Propagation of pulses in a three-level medium at exact two-photon resonance

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Propagation of a pulse pair in a nondissipative three-level medium is investigated in the adiabatic-following approximation to a trapped state. The general case of unequal oscillator strengths of two electric dipole transitions in an atom is studied analytically, and the adiabaticity criterion for the matter-field interaction is derived. It is shown that the interaction adiabaticity strongly depends on the relationship between oscillator strengths. A simple expression specifying the critical propagation length at which the stimulated Raman adiabatic passage process is still effective is derived. An estimate of the propagation distance at which a complete energy transfer from the pump pulse into the Stokes pulse occurs is made.

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I. INTRODUCTION

Coherent interaction of two laser pulses with three-level media has exhibited a rich variety of interesting phenomena, such as electromagnetically induced transparency, population transfer, lasing without inversion, and others [1]. The mechanism underlying these coherent phenomena is the existence of a so-called ''trapped'' superposition state [2] that does not involve an intermediate bare state. Such a state can be realized, for example, by using a counterintuitive sequence of pulses (the Stokes pulse is switched on and off before the pump pulse) interacting with a three-level Λ system that is initially in the ground state. If the pulses meet certain adiabaticity conditions [1], after the interaction, the system is found to be completely inverted. This method of population transfer is known in the literature as stimulated Raman adiabatic passage (STIRAP) and has been successfully demonstrated for atomic [3] and molecular [4] beams. The efficiency of using a counterintuitive pulse sequence in nonlinear interaction of radiation with a medium has been shown experimentally recently in Ref. [5] where the process of nonlinear four-wave sum mixing has been investigated and the enhancement of the generation efficiency by a factor of 2 and more has been shown. In this connection it is considered to be important and interesting to investigate the interaction of pulses with a medium under the condition of a population trapped state in the presence of energy transfer from one pulse into another.

Much attention in the previous study of the propagation problem has been focused on soliton-wave propagation (without loss and dispersion), for example, propagation of ''matched pulses'' [6], ''adiabatons'' [7], and others [8,9]. However, the processes of energy transfer from one pulse into another lead to significant pulse-shape changes in an optically thick medium. Such processes have been investigated in a number of numerical studies, for example, in Ref. [10], but, to our knowledge, there are no analytic results. It is obvious that pulse-shape changes accumulated during propagation can result, for example, in breaking down the adiabaticity condition, which is very important in most methods of creating a population trapped state, for example, in Ref. [11]. The adiabaticity condition for a single atom has been well investigated [12] and requires both field intensities to vary slowly as compared to the Rabi frequencies. However, for a gaseous medium, the adiabaticity condition should differ from that for a single atom and contain the parameters of the medium. It is difficult to obtain such a criterion from numerical analysis. It is shown in Ref. [7] that the adiabaticity of the system does not break down during propagation in the case of equal oscillator strengths of two electric dipole transitions in an atom. The equality of the oscillator strengths means that the linear absorption coefficients of the both fields are equal. It is not so easy to realize such a condition in a real experiment. The case of unequal oscillator strengths has been investigated in Ref. [13]. The investigations performed showed that in such a situation nonlinear adiabaton-like waves are not shape-preserving pulses but undergo a front sharpening and the authors state that the inequality of the oscillator strengths results in the appearance of strong nonadiabatic effects in the ''atoms + field'' system. However, a more detailed analysis performed in the present paper shows that the interaction adiabaticity strongly depends on the relation between the oscillator strengths and can be preserved under certain conditions.

The goal of the present paper is to investigate the propagation of two optical pulses in a three-level medium with unequal oscillator strengths without restriction on pulse shapes in the adiabatic-following approximation [14] to the trapped state. The pulse durations are assumed to be short as compared to all relaxation times. The effects of inhomogeneous broadening are not taken into account in this paper [15]. Here we ask what happens with the ''trapped'' state as the pulses propagate into a medium, namely whether it is destroyed or preserved during propagation and what are the corresponding conditions. It is shown that the trapped state destroys rather quickly during propagation in some cases and is preserved in other cases. We present exact analytic solutions to propagation equations by taking into account the first nonadiabatic corrections to the trapped state. From the solutions obtained we derive the conditions for loss-free propagation. The process of adiabaticity breaking down is analyzed and the adiabaticity criterion is obtained.

We investigate in detail the propagation dynamics of a

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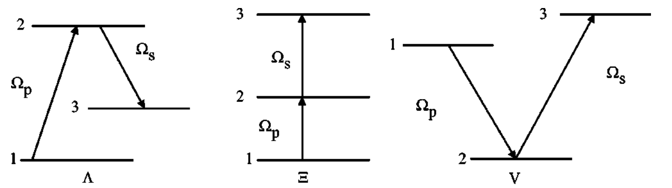

FIG. 1. Three-level systems coupled by two resonant pulses with Rabi frequencies Ω_p and Ω_s.

counterintuitive pulse sequence in a Λ system. It is shown that both pulses experience a considerable reshaping as they propagate into the medium, namely, the leading edge of the pump pulse is continuously depleted and its magnitude decreases, while at the tail of the Stokes pulse there appears an additional peak, which is correspondingly amplified. Such pulse-shape change during propagation leads to the decrease of the population transfer efficiency. We derive a simple expression for the propagation length over which population transfer is still effective. The possibility of a complete energy transfer from the pump pulse to the Stokes pulse is shown and an estimate of the propagation distance at which this occurs is made.

The paper is organized as follows. In Sec. II we formulate the problem and present the propagation equations in the adiabatic-following approximation. The solutions to the propagation equations are presented in Sec. III. In Sec. IV we focus on the adiabatic behavior of the system and derive the adiabaticity criterion specifying the propagation length at which the interaction adiabaticity is preserved. In Sec. V we study the propagation of a counterintuitive pulse sequence in the STIRAP regime and analyze the evolution of the pulses for different propagation dynamics. The propagation length, specifying the critical length at which an effective STIRAP is still possible in a medium, and the length at which a complete energy transfer from the pump pulse to the Stokes pulse occurs, are derived. The results are summarized in Sec. VI.

II. BASIC FORMULAS

The three-level systems under consideration are presented in Fig. 1. States $|1\rangle$ and $|2\rangle$, $|2\rangle$ and $|3\rangle$ are connected by the laser radiations $E_p = A_p \cos(k_p x - \omega_p t + \varphi_p)$ and $E_s = A_s \cos(k_s x - \omega_s t + \varphi_s)$, respectively. The direct transition $|1\rangle \to |3\rangle$ is electric dipole forbidden.

The probability amplitudes of the three states obey the time-dependent Shröedinger equation, which in the rotating-wave (or resonant) approximation takes the form [1]

$$i\frac{d}{dt}\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 & \Omega_p & 0 \\ \Omega_p & \Delta_p & \Omega_s \\ 0 & \Omega_s & \Delta_p + \Delta_s \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}. \tag{1}$$

Here $\Omega_p = -|A_p d_p|/\hbar$, $\Omega_s = -|A_s d_s|/\hbar$ are the Rabi frequencies of the corresponding fields, with d_p and d_s being the transition dipole moments between states $|1\rangle$ and $|2\rangle$, $|2\rangle$ and $|3\rangle$, respectively. The detunings off resonance are defined as

$$\begin{array}{lll} \Lambda & V & \Xi \\ \hline \Delta_p = \omega_{21} - \omega_p + \dot{\varphi}_p & \Delta_p = \omega_{21} + \omega_p - \dot{\varphi}_p & \Delta_p = \omega_{21} - \omega_p + \dot{\varphi}_p \\ \Delta_s = \omega_{32} + \omega_s - \dot{\varphi}_s & \Delta_s = \omega_{32} - \omega_s + \dot{\varphi}_s & \Delta_s = \omega_{32} - \omega_s + \dot{\varphi}_s. \end{array} \tag{2}$$

The instantaneous Hamiltonian for all three systems has a zero eigenvalue under the exact two-photon resonance condition, i.e., when $\Delta_p + \Delta_s = 0$. The corresponding eigenstates are referred to as trapped states and realized under the following initial conditions:

$$\frac{b_3(-\infty)}{b_1(-\infty)} = -\frac{\Omega_p(-\infty)}{\Omega_s(-\infty)}, \quad b_2(-\infty) = 0. \tag{3}$$

For example, if the atom is initially in the ground state, i.e., $b_2(-\infty) = b_3(-\infty) = 0$, $b_1(-\infty) = 1$, a counterintuitive pulse sequence should be applied to realize a trapped state.

In the adiabatic following approximation (taking into account the first nonadiabatic corrections to the trapped state) the atomic state populations are as follows [12]:

$$b_1 = \cos\theta + i\frac{\dot{\theta}}{\Omega}\frac{2\sin\theta}{\tan 2\psi}, \quad b_2 = -i\frac{\dot{\theta}}{\Omega},$$

$$b_3 = -\sin\theta + i\frac{\dot{\theta}}{\Omega}\frac{2\cos\theta}{\tan 2\psi}, \tag{4}$$

where $\Omega = \sqrt{\Omega_p^2 + \Omega_s^2}$, $\tan\theta = \Omega_p/\Omega_s$, $\tan 2\psi = 2\Omega/\Delta_p$ ($\psi \to \pi/4$ at $\Delta_p \to 0$). Equations (4) are obtained under the condition $\dot{\theta}/\Omega \approx (\Omega T)^{-1} \ll 1$, which is the adiabaticity criterion for a single atom [1].

For the Λ system in the general case, the reduced propagation equations in terms of wave variables x, $\tau = t - x/c$ reads [1]

$$2\Omega_p \frac{\partial \varphi_p}{\partial x} = -q_p(b_1^* b_2 + b_1 b_2^*);$$

$$2\Omega_s \frac{\partial \varphi_s}{\partial x} = -q_s(b_3^* b_2 + b_3 b_2^*);$$

$$2\frac{\partial \Omega_p}{\partial x} = -iq_p(b_1^* b_2 - b_1 b_2^*);$$

$$2\frac{\partial \Omega_s}{\partial x} = -iq_s(b_3^* b_2 - b_3 b_2^*), \tag{5}$$

where the coefficients $q_{p,s} = 2\pi N\omega_{p,s} d_{p,s}^2/\hbar c$ are proportional to the products of the atomic number density and the oscillator strengths for the transitions $|1\rangle \to |2\rangle$ and $|2\rangle \to |3\rangle$, respectively.

In Eqs. (5) and all the posterior formulas for a Ξ system, q_s should be replaced by $-q_s$, and for a V system q_p should be replaced by $-q_p$ and q_s by $-q_s$.

It follows from Eqs. (5) that in the ideal adiabatic limit ($b_2=0$) the pulses propagate through the medium without the shape and phase change (both real and imaginary parts of the dipole moments induced in the medium are equal to zero). However, due to a small nonadiabatic coupling between the trapped state and the two other eigenstates, the upper level is populated slightly. This leads to nonzero induced dipole moments between states $|1\rangle$ and $|2\rangle$ and $|2\rangle$ and $|3\rangle$, which in turn causes a consequent alteration of both fields. These changes, accumulated during the propagation process, can affect the propagation dynamics remarkably.

III. SOLUTIONS TO PROPAGATION EQUATIONS IN THE ADIABATIC-FOLLOWING APPROXIMATION

Using the definitions of the functions $\theta(x,\tau)$ and $\Omega(x,\tau)$ we have

$$\Omega_p(x,\tau)=\Omega(x,\tau)\sin\theta(x,\tau),$$

$$\Omega_s(x,\tau)=\Omega(x,\tau)\cos\theta(x,\tau). \tag{6}$$

Substituting Eqs. (4) into Eqs. (5), we obtain the following equations for the functions $\theta(x,\tau)$ and $\Omega(x,\tau)$:

$$\frac{\partial\Omega^2(x,\tau)}{\partial x}=(q_s-q_p)\,\dot{\theta}(x,\tau)\sin 2\theta(x,\tau), \tag{7}$$

$$\frac{\partial\theta(x,\tau)}{\partial x}+\frac{q_p\cos^2\theta(x,\tau)+q_s\sin^2\theta(x,\tau)}{\Omega^2(x,\tau)}\,\frac{\partial\theta(x,\tau)}{\partial\tau}=0. \tag{8}$$

It is seen from Eq. (7) that in the case of equal oscillator strengths, $q_p=q_s$, the function $\Omega(x,\tau)$ does not change during propagation, $\Omega^2(x,\tau)=\Omega_0^2(\tau)$, and represents the integral of motion. Note that in this case Eq. (8) is simplified substantially. In the general case of unequal oscillator strengths it is the total photon number density that represents the integral of motion. Indeed, introducing the photon number densities $n_{p,s}=A_{p,s}^2/\hbar\omega_{p,s}=2\pi N\Omega_{p,s}^2/cq_{p,s}$ (the dimension of $n_{p,s}$ coincides with that of atomic number density) from Eqs. (5) for the total photon number density (n_p+n_s) we have

$$\frac{\partial(n_p+n_s)}{\partial x}=\frac{2\pi N}{c}\,\frac{d}{d\tau}|b_2|^2. \tag{9}$$

In the adiabatic-following approximation, as seen from Eq. (4), the magnitude of $|b_2|^2$ is of the second order in the adiabaticity parameter $\dot{\theta}/\Omega$, and we may neglect the right-hand side of Eq. (9). Equation (9) presents the conservation law of the total photon number density under the trapped state condition during propagation.

For a Ξ system an analogous relationship is obtained for the difference of the photon number densities

$$\frac{\partial(n_p-n_s)}{\partial x}=\frac{2\pi N}{c}\,\frac{d}{d\tau}|b_2|^2. \tag{10}$$

For the Λ system, from the law of conservation of the photon number density we have the following expression for the function $\Omega(x,\tau)$:

$$\Omega^2(x,\tau)=\Omega_0^2(\tau)\,\frac{q_s\sin^2\theta_0(\tau)+q_p\cos^2\theta_0(\tau)}{q_s\sin^2\theta(x,\tau)+q_p\cos^2\theta(x,\tau)}, \tag{11}$$

where $\Omega_0^2(\tau)=\Omega_{p0}^2+\Omega_{s0}^2$, with Ω_{p0} and Ω_{s0} being the values of the corresponding functions at the medium entrance.

It follows from Eqs. (4), (6), and (11) that in the adiabatic-following approximation the dynamics of the system (both the atomic state population change and the pulse-shape evolution) is completely determined by the function $\theta(x,\tau)$ satisfying Eq. (8).

Using Eq. (8) and introducing ''nonlinear group'' velocity $u(x,\tau)$ and ''nonlinear time'' $\xi=\tau-x/u(x,\tau)$, for the function $\theta(x,\tau)$ we find

$$\frac{\partial\theta(x,\xi)}{\partial x}=0. \tag{12}$$

The solution to Eq. (12) is straightforward and reads [16]

$$\theta(x,\xi)=\theta_0(\xi), \tag{13}$$

where $\theta_0(\xi)$ is the value of $\theta(\xi)$ at the medium entrance and the function ξ is determined from the implicit equation

$$\int_\xi^\tau n_0(t')\,dt'=\frac{x}{q_pq_s}\,\frac{2\pi N}{c}\,f^2(\theta_0(\xi)). \tag{14}$$

Here $n_0(t')$ is the total photon number density at the medium entrance, $n_0=n_{p0}+n_{s0}$.

Given any explicit integrable expressions for the time dependence of the photon number density $n_0(t)$ at the medium entrance, we can directly evaluate the integral in Eq. (14) and obtain a set of reasonably simple analytic solutions to the problem considered. Indeed, evaluating the integral in Eq. (14) for a fixed x, we obtain the function $\tau(\xi)$ in an explicit form, and after that there is no difficulty in the determination of the inverse function $\xi(x,\tau)$. For example, substituting instead of the functions n_{p0} and n_{s0} in Eq. (14), the functions used in Ref. [13] or the Jacobi elliptic functions of Ref. [8], we find that Eq. (14) takes the form

$$\text{const}\times(\tau-\xi)=\frac{x}{q_pq_s}\,\frac{2\pi N}{c}\,f^2(\theta_0(\xi)).$$

One can easily obtain from this expression the function $\tau(\xi)$ and, hence, the function $\xi(x,\tau)$. Thus, formulas (6), (11), (13), and (14) represent exact analytical solutions of the task considered.

In the case of equal oscillator strengths $f^2(\theta)/q_pq_s=1$ and, as follows from Eq. (14), the nonlinear propagation velocity $u(x,\tau)$ is constant if the total photon number density $n_0(\tau)$ does not depend on time. This propagation regime has been called ''adiabatons.'' In the general case of unequal oscillator strengths for soliton wave propagation, it is necessary that the function $\xi(x,\tau)$, determined by Eq. (14), be a linear function of τ and x, which means constancy of the propagation velocity. Evaluating the corresponding derivatives of the function $\xi(x,\tau)$ we have

$$\frac{\partial\xi}{\partial\tau}=\frac{n_0(\tau)}{n_0(\xi)+(2x/q_sq_p)(2\pi N/c)f(df/d\theta_0(\xi))(d\theta_0(\xi)/d\xi)}=\text{const}, \tag{15}$$

$$\frac{\partial\xi}{\partial x}=-\frac{1}{q_sq_p}\frac{2\pi N}{c}\frac{f^2[\theta_0(\xi)]}{n_0(\xi)+(2x/q_sq_p)(2\pi N/c)f(df/d\theta_0(\xi))(d\theta_0(\xi)/d\xi)}=\text{const}. \tag{16}$$

As x enters the denominators of Eqs. (15) and (16), in order for $\xi(x,\tau)$ to be a linear function of τ and x, it is necessary and sufficient that the following two conditions be satisfied: (i) $(df/d\theta_0(\xi))(d\theta_0(\xi)/d\xi)=0$, and (ii) $n_0(\xi)=\text{const}$. The first of these conditions is fulfilled in the case of equal oscillator strengths (adiabatons) or in the case of a constant θ (identical envelopes). The second condition means constancy of photon number density.

It should be noted that the regime of propagation of simultaneous different-wavelength optical solitons [17] is not possible under the conditions considered here.

Note that, as follows from Eqs. (4), (6), and (14), two waves, ''population wave'' and ''polarization wave,'' propagate in the medium with the nonlinear group velocity determined by Eq. (14).

IV. ADIABATICITY CRITERION

The analytic solution obtained has been derived under the condition $\dot{\theta}/\Omega \ll 1$. In the medium, this condition takes the form $(1/\Omega)(d\theta/d\xi)(\partial\xi(x,\tau)/\partial\tau)\ll 1$. Even though we provide that the derivative $d\theta/d\xi$ be small, the adiabaticity condition can break down, since during propagation in the medium, the derivative $\partial\xi(x,\tau)/\partial\tau$ can become considerably large. We find now an expression specifying the critical propagation length at which the interaction adiabaticity met at the medium entrance is preserved during propagation.

It follows from Eq. (15) that $\partial\xi/\partial\tau\to\infty$ when the denominator in the right-hand side tends to zero. Using the definition of the function f we find for the derivative $df/d\theta_0$

$$\frac{df}{d\theta_0}=\sin 2\theta_0(\xi)(q_s-q_p). \tag{17}$$

Substituting Eq. (17) into Eq. (15), we find that in order for the denominator in the right-hand side of Eq. (15) to be zero, the following condition should be satisfied:

$$1-\frac{2(q_p-q_s)x}{\Omega_0^2}\frac{d\theta_0(\xi)}{d\xi}\sin 2\theta_0(\xi)\to 0. \tag{18}$$

As $\sin 2\theta_0(\xi)$ remains positive over the change range of θ, expression (18) can be fulfilled only under the following conditions:

$$(q_p-q_s)\frac{d\theta_0(\xi)}{d\xi}>0;\quad \frac{2(q_p-q_s)x}{\Omega_0^2T}\approx 1. \tag{19}$$

Conditions (19) represent the interaction adiabaticity criterion specifying the critical length at which the interaction adiabaticity, met at the medium entrance, breaks down during propagation.

Consider this condition in detail. For a Λ system at a counterintuitive pulse sequence, corresponding to the initial conditions $b_1(-\infty)=1, b_3(-\infty)=0$, we have $\dot{\theta}>0$. In this case, as follows from Eq. (19), the interaction adiabaticity breaks down when $q_p>q_s$. The condition $q_p>q_s$ means that the probability of the transition $|1\rangle\to|2\rangle$ is greater than that of the transition $|2\rangle\to|3\rangle$ and, thus, the population transfer $|1\rangle\to|2\rangle$ dominates the depletion of level $|2\rangle$, i.e., the interaction adiabaticity breaks down. When $q_p\leqslant q_s$, condition (19) is never fulfilled. In this case $\partial\xi/\partial\tau$ remains finite and, thus, the condition $\Omega T\gg 1$ is sufficient to provide the adiabaticity of the system.

The condition stated above is illustrated by Fig. 2, which presents the time evolution of the mixing parameter θ (solid curves) for $q=0.5;1$ and 2 (here $q=q_p/q_s$) at the normalized propagation length $z=0.03$ ($z\equiv xq_s/\Omega_0^2T$). The dashed

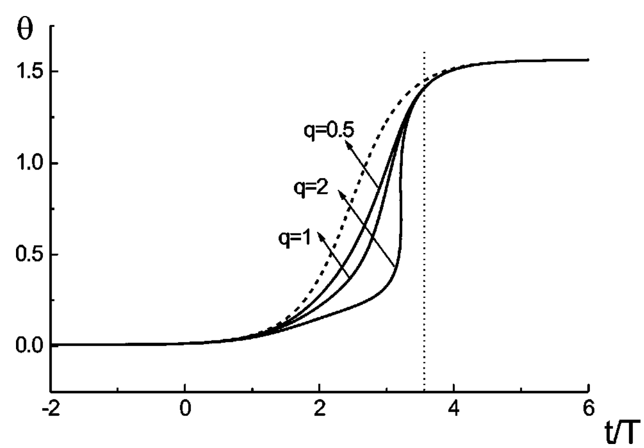

FIG. 2. Time evolution of the mixing parameter θ (solid curves) for different relationships between q_p and q_s ($q=0.5;1$ and 2) at propagation length $z=0.03$. The dashed curve corresponds to the case of $z=0$. The input pulses have been chosen in the form $\Omega_p(0,t)=\Omega_{\max}/\cosh[(t-\tau_d)T]$, $\Omega_s(0,t)=\Omega_{\max}/\cosh(t/T)$, where τ_d is the delay time between the pulses.

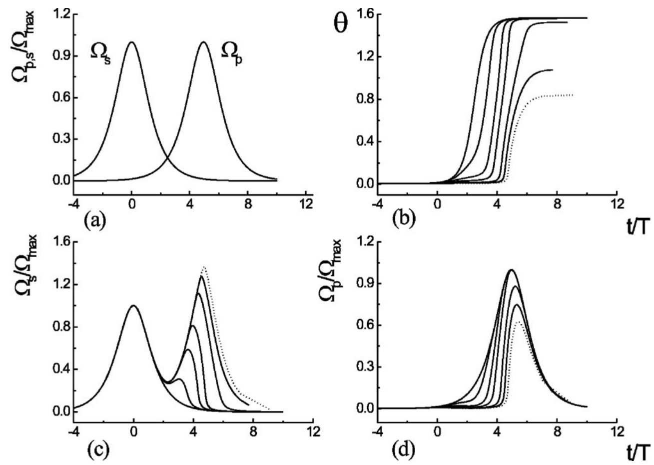

FIG. 3. Time evolution of (a) the normalized Rabi frequencies of the pump and Stokes pulses at the medium entrance $z=0$, (b) the mixing parameter θ, (c) the normalized Rabi frequencies of the Stokes pulse for different propagation lengths (z changes from 0 to 3.3), and (d) the normalized Rabi frequencies of the pump pulse for different propagation lengths (z changes from 0 to 3.3). The relationship between q_p and q_s is equal to 0.5.

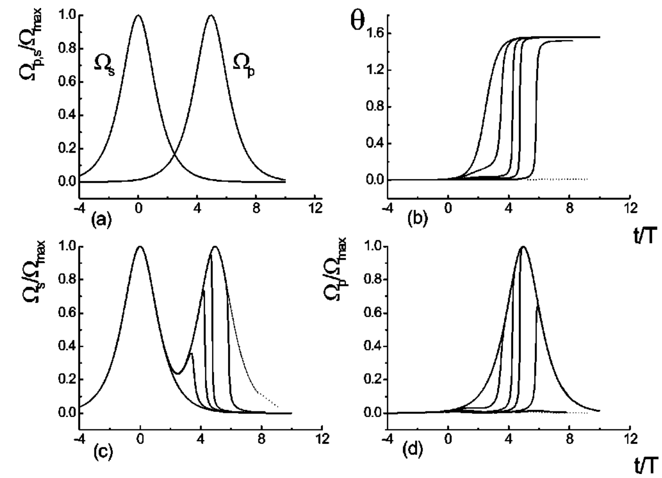

FIG. 4. Time evolution of (a) the normalized Rabi frequencies of the pump and Stokes pulses at the medium entrance $z=0$, (b) the mixing parameter θ, (c) the normalized Rabi frequencies of the Stokes pulse for different propagation lengths (z changes from 0 to 3.3), and (d) the normalized Rabi frequencies of the pump pulse for different propagation lengths (z changes from 0 to 3.3). The relationship between q_p and q_s is equal to 1.

line has been obtained at the entry surface of the medium at $z=0$. The dotted curve corresponds to the case of $\dot{\theta}\to\infty$, i.e., when the adiabatic-following approximation breaks down. It is seen from the figure that for $q=2$ the evolution of the mixing parameter differs from the adiabatic one (i.e., the trapped state is destroyed) already at this propagation length, while in the cases of $q=0.5$ and 1 the adiabatic evolution of the mixing parameter is preserved with propagation.

An intuitive pulse sequence ($\dot{\theta}<0$), which can be applied under the initial conditions $b_1(-\infty)=0$, $b_3(-\infty)=1$, results in population transfer from state $|3\rangle$ to state $|1\rangle$ via state $|2\rangle$. Thus we see that the interaction adiabaticity is rather sensitive to the relationship between oscillator strengths q.

For a Ξ system, replacing q_s by $-q_s$, we find that at a counterintuitive pulse sequence the interaction remains adiabatic at any propagation length, but for an intuitive pulse sequence it can break down at $2x(q_p-q_s)/\Omega_0^2T\approx 1$. For a V system, replacing q_s by $-q_s$ and q_p by $-q_s$, we have the case just opposite to that of a Λ system.

V. PROPAGATION IN THE STIRAP REGIME

Let us now investigate the propagation of a counterintuitive pulse sequence in a Λ system in the STIRAP regime for different relationships between the oscillator strengths ($q \leqslant 1$).

In Figs. 3(c) and 3(d) and 4(c) and 4(d) we present the time evolution of the envelopes of both pulses as they propagate through the medium for $q=0.5$ and $q=1$, respectively. We consider the hyperbolic secant pulse shapes: $\Omega_p(0,t)=\Omega_{\max}/\cosh[(t-\tau_d)/T]$, $\Omega_s(0,t)=\Omega_{\max}/\cosh(t/T)$, where τ_d is the delay time between the pulses.

It is seen that both pulses experience significant reshaping during propagation. The leading edge of the pump pulse undergoes gradual depletion. At the tailing edge of the Stokes pulse there appears an additional peak, the amplitude of which increases continuously. Thus, there occurs a significant dynamical redistribution of the number of photons in the overlapping range. Comparisons clearly show that the redistribution dynamics depends on the value of q. Indeed, for the case of $q=0.5$ [Fig. 3(d)] at the propagation length of $z=3.3$ (the dotted curve in the figures) the pump pulse is still intense enough, while at the same length for the case of $q=1$ [Fig. 4(d)] the energy of the pump pulse is completely transferred to the Stokes pulse.

It is known that for an effective STIRAP process not only a counterintuitive pulse switching on $[\theta(-\infty)\to 0]$ but also a corresponding switching off $[\theta(+\infty)\to\pi/2]$ is required, which provides the corresponding change of θ, namely, from 0 to $\pi/2$. For such a switching off it is necessary that at $t\to+\infty$, $\Omega_p\neq 0$ but $\Omega_s\to 0$. The time evolution of the mixing angle $\theta(\xi,\tau)$ is presented in Figs. 3(b) and 4(b). It is seen from the figures that at the initial stage of propagation the needed condition is fulfilled and the final value of θ is close to $\pi/2$. However, after a certain propagation length, there occurs a noticeable decrease in the amplitude of the pump pulse and the corresponding amplification of the Stokes pulse at its tail. Thus, after this length, the conditions $\Omega_p(+\infty)\neq 0$, $\Omega_s(+\infty)\to 0$, required for effective STIRAP, break down and the final value of θ differs from $\pi/2$, which means that the transfer process is not complete.

We see that even if all the conditions for effective STIRAP are met at the entry surface of the medium, they may break down during propagation due to the shape change of both pulses.

The analytic solution obtained enables us to find a simple expression specifying the propagation length at which effective population transfer is still possible in the medium. After the interaction with the pulses the atoms will be in the final state if $\sin^2\theta_0(\xi)=1$ and $\cos^2\theta_0(\xi)=0$, which corresponds to $\xi\geqslant T$. Then for Eq. (14) we have

$$\int_T^{T+\tau_d}(n_{p0}+n_{s0})dt'=\frac{q_s}{q_p}\frac{2\pi N}{c}x \qquad (20)$$

or

$$\overline{n}_0\tau_d=\frac{q_s}{q_p}\frac{2\pi N}{c}x, \qquad (21)$$

where $\overline{n}_0$ is the mean photon number density in the pump pulse after the Stokes pulse is switched off, which can be estimated as

$$\overline{n}_0=\frac{1}{2}\frac{2\pi N}{c}\frac{\Omega_{p,s}^2}{q_{p,s}}. \qquad (22)$$

Substituting Eq. (22) into Eq. (21), one can easily obtain the following expression for the normalized propagation length at which effective population transfer occurs

$$z_{\mathrm{STIRAP}}\sim\tau_d/2T. \qquad (23)$$

It follows from Eq. (21) that for the given time delay τ_d between the pulses, the more the number of photons in the time interval between T and $T+\tau_d$, the larger this length. This means that the Stokes pulse should be switched off more sharply, while the pump pulse should be switched off more smoothly. Thus, by a corresponding, namely, asymmetric pulse switching off, one can provide more photons in the pump pulse, which results in the longer penetration length for an effective STIRAP process.

On the other hand, as follows from Eq. (23), the more τ_d, the longer the STIRAP penetration length. It is obvious that τ_d should not be too large in order to provide corresponding overlap between the pulses [1].

It is seen from Figs. 4(c) and 4(d) that an effective photon transfer from the pump pulse leading edge to the Stokes pulse tail edge is possible during propagation. The complete transfer occurs when $\sin^2\theta_0(\xi)=0$, i.e., when $\xi\leqslant -T+\tau_d$. Using Eq. (14) we obtain the following simple expression specifying the critical length at which a complete energy transfer from the pump pulse into the Stokes pulse occurs:

$$z_{pump}\sim\frac{2(1+q)}{q^2}. \qquad (24)$$

As follows from Eq. (24), the more q is, the quicker the regime is set. In particular, for the case $q=1$, a complete energy transfer from the pump pulse into the Stokes pulse occurs at the length $z\sim 4$, which agrees with Fig. 4. Let us now estimate the value of z. For standard experimental parameters: $N=10^{15}$ atoms/cm^{-3}, $\omega=10^{15}$ s^{-1},$d^2=10^{-35}$ esu, $\Omega_0 T=10, T=10^{-12}$ s estimations show that $z=1$ corresponds to ~ 50 cm. Thus, for $q=1$ at the propagation length of 200 cm, the energy of the pump pulse is completely transferred to the Stokes pulse.

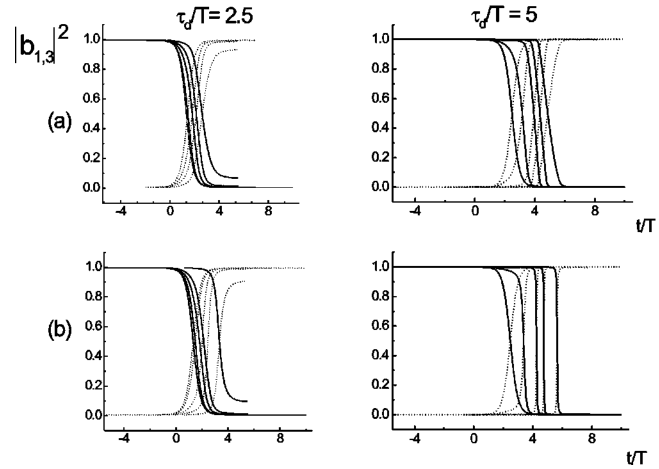

FIG. 5. Population transfer for (a) $q=0.5$ and (b) $q=1$. Shown are the populations of the ground and final states for $\tau_d/T=2.5$ and 5. Different curves correspond to different propagation lengths: $z=0$, 0.08, 0.4, 0.8, and 1.6.

The propagation length at which effective STIRAP is possible depends on the time delay between the pulses but not on the relationship between q_p and q_s. This conclusion is confirmed by Fig. 5. This figure presents the time evolution of the ground- and final-state populations for different time delays τ_d between the pulses for $q=0.5$ and $q=1$. Comparisons show that the penetration length of the STIRAP process increases with the increase of τ_d. Indeed, as seen from the figure, at the propagation length of $z=1.6$ the efficiency of the STIRAP process for $\tau_d/T=2.5$ is less than that for the case of $\tau_d/T=5$, i.e., according to Eq. (23) the more τ_d, the more the efficiency of the STIRAP process. For example, for the experimental parameters given above in the case of $\tau_d/T=5$, one can provide an effective STIRAP process at the propagation length of ~ 125 cm.

It is seen from the figure that the final values of the ground- and final-state populations do not differ for $q=0.5$ and $q=1$, i.e., the population transfer process does not depend on the relationship between the oscillator strengths. Some difference between the population dynamics is explained by the fact that the time evolution of the pulses depends on the value of q.

It should be noted that z_{stirap} should not exceed z_{pump}, as at this length the pump pulse, actually, is completely depleted and the STIRAP process is not possible.

Figure 6 presents the case where $q=0.001$. It is seen from the figure that the pump pulse propagates without the change of its shape while the Stokes pulse changes its shape significantly, which is not surprising as the number of photons is preserved during propagation, which is confirmed by Fig. 6(c). The figure presents the change of the number of photons as the pulses propagate into the medium. Some amplification of the pump pulse, observed at the tailing edge, indicates the beginning of the reverse process of the photon transfer from the Stokes pulse to the pump pulse. This is the beginning of the process of adiabaton formation. However,

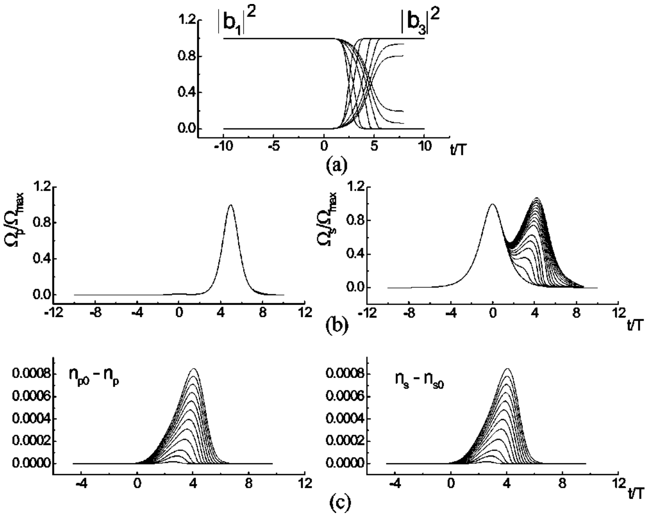

FIG. 6. Time evolution of (a) the ground- and final-state populations for different propagation lengths, (b) the normalized Rabi frequencies of the pump and Stokes pulses for different propagation lengths (z changes from 0 to 3), and (c) the change of the photon number in the pump and Stokes pulses for different propagation lengths (z changes from 0 to 3). The relationship between q_p and q_s is equal to 0.001.

in the case considered, adiabatons cannot be formed as the Stokes pulse is switched off and the process stops.

VI. SUMMARY

In the present paper we have studied the propagation of a pulse pair through a nondissipative three-level medium of Λ, Ξ, and V types for the general case of unequal oscillator strengths without restriction on the pulse shapes. We have obtained exact analytical solutions to the propagation equations under the adiabatic-following approximation and have shown that two waves, ''population wave'' and ''polarization wave,'' propagate in the medium with nonlinear group velocity. The investigation performed shows that propagation dynamics are strongly affected by the relationship between oscillator strengths.

We have derived the adiabaticity criterion for the matter-field interaction depending on the parameters of a medium. In the case of a counterintuitive pulse sequence in a Λ system, the interaction adiabaticity, provided at the medium entrance, is preserved for any value of propagation length when $q \leqslant 1$ and breaks down rather quickly with propagation when $q > 1$.

Next we have studied the spatial evolution of a counterintuitive pulse sequence during propagation for parameters at which the interaction adiabaticity is met. The analysis performed shows that during propagation under the adiabatic-following approximation there occurs a considerable pulse reshaping, which lies in gradual depletion of the leading edge of the pump pulse and corresponding amplification of the tailing edge of the Stokes pulse. It is found that during propagation, pulse shape change leads to the decrease of the efficiency of the STIRAP process in the medium. From the analytical solutions we have derived a simple expression specifying the critical length at which the population transfer process is still effective in a medium. It follows from the analysis that the efficiency of the STIRAP process can be increased by a corresponding pulse switching off, namely, sharper for the Stokes pulse and smoother for the pump pulse.

Our investigation shows that during propagation a complete energy transfer from the pump pulse to the Stokes pulse is possible. A simple expression for the propagation length at which this occurs has been obtained.

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