Optical transition and momentum transfer in atomic wave packets

A. Zh. Muradyan[1,2,*] and V. A. Poghosyan[1]
[1]*Department of Physics, Yerevan State University, 1 Alex Manukian, Yerevan, Armenia*
[2]*Engineering Center of Armenian National Academy of Sciences, Ashtarak-2, 378410 Armenia*
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It is shown that the population Rabi oscillations in a lossless two-level atom, interacting with a monochromatic electromagnetic field, in general, are convergent in time. The well-known continuous oscillations take place because the restricted choosing of initial conditions, that is, when the atom is chosen on ground or excited level before the interaction, simultaneously having a definite value of momentum there. The convergence of Rabi oscillations in atomic wave-packet states is a direct consequence of the Doppler effect on optical transition rates (Rabi frequencies): it gradually leads to ''irregular'' chaotic-type distributions of momentum in ground and excited energy levels, smearing the amplitudes of Rabi oscillations. Conjointly with Rabi oscillations, the coherent accumulation of momentum on each internal energy level monotonically diminishes too.

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I. INTRODUCTION

It is well known that due to interaction with a plane traveling wave the atomic momentum may be changed in limits of one photon momentum $\hbar k$. This limitation follows from momentum conservation law, and, concerns total atomic momentum. As to momentum per each internal energy level, as was shown recently [1], its change may be great and may even further surpass the coupling photon's momentum $\hbar k$. This is the case when the atom initially is in a superposition state of lower- and upper-energy levels with some (different in general) momentum distributions there. In other words, optical transition between atomic wave packet-states is accompanied by large-scale coherent accumulation of momentum in internal energy levels (CAMEL).

This phenomenon, which, as we hope, will have far-reaching consequences for atomic and molecular physics, is presented in Ref. [1] in the form as simple as possible. In particular, the operator of kinetic energy of atomic transitional motion was not included in the Hamiltonian of the atom-field system. Nevertheless, this operator has not only a quantitative, but also a qualitative contribution into the picture of interaction. For instance, such an important phenomenon as the Doppler shift of frequencies is introduced into the theory (in laboratory frame) by means of beforementioned operator. Therefore, in all cases, the more logical theory of atom-field interaction, concerning the atomic wave packet-states, should contain the atomic kinetic energy operator, unquestionably. This is done in the present paper. Here we consider an optical transition in the two-level atom, which has been prepared in general quantum-mechanical transitional states for lower- and upper-internal energy levels. The behavior of level population and momentum transfer between energy levels is considered in details. It is shown that the CAMEL phenomenon, which exhibits regular periodic behavior in the time when the kinetic energy operator is not taken into account, really has a damping periodic behavior and it is due to the influence of the Doppler shift of frequencies on the rate of optical transitions (Rabi frequencies). Simultaneously, a strictly important result has been obtained for atomic internal levels populations (population amplitudes), according to which the Rabi oscillations of populations for wave-packet atomic states have a damping behavior in general. It is worthwhile to remember that the well-known continuous periodic behavior takes place for ''ordinary,'' fully unexcited, or fully excited pure initial states, that is when the time evolution of populations begins from only one populated internal energy level, and, in addition, this populated state has definite value of momentum.

The quantum-mechanical behavior of a two-level atom in the near-resonant, plane-wave monochromatic radiation, taking into account the atomic kinetic energy operator, was considered earlier many times [2], very similar to ours were in Refs. [3,4]. In Ref. [3] the atom initially is on one energy level and the analysis is limited by narrow momentum distributions and short times of interaction. As a consequence, a splitting of an extra narrow wave packet into two subpackets has been created due to interaction. In Ref. [4] the authors restricted the analysis by the definite momentum and one energy-level population case. As a consequence, only the continuous periodic behavior, taking into account the energy-level splittings due to photon momentum exchange, has been obtained there for energy level's populations.

Taking into account the results of this and previous papers [1], the following may be stated about the role of initial conditions. For more general quantum-mechanical initial states, including the atomic wave-packet transitional states, (a) the Rabi oscillations have damping in time oscillatory character and (b) the optical transitions are accompanied by saturating accumulation of momentum on the internal energy levels. Choosing the initial population only on the one internal energy level annihilates the possibility of CAMEL phenomenon. If, in addition, the populated state has only a definite value of momentum, then the damping behavior of Rabi oscillations disappears too. Only for the simplest initial conditions the behavior of interaction obtains continuous periodic nature and exchanges one photon momentum $\hbar k$ between internal energy levels.

This paper is organized as follows. The Schrödinger equa-

*Email address: muradyan@ec.sci.am; yndanfiz@ysu.am

tion and the stationary solutions in momentum representation are presented in Sec. II. The time evolution of level populations and the transfer of momentum and kinetic energy between internal energy levels are examined in Secs. III and IV, respectively.

II. ROTATING-WAVE APPROXIMATION STATIONARY SOLUTIONS OF SCHRÖDINGER EQUATION IN MOMENTUM REPRESENTATION

Let us consider the near-resonant interaction of a two-level atom with a plane-wave radiation field [5]. The Hamiltonian of this system is well known and, taking into account the translational motion of atom, can be presented in the form

$$\hat{H}=\frac{\hat{p}^2}{2M}+\frac{\hbar\omega_0}{2}\hat{\sigma}_3+\hat{V}, \tag{1}$$

where M and ω_0 denote, respectively, the atomic mass and optical transition frequency, $\hat{\sigma}_3$ is quasispin (Pauli) operator. Second term presents the free atom with $-\hbar\omega_0/2$ and $\hbar\omega_0/2$ energies in lower- and upper-energy levels. The last term $\hat{V}$ presents the interaction of the atom with the external traveling-wave field, and can be written in the dipole approximation as

$$\hat{V}=-\hat{d}E(t,z), \tag{2}$$

where $\hat{d}$ is the dipole moment operator for optical transition. We will present the intensity of the plane traveling wave in the form

$$E(t,z)=E_0\ \exp(ikz-i\omega t)+\text{c.c.}, \tag{3}$$

where E_0 is constant, ω and $k=\omega/c$ represent the wave frequency and the wave number. Polarization effects are not included into the field of investigation. Such an approach is valid, as is well known, for purely linear or circular polarizations of the wave.

Denoting by $\varphi_g(\vec{\rho},t)$ and $\varphi_e(\vec{\rho},t)$ the wave functions of ground (g) and excited (e) energy levels ($\vec{\rho}$ is atomic internal coordinate, i.e., the radius vector of optical electron relative to atomic center-of-mass), the wave function of the interacting atom may be written in the following form:

$$\Psi(\vec{\rho},z,t)=A(z,t)\varphi_g(\vec{\rho},t)+B(z,t)\varphi_e(\vec{\rho},t), \tag{4}$$

where $A(z,t)$ represents the atomic probability amplitude to be on the lower level and have a space-coordinate z at the time moment t; and the other coefficient $B(z,t)$ represents the same for the upper level atom. Note, that the coordinate z in Eq. (4) represents the atomic center-of-mass position in the wave direction; and hence, the plane wave (3) includes only this single variable z, the case can be considered as a question of one dimension.

In this paper, our attention will be focused onto the time evolution of atomic populations and momentum distributions on lower and upper internal energy levels and their physical consequences. So, hereafter, it is worthwhile to deal with atomic amplitudes in the momentum representation. Moreover, just in momentum representation, the eigenvalue problem for the system under consideration has analytic solutions [3,4].

Expanding $A(z,t)$ and $B(z,t)$ amplitudes into momentum space on the basis of definite momentum states

$$\chi(p)=\frac{1}{\sqrt{2\pi\hbar}}\exp\left(\frac{i}{\hbar}pz\right), \tag{5}$$

that is

$$A(z,t)=\int a(p,t)\chi(p)dp;\quad B(z,t)=\int b(p,t)\chi(p)dp, \tag{6}$$

we substitute all related quantities (1)–(6) into the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t}=\hat{H}\Psi. \tag{7}$$

After standard transformations, we arrive for seeking amplitudes $a(p,t)$ and $b(p,t)$ to

$$i\hbar\frac{da(p,t)}{dt}=\left(\frac{p^2}{2M}+\frac{\hbar\omega_0}{2}\right)a(p,t)-\frac{\hbar\Omega}{2}e^{-i\omega t}b(p-\hbar k,t), \tag{8}$$

$$i\hbar\frac{db(p,t)}{dt}=\left(\frac{p^2}{2M}-\frac{\hbar\omega_0}{2}\right)b(p,t)-\frac{\hbar\Omega}{2}e^{i\omega t}a(p+\hbar k,t), \tag{9}$$

where $\Omega=2dE_0/\hbar$ is the parameter of induced transitions and commonly referred to as Rabi frequency [6]. This system fully coincides in form with the system of Eqs. (8), (9) in Ref. [4], and its stationary form should be coincide with the system (5) in Ref. [3].

A general solution of Eqs. (8) and (9) is

$$\begin{aligned}a(p,t)=&-\left(\frac{\alpha(p)-\beta(p)}{2\beta(p)}a(p,0)+\frac{\Omega}{2\beta(p)}b(p+\hbar k,0)\right)\\&\times e^{-i\omega_g(p)t}+\left(\frac{\alpha(p)+\beta(p)}{2\beta(p)}a(p,0)\right.\\&\left.+\frac{\Omega}{2\beta(p)}b(p+\hbar k,0)\right)e^{-i\omega_g'(p)t},\end{aligned} \tag{10}$$

$$\begin{aligned}b(p+\hbar k,t)=&\left(-\frac{\Omega}{2\beta(p)}a(p,0)+\frac{\alpha(p)+\beta(p)}{2\beta(p)}\right.\\&\left.\times b(p+\hbar k,0)\right)e^{-i\omega_e(p)t}+\left(\frac{\Omega}{2\beta(p)}a(p,0)\right.\\&\left.-\frac{\alpha(p)-\beta(p)}{2\beta(p)}b(p+\hbar k,0)\right)e^{-i\omega_e'(p)t}.\end{aligned} \tag{11}$$

Here,

$$\alpha(p)=\frac{\hbar k^2}{2M}+\frac{pk}{M}+\Delta, \tag{12}$$

and may be viewed as a generalized detuning, which involves the field-atom detuning $\Delta=\omega_0-\omega$, Doppler and recoil shifts pk/M and $\hbar k^2/2M$, respectively. It really represents the usual field-atom detuning viewed from atomic center-of-mass frame of reference. The second term

$$\beta(p)=\sqrt{\left(\frac{\hbar k^2}{2M}+\frac{pk}{M}+\Delta\right)^2+\Omega^2}, \tag{13}$$

and represents merely the so-called generalized Rabi frequency, including the generalized detuning $\alpha(p)$ instead of common frequency detuning Δ. Primed and nonprimed frequencies in exponents are

$$\omega'_{g,e}(p)=\frac{1}{2\hbar}\left(\frac{p^2}{2M}+\frac{(p+\hbar k)^2}{2M}\mp\hbar\omega\right)-\frac{\beta(p)}{2}, \tag{14}$$

$$\omega_{g,e}(p)=\frac{1}{2\hbar}\left(\frac{p^2}{2M}+\frac{(p+\hbar k)^2}{2M}\mp\hbar\omega\right)+\frac{\beta(p)}{2}, \tag{15}$$

and represent the energies of system quasistationary states, double-splitted, as in familiar theory, in both excited and ground levels. The size of splittry is $\omega_g-\omega'_g=\omega_e-\omega'_e=\beta(p)$. Note, that plugging in Eqs. (10), (11) $b(p,0)=0$, $a(p,0)=\delta(p-p_0)$, we arrive at the case, analyzed in Ref. [4].

III. POPULATION AND MOMENTUM PER INTERNAL GROUND AND EXCITED ENERGY LEVELS

Let us now proceed to the calculation of such physical quantities as population and mean momentum in each atomic internal energy level, and to their distributions in momentum space. Time evolution of population distributions has been determined and is presented by Eqs. (10) and (11). The main peculiarity of these formulas is their p dependence due to the Doppler effect, which will play a key role in further presenting results. It, first of all, disturbs the population distribution in momentum space and gradually transforms the initial smooth distribution into the modulated, chaoticlike one. A behavior of time evolution is illustrated in Fig. 1. Curve *a* represents the initial distribution, curve *b*—after four Rabi-oscillations (for central range of distribution) and curve *c*—after 12 Rabi-oscillations. For simplicity, the excited level was assumed to be initially empty, and the preserving symmetry about $p=0$ value is conditioned by this assumption. In general, when both energy levels are populated, even symmetric with respect to some values of momentum, any symmetry in distribution is being lost rapidly.

Total population of internal energy level is

$$n_g=\int |a(p,t)|^2dp, \tag{16}$$

for ground energy level, and

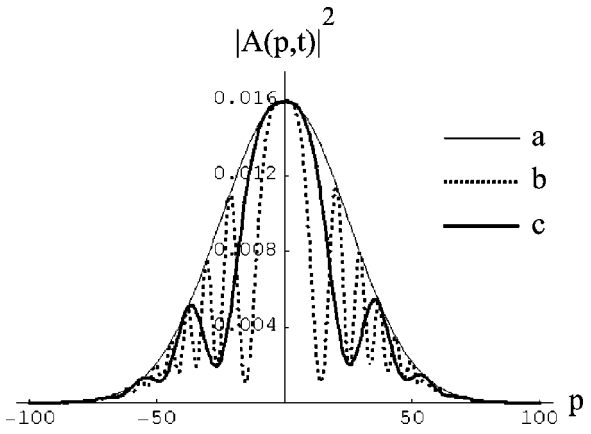

FIG. 1. Wave-packet momentum distribution in internal atomic energy level at three consecutive instances of time: initial (a), after 4 (b), and after 12 (c) Rabi oscillations of central part of distribution. The tendency to chaotization due to interaction is obvious. The ordinate, probability density in momentum space, is scaled to inverse photon momentum $(\hbar k)^{-1}$, following from normalization condition $\int|A(p,t)|^2dp=1$. Momentum p (abscissa) is scaled to photon momentum $\hbar k$.

$$n_e=\int |b(p,t)|^2dp, \tag{17}$$

for excited energy level. The typical form of time evolution for these populations is presented in Fig. 2. As is seen, the Rabi oscillations are gradually flattened due to redistributions of momentum states for interaction time. So, the momentum dependence of probabilities for optical transitions (Rabi oscillations), arising due to a Doppler shift of frequencies, leads, in general, to damping in population oscillations and to the establishment of definite-value populations in internal energy levels without any mechanism of relaxation.

The expression of atomic momentum

$$\langle p\rangle=\int \Psi^*\hat{p}\Psi d\vec{\rho}dz, \tag{18}$$

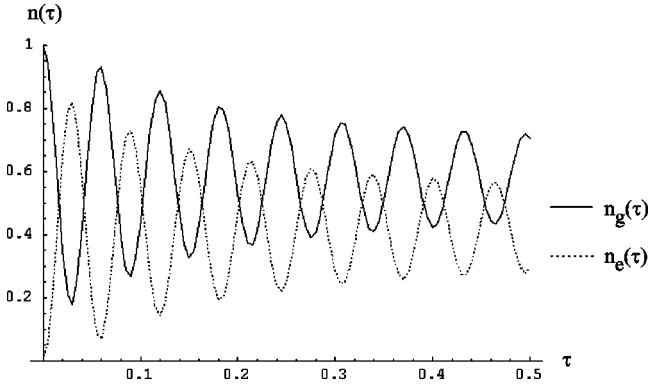

FIG. 2. Time evolution of internal energy levels' total populations. Ground level is initially populated in wave-packet state, but the excited level is initially empty. The scaled time $\tau=\omega_r t$, where $\omega_r=\hbar k^2/2M$ is recoil frequency. This scaling is preserved in following figures, too.

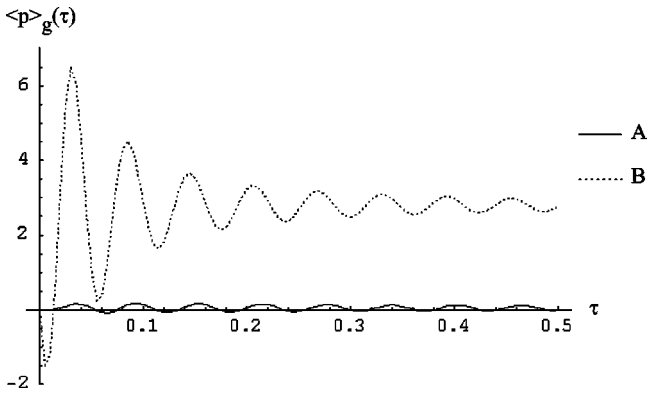

FIG. 3. Time evolution of mean momenta of internal ground energy level. Curve *A* exhibits the case, when only ground energy level is populated in wave-packet state (see Fig. 1). Curve *B*, in contrary, exhibits a more general, wave-packet case, when both internal energy levels are initially populated. Level momentum distributions are shifted with each other and this shift amounts to 20 $\hbar k$. Mean momentum (ordinate) is scaled to photon momentum $\hbar k$. This scaling is preserved in following figures, too (up to Fig. 6).

after elementary substitution of general expression (4) and respective standard transformations can be expressed as a sum of two terms [1]

$$\langle p\rangle=\langle p\rangle_g+\langle p\rangle_e, \tag{19}$$

first of which represents the amount of contribution of ground level states into the atomic momentum and is presented in general as

$$\langle p\rangle_g=\int |a(p,t)|^2 p\,dp. \tag{20}$$

The second term has the same sense for excited level and is

$$\langle p\rangle_e=\int |b(p,t)|^2 p\,dp. \tag{21}$$

It should be mentioned that these quantities, besides being the ingredients of total atomic momentum, in accordance with first principles of quantum mechanics, have their own physical meaning and are measurable quantities [1].

A typical form of momentum time evolutions is illustrated in Fig. 3 and Fig. 4; first, for ground energy level and second, for excited energy level. For thorough representation we distinguish the cases, when only one (ground) energy level is initially populated (solid line) and when both energy levels are populated (dashed line). In the last case, the momentum distributions on each internal energy level have the same form and overlap each other partly. As is seen, the character of momentum time evolution is independent on concrete conditions of state preparation and behaves as a damping of oscillations in all cases.

To turn to second goal of this paper, to uncover the time evolution for CAMEL phenomenon, it is necessary to pick out from level momentums $\langle p\rangle_g$ and $\langle p\rangle_e$ the parts, conditioned by population evolution. To this end, a pair of normalized momentums can be introduced into the theory [1],

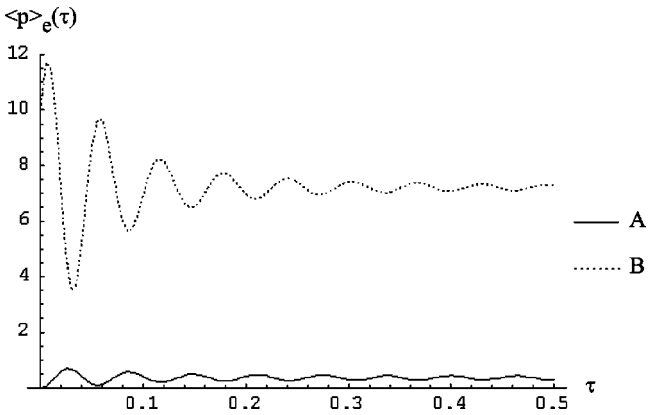

FIG. 4. The same, as in previous figure, dependencies for excited internal energy level.

$$p_g=\langle p\rangle_g/n_g \quad \text{and} \quad p_e=\langle p\rangle_e/n_e, \tag{22}$$

the time evolution of which should be solely conditioned by redistributions in wave-packet momentum states (referred to as coherent accumulation of momentum on an internal energy level). For familiar optical transitions with continuous periodic Rabi oscillations, new introduced momentums p_g and p_e are constant in the course of time and emerge as mean values of normalized momentum distribution per each energy level. Indeed, it is readily simple to verify it by direct substitution and the reason is the fact that $\langle p\rangle_{g,e}$ momentums in nominators and $n_{g,e}$ in denominators in Eq. (22) have the same form of time dependence for familiar optical transitions, with continuous periodic Rabi oscillations.

The typical behavior of p_g and p_e for wave-packet initial states is presented in Fig. 5 and Fig. 6. First, one of them elucidates the so-called ''intermediate'' case, when one internal energy level is initially populated by wave-packet states, but the other energy level is empty. As is seen, p_g and p_e are not already constant, but their variations are suppressed in limits of one photon momentum $\hbar k$ and, probably, do not present any special interest. A strictly different behavior manifests the more general case (Fig. 6), when both internal energy levels are initially populated by wave-packet states. Now p_g and p_e momentums are being changed in

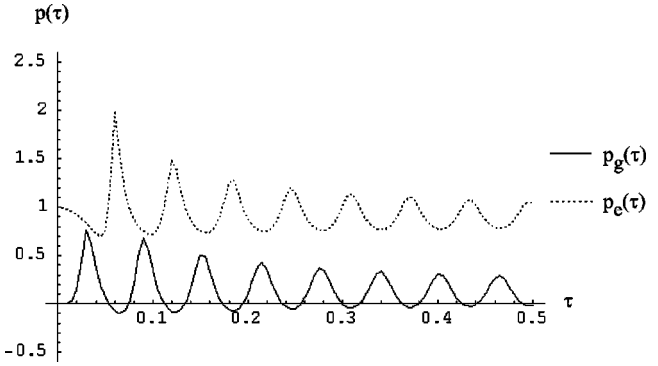

FIG. 5. Time evolution of normalized momenta p_g and p_e when only one of the internal energy levels (ground) is populated initially in wave-packet state.

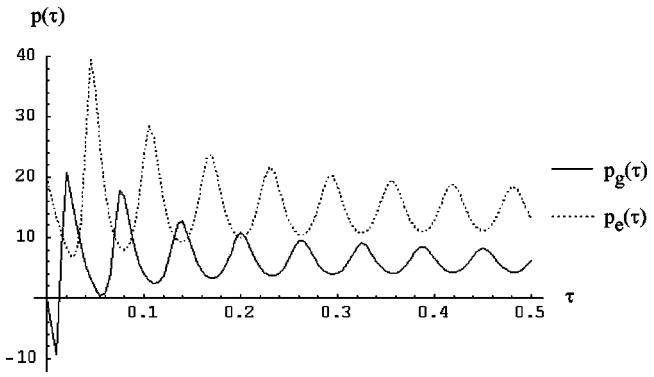

FIG. 6. Time evolution of normalized momenta p_g and p_e for more general superpositional case, when both internal energy levels are initially populated in wave-packet states.

essentially exceeding the $\hbar k$ limits, directly exhibiting the essence of CAMEL phenomenon. For initial intervals of interaction time it has essential oscillations, which later, in full analogy with Rabi oscillations, is depressed. It is worthwhile, probably, to note that the depressing concerns to oscillations only, but not to CAMEL phenomenon on its own; p_g and p_e tend to definite values, different from their initial values. The reason for this convergence is the gradual chaotization of momentum distribution in each internal energy level (see Fig. 1), when the further evolution leaves less and less significant influence on the system behavior, in particular, on the CAMEL behavior.

IV. TRANSLATIONAL ENERGY PER INTERNAL GROUND AND EXCITED ENERGY LEVELS

We have also made an analogous set of calculations for the kinetic energy of atom. It can be readily verified by standard calculations, that the (19)-type splitting is true also for kinetic energy:

$$\langle E_{kin}\rangle = \langle E_{kin}\rangle_g + \langle E_{kin}\rangle_e , \tag{23}$$

with

$$\langle E_{kin}\rangle_g = \int |a(p,t)|^2 \frac{p^2}{2M} dp, \tag{24}$$

$$\langle E_{kin}\rangle_e = \int |b(p,t)|^2 \frac{p^2}{2M} dp, \tag{25}$$

presenting the part contribution of ground and excited internal energy levels into the atomic kinetic energy. Time behavior of Eqs. (24) and (25) for one-level initial population case $[a(p,0)\neq 0, b(p,0)=0]$ exhibits, of course, continuously flopping behavior for familiar one-state population case and oscillatory converging behavior for one-level wave-packet populated case (Fig. 7). The general, both-level population case with different wave-packet distributions, is presented in Fig. 8. For comparison, there we also present the total kinetic energy of atom (thick lines). Note that they have some fluctuations after instant switching on of the interaction (because of the principle of time-energy indeterminacy), which gradually disappears.

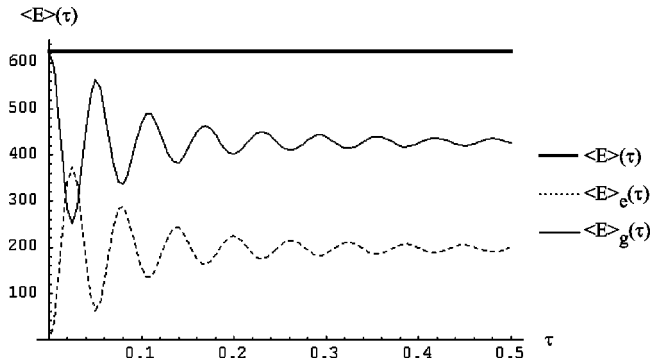

FIG. 7. Time evolution of kinetic energy per each internal energy level and per total atom. The atomic population initially was in one energy level and had some momentum distribution (''intermediate'' wave-packet state). Hereafter, the kinetic energy (ordinate) is scaled to recoil energy $E_r=\hbar^2k^2/2M$.

The behavior of normalized quantities,

$$E_{kin,g} = \langle E_{kin}\rangle_g / n_g , \tag{26}$$

$$E_{kin,e} = \langle E_{kin}\rangle_e / n_e , \tag{27}$$

are presented in Fig. 9 and Fig. 10. The role of initial-state preparation is obvious for kinetic energy, too and is totally consonant to the case of momentum behavior.

Nevertheless, there is a very important difference between the behaviors of momentum and energy, if the internal and the external degrees of atom are discriminated for the interaction. The photon momentum can be transferred to (or be obtained from) external translational degrees of atom only, while the photonic energy deals with both internal and external degrees of freedom, herewith, the internal part is the main one. As a consequence, a logically correct scale, as the photon momentum is for atomic translational motion momentum, absent for atomic translational motion (kinetic) energy. That is why the question about CAMEL-type phenom-

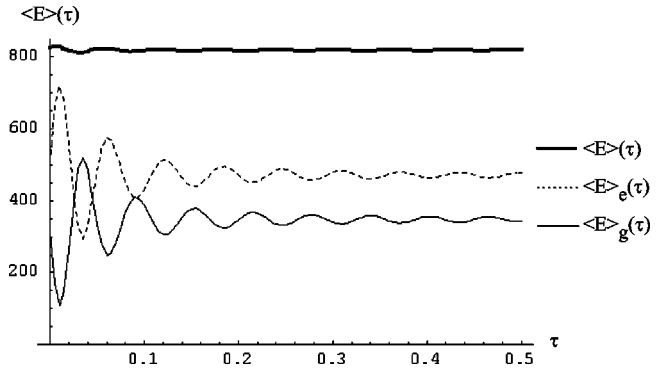

FIG. 8. Time evolution of kinetic energy per each internal energy level and per total atom in general quantum-mechanical case, when both energy levels are populated in mutually different wave-packet states, as in Fig. 3.

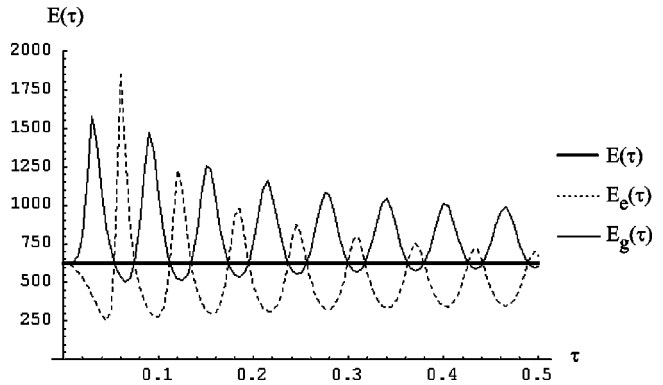

FIG. 9. Time evolution of kinetic energy, normalized by respective level populations. The initial state is ''intermediate,'' as in Fig. 7.

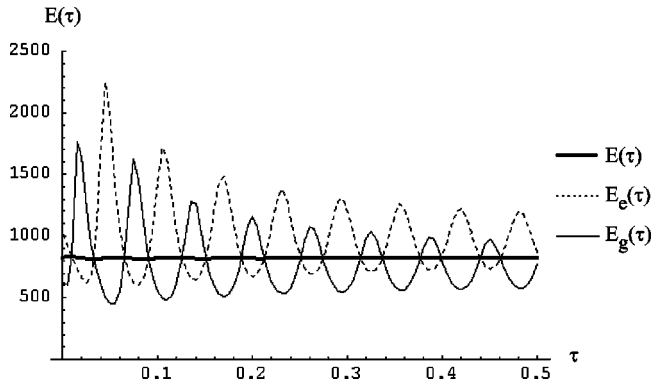

FIG. 10. Time evolution of kinetic energy, normalized by respective level populations. The initial state is general, as in Fig. 8.

enon quantitatively in the translational (kinetic) energy space looks to be problematic, at least hitherto.

V. CONCLUSIONS

The detailed analysis of a two-level atom behavior in the field of near-resonant monochromatic radiation shows that in quantum-mechanical wave-packet states, the Rabi oscillations between internal energy levels monotonically decrease for long times of interaction. The reason for decreasing is the momentum dependence of optical transition probabilities, or, in more usual laser physics terminology, due to the Doppler-shift of Rabi-oscillation frequencies.

It deserves special attention that the mentioned smoothing out of oscillations in Rabi oscillations appears, at first glance, as a repetition of nutation of population, a familiar phenomenon in nonlinear coherent optics. Really, the similarity is only in appearance; the presented phenomenon is conceptually different from one of nutation of population. The presented one is a quantum attribute of a single atom and is directly conditioned by quantum mechanical principle of superposition (for translational states), while at the phenomenon of nutation of population each atom has a definite value of momentum (velocity) and the damping of oscillations appears to be due to momentum (velocity) distribution over the atomic ensemble, for instance, Maxwellian distribution in gas sample. The nutation of population has not any relation to quantum-mechanical principle of superposition and, thereby, is not quantum-mechanical phenomenon in that sense.

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