathcal{H}}{2I_s}=-(\Omega-\nu_s), \tag{18b}$$

$$\frac{\mathcal{P}}{2I_s}=\kappa, \tag{18c}$$

and consequently $\mathcal{G}=\mathcal{P}$.

We can now proceed with the linearization of the operatorial Eqs. (1) around the stationary state, obtaining

$$\delta\dot{\hat{A}}=-[\kappa+i(\Omega-\nu_s)]\delta\hat{A}-i\sum_k g_k^*\delta\hat{\sigma}_k+\hat{F}_{\hat{A}}, \tag{19a}$$

$$\delta\dot{\hat{\sigma}}_k=-[\gamma+i(\omega_k-\nu_s)]\delta\hat{\sigma}_k+ig_k(2f_k-1)\delta\hat{A} + ig_k\sqrt{I_s}(\delta\hat{n}_{ek}+\delta\hat{n}_{hk})+\hat{F}_{\hat{\sigma}_k}, \tag{19b}$$

$$\delta\dot{\hat{n}}_{\alpha k}=-\left[\gamma_p\frac{f_{pk}}{N_p}(1-f_k)\right](\delta\hat{n}_{ek}+\delta\hat{n}_{hk})-B_kf_k(\delta\hat{n}_{ek} + \delta\hat{n}_{hk})-\gamma(\delta\hat{n}_{\alpha k}-f_k'\delta\hat{N}_\alpha)+i\sqrt{I_s}(g_k^*\delta\hat{\sigma}_k-g_k\delta\hat{\sigma}_k^\dagger) + i(g_k^*\sigma_k\delta A^\dagger-g_k\sigma_k^\dagger\delta A)+\hat{F}_{\hat{n}_{\alpha k}}, \tag{19c}$$

$$\delta\dot{\hat{N}}_\alpha=-\sum_k\left[\gamma_p\frac{f_{pk}}{N_p}(1-f_k)\right](\delta\hat{n}_{ek}+\delta\hat{n}_{hk}) -\sum_k B_kf_k(\delta\hat{n}_{ek}+\delta\hat{n}_{hk})+i\sqrt{I_s}\sum_k(g_k^*\delta\hat{\sigma}_k-g_k\delta\hat{\sigma}_k^\dagger) +i\sum_k(g_k^*\sigma_k\delta A^\dagger-g_k\sigma_k^\dagger\delta A)+\hat{F}_{\hat{N}_\alpha}. \tag{19d}$$

It can be noted that in this set of equations we have macroscopic variables ($\delta\hat{A}$ and $\delta\hat{N}_\alpha$) and microscopic variables ($\delta\hat{\sigma}_k$ and $\delta\hat{n}_{\alpha k}$). The microscopic variables evolve much faster than the macroscopic ones, due to the presence of the polarization decay rate and of the carrier scattering rate γ in Eqs. (19b,c). We can therefore adiabatically eliminate Eqs. (19b,c) without losing anything of the physics of the problem. This will be done in the next section.

III. LINEARIZED MACROSCOPIC DYNAMICAL EQUATIONS

To calculate the phase noise spectrum, we have to introduce the linearized fluctuating operator $\delta\hat{\phi}$. It is well known that the definition of an Hermitian operator associated with the phase variable is still an open question in quantum mechanics, and up to date has found only *ad hoc* solutions. The convenience to use one or the other of these ''operative'' definitions depends on the particular problem under investigation, as reviewed, for example, in [26]. Our definition of the linearized phase operator follows from a simple and commonly used procedure (see [27]), which will be briefly described in the following. Let us start with the usual decomposition

$$\hat{A}=\langle\hat{A}_s\rangle+\delta\hat{A}, \tag{20a}$$

$$\hat{A}^\dagger=\langle\hat{A}_s^\dagger\rangle+\delta\hat{A}^\dagger, \tag{20b}$$

where $\langle\hat{A}_s\rangle$ and $\langle\hat{A}_s^\dagger\rangle$ indicate the stationary values of the field operators, and recall the semiclassical relations

$$\langle\hat{A}\rangle=\sqrt{I}e^{i\phi/2}, \tag{21a}$$

$$\langle\hat{A}^\dagger\rangle=\sqrt{I}e^{-i\phi/2}, \tag{21b}$$

which are certainly valid at high intensity. According to Eqs. (21), the semiclassical variable ϕ can be written as

$$\phi=i(\ln\langle\hat{A}^\dagger\rangle-\ln\langle\hat{A}\rangle), \tag{22}$$

and substituting Eqs. (20) into Eqs. (22), we obtain

$$\delta\phi=\frac{i}{\sqrt{I_s}}(\langle\delta\hat{A}^\dagger\rangle-\langle\delta\hat{A}\rangle), \tag{23}$$

where the arbitrary stationary value of the field phase has been set to zero, so that $\langle\hat{A}_s\rangle\approx\langle\hat{A}_s^\dagger\rangle\approx\sqrt{I_s}$. Since the semiclassical relation Eq. (23) is linear, it can be safely extended into an operatorial one, which will be used as definition of the linearized phase operator $\delta\hat{\phi}$. The expressions of the linearized operators $\delta\hat{I}$ and $\delta\hat{N}$ straightfully follows from Eqs. (6), so that in the end our macroscopic linearized operators will be given by

$$\delta\hat{I} \equiv \sqrt{I_s}(\delta\hat{A}^{\dagger} + \delta\hat{A}), \tag{24a}$$

$$I_s\delta\hat{\phi} \equiv i\sqrt{I_s}(\delta\hat{A}^{\dagger} - \delta\hat{A}), \tag{24b}$$

$$\delta\hat{N} \equiv (\delta\hat{N}_e + \delta\hat{N}_h)/2, \tag{24c}$$

where the stationary value of the intensity I_s is determined by Eq. (13).

In [1] the linearized macroscopic operators under analysis are the field amplitude, the field phase, and the carrier number. Instead of the field amplitude and phase we will use $\delta\hat{I}$ and $I_s\delta\hat{\phi}$, that is the quadrature components of the linearized intensity operator. This choice has the advantages that the dynamical equations for the quadrature components are characterized by a higher degree of symmetry, and that the intensity and phase noise spectra can be determined independently.

Adiabatically eliminating from Eqs. (19) the fast-evolving variables $\delta\hat{\sigma}_k$ and $\delta\hat{n}_{\alpha k}$ we are left with the macroscopic linearized set of equations

$$\delta\dot{\hat{I}} = -\sum_k \frac{a_k c_k}{b_k}\delta\hat{I} + \sum_k \frac{a_k\gamma f_k'}{b_k}\delta\hat{N} + \sqrt{I_s}(\hat{F}_{\hat{A}^{\dagger}} + \hat{F}_{\hat{A}}) + i\sqrt{I_s}\sum_k (g_k h_k^*\hat{F}_{\hat{\sigma}_k^{\dagger}} - g_k^* h_k\hat{F}_{\hat{\sigma}_k}) - i\sqrt{I_s}\sum_k \frac{a_k}{b_k}(g_k h_k^*\hat{F}_{\hat{\sigma}_k^{\dagger}} - g_k^* h_k\hat{F}_{\hat{\sigma}_k}) + \sum_k \frac{a_k}{b_k}\left(\frac{\hat{F}_{\hat{n}_{ek}} + \hat{F}_{\hat{n}_{hk}}}{2}\right), \tag{25a}$$

$$I_s\delta\dot{\hat{\phi}} = +\sum_k \frac{\tilde{a}_k c_k}{b_k}\delta\hat{I} - \sum_k \frac{\tilde{a}_k\gamma f_k'}{b_k}\delta\hat{N} + i\sqrt{I_s}(\hat{F}_{\hat{A}^{\dagger}} - \hat{F}_{\hat{A}}) - \sqrt{I_s}\sum_k (g_k h_k^*\hat{F}_{\hat{\sigma}_k^{\dagger}} + g_k^* h_k\hat{F}_{\hat{\sigma}_k}) + i\sqrt{I_s}\sum_k \frac{\tilde{a}_k}{b_k}(g_k h_k^*\hat{F}_{\hat{\sigma}_k^{\dagger}} - g_k^* h_k\hat{F}_{\hat{\sigma}_k}) - \sum_k \frac{\tilde{a}_k}{b_k}\left(\frac{\hat{F}_{\hat{n}_{ek}} + \hat{F}_{\hat{n}_{hk}}}{2}\right), \tag{25b}$$

$$\delta\dot{\hat{N}} = -\sum_k \frac{(b_k - \gamma)\gamma f_k'}{b_k}\delta\hat{N} - \sum_k \frac{\gamma c_k}{b_k}\delta\hat{I} - i\sqrt{I_s}\sum_k \frac{\gamma}{b_k}(g_k h_k^*\hat{F}_{\hat{\sigma}_k^{\dagger}} - g_k^* h_k\hat{F}_{\hat{\sigma}_k}) + \sum_k \frac{\gamma}{b_k}\left(\frac{\hat{F}_{\hat{n}_{ek}} + \hat{F}_{\hat{n}_{hk}}}{2}\right), \tag{25c}$$

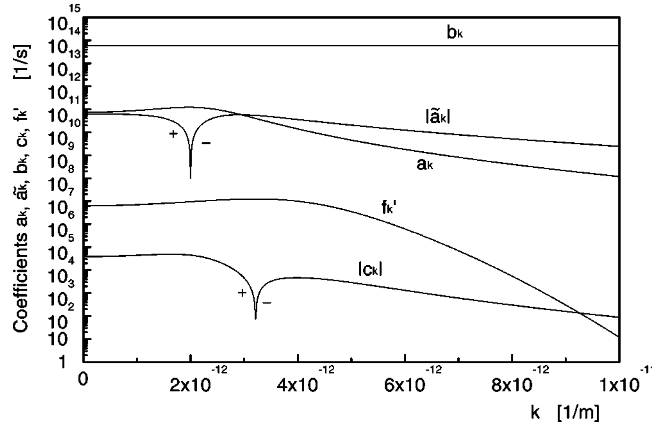

FIG. 2. Functional dependence of the coefficients a_k, $\tilde{a}_k$, b_k, c_k, and f_k' on the wave vector k, with $N_p = N_s$ and $\Lambda \approx 20\Lambda_{th}$. The other parameters have the same values as in Fig. 1.

where $h_k \equiv 1/(\gamma + i\Delta_k)$, and the coefficients a_k, $\tilde{a}_k$, b_k, and c_k are, respectively, defined by

$$a_k = \frac{2\gamma}{\gamma^2 + \Delta_k^2} 2I_s|g_k|^2, \tag{26a}$$

$$\tilde{a}_k = \frac{2\Delta_k}{\gamma^2 + \Delta_k^2} 2I_s|g_k|^2, \tag{26b}$$

$$b_k = \gamma + 2B_k f_k + 2\gamma_p \frac{f_{pk}}{N_p}(1 - f_k) + \frac{2\gamma}{\gamma^2 + \Delta_k^2} 2I_s|g_k|^2, \tag{26c}$$

$$c_k = \frac{2\gamma}{\gamma^2 + \Delta_k^2}|g_k|^2(2f_k - 1). \tag{26d}$$

These coefficients have been plotted, together with f_k', in Fig. 2. At a first glance it is apparent that b_k is the largest among them, and it is practically independent of the wave vector k. This is because, in semiconductor lasers the scattering rate γ is by far larger than any other time rate. We can therefore define the vanishingly small parameters

$$\epsilon_{ak} \equiv \frac{a_k}{\gamma} \ll 1, \quad \epsilon_{\tilde{a}k} \equiv \frac{\tilde{a}_k}{\gamma} \ll 1, \quad \epsilon_{bk} \equiv \frac{b_k}{\gamma} - 1 \ll 1,$$

$$\epsilon_{ck} \equiv \frac{c_k}{\gamma} \ll 1, \tag{27}$$

and use them to rewrite the various terms that appear in Eqs. (25). We see that the following approximations can be done.

$$\frac{a_k c_k}{b_k} = \frac{a_k \epsilon_{ck}}{1 + \epsilon_{bk}} \approx a_k \epsilon_{ck} \approx 0, \tag{28a}$$

$$\frac{a_k \gamma f_k'}{b_k} = \frac{a_k f_k'}{1 + \epsilon_{bk}} \approx a_k f_k', \tag{28b}$$

$$\frac{a_k}{b_k}=\frac{\epsilon_{ak}}{1+\epsilon_{bk}}\approx\epsilon_{ak}\approx 0, \tag{28c}$$

$$\frac{\tilde{a}_k c_k}{b_k}=\frac{\tilde{a}_k\epsilon_{ck}}{1+\epsilon_{bk}}\approx\tilde{a}_k\epsilon_{ck}\approx 0, \tag{28d}$$

$$\frac{\tilde{a}_k\gamma f'_k}{b_k}=\frac{\tilde{a}_k f'_k}{1+\epsilon_{bk}}\approx\tilde{a}_k f'_k, \tag{28e}$$

$$\frac{\tilde{a}_k}{b_k}=\frac{\epsilon_{\tilde{a}k}}{1+\epsilon_{bk}}\approx\epsilon_{\tilde{a}k}\approx 0, \tag{28f}$$

$$\frac{(b_k-\gamma)\gamma f'_k}{b_k}=\frac{(b_k-\gamma)f'_k}{1+\epsilon_{bk}}\approx(b_k-\gamma)f'_k, \tag{28g}$$

$$\frac{\gamma c_k}{b_k}=\frac{c_k}{1+\epsilon_{bk}}\approx c_k, \tag{28h}$$

$$\frac{\gamma}{b_k}=\frac{1}{1+\epsilon_{bk}}\approx 1. \tag{28i}$$

The only nonzero sums that remain in Eqs. (25) can be written in the form

$$\sum_k a_k f'_k=\mathcal{G}_N, \tag{29a}$$

$$\sum_k \tilde{a}_k f'_k=\mathcal{H}_N, \tag{29b}$$

$$\sum_k (b_k-\gamma)f'_k=-\mathcal{P}_N+\mathcal{G}_N, \tag{29c}$$

$$\sum_k c_k=2\kappa, \tag{29d}$$

where in Eqs. (29a,b,c) $\mathcal{G}_N$, $\mathcal{H}_N$, and $\mathcal{P}_N$ are the derivatives with respect to the carriers number N of the quantities defined by Eq. (17), calculated in the stationary state, while Eqs. (29d) is a direct consequence of the solution of the stationary state determined by Eq. (10a). The linearized set Eqs. (25) can therefore be rewritten in the much simpler form

$$\delta\dot{\hat{I}}=\mathcal{G}_N\delta\hat{N}+\sqrt{I_s}(\hat{F}_{\hat{A}^\dagger}+\hat{F}_{\hat{A}}) + i\sqrt{I_s}\sum_k (g_k h_k^*\hat{F}_{\hat{\sigma}_k^\dagger}-g_k^* h_k\hat{F}_{\hat{\sigma}_k}), \tag{30a}$$

$$I_s\delta\dot{\hat{\phi}}=-\mathcal{H}_N\delta\hat{N}+i\sqrt{I_s}(\hat{F}_{\hat{A}^\dagger}-\hat{F}_{\hat{A}}) - \sqrt{I_s}\sum_k (g_k h_k^*\hat{F}_{\hat{\sigma}_k^\dagger}+g_k^* h_k\hat{F}_{\hat{\sigma}_k}), \tag{30b}$$

$$\delta\dot{\hat{N}}=\mathcal{P}_N\delta\hat{N}-\mathcal{G}_N\delta\hat{N}-2\kappa\delta\hat{I}+\hat{F}_{\hat{N}} - i\sqrt{I_s}\sum_k (g_k h_k^*\hat{F}_{\hat{\sigma}_k^\dagger}-g_k^* h_k\hat{F}_{\hat{\sigma}_k}), \tag{30c}$$

where the definition

$$\hat{F}_{\hat{N}}\equiv\frac{\hat{F}_{\hat{N}_e}+\hat{F}_{\hat{N}_h}}{2} \tag{31}$$

has been used, in agreement with Eq. (6b) and Eq. (24c).

Let us now introduce the ratios [22]

$$\tau\equiv\left|\frac{\mathcal{P}_N}{\mathcal{G}_N}\right|=\frac{\sum_k\left\{\gamma_p\frac{f_{pk}}{N_p}(1-f_k)+B_k f_k\right\}f'_k}{\sum_k\left\{\frac{2\gamma}{\gamma^2+\Delta_k^2}I_s|g_k|^2\right\}f'_k}, \tag{32a}$$

$$\alpha\equiv\left|\frac{\mathcal{H}_N}{\mathcal{G}_N}\right|=\frac{\sum_k\left\{\frac{2\Delta_k}{\gamma^2+\Delta_k^2}|g_k|^2\right\}f'_k}{\sum_k\left\{\frac{2\gamma}{\gamma^2+\Delta_k^2}|g_k|^2\right\}f'_k}, \tag{32b}$$

and rewrite Eqs. (30) in the form

$$\delta\dot{\hat{I}}=\mathcal{G}_N\delta\hat{N}+\sqrt{I_s}(\hat{F}_{\hat{A}^\dagger}+\hat{F}_{\hat{A}}) + i\sqrt{I_s}\sum_k (g_k h_k^*\hat{F}_{\hat{\sigma}_k^\dagger}-g_k^* h_k\hat{F}_{\hat{\sigma}_k}), \tag{33a}$$

$$I_s\delta\dot{\hat{\phi}}=\alpha\mathcal{G}_N\delta\hat{N}+i\sqrt{I_s}(\hat{F}_{\hat{A}^\dagger}-\hat{F}_{\hat{A}}) - \sqrt{I_s}\sum_k (g_k h_k^*\hat{F}_{\hat{\sigma}_k^\dagger}+g_k^* h_k\hat{F}_{\hat{\sigma}_k}), \tag{33b}$$

$$\delta\dot{\hat{N}}=-\tau\mathcal{G}_N\delta\hat{N}-\mathcal{G}_N\delta\hat{N}-2\kappa\delta\hat{I}+\hat{F}_{\hat{N}} - i\sqrt{I_s}\sum_k (g_k h_k^*\hat{F}_{\hat{\sigma}_k^\dagger}-g_k^* h_k\hat{F}_{\hat{\sigma}_k}), \tag{33c}$$

which will be the basis for the determination of the intensity and phase noise spectra. These equations are the equivalent of Eqs. (A1)–(A3) in the Appendix of [1]. In Eq. (33b) α is the nonlinear dispersion parameter, and it transfers carriers fluctuations into phase fluctuations. In the same way the pump-blocking parameter τ transfers carriers fluctuations into pump fluctuations and then into carrier fluctuations again. If we have, for example, $\delta\hat{N}>0$ then the pump-blocking effect increases, less pump carriers will enter the active layer, and $\delta\hat{N}$ will diminish: all this process is taken into account by the term $-\tau\mathcal{G}_N\delta\hat{N}$ in Eq. (33c). In the following section we will show the impact of τ and α on the intensity and phase noise spectra.

IV. EXTRACAVITY INTENSITY AND PHASE NOISE SPECTRA

To determine the intensity and noise spectra of the laser we proceed as in [21], that is, we Fourier transform Eqs. (33) and we obtain the frequency-dependent fluctuations $\delta\hat{I}(\omega)$ and $I_s\delta\hat{\phi}(\omega)$ inside the cavity. These turn out to be

$$\delta\hat{I}(\omega)=\left(\frac{i\omega+\mathcal{G}_N(1+\tau)}{2\kappa\mathcal{G}_N-\omega^2+i\omega\mathcal{G}_N(1+\tau)}\right)\sqrt{I_s}(\hat{F}_{\hat{A}^\dagger}+\hat{F}_{\hat{A}})+\left(\frac{i\omega+\tau\mathcal{G}_N}{2\kappa\mathcal{G}_N-\omega^2+i\omega\mathcal{G}_N(1+\tau)}\right)i\sqrt{I_s}\sum_k(g_kh_k^*\hat{F}_{\hat{\sigma}_k^\dagger}-g_k^*h_k\hat{F}_{\hat{\sigma}_k})+\left(\frac{\mathcal{G}_N}{2\kappa\mathcal{G}_N-\omega^2+i\omega\mathcal{G}_N(1+\tau)}\right)\hat{F}_{\hat{N}},\tag{34a}$$

$$I_s\delta\hat{\phi}(\omega)=\left(\frac{1}{i\omega}\right)i\sqrt{I_s}(\hat{F}_{\hat{A}^\dagger}-\hat{F}_{\hat{A}})+\left(\frac{1}{i\omega}\right)\times\left(\frac{\alpha 2\kappa\mathcal{G}_N}{2\kappa\mathcal{G}_N-\omega^2+i\omega\mathcal{G}_N(1+\tau)}\right)\sqrt{I_s}(\hat{F}_{\hat{A}^\dagger}+\hat{F}_{\hat{A}})-\left(\frac{1}{i\omega}\right)\sqrt{I_s}\sum_k(g_kh_k^*\hat{F}_{\hat{\sigma}_k^\dagger}+g_k^*h_k\hat{F}_{\hat{\sigma}_k})+\left(\frac{1}{i\omega}\right)\times\left(\frac{\alpha\mathcal{G}_N(2\kappa+i\omega)}{2\kappa\mathcal{G}_N-\omega^2+i\omega\mathcal{G}_N(1+\tau)}\right)i\sqrt{I_s}\sum_k(g_kh_k^*\hat{F}_{\hat{\sigma}_k^\dagger}-g_k^*h_k\hat{F}_{\hat{\sigma}_k})-\left(\frac{\alpha\mathcal{G}_NI_s}{2\kappa\mathcal{G}_N-\omega^2+i\omega\mathcal{G}_N(1+\tau)}\right)\hat{F}_{\hat{N}}.\tag{34b}$$

For clarity, the contributions of the various noise operators have been written in different lines. Note that, due to the presence of α, the phase noise receives contributions not only from the noise operators appearing in Eq. (33b), but also from those appearing in Eq. (33a) and Eq. (33c).

The field input-output relation is given by [28]

$$\hat{A}^{out}=\sqrt{2\kappa_m}\hat{A}-\frac{\hat{F}^m_{\hat{A}}}{\sqrt{2\kappa_m}},\tag{35}$$

and its linearization gives

$$\delta\hat{A}^{out}=\sqrt{2\kappa_m}\delta\hat{A}-\frac{\hat{F}^m_{\hat{A}}}{\sqrt{2\kappa_m}}.\tag{36}$$

In terms of the quadrature components of the intensity, and defining the output photon flux as $I^{out}=2\kappa_mI_s$, we obtain

$$\delta\hat{I}^{out}=\sqrt{I^{out}}[(\delta\hat{A}^{out})^\dagger+\delta\hat{A}^{out}]=2\kappa_m\delta\hat{I}-\hat{F}^m_{r\hat{I}},\tag{37a}$$

$$I_s\delta\hat{\phi}^{out}=i\sqrt{I^{out}}[(\delta\hat{A}^{out})^\dagger-\delta\hat{A}^{out}]=2\kappa_mI_s\delta\hat{\phi}-\hat{F}^m_{i\hat{I}},\tag{37b}$$

where for brevity we have defined

$$\hat{F}^m_{r\hat{I}}=\sqrt{I_s}(\hat{F}^m_{\hat{A}^\dagger}+\hat{F}^m_{\hat{A}}),\tag{38a}$$

$$\hat{F}^m_{i\hat{I}}=i\sqrt{I_s}(\hat{F}^m_{\hat{A}^\dagger}-\hat{F}^m_{\hat{A}}).\tag{38b}$$

The output spectra, normalized with respect to the standard quantum limit $2\kappa_mI_s$, are determined by the relations

$$S_I(\omega)=\frac{1}{2\kappa_mI_s}\frac{1}{2\pi}\{\langle[\delta\hat{I}^{\rm out}(\omega)]^\dagger\delta\hat{I}^{\rm out}(\omega)\rangle\}=\frac{1}{2\kappa_mI_s}\frac{1}{2\pi}\{4\kappa_m^2\langle[\delta\hat{I}(\omega)]^\dagger\delta\hat{I}(\omega)\rangle-2\kappa_m\{\langle[\delta\hat{I}(\omega)]^\dagger\hat{F}^m_{r\hat{I}}(\omega)\rangle+\langle[\hat{F}^m_{r\hat{I}}(\omega)]^\dagger\delta\hat{I}(\omega)\rangle\}+\langle[\hat{F}^m_{r\hat{I}}(\omega)]^\dagger\hat{F}^m_{r\hat{I}}(\omega)\rangle\},\tag{39a}$$

$$S_\phi(\omega)=\frac{1}{2\kappa_mI_s}\frac{1}{2\pi}\{\langle[I_s\delta\hat{\phi}^{\rm out}(\omega)]^\dagger I_s\delta\hat{\phi}^{\rm out}(\omega)\rangle\}=\frac{1}{2\kappa_mI_s}\frac{1}{2\pi}\{4\kappa_m^2\langle[I_s\delta\hat{\phi}(\omega)]^\dagger I_s\delta\hat{\phi}(\omega)\rangle-2\kappa_m\{[\langle[I_s\delta\hat{\phi}(\omega)]^\dagger\hat{F}^m_{i\hat{I}}(\omega)\rangle+\langle[\hat{F}^m_{i\hat{I}}(\omega)]^\dagger I_s\delta\hat{\phi}(\omega)\rangle\}+\langle[\hat{F}^m_{i\hat{I}}(\omega)]^\dagger\hat{F}^m_{i\hat{I}}(\omega)\rangle\}.\tag{39b}$$

The three terms appearing on the right-hand side of Eqs. (39a,b) represent, in the order, the noise contribution of (i) the internal fluctuations exiting from the outcoupling mirror, (ii) the interference between the exiting internal fluctuations and the external zero-point fluctuations reflected by the mirror, (iii) the external fluctuations reflected by the mirror. For the intensity noise spectrum we have

$$\frac{1}{2\kappa_mI_s}\frac{1}{2\pi}\{4\kappa_m^2\langle[\delta\hat{I}(\omega)]^\dagger\delta\hat{I}(\omega)\rangle\}=\frac{2\kappa_m}{I_s}\frac{[\omega^2+(1+\tau)^2\mathcal{G}_N^2]\mathcal{D}^f+[\omega^2+\tau^2\mathcal{G}_N^2]\mathcal{D}^p+\mathcal{G}_N^2\mathcal{D}^c}{(2\kappa\mathcal{G}_N-\omega^2)^2+\omega^2\mathcal{G}_N^2(1+\tau)^2}\tag{40a}$$

$$\frac{1}{2\kappa_mI_s}\frac{1}{2\pi}\{-2\kappa_m\{\langle[\delta\hat{I}(\omega)]^\dagger\hat{F}^m_{r\hat{I}}(\omega)\rangle+\langle[\hat{F}^m_{r\hat{I}}(\omega)]^\dagger\delta\hat{I}(\omega)\rangle\}\}=-\frac{2\kappa_m}{I_s}\frac{2(1+\tau)\mathcal{G}_N^2\mathcal{D}^f}{(2\kappa\mathcal{G}_N-\omega^2)^2+\omega^2\mathcal{G}_N^2(1+\tau)^2}\tag{40b}$$

$$\frac{1}{2\kappa_mI_s}\frac{1}{2\pi}\{\langle[\hat{F}^m_{r\hat{I}}(\omega)]^\dagger\hat{F}^m_{r\hat{I}}(\omega)\rangle\}=1\tag{40c}$$

from which we can see that the noise exiting from the cavity gives a positive contribution to the spectrum, and the noise reflected by the mirror interferes partially destructively with it. In the phase noise case we have

$$\frac{1}{2\kappa_m I_s}\frac{1}{2\pi}\{4\kappa_m^2\langle[I_s\delta\hat{\phi}(\omega)]^\dagger I_s\delta\hat{\phi}(\omega)\rangle\} = \frac{2\kappa_m}{I_s\omega^2}\Bigg\{(\mathcal{D}^f+\mathcal{D}^p)+\alpha^2\mathcal{G}_N^2 \times\frac{(2\kappa)^2\mathcal{D}^f+[\omega^2+(2\kappa)^2]\mathcal{D}^p+\omega^2\mathcal{D}^c}{(2\kappa\mathcal{G}_N-\omega^2)^2+\omega^2\mathcal{G}_N^2(1+\tau)^2}\Bigg\} \tag{41a}$$

$$\frac{1}{2\kappa_m I_s}\frac{1}{2\pi}\{-2\kappa_m\{\langle[I_s\delta\hat{\phi}(\omega)]^\dagger\hat{F}_{i\hat{I}}^m(\omega)\rangle +\langle[\hat{F}_{i\hat{I}}^m(\omega)]^\dagger I_s\delta\hat{\phi}(\omega)\rangle\}\}=0 \tag{41b}$$

$$\frac{1}{2\kappa_m I_s}\frac{1}{2\pi}\{\langle[\hat{F}_{i\hat{I}}^m(\omega)]^\dagger\hat{F}_{i\hat{I}}^m(\omega)\rangle\}=1 \tag{41c}$$

and we can see that there is not any destructive interference, so that the output phase spectra is essentially identical to the phase spectra inside the laser cavity.

The noise coefficients for the field $\mathcal{D}^f$, the polarization $\mathcal{D}^p$, and the carriers $\mathcal{D}^c$, have been determined in [21]. They are given by

$$\mathcal{D}^f=\mathcal{D}_m^f+\mathcal{D}_l^f=2\kappa_m I_s+2\kappa_l I_s=2\kappa I_s, \tag{42a}$$

$$\mathcal{D}^p=\sum_k \frac{2\gamma}{\gamma^2+\Delta_k^2}I_s|g_k|^2(2f_k^2-2f_k+1), \tag{42b}$$

$$\mathcal{D}^c=\xi\sum_k \gamma_p\frac{f_{pk}}{N_p}(1-f_k)^2+\sum_k B_k f_k^2\equiv\xi\Lambda+\Lambda_{th}, \tag{42c}$$

where the parameter ξ is added *ex post* and must be set equal to 1 or to 0 in the case of Poissonian or quiet pump, respectively.

Summing up the three terms of Eq. (40) and of Eq. (41), we obtain, respectively,

$$S_I(\omega)=1+\frac{2\kappa_m}{I_s} \times\frac{[\omega^2-\mathcal{G}_N^2(1-\tau^2)]\mathcal{D}^f+[\omega^2+\tau^2\mathcal{G}_N^2]\mathcal{D}^p+\mathcal{G}_N^2\mathcal{D}^c}{(2\kappa\mathcal{G}_N-\omega^2)^2+\omega^2\mathcal{G}_N^2(1+\tau)^2}, \tag{43a}$$

$$S_\phi(\omega)=1+\frac{2\kappa_m}{I_s\omega^2}\Bigg\{(\mathcal{D}^f+\mathcal{D}^p)+\alpha^2\mathcal{G}_N^2 \times\frac{(2\kappa)^2\mathcal{D}^f+[\omega^2+(2\kappa)^2]\mathcal{D}^p+\omega^2\mathcal{D}^c}{(2\kappa\mathcal{G}_N-\omega^2)^2+\omega^2\mathcal{G}_N^2(1+\tau)^2}\Bigg\}. \tag{43b}$$

These expressions will be plotted in the next section, after having derived a formal comparison between Yamamoto's and ours results for the intensity and phase noise spectra.

V. COMPARISON WITH YAMAMOTO'S THEORY

Let us rewrite the final expressions obtained by Yamamoto's treatment of the intensity and phase spectra, that is Eq. (3.19) and Eq. (3.20) of [1]. In the original notation, we have

$$P_{\Delta\hat{r}}(\Omega)=\frac{\frac{\omega}{Q_e}[A_3^2\langle\Gamma_p^2+\Gamma_{sp}^2+\Gamma^2\rangle+(A_1^2+\Omega^2)\langle G_r^2+g_r^2\rangle-2A_1A_3\langle\Gamma G_r\rangle]+\left[\left(\frac{\omega}{Q_e}A_1-A_2A_3-\Omega^2\right)^2+\Omega^2\left(\frac{\omega}{Q_e}+A_1\right)^2\right]\frac{\langle f_r^2\rangle}{\frac{\omega}{Q_e}}}{[(A_2A_3-\Omega^2)^2+A_1^2\Omega^2]}, \tag{44a}$$

$$P_{\Delta\hat{\psi}}(\Omega)=\frac{\frac{\omega}{Q_e}}{r_0^2}\left\{\left[\left(\frac{\omega}{Q_e}\right)^2\right]^{-1}+\frac{1}{\Omega^2}\right\}+\frac{\frac{\omega}{Q_e}}{r_0^2\Omega^2}\langle G_i^2+g_i^2\rangle+\frac{A_4^2\left[\langle\Gamma_p^2+\Gamma_{sp}^2+\Gamma^2\rangle+\left(\frac{A_2}{\Omega}\right)^2\langle G_r^2+g_r^2+f_r^2\rangle\right]}{[(A_2A_3-\Omega^2)^2+A_1^2\Omega^2]}. \tag{44b}$$

With respect to the original, we have corrected the following misprints (i) in both Eq. (44a) and Eq. (44b) the first term in the denominator is $(A_2A_3-\Omega^2)^2$, and not $(A_2A_3+\Omega^2)^2$ (ii) in Eq. (44b) the term $A_4^2[\langle\Gamma_p^2+\Gamma_{sp}^2+\Gamma^2\rangle$ was missing, and has been added.

The coefficients A_1, A_2, A_3, A_4 are given by

$$A_1=\frac{1}{\tau_{sp}}+\frac{1}{\tau_{st}}, \tag{45a}$$

$$A_2=2\frac{\omega}{Q}, \tag{45b}$$

$$A_3=\frac{1}{2A_0\tau_{sp}}, \tag{45c}$$

$$A_4=\frac{\alpha}{2A_0^2\tau_{st}}. \tag{45d}$$

With the following connections between Yamamoto's and our notation

$$\omega/Q\rightarrow 2\kappa, \tag{46a}$$

$$\omega/Q_e\rightarrow 2\kappa_m, \tag{46b}$$

$$\langle f_r^2\rangle\rightarrow 2\kappa_m, \tag{46c}$$

$$\langle f_r^2+g_r^2\rangle\rightarrow 2\kappa, \tag{46d}$$

$$(2A_0)^2\rightarrow I_s, \tag{46e}$$

$$\langle\Gamma_p^2+\Gamma_{sp}^2\rangle\rightarrow\mathcal{D}^c, \tag{46f}$$

$$\langle\Gamma^2\rangle,(2A_0)^2\langle G_r^2\rangle,(2A_0)\langle\Gamma G_r\rangle\rightarrow\mathcal{D}^p, \tag{46g}$$

$$(2A_0)^2\langle f_r^2+g_r^2\rangle\rightarrow\mathcal{D}^f, \tag{46h}$$

$$\Omega\rightarrow\omega \tag{46i}$$

we obtain

$$P_{\Delta\hat{r}}(\omega)=1+\frac{2\kappa_m}{I_s}\frac{\left[\omega^2+\left(\frac{1}{\tau_{sp}}\right)^2-\left(\frac{1}{\tau_{st}}\right)^2\right]\mathcal{D}^f+\left[\omega^2+\left(\frac{1}{\tau_{sp}}\right)^2\right]\mathcal{D}^p+\left(\frac{1}{\tau_{st}}\right)^2\mathcal{D}^c}{\left(\frac{2\kappa}{\tau_{st}}-\omega^2\right)^2+\left(\frac{1}{\tau_{st}}+\frac{1}{\tau_{sp}}\right)^2\omega^2}, \tag{47a}$$

$$r_0^2P_{\Delta\hat{\psi}}(\omega)=1+\frac{2\kappa_m}{I_s\omega^2}\left\{(\mathcal{D}^f+\mathcal{D}^p)+\alpha^2\left(\frac{1}{\tau_{st}}\right)^2\frac{(2\kappa)^2\mathcal{D}^f+[(2k)^2+\omega^2]\mathcal{D}^p+\omega^2\mathcal{D}^c}{\left(\frac{2\kappa}{\tau_{st}}-\omega^2\right)^2+\left(\frac{1}{\tau_{st}}+\frac{1}{\tau_{sp}}\right)^2\omega^2}\right\}. \tag{47b}$$

Under the assumption that the photon absorption rate $\langle E_{vc}\rangle$ is much smaller than the stimulated emission rate $\langle E_{cv}\rangle$ the population inversion parameter n_{sp}, defined by Eq. (3.10) of [1], is ≈ 1, and one obtains

$$\frac{1}{\tau_{sp}}=\frac{x}{\tau_{st}}, \tag{48}$$

where

$$x\equiv\frac{p_{th}}{p-p_{th}}, \tag{49}$$

with p and p_{th} being, respectively, the pump level and the pump threshold. If we assume the correspondence

$$\frac{1}{\tau_{st}}\rightarrow\mathcal{G}_N, \tag{50}$$

and introduce Eq. (48) and Eq. (50) in Eqs. (47a,b), we obtain

$$P_{\Delta\hat{r}}(\omega)=1+\frac{2\kappa_m}{I_s}\times\frac{[\omega^2-\mathcal{G}_N^2(1-x^2)]\mathcal{D}^f+(\omega^2+x^2\mathcal{G}_N^2)\mathcal{D}^p+\mathcal{G}_N^2\mathcal{D}^c}{(2\kappa\mathcal{G}_N-\omega^2)^2+\omega^2\mathcal{G}_N^2(1+x)^2}, \tag{51a}$$

$$r_0^2P_{\Delta\hat{\psi}}(\omega)=1+\frac{2\kappa_m}{I_s\omega^2}\left\{(\mathcal{D}^f+\mathcal{D}^p)+\alpha^2\mathcal{G}_N^2\frac{(2\kappa)^2\mathcal{D}^f+[\omega^2+(2k)^2]\mathcal{D}^p+\omega^2\mathcal{D}^c}{(2\kappa\mathcal{G}_N-\omega^2)^2+\omega^2\mathcal{G}_N^2(1+x)^2}\right\}. \tag{51b}$$

Comparing our noise spectra [Eqs. (43)] and Yamamoto's ones [Eqs. (51)], we can see that a formal identity can be established between them if the correspondence

$$x\rightarrow\tau \tag{52}$$

is assumed. In our notation x can be rewritten as

$$x=\frac{\Lambda^{th}}{\Lambda-\Lambda^{th}}=\frac{\Lambda^{th}}{\mathcal{G}}, \tag{53}$$

while τ is defined as

$$\tau=\frac{|\Lambda_N-\Lambda_N^{th}|}{\mathcal{G}_N}. \tag{54}$$

We can therefore say that our model reduces to that of Yamamoto if τ reduces to x, and this happens if we assume: (i) linear dependence of the threshold and of the gain on the carrier number; (ii) no pump blocking. As already pointed

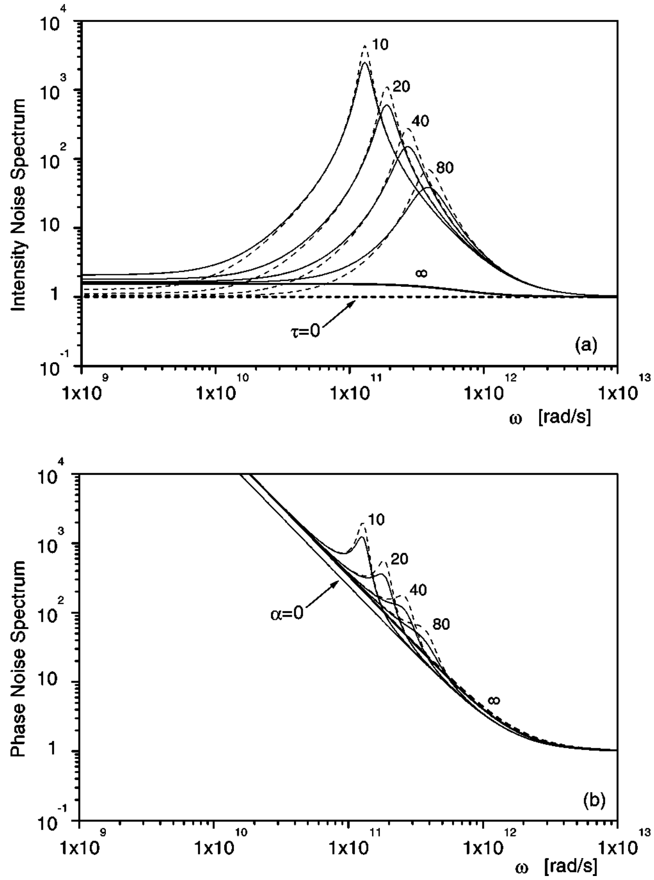

FIG. 3. Intensity (a) and phase (b) noise spectra for increasing value of the pump, with $N_p = N_s$, $\kappa_l = 0$, $\xi = 1$, and the other parameters as in Fig. 1. The full and dashed lines correspond to our Eqs. (43) and Yamamoto's Eqs. (51), respectively. The numbers 10, 20, 40, 80, and the symbol ∞ represent the ratio Λ/Λ_{th}.

out in [20], our treatment is different due to the presence of Λ_N in the numerator of Eq. (54), which prevents the vanishing of τ as the pumping increases, in sharp contrast with what happens to x. In Fig. 3 we have plotted the intensity and phase spectra given by our analysis and by Yamamoto's one for increasing value of the pumping. Note that the presence of pump blocking reduces the noise in correspondence of the carrier relaxation oscillation peak and, in the intensity case, increases the noise at the left side of the peak. In other terms, pump blocking increases intensity noise in all the frequency range where we expect to obtain squeezing if we quietly pump the laser. Let us therefore concentrate on the low-frequency branch of the intensity noise spectrum, and rewrite the intensity noise spectra at $\omega = 0$:

$$P_{\Delta\hat{r}}(0) = \frac{\kappa_m}{\kappa}\left[\left(\frac{\kappa}{\kappa_m} - 1\right)\frac{\mathcal{D}^f}{2\kappa I_s} + \frac{\mathcal{D}^c}{2\kappa I_s} + \frac{\mathcal{D}^f + \mathcal{D}^p}{2\kappa I_s}x^2\right], \tag{55a}$$

$$S_I(0) = \frac{\kappa_m}{\kappa}\left[\left(\frac{\kappa}{\kappa_m} - 1\right)\frac{\mathcal{D}^f}{2\kappa I_s} + \frac{\mathcal{D}^c}{2\kappa I_s} + \frac{\mathcal{D}^f + \mathcal{D}^p}{2\kappa I_s}\tau^2\right]. \tag{55b}$$

From Eqs. (42) we have

$$\frac{\mathcal{D}^f}{2\kappa I_s} = 1, \tag{56a}$$

$$\frac{\mathcal{D}^c}{2\kappa I_s} = x + (1+x)\xi, \tag{56b}$$

$$\frac{\mathcal{D}^f + \mathcal{D}^p}{2\kappa I_s} = 2\left(\frac{\sum_k L_k|g_k|^2 f_k^2}{\sum_k L_k|g_k|^2(2f_k - 1)}\right), \tag{56c}$$

so that from Eqs. (55) we obtain

$$P_{\Delta\hat{r}}(0) = \frac{\kappa_m}{\kappa}\left[\left(\frac{\kappa_m}{\kappa} - 1\right) + x + (1+x)\xi + 2\left(\frac{\sum_k L_k|g_k|^2 f_k^2}{\sum_k L_k|g_k|^2(2f_k - 1)}\right)x^2\right], \tag{57a}$$

$$S_I(0) = \frac{\kappa_m}{\kappa}\left[\left(\frac{\kappa}{\kappa_m} - 1\right) + x + (1+x)\xi + 2\left(\frac{\sum_k L_k|g_k|^2 f_k^2}{\sum_k L_k|g_k|^2(2f_k - 1)}\right)\tau^2\right]. \tag{57b}$$

Let us demonstrate that the coefficient in the parenthesis which precedes x^2 and τ^2 is always bigger than 1. From Eq. (10b) we have indeed

$$\sum_k L_k|g_k|^2(2f_k - 1) = \kappa\gamma > 0, \tag{58}$$

and therefore

$$\frac{\sum_k L_k|g_k|^2 f_k^2}{\sum_k L_k|g_k|^2(2f_k - 1)} = 1 + \frac{\sum_k L_k|g_k|^2(f_k - 1)^2}{\sum_k L_k|g_k|^2(2f_k - 1)} > 1. \tag{59}$$

Note however that the second term tends to 0 if f_k becomes approximately equal to one for all the values of k where $L_k|g_k|^2$ is appreciably different from zero, and this happens as the stationary carrier number N_s increases. By inspecting Eqs. (56a,c) and Eq. (59) we conclude that it is always $\mathcal{D}^p > \mathcal{D}^f$, and it becomes $\mathcal{D}^p \approx \mathcal{D}^f$ only for high values of the carrier density.

Let us now define the optical and the internal efficiencies

$$\eta_{opt} = \frac{\kappa_m}{\kappa}, \tag{60a}$$

$$\eta_{int} = 1 - x, \tag{60b}$$

so that the zero-frequency noise can be rewritten as

$$P_{\Delta\hat{r}}(0) = 1 - \eta_{opt}\eta_{int} + \eta_{opt}(2-\eta_{int})\xi + \eta_{opt} 2 \left(\frac{\sum_k L_k |g_k|^2 f_k^2}{\sum_k L_k |g_k|^2 (2f_k - 1)} \right)(1-\eta_{int})^2, \tag{61a}$$

$$S_I(0) = 1 - \eta_{opt}\eta_{int} + \eta_{opt}(2-\eta_{int})\xi + \eta_{opt} 2 \left(\frac{\sum_k L_k |g_k|^2 f_k^2}{\sum_k L_k |g_k|^2 (2f_k - 1)} \right)\tau^2. \tag{61b}$$

If the pump level is risen up to infinity, then $\eta_{int} \to 1$, and we obtain

$$P^{\infty}_{\Delta\hat{r}}(0) = 1 - \eta_{opt} + \eta_{opt}\xi, \tag{62a}$$

$$S_I^{\infty}(0) = 1 - \eta_{opt} + \eta_{opt}\xi + \eta_{opt} 2 \left(\frac{\sum_k L_k |g_k|^2 f_k^2}{\sum_k L_k |g_k|^2 (2f_k - 1)} \right)\tau_\infty^2, \tag{62b}$$

where, using the relation $f'_k \propto f_k(1-f_k)$, it results [21]

$$\tau_\infty = \frac{\sum_k f_{pk} f_k (1-f_k)^2}{\sum_k f_{pk}(1-f_k)^2} \frac{\sum_k L_k |g_k|^2 (2f_k - 1)}{\sum_k L_k |g_k|^2 f_k (1-f_k)}. \tag{63}$$

If $\eta_{opt} = 1$ then

$$P^{\infty}_{\Delta\hat{r}}(0) = \xi, \tag{64a}$$

$$S_I^{\infty}(0) = \xi + 2 \left(\frac{\sum_k L_k |g_k|^2 (2f_k - 1)}{\sum_k L_k |g_k|^2 f_k^2} \right) \times \frac{\left(\frac{\sum_k f_{pk} f_k (1-f_k)^2}{\sum_k f_{pk}(1-f_k)^2} \right)^2}{\left(\frac{\sum_k L_k |g_k|^2 f_k}{\sum_k L_k |g_k|^2 f_k^2} - 1 \right)^2}. \tag{64b}$$

Let us now define the following expressions

$$B_1 \equiv \frac{\sum_k L_k |g_k|^2 (2f_k - 1)}{\sum_k L_k |g_k|^2 f_k^2}, \tag{65a}$$

$$B_2 \equiv \frac{\sum_k f_{pk} f_k (1-f_k)^2}{\sum_k f_{pk}(1-f_k)^2}, \tag{65b}$$

$$B_3 \equiv \frac{\sum_k L_k |g_k|^2 f_k}{\sum_k L_k |g_k|^2 f_k^2}, \tag{65c}$$

so that Eq. (64b) can be rewritten in the compact form

$$S_I^{\infty}(0) = \xi + 2B_1 \frac{B_2^2}{(B_3 - 1)^2}. \tag{66}$$

In Fig. 4 we have represented the functions f_k, f_k^2, and $L_k|g_k|^2$ that appear in B_3 for two different values of the stationary carriers density. Since for increasing density f_k and f_k^2 tends to be equal wherever $L_k|g_k|^2$ is appreciably different from zero, we can deduce that $B_3 \to 1^+$ as the density increases. This has a major impact on the zero-frequency noise, because it means that the denominator of the second term in Eq. (66) vanishes, and therefore the noise might become very large as the density is increased. To confirm this possibility, it is necessary to describe also the behavior of the other blocks that appear in Eq. (66). This has been done in Figs. 5(a) and 5(b), where the density increase has been obtained by reducing the confinement factor from 1 to 0.025. We can see that, as the density increases, $B_1 \to 1^-$, B_2

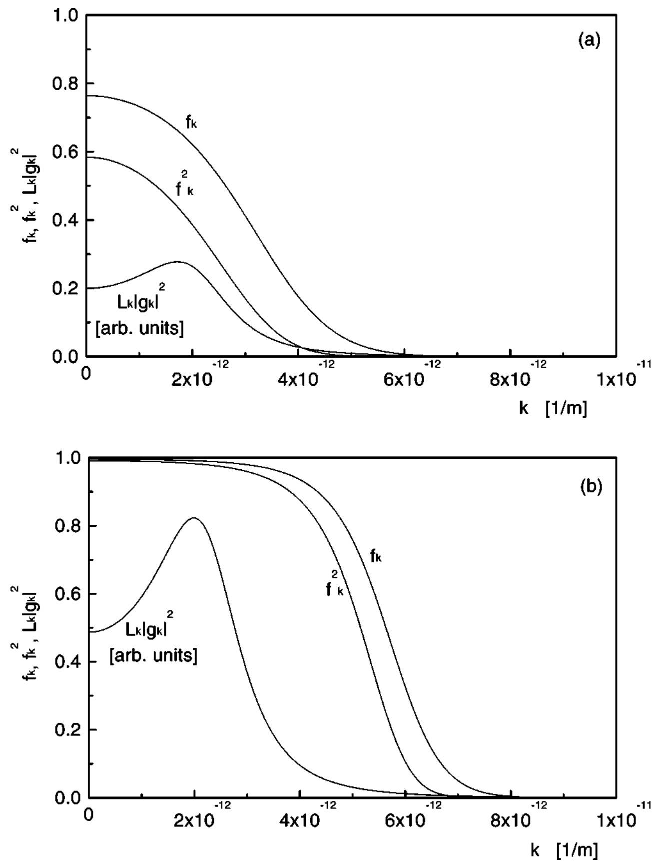

FIG. 4. Behavior of the functions f_k, f_k^2, and $L_k|g_k|^2$ for (a) $c_f=1 \rightarrow N_s/(L_xL_y)=1.35\times 10^{12}$ cm^{-2} (b) $c_f=0.025 \rightarrow N_s/(L_xL_y)=5.0\times 10^{12}$ cm^{-2}. The values of the other parameters are as in Fig. 1.

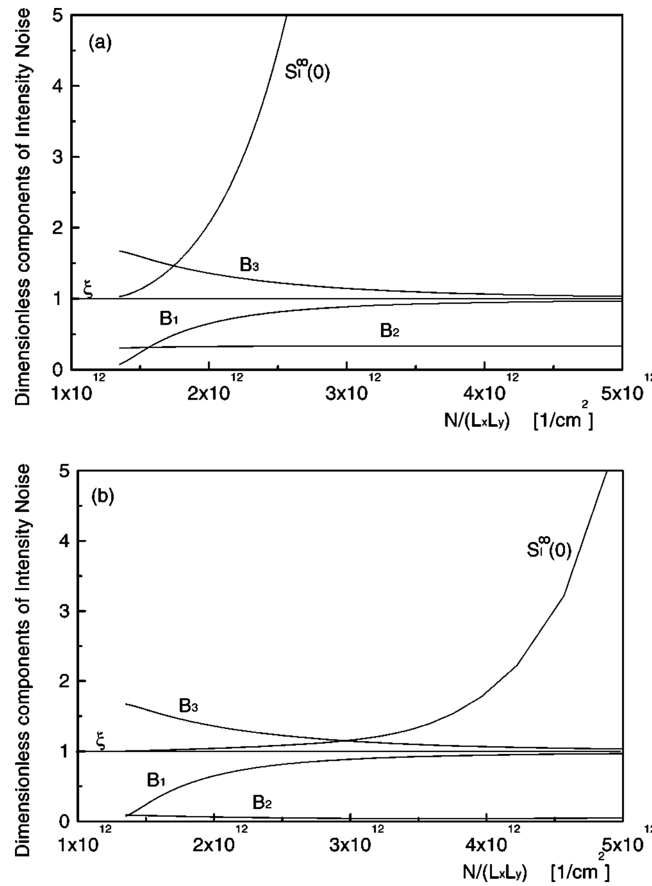

FIG. 5. Dependence of the various terms appearing in Eq. (66) on the carrier density for the cases (a) $N_p=N_s$ and (b) $N_p=5N_s$. The flat line at 1 is the Poissonian pump noise ξ, while the meaning of the lines marked by B_1, B_2, and B_3 is explained in the text.

remains approximately constant, and $B_3 \rightarrow 1^+$. As a consequence, the noise increases well above the pump contribution ξ. Figure 4(b) differs from Fig. 4(a) because the amount of pump blocking is reduced; this is obtained, as explained in [20,21], by increasing the ratio N_p/N_s. The only difference with respect to Fig. 4(a) is a smaller value of B_2, but again the noise increases well above the standard quantum limit if the carrier density is increased. This brings us to the Conclusions.

VI. CONCLUSIONS

The main result of this paper is that in the presence of even a small amount of pump blocking, the zero-frequency intensity noise steeply increases with the stationary value of the carrier density in the active layer [see Fig. 5]. Since the carrier density depends on a lot of laser parameters [see Fig. 1], this finding can explain the great dependence of the noise performance of semiconductor lasers on the specific device under examination, even when single-mode operation is ensured.

To show the origin of this pump-blocking-induced noise we have demonstrated that a formal identity between the paradigmatic noise spectra Eqs. (51) calculated in [1] and the spectra Eqs. (43) determined in this paper can be established. The only essential difference is that the parameter x defined by Eq. (49) is substituted by the parameter τ defined by Eq. (32a), and the central point is that, because of pump blocking, τ does not go to zero as the pump level is increased, whereas x does. As an aside comment, we note that this suggests that the noise effects that we have ascribed to pump blocking could actually derive from any other mechanism capable of introducing a dependence of the pump rate on the carrier density in the active layer.

The consequence of a carrier-dependent pump rate is that at zero frequency the coefficient that multiplies the field and the polarization noise terms do not disappear as the pump is increased. Even if ideal optical and internal efficiency are assumed, so that we are guaranteed that any pump carrier is transformed in a photon in the output laser beam, it results that, in contrast with Yamamoto predictions, the low-frequency intensity noise is not given only by the pump noise, but there are unexpected contributions from field and polarization noise. To explain the presence of these noise contributions we observe that if the pump rate depends on the carrier density, then the stimulated recombination rate will depend on it. As a consequence, any carrier fluctuations will cause a stimulated recombination rate fluctuation, and it is precisely this fluctuation that causes the additional field and polarization noise. It is natural to expect that this effect increases with the increasing of the carrier density. To summarize, we can say that in the presence of pump blocking a semiconductor laser does not behave as a shot noise limited device neither from the point of view of phase noise (in agreement with [1]) nor from the point of view of intensity noise (in disagreement with [1]), and consequently we can not expect perfect squeezing for asymptotically high pump, even if the pump noise is completely removed.

[1] Y. Yamamoto, S. Machida, and O. Nilsson, Phys. Rev. A **34**, 4025 (1986).

[2] Y. Yamamoto and S. Machida, Phys. Rev. A **35**, 5114 (1987).

[3] S. Machida, Y. Yamamoto, and Y. Itaya, Phys. Rev. Lett. **58**, 1000 (1987).

[4] W. H. Richardson and R. M. Shelby, Phys. Rev. Lett. **64**, 400 (1990).

[5] W. H. Richardson, S. Machida, and Y. Yamamoto, Phys. Rev. Lett. **66**, 2867 (1991).

[6] H. Wang, M. J. Freeman, and D. J. Steel, Phys. Rev. Lett. **71**, 3951 (1993).

[7] T. C. Zhang, J. Ph. Poizat, P. Grelu, J-F. Roch, P. Grangier, F. Marin, A. Bramati, V. Jost, M. D. Levenson, and E. Giacobino, Quantum Semiclassic. Opt. **7**, 601 (1993).

[8] J. Kitching, A. Yariv, and Y. Shevy, Phys. Rev. Lett. **74**, 3372 (1995).

[9] F. Marin, A. Bramati, E. Giacobino, T. C. Zhang, J. P. Poizat, J. F. Roch, and P. Grangier, Phys. Rev. Lett. **75**, 4606 (1995).

[10] D. C. Kilper, D. G. Steel, R. Craig, and D. R. Scifres, Opt. Lett. **21**, 1283 (1996).

[11] D. C. Kilper, P. A. Roos, J. L. Carlsten, and K. L. Lear, Phys. Rev. A **55**, R3323 (1997).

[12] S. Inoue, S. Lathi, and Y. Yamamoto, J. Opt. Soc. Am. B **14**, 2761 (1997).

[13] A. Bramati, V. Jost, F. Marin, and E. Giacobino, J. Mod. Opt. **44**, 1929 (1997).

[14] C. Becher, E. Gehrig, and K.-J. Boiler, Phys. Rev. A **57**, 3952 (1998).

[15] S. Lathi and Y. Yamamoto, Phys. Rev. A **59**, 819 (1999).

[16] Ph. Grangier and J.-Ph. Poizat, Eur. Phys. J. D **1**, 97 (1998); **7**, 99 (1999).

[17] J.-Ph. Poizat, T. Chang, and Ph. Grangier, Phys. Rev. A **61**, 043807 (2000).

[18] K. Petermann, IEEE J. Quantum Electron. **15**, 566 (1979).

[19] A. M. van der Lee, N. J. van Druten, M. P. van Exter, J. P. Woerdman, J.-Ph. Poizat, and Ph. Grangier, Phys. Rev. Lett. **85**, 4711 (2000).

[20] M. Travagnin, F. Castelli, and L. A. Lugiato, Europhys. Lett. **50**, 312 (2000).

[21] M. Travagnin and L. A. Lugiato, Phys. Rev. A **62**, 43813 (2000).

[22] M. Travagnin, J. Opt. B: Quantum Semiclassical Opt. **2**, L25 (2000).

[23] W. W. Chow, S. W. Koch, and M. Sargent III, *Semiconductor—Laser Physics* (Springer-Verlag, Berlin, 1993).

[24] S. Schuster and H. Haug, Semicond. Sci. Technol. **10**, 281 (1995); S. Schuster and H. Haug, J. Opt. Soc. Am. B **13**, 1605 (1996).

[25] K. W. Boer, *Survey of Semiconductor Physics* (Van Nostrand Reinhold, New York, 1990); G. Bastard, *Wave Mechanics Applied to Semiconductor Heterostructures* (Editions de Physique, Les Ulis, Paris, 1988).

[26] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).

[27] F. Castelli and L. A. Lugiato, J. Mod. Opt. **44**, 765 (1997).

[28] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).