# Three-level $\Lambda$ atom in the field of frequency-chirped bichromatic laser pulses: Writing and storage of optical phase information

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We investigate the behavior of a three-level atomic system with  $\Lambda$  structure of levels in the field of a sequence of short frequency-chirped bichromatic laser pulses (BLP's) in the adiabatic passage regime of interaction. We show that the efficiency of the population transfer in this atomic system is sensitive to the relative phase of the pulses forming the BLP when the atom is prepared in a coherent superposition of its two ground states initially. We propose to use this sensitivity for coherent fast and robust writing and reading of optical phase information. The initial preparation of the atom in the "dark" superposition state is suggested through the action of a sequence of a few frequency-chirped BLP's with duration longer than the atomic relaxation time.

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#### I. INTRODUCTION

The coherent superposition of quantum states plays an important role in quantum optics, quantum chemistry, and mechanical manipulation and cooling of atoms by laser radiation, as well as in other fields of contemporary science. Population trapping [1] and its use for laser cooling below the recoil limit in the scheme of velocity-selective population trapping [2], construction of atomic beam splitters based on "'dark'' states [3], use of the dressed state with zero energy for complete transfer of the atomic population from one ground state into another without population of the intermediate excited state in the scheme of stimulated Raman adiabatic passage (STIRAP) [4], enhancement of the refractive index of a resonant medium using the appropriate coherent superposition of ground states [5], and quantum computing [6] are some of the most important applications of superposition of quantum states of atoms and molecules. The question of how to produce a desired superposition of atomic (molecular) quantum states and govern them in a fast and robust way is of great practical interest in several fields of contemporary science.

A modification of the STIRAP technique in a three-level  $\Lambda$  atom to create a definite superposition of its ground states by maintaining a fixed ratio of Stokes and pump pulse amplitudes at the end of the interaction was proposed in Refs. [7,8]. This method, however, requires accurate control of the relative strength of Stokes and pump pulses. Inhomogeneous transverse intensity distribution of the laser beams, for example, may cause problems for application of this method. Another method was suggested in Ref. [9] for generation of a coherent superposition of long-lived quantum states robust against small variations of the laser parameters. This method is based on the usual three-level STIRAP scheme with coupling of the intermediate excited state to an additional, fourth metastable state of the atom by a third (control) laser pulse. The final coherent superposition of the ground states in this tripod-linked system can be governed by adjusting the relative delay of the control pulse with respect to the pump and Stokes pulses. The relative phase of the resulting quantum

superposition state is determined by the relative phase of the Stokes and pump pulses.

We propose in this paper an alternative and simpler scheme for generation of arbitrary coherent superposition of two ground states of a three-level  $\Lambda$  atom in a robust and fast way by using frequency-chirped bichromatic laser pulses (BLP's). Each BLP is a superposition of two pulses of the same shape with different carrier (and, in general, different Rabi) frequencies in Raman resonance with the atom. The conditions for the adiabatic passage (AP) regime of interaction between the  $\Lambda$  atom and the frequency-chirped BLP's are assumed to occur in our consideration (see, for example, Ref. [10]).

Each BLP couples the corresponding ground state of the  $\Lambda$  atom to a common excited state, as shown in Fig. 1. Due to the identical shape and chirp of the pulses forming the BLP and the assumed condition of Raman resonance, the system under consideration is equivalent to a two-state quantum system. The ground state of this system consists of a superposition of orthogonal "bright" and "dark" states. The "bright" state is a superposition of two ground states of the initial  $\Lambda$  atom and is coupled by the laser field to the excited state (which is the same as for the initial  $\Lambda$  atom). At the same time, the "dark" superposition of the ground states, which is decoupled from the excited state, does not interact with the laser field.

As is well known, a frequency-chirped laser pulse with duration much shorter than the relaxation times of the quantum system may produce complete transfer of populations between the states of a two-state atom in the AP regime of interaction [10]. The population transfer is not complete if



FIG. 1. The scheme of the  $\Lambda$  atomic system. On the right, the BLP's envelope and the frequency chirp are shown.

the pulse duration is of the order of or larger than the decay time of the excited state.

In our case, the frequency-chirped BLP produces population transfer from the bright superposition of the ground states to the excited state and does not interact with the dark superposition state. Since the bright and dark superposition states depend on the relative phase and relative strength of the pulses forming the BLP, this dependence will be transferred to the population of the excited state. This phase sensitivity may be used for generation of a desired superposition of quantum states (an arbitrary Raman or Zeeman coherence) by controlling the relative (difference) phase of the BLP's.

The writing of optical phase information into the superposition states may be considered as another important application of the phase sensitivity of excitation of the  $\Lambda$  atom. All atoms of the ensemble interacting with the frequencychirped BLP have to be in the same initial superposition of the ground states for coherent optical information writing to be successful [11].

The relaxation of an atom from the excited state results in decoherence, disturbance of the phase relations, and concentration of the atoms in the dark superposition state. To investigate the influence of the atomic relaxation on the processes mentioned above, we use the set of Bloch equations to describe interaction of the three-level  $\Lambda$  atom with a sequence of frequency-chirped BLP's. We show that the initial coherent superposition of the ground states (a dark one) in the ensemble of  $\Lambda$  atoms necessary for phase information writing may be effectively generated by a sequence of frequency-chirped BLP's with duration of the order of the decay time of the excited state of the  $\Lambda$  atom.

The problem of interaction of a short frequency-chirped laser pulse with a two-level quantum system may be solved analytically in the AP regime of interaction [10]. Note that there exist a large number of exact analytic solutions to this problem for certain pulse shapes and modulations in time of the frequency of the laser pulses [12]. We use here simpler approximate solutions corresponding to the AP regime of interaction. These solutions describe with high accuracy the effects of interaction of short frequency-chirped laser pulses with two-level atoms in the case when the laser pulse amplitudes and phases have slow enough modulations in time (for the exact conditions for the AP regime to occur, see Refs. [10,13,14]).

The remainder of this paper is organized as follows. In Sec. II we transform the Schrödinger equation for the probability state amplitudes of the three-level  $\Lambda$  atom into a set of equations for an equivalent two-state atom and analyze the approximate solutions of these equations in the AP regime. In Sec. III, the dressed-state picture of the problem under consideration is developed and analyzed. In Sec. IV we discuss the possibility of generation of a coherent superposition of the ground states in the  $\Lambda$  atom by frequency-chirped BLP's. A reversible population transfer between the ground states of the  $\Lambda$  atom is discussed in Sec. V. In Sec. VI, we discuss the schemes for writing, storage, and reading of the phase and intensity information using short frequencychirped BLP's. Generation of the dark superposition of the ground states in the  $\Lambda$  atom for optical phase information writing is analyzed in Sec. VII by numerical solution of the set of Bloch equations, taking into account the decay of the excited state. This dark-state generation is performed by a sequence of frequency-chirped BLP's having durations longer than the decay time of the excited state. The robustness of the quantum transition schemes analyzed in this paper is discussed in Sec. VIII. A summary of the main results obtained in this paper is presented in Sec. IX.

### II. THE MATHEMATICAL FORMALISM FOR INTERACTION OF SHORT FREQUENCY-CHIRPED BLP's WITH Λ ATOMS

We begin with the Schrödinger equation for the probability amplitudes  $c_i$ , i=1,2,3, of the atomic states of a threelevel  $\Lambda$  atom (see Fig. 1) interacting with a frequencychirped BLP having duration much shorter than all relaxation times of the atomic system. The BLP consists of two linearly polarized laser pulses having the same shapes (with, in general, different amplitudes and Rabi frequencies) with the relative phase  $\Delta \Phi_{12} = \phi_1 - \phi_2$  ( $\phi_1$  and  $\phi_2$  are the phases of the laser pulses forming the BLP). The carrier frequencies of the pulses forming the BLP are in Raman resonance with the  $\Lambda$  atom. The laser pulses are frequency chirped in the same way. A linear chirp of the carrier frequencies is considered in this paper for the sake of simplicity (see the right panel in Fig. 1).

We neglect all relaxation processes in the atomic system at this point due to the short duration of the BLP. This allows us to deal with the Schrödinger equation for the state vector  $\mathbf{c} = (c_1, c_2, c_3)$  [10]:

$$\frac{d}{dt}\mathbf{c} = i\hat{H}_{\mathbf{c}}.$$
 (1)

The rotating-wave-approximation Hamiltonian  $\hat{H}$  is

$$\hat{H} = \begin{pmatrix} 0 & \Omega_1 & 0 \\ \Omega_1^* & \epsilon(t) & \Omega_2^* \\ 0 & \Omega_2 & 0 \end{pmatrix}.$$

The time-dependent Rabi frequencies  $2\Omega_{12}(t)$  and  $2\Omega_{32}(t)$  with maximum values  $W_1$  and  $W_2$  are, respectively,

$$\Omega_1 = \Omega_{12} = \Omega_{21}^* = f(t) W_1 = \frac{1}{2\bar{h}} d_{12} A(t);$$
  
$$\Omega_2 = \Omega_{23} = \Omega_{32}^* = f(t) W_2 = \frac{1}{2\bar{h}} d_{23} A(t).$$

 $d_{ij}$  is the dipole moment matrix element for laser-induced transition from the state  $|j\rangle$  to the state  $|i\rangle$  (i, j = 1, 2, 3). f(t) is the (common) envelope function of the pulses forming the BLP with  $A_j = |A_j| \exp(i\phi_j)$  and  $\omega_{Lj}(t)$  (j = 1, 2) being the constant complex amplitudes and time-dependent carrier frequencies of these pulses. The assumed condition of Raman resonance states that  $\epsilon(t) = \epsilon_{21}(t) \equiv \epsilon_{23}(t)$ , where  $\epsilon_{21} = \omega_{L1}(t) - \omega_{21}$ ,  $\epsilon_{23} = \omega_{L2}(t) - \omega_{23}$  are detunings from one-

photon resonance, and  $\omega_{21}, \omega_{23}$  are the resonant transition frequencies between the corresponding states. The linear chirp of the carrier frequencies of the pulses forming the BLP is  $\omega_{Lj} = \omega_{Lj(0)} + 2\beta t$ , where  $\omega_{Lj(0)}$  (*j*=1,2) are the central frequencies of the pulses and  $2\beta$  is the speed of the chirp.

It is easy to show that an equivalent two-level system may be introduced with the probability amplitudes of the ground and the excited states  $g_b$  and e, respectively, which fully describes the three-level system under consideration:

$$\frac{d}{dt}g_b = iF(t)e, \quad \frac{d}{dt}e - i\epsilon(t)e = iF(t)g_b, \qquad (2)$$

where  $g_b(t) = [W_1^* c_1(t) + W_2^* c_3(t)] / \sqrt{|W_1|^2 + |W_2|^2}$ ,  $e(t) = c_2(t) \exp[i\epsilon(t)t]$ , and  $F(t) = f(t) \sqrt{|W_1|^2 + |W_2|^2}$ .

The amplitude  $g_b(t)$  corresponds to the bright superposition of the ground states that is coupled with the excited state by the frequency-chirped BLP, as follows from Eq. (2). The amplitude of the dark superposition of the ground states is  $g_d(t) \equiv [W_2c_1(t) - W_1c_3(t)]/\sqrt{|W_1|^2 + |W_2|^2}$ . This superposition state is uncoupled from the excited state, as follows from Eq. (1):

$$\frac{d}{dt}g_d = 0. \tag{3}$$

The solution to the two-state problem of Eq. (2) for the population difference  $Z(t) = |e(t)|^2 - |g(t)|^2$  has the following form in the AP regime of interaction [10,13]:

$$Z(t) \cong -Z(-\infty) \frac{\epsilon(t)}{\sqrt{\epsilon^2(t) + F^2(t)}}.$$
(4)

It is well known [see also the solution (4) for a linearly chirped BLP] that complete transfer of the populations of a two-level atom takes place as a result of interaction with a frequency-chirped laser pulse in the AP regime [10]. This means that we have for the final amplitude  $g_{b(fin)}$  of the bright superposition state at the end of the laser pulse

$$g_{b(\text{fin})} = W_1^* c_{1(\text{fin})} + W_2^* c_{3(\text{fin})} = 0$$
(5)

in the case when the atom was in the ground state initially  $(e_{in}=c_{2(in)}=0)$ . The subscripts "in" and "fin" stand for the initial  $(t \rightarrow -\infty)$  and final  $(t \rightarrow \infty)$  values of the populations and phases.

We have for the final populations  $n_{j(\text{fin})} = |c_{j(\text{fin})}|^2$  and phases  $\phi_{j(\text{fin})}$  of the ground states (with the complex probability amplitudes  $c_{j(\text{fin})} = |c_{j(\text{fin})}| \exp[i\phi_{j(\text{fin})}]$ , j = 1,3) in the simplest case of the same Rabi frequencies ( $|W_1| = |W_2|$ ) of the laser pulses

$$n_{1(\text{fin})} = n_{3(\text{fin})}, \quad \phi_{1(\text{fin})} - \phi_{3(\text{fin})} = \Delta \phi_{13(\text{fin})} = \pi + \Delta \Phi_{12}.$$
(6)

The final population  $n_{2(\text{fin})}$  of the excited state of the  $\Lambda$  atom initially in the ground state is (in the case of  $|W_1| = |W_2|$ )



FIG. 2. (a) Dependence of the excited-state population  $n_{2(\text{fin})}$  on the relative phase  $\Delta \Phi_{12}$  of the pulses forming the BLP and on the ground-state population  $n_{1(\text{in})}$ . The initial phase of the Raman coherence is  $\Delta \phi_{13(\text{in})}=0$ . (b) The same as in (a) at different values of the initial population  $n_{1(\text{in})}$ :  $n_{1(\text{in})}=1(1)$ ; 0.8(2); 0.6(3); 0.5(4).

$$n_{2(\text{fin})} = \frac{1}{2} \left[ 1 + 2n_{1(\text{in})} \sqrt{1 - n_{1(\text{in})} \cos(\Delta \phi_{13(\text{in})} + \Delta \Phi_{12})} \right].$$
(7)

Note that the population  $n_{2(\text{fin})} = |c_{2(\text{fin})}|^2$  does not depend on the phases:  $n_{2(\text{fin})} = \frac{1}{2}$  for equal Rabi frequencies  $|W_1|$  $= |W_2|$  in the case of a  $\Lambda$  atom in a single ground state (for example,  $n_{1(\text{in})} = 1$ ) initially, as follows from Eq. (7).

The dependence of the excited-state population on the relative phase  $\Delta \Phi_{12}$  of the pulses forming the BLP and on the initial population of the ground states is depicted in Fig. 2 for zero value of the initial phase  $\Delta \Phi_{13(in)}$  of the Raman coherence. Note complete excitation of the atom at  $\Delta \Phi_{12} = 0$ , and complete suppression of the excitation when  $\Delta \Phi_{12} = \pi$  at the same initial population of the ground states:  $n_{1in} = n_{3in} = \frac{1}{2}$ . These two extreme cases correspond to the atom prepared initially in the bright or dark ground superposition state, respectively.

The peculiarities of excitation of the  $\Lambda$  atom by the frequency-chirped BLP described by Eq. (7) (see also Fig. 2) are the basis for the coherent writing of optical phase information proposed and analyzed in this paper (see below).

#### **III. THE DRESSED-STATE PICTURE**

The peculiarities of population transfer in the three-level  $\Lambda$  atom in the field of a frequency-chirped BLP may also be



FIG. 3. The evolution in time of the dressed states' quasienergies for different values of the normalized Rabi frequency  $\Omega \tau_L = 1(1)$ ; 5(2); 10(3); 15(4).

derived from the dressed-state picture (see, for example [14]). The solution  $\mathbf{c}(t)$  of Eq. (1) can be represented in the basis of the adiabatic dressed states  $\mathbf{b}^{(k)}(t)$ :

$$\mathbf{c}(t) = \sum_{k} r_{k}(t) \mathbf{b}^{(k)}(t) \exp\left[-i \int_{-\infty}^{t} w_{k}(t') dt'\right]$$

with the initial condition at  $t \rightarrow -\infty$ 

$$\mathbf{c}(-\infty) = \sum_{k} r_{k}(-\infty) \mathbf{b}^{(k)}(-\infty),$$

where  $\mathbf{b}^{(k)}(t)$  is the dressed-state eigenvector corresponding to the *k*th eigenvalue (quasienergy)  $w_k$  of the Hamiltonian  $\hat{H}$ :

$$\hat{H}\mathbf{b}^{(k)} = w_k \mathbf{b}^{(k)}.$$
(8)

The quantity  $r_k(t)$  is the statistical weight of the adiabatic dressed-state vector  $\mathbf{b}^{(k)}(t)$  in the (bare) state vector  $\mathbf{c}(t)$ . According to the adiabatic theorem [14],  $r_k(t) \equiv r_k(-\infty)$ .

We obtain for the eigenvalues  $w_k$  from Eq. (8)

$$w_{1} \equiv 0,$$

$$w_{2} = \epsilon(t)/2 + \sqrt{\epsilon^{2}(t)/4 + |\Omega_{1}|^{2} + |\Omega_{2}|^{2}},$$

$$w_{3} = \epsilon(t)/2 - \sqrt{\epsilon^{2}(t)/4 + |\Omega_{1}|^{2} + |\Omega_{2}|^{2}}.$$
(9)

The time dependencies of the three eigenvalues  $w_k$ , k = 1,2,3, are shown in Fig. 3 in the case of a linearly chirped BLP for different values of the generalized Rabi frequency  $\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2}$ . The diabatic lines (the eigenvalues  $w_k$ , k = 1,2,3, versus time in the absence of the laser field when  $\Omega \rightarrow 0$ ) are depicted in Fig. 3 as dashed lines.

We obtain the following solution for the components  $b_j^{(k)}$ (*j*,*k*=1,2,3) of the dressed-state vectors using Eq. (8):

$$b_{1}^{(k)} = \Omega_{1} / N_{k}; \quad b_{2}^{(k)} = w_{k} / N_{k};$$

$$b_{3}^{(k)} = [w_{k}(w_{k} - \epsilon) - |\Omega_{1}|^{2}] / (\Omega_{2}^{*}N_{k}); \quad (10)$$

$$N_{k} = \sqrt{w_{k}^{2} + |\Omega_{1}|^{2} + |w_{k}(w_{k} - \epsilon) - |\Omega_{1}|^{2}|^{2} / |\Omega_{2}|^{2}},$$

$$(k = 1, 2, 3).$$

It follows from Eq. (9), that  $w_2 \rightarrow 0$  and  $w_3 \rightarrow \epsilon$  when  $t \rightarrow -\infty$ . This means that the dressed states  $\mathbf{b}^{(1)}$  and  $\mathbf{b}^{(2)}$  are those coinciding with the bare ground states when the dressed state  $\mathbf{b}^{(3)}$  coincides initially with the bare excited state of the  $\Lambda$  atom.

We have the following relation between the dressed-state and the bare-state vectors at the beginning of the interaction at  $t \rightarrow -\infty$  using Eqs. (9) and (10):

$$\mathbf{c}(-\infty) = R_1 \mathbf{b}^{(1)}(-\infty) + R_2 \mathbf{b}^{(2)}(-\infty) + R_3 \mathbf{b}^{(3)}(-\infty),$$
(11)

where  $R_{i} = r_{i}(-\infty)$ , j = 1, 2, 3, and

$$\mathbf{b}^{(1)}(-\infty) = \frac{1}{\sqrt{|W_1|^2 + |W_2|^2}} \begin{pmatrix} W_2 \\ 0 \\ -W_1 \end{pmatrix},$$
$$\mathbf{b}^{(2)}(-\infty) = \frac{1}{\sqrt{|W_1|^2 + |W_2|^2}} \begin{pmatrix} W_1 \\ 0 \\ W_2 \end{pmatrix},$$
$$\mathbf{b}^{(3)}(-\infty) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$
(12)

The dressed-state vector  $\mathbf{b}^{(1)}(t) \equiv \mathbf{b}^{(1)}(-\infty)$  describes the dark superposition of the ground states. This state vector does not change during the interaction of the  $\Lambda$  atom with the frequency-chirped BLP. In contrast, the dressed-state vector  $\mathbf{b}^{(2)}(-\infty)$  corresponds to the bright superposition of the ground states and is orthogonal to the dark one. It evolves into the bare excited-state vector of the  $\Lambda$  atom at the end of the interaction (see below).

In the case of the  $\Lambda$  atom in the ground state initially,  $R_3=0$ , and we have from Eq. (11) using orthonormality of the vectors  $\mathbf{b}^{(1)}(-\infty)$  and  $\mathbf{b}^{(2)}(-\infty)$ 

$$R_{1,2} = \mathbf{c}(-\infty) \cdot \mathbf{b}^{(1,2)}(-\infty), \qquad (13)$$

where the center dot means a scalar product of vectors. We have at the end of the interaction (at  $t \rightarrow \infty$ )

$$\mathbf{c}(\infty) = R_1 \mathbf{b}^{(1)}(\infty) + R_2 \mathbf{b}^{(2)}(\infty)$$

with  $R_1$  and  $R_2$  being defined by Eq. (13) and the dressedstate vectors of the following form:

$$\mathbf{b}^{(1)}(\infty) \equiv \mathbf{b}^{(1)}(-\infty), \quad \mathbf{b}^{(2)}(\infty) = \begin{pmatrix} 0\\1\\0 \end{pmatrix}.$$

In the case when the  $\Lambda$  atom is in a single ground state initially,

$$\mathbf{c}(-\infty) = \begin{pmatrix} 1\\0\\0 \end{pmatrix}, \quad R_1 = W_2 / \sqrt{|W_1|^2 + |W_2|^2},$$
$$R_2 = W_1 / \sqrt{|W_1|^2 + |W_2|^2}.$$

We obtain at the end of the interaction  $(t \rightarrow \infty)$ 

$$\mathbf{c}^{(\infty)} = \frac{W_2}{|W_1|^2 + |W_2|^2} \begin{pmatrix} W_2 \\ 0 \\ -W_1 \end{pmatrix} + \frac{W_1}{\sqrt{|W_1|^2 + |W_2|^2}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$
(14)

As follows from the expression obtained, exactly one-half of the  $\Lambda$  atom is excited and the other half is divided in equal parts between the ground states as a result of interaction with the frequency-chirped BLP with equal Rabi frequencies of its pulses. Note that the same result follows from exact numerical solution of the equations and is discussed in Sec. V.

It is important to note that the dressed-state vectors of Eq. (12) with the replacements  $W_{1,2} \rightarrow \Omega_{1,2}(t)$  coincide with those used in the scheme of population transfer from one ground state to the other without excitation of the atom referred to as STIRAP. In this scheme, two laser pulses are delayed with respect to each other in such a way that the laser pulse acting first on the  $\Lambda$  atom is resonant with the empty transition. The second pulse overlapping with the first one is resonant with the transition from the initially populated ground state. In this scheme, the dressed-state vector  $\mathbf{b}^{(1)}(t)$  with the eigenvalue  $w_1 \equiv 0$  starts from a bare darkstate vector. This state is dark due to the fact that the first laser pulse is acting on an initially empty transition. This dressed-state vector remains a dark one during the whole interaction and evolves into a bare final-state vector that is dark. too.

The situation is different in the case considered in this paper of no delay between the pulses of the BLP. The state of the  $\Lambda$  atom in a single ground state initially is not dark. It has a bright component, as follows from Eq. (11). The interaction with the frequency-chirped BLP leaves unchanged the dark component of the initial-state vector. The bright component evolves into a state vector corresponding to the bare excited state. Which part of the atom will be excited and which will be transferred into the superposition of the ground states at the end of the interaction with the BLP depends on the initial population distribution among the states as well as on the initial phase difference between the probability amplitudes of the ground states. It also depends on the relative (difference) phase of the pulses forming the BLP. In particular, we have excitation of exactly one-half of an atom initially in a single ground state after interaction with a BLP having the same Rabi frequencies and same phases of the laser pulses forming the BLP, as follows from Eq. (14). The other half of the atomic population evolves in to a coherent superposition of the two ground states, having the same populations and the phase difference of the probability amplitudes (the phase of the Raman coherence) equal to  $\pi$ .

In the case of an atom initially in the excited state  $[R_3 = 1; R_1 = R_2 = 0$  in Eq. (11)], the interaction with the frequency-chirped BLP leads to transfer of the excited-state population into the dark superposition of the ground states corresponding to the eigenvalue  $w_1 = 0$  (see the evolution of the eigenvalue  $w_3$  in Fig. 3):

$$\mathbf{c}(\infty) = \frac{1}{\sqrt{|W_1|^2 + |W_2|^2}} \begin{pmatrix} W_2 \\ 0 \\ -W_1 \end{pmatrix}.$$

#### IV. GENERATION OF COHERENT SUPERPOSITION OF THE GROUND STATES

The atomic system under consideration may be used for construction of a desired coherent superposition of the ground states and for coherent writing, storage, and reading of phase information contained in the BLP. As follows from Eq. (6), the relative phase of the probability amplitudes of the ground states at the end of the BLP is equal to  $\Delta \phi_{13\text{fin}}$  $= \pi + \Delta \Phi_{12}$  with  $\Delta \Phi_{12}$  being the phase difference of the pulses forming the BLP. So the desired phase difference  $\Delta \phi_{13\text{(fin)}}$  and populations of the ground states (desired values of the Raman or Zeeman coherence) may be generated by controlling the difference  $\Delta \Phi_{12}$  of the phases of the pulses forming the BLP.

## V. REVERSIBLE POPULATION TRANSFER BETWEEN GROUND STATES AND OPTICAL PUMPING OF THE Λ ATOM

An interesting property of the system under consideration is the possibility of controllable and reversible redistribution of the atomic populations between the three levels of the  $\Lambda$ atom by action with a sequence of bichromatic frequencychirped laser pulses. In the case of the same Rabi frequencies  $(|W_1| = |W_2|)$  of the BLP's, we have excitation of exactly one-half of the atom after action of the first BLP with an atom in the state  $|1\rangle$  initially. The second half is divided in equal parts between the two ground states. The second BLP results in deexcitation of the atom with all population being concentrated in the initially empty ground state  $|3\rangle$  (see Fig. 4). The action of the third BLP leads to the same population distribution as after the action of the first one. The action of the fourth BLP results in reconstruction of the initial population distribution.

The envelope of the BLP's was taken as a Gaussian function in simulations,  $A(t) = A_0 \exp[-t^2/2\tau_L^2]$ , and the same linear chirp was assumed for the pulses forming the BLP's  $\epsilon_{21}(t) = 2\beta t$ .

Another interesting possibility of using frequency-chirped BLP's may be considered in the coherent optical pumping of the atomic populations into a single quantum state. First, the population of the  $\Lambda$  atom in the dark superposition of the ground states is collected into the excited state by the action of a frequency-chirped BLP having the relative phase of its pulses equal to  $\pi$ . This population in the second step is trans-



FIG. 4. Reversible population transfer between the levels of the  $\Lambda$  atom. The populations after the action of the first (a) and of the second (b) frequency-chirped BLP's. The parameters applied are  $W\tau_L=5$ ,  $\beta\tau_L^2=5$ . The atoms are assumed to be in a single ground state initially:  $n_{1(in)}=1$ .

ferred into a long-living state by the action of a subsequent frequency-chirped laser pulse in the AP regime of interaction.

### VI. WRITING, STORAGE, AND READING OF PHASE INFORMATION

It follows from Eq. (7) that the population  $n_{2(\text{fin})}$  of the excited state is a function of the relative phase  $\Delta \Phi_{12}$  of the pulses forming the BLP. So the phase information contained in the relative phase  $\Delta \Phi_{12}$  may be written in the population  $n_{2(\text{fin})}$  of the excited state. Since the population of the excited state depends on the initial phase difference of the ground states' probability amplitudes, it is obvious that the atoms have to be prepared initially in the same coherent superposition of the ground states, with the same initial populations  $n_{1(\text{in})}$ ,  $n_{3(\text{in})}$ , and phase  $\Delta \phi_{13(\text{in})}$ . This superposition state may be a dark one.

The information written in the population of the excited state, however, will be distorted due to spontaneous decay from the excited state during the decay time of this state. One of the ways to preserve this information is to transfer all the population of this state to a long-lived state (the state  $|4\rangle$  in Fig. 1), by the action of a subsequent frequency-chirped short laser pulse. Reading of the phase information stored in this long-lived state may be done by action of the same chirped laser pulse, which transforms the phase information into the population distribution of the excited atomic state. The latter may be detected, for example, by analyzing the



FIG. 5. (a) Transverse distribution of the relative phase  $\Delta \Phi_{12}$  of the pulses forming the BLP and (b) the corresponding transverse distribution of the excited-state population after the interaction of this BLP with the  $\Lambda$  atom.

spontaneous or stimulated emission from this state. An example of the transfer of the space distribution of the relative phase  $\Delta \Phi_{12}$  of the pulses forming the BLP into the population of the excited state is shown in Fig. 5.

### VII. DARK-STATE PREPARATION BY A FREQUENCY-CHIRPED BLP

Initial preparation of atoms in the same superposition of the ground states (e.g., in a dark one) is a necessary condition for the coherent writing of phase information in atomic quantum states. One of the commonly used methods to obtain a trapped or dark state in the  $\Lambda$  atom is the action by two cw laser beams each resonant with a corresponding transition of the  $\Lambda$  atom. Stimulated excitation and deexcitation of the atom by the laser radiation accompanied by spontaneous relaxation from the excited state result in generation of the dark superposition of the ground states in the  $\Lambda$  atom. The atom in this superposition state does not interact with the laser field. In a similar way, by using only one laser beam, it is possible to optically pump the atom into a single ground state.

The analysis shows that the efficiency of generation of the

dark superposition states in the  $\Lambda$  atom under consideration does not depend significantly on the frequency chirp of the laser pulses whose duration has to be longer than the decay time of the excited state. However, using frequency-chirped laser pulses seems to be more advantageous for dark-state generation in the case when the levels of the  $\Lambda$  atom have substructure (not considered here, being the matter of our forthcoming paper). Indeed, when using two cw laser beams, for example, in the case of spectrally resolved magnetic (Zeeman) substructure of the atomic states, it is necessary to tune the frequency of the laser beams into resonance with transitions from each individual pair of the ground magnetic sublevels to the excited state to generate dark superposition of these sublevels. However, the spontaneous decay of the excited state, which plays an important role in generation of the dark states, will induce destruction of the previously generated dark superpositions of the sublevels, which are off resonance with the laser beams. A similar effect in optical pumping was observed experimentally in Ref. [15] when the frequency tuning of a (single) cw pumping laser resulted in reversal of direction of the optical pumping in an atomic system having substructure of the levels.

In contrast, dark-state generation and optical pumping by a sequence of frequency-chirped laser pulses do not require the tuning of the pulse frequency as in the case of pulses without frequency chirp. The frequency of *each* frequencychirped laser pulse is swept through the resonance with *each* individual transition from the sublevels of corresponding atomic levels having spectrally resolved substructure. The interaction of each subsequent frequency-chirped laser pulse with each corresponding resonant transition in the atoms eliminates the destruction of the previously generated dark superposition states, or the reversal of the optical pumping direction observed in the case of laser pulses without frequency chirp.

To examine the generation of the dark superposition of the ground states of the  $\Lambda$  atom, we have numerically solved the set of Bloch equations for the density matrix elements in the field of a sequence of three frequency-chirped BLP's including spontaneous decay from the excited atomic state. The results of the simulations are shown in Fig. 6. The generation of a dark state is clearly seen in Fig. 6(c) after the action of the third frequency-chirped BLP.

#### VIII. DISCUSSION

Let us consider as a model for the  $\Lambda$  atom (see Fig. 1) the <sup>85</sup>Rb atom with two hyperfine sublevels of the 5  ${}^{2}S_{1/2}$  state as two ground states and one of the hyperfine sublevels of the 5  ${}^{2}P_{3/2}$  state as an excited state. The decay time of the excited state is about 27 ns [16]. To avoid the dephasing action of the spontaneous transitions, the laser pulses have to be shorter than this decay time. Also, there is another restriction on the pulse duration from below connected with fulfilment of conditions for the AP regime assumed in this paper [10,13,14]. It states that the speed of variation in time of the Rabi frequency must be sufficiently slow with respect to the pulse's Rabi frequency. To avoid the necessity of very large

pulse peak intensities, we assume that our pulses are in the nanosecond duration range. Note that, while nowadays there is a very good and routine technique for chirping laser pulses with durations shorter than 1 ps by using a pair of diffraction gratings [17,18], it is more difficult to chirp laser pulses of nanosecond duration because of their insufficiently broad frequency spectrum. However, this may be achieved by using lasers with frequency-shifted feedback [19,20] or by using diode lasers with time-modulated pumping current along with an external Fabry-Pérot resonator.

For experimental realization of the quantum transition schemes proposed in this paper, it is important to test the robustness of these schemes against fluctuations of the pumping pulses' parameters. To model fluctuations of the Rabi frequency, we include a harmonic modulation in time (with depth equal to 30% of the average value) of the Rabi frequency of pumping pulses forming the BLP. The resulting quantum state populations of the  $\Lambda$  atom (initially in one of its ground states) are shown in Fig. 7 after the action of a sequence of nine frequency-chirped BLP's. Comparison with Fig. 4(a) shows a negligibly small difference between the results obtained and those corresponding to the case of no modulation of the Rabi frequency. This may be considered as a demonstration of the robustness of the transition schemes against intensity fluctuations of the frequencychirped BLP's.

To model phase fluctuations, we include a modulation in time of the chirp speed of the BLP. As follows from the results of our numerical analysis, the speed of the chirp of the pumping pulses is more sensitive to fluctuations. However, the influence of the modulation of the chirp speed on the resulting quantum state population distribution is negligibly small when the modulation depth is less than 20% for a pulse sequence of up to ten frequency-chirped BLP's. We conclude that the schemes proposed in this paper are still stable against phase fluctuations with amplitude less than 20% of the average values. It is not surprising because all effects described in this paper are assumed to take place in the AP regime of interaction, which is robust against fluctuations of the pump beam parameters [10].

#### **IX. CONCLUSIONS**

In conclusion, the results of analysis of a bichromatic frequency-chirped laser pulse interaction with three-level  $\Lambda$  atoms are presented and a number of applications of this interaction are considered in this paper.

We have shown that an arbitrary desired Raman or Zeemann coherence may be generated in  $\Lambda$  atoms by governing the relative phase of the pulses forming the BLP.

The dependence of the excitation probability of the  $\Lambda$  atom on the relative phase of the pulses forming the BLP has been used for coherent writing, storage, and reading of optical phase information. The information written in the relative phase of the pulses forming the BLP is transferred into the population of the excited state, and a subsequent frequency-chirped short laser pulse is applied to store the phase information. This pulse transfers the population of the excited state quickly into a long-lived quantum state of the atom to



FIG. 6. Time dependence of the atomic populations in the field of a frequency-chirped BLP with duration  $\tau_L = 2T_{rel}$  ( $T_{rel}$  is the decay time of the excited state) after the action of the first (a), the second (b), and the third (c) frequency-chirped BLP. The parameters applied are  $W_1\tau_L = W_2\tau_L = 5$ ,  $\beta\tau_L^2 = 5$ . Atoms are assumed to be in one of the ground states initially.

prevent its distortion due to relaxation processes. Reading of the stored information may be produced by excitation of the atoms by the same frequency-chirped short laser pulse in the AP regime of interaction, and subsequent analysis of the spontaneous or stimulated fluorescence from the excited state.

The atoms have to be prepared in a coherent superposition of the ground states initially to use the proposed method of



FIG. 7. Reversible population transfer between the levels of the  $\Lambda$  atom: the populations after the action of a sequence of nine frequency-chirped BLP's with time-modulated Rabi frequency. The depth of the modulation is 30%. The other parameters applied are  $W\tau_L=5$ ,  $\beta\tau_L^2=5$ . The atoms are assumed to be in a single ground state initially,  $n_{1(in)}=1$ .

writing the phase information. The possibility of preparation of the  $\Lambda$  atoms in the dark superposition of the ground states has been analyzed using the set of Bloch equations, taking into account the decay of the excited atomic state. It is shown that such a dark superposition state may be generated by a sequence of a few frequency-chirped BLP's having durations of the order of or longer than the decay time of the excited state.

The possibilities of reversible population transfer between the ground states and coherent optical pumping of the population of the  $\Lambda$  atom into a single quantum state have also been shown using frequency-chirped BLP's.

It is worth noting that information writing and reading as well as reversible population transfer between the quantum states of the  $\Lambda$  atom are *fast* and *robust* in the proposed schemes. These processes have time scales equal to those of the laser pulses, whose duration may be chosen to be very short. The restriction on the duration of the laser pulses is connected mainly with the conditions for the AP regime of interaction [10,13,14]. The robustness of these processes comes from the robustness of the population transfer in quantum systems produced by frequency-chirped laser pulses in the AP regime of interaction. It is well known that the efficiency of such population transfer may reach 100% and is insensitive to the shape and transverse intensity distribution of the laser pulses, as well as to the exact resonance conditions.

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