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Concentration and purification scheme for two partially entangled photon pairs

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An experimental scheme for concentrating entanglement in partially entangled photon pairs is proposed. In this scheme, two separated parties obtain one maximally entangled photon pair from two previously shared partially entangled photon pairs by local operations and classical communication. A practical realization of the proposed scheme is discussed, which uses imperfect photon detectors and spontaneous parametric down-conversion as a photon source. This scheme also works for purifying a class of mixed states.

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I. INTRODUCTION

In many applications in quantum information processing, such as quantum teleportation [1–3] and entanglement based quantum key distribution [4,5], it is essential that two separated parties, Alice and Bob, share the maximally entangled particles in advance. In practice, a quantum channel, to be used to distribute the pairs, is usually noisy. It is thus important that Alice and Bob share maximally entangled pairs even through such channels. Entanglement concentration [6] and purification (or distillation) [7] were originally proposed for that purpose. In these schemes, previously shared less entangled pairs can be transformed into a smaller number of maximally entangled pairs by local operations and classical communication (LOCC). Many schemes to obtain maximally entangled particles by LOCC have been proposed [8–11].

In this paper we propose an experimentally feasible concentration and purification scheme, in which a maximally entangled photon pair is obtained from two photon pairs in identical partially entangled states. The essential idea of this paper is based on the concentration scheme proposed by Bennett *et al.* [6]. In their proposal, Alice or Bob performs a collective measurement for the joint state of n pairs of particles (called the Schmidt projection method), and then they convert the projected state into a smaller number of maximally entangled pairs. For polarization entangled photons, however, the Schmidt projection method is difficult to perform because collective and nondestructive measurements for photons are not feasible today. In our scheme, Alice and Bob use only linear optical elements and photon detectors, in which destructive detection of two photons realizes the required projection and the conversion at the same time. In a similar scheme [8], which uses entanglement swapping for two pairs of entangled photons, it is assumed that initially Alice has both photons of one pair and Alice and Bob share photons of the other pair. In our scheme, in contrast, we assume that the two pairs are distributed in the same way, namely, Alice obtains one member of each photon pair, and Bob obtains the other member of each photon pair, as shown in Fig. 1. This feature makes the proposed scheme applicable to quantum channels with unknown fluctuations, namely, the proposed scheme also works for purifying a class of mixed states. In the following, therefore, we use ‘‘purification’’ instead of ‘‘concentration and purification’’ for simplicity.

This paper is organized as follows. In Sec. II, we explain our purification scheme in an ideal situation. In Sec. III, we discuss two types of imperfect detector and analyze the state after purification. In Sec. IV, we consider the use of spontaneous parametric down-conversion (PDC) as a photon pair source and the effect of dark counts of the detectors. Finally, we discuss in Sec. V the required property of fluctuating quantum channels for our scheme to be applicable.

II. BASIC IDEA

In this section, we show how the two separated parties Alice and Bob can purify a maximally entangled photon pair from two identical partially entangled photon pairs by LOCC. Let us assume that Alice and Bob are given two pairs of photons in the following polarization entangled states (we will describe a method creating this state in Sec. IV):

$$|\alpha,\beta\rangle_{12}|\alpha,\beta\rangle_{34}\equiv(\alpha|1\rangle_{1\mathrm{H}}|1\rangle_{2\mathrm{H}}+\beta|1\rangle_{1\mathrm{V}}|1\rangle_{2\mathrm{V}})\otimes(\alpha|1\rangle_{3\mathrm{H}}|1\rangle_{4\mathrm{H}}+\beta|1\rangle_{3\mathrm{V}}|1\rangle_{4\mathrm{V}}), \quad (1)$$

where α and β are complex numbers satisfying $|\alpha|^2+|\beta|^2=1$ and $|n\rangle$ is the normalized n-photon number state. The subscript numbers represent the spatial modes, and H and V represent horizontal and vertical polarization modes, respectively. As shown in Fig. 1, Alice receives photons in modes 1 and 3, and Bob receives photons in modes 2 and 4. For simplicity, we omit the modes in the vacuum, using abbreviations such as $|1\rangle_{1\mathrm{H}}|1\rangle_{2\mathrm{H}}|0\rangle_{1\mathrm{V}}|0\rangle_{2\mathrm{V}}\rightarrow|1\rangle_{1\mathrm{H}}|1\rangle_{2\mathrm{H}}$. Alice

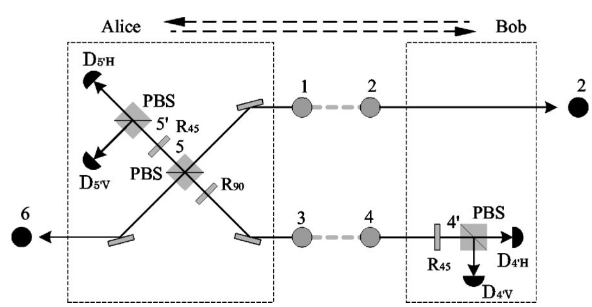

FIG. 1. The schematic diagram of the proposed purification scheme. Polarization beam splitters (PBS) transmit H photons and reflect V photons. $\lambda/2$ wave plates R_{45} and R_{90} rotate the polarization by 45° and 90°, respectively.

and Bob can transform these photons into a maximally entangled photon pair in modes 6 and 2, in the following way. Equation (1) is expanded as

$$\alpha^2|1\rangle_{1\mathrm{H}}|1\rangle_{3\mathrm{H}}|1\rangle_{2\mathrm{H}}|1\rangle_{4\mathrm{H}}+\beta^2|1\rangle_{1\mathrm{V}}|1\rangle_{3\mathrm{V}}|1\rangle_{2\mathrm{V}}|1\rangle_{4\mathrm{V}} + \alpha\beta(|1\rangle_{1\mathrm{H}}|1\rangle_{3\mathrm{V}}|1\rangle_{2\mathrm{H}}|1\rangle_{4\mathrm{V}}+|1\rangle_{1\mathrm{V}}|1\rangle_{3\mathrm{H}}|1\rangle_{2\mathrm{V}}|1\rangle_{4\mathrm{H}}). \quad (2)$$

Note that the third and fourth terms in Eq. (2) have the same coefficients $\alpha\beta$. Alice rotates the polarization of the photon in mode 3 by 90° using a λ/2 wave plate (R_{90}) and sends it to one port of a polarization beam splitter (PBS). The photon in mode 1 is sent to another port of the PBS. After the PBS, the state of Eq. (2) is transformed into

$$\alpha^2|1\rangle_{6\mathrm{H}}|1\rangle_{6\mathrm{V}}|1\rangle_{2\mathrm{H}}|1\rangle_{4\mathrm{H}}+\beta^2|1\rangle_{5\mathrm{V}}|1\rangle_{5\mathrm{H}}|1\rangle_{2\mathrm{V}}|1\rangle_{4\mathrm{V}} + \alpha\beta(|1\rangle_{5\mathrm{H}}|1\rangle_{6\mathrm{H}}|1\rangle_{2\mathrm{H}}|1\rangle_{4\mathrm{V}}+|1\rangle_{5\mathrm{V}}|1\rangle_{6\mathrm{V}}|1\rangle_{2\mathrm{V}}|1\rangle_{4\mathrm{H}}). \quad (3)$$

Note that there are two photons in the same spatial modes for the first two terms. Alice and Bob rotate the polarizations of their photons in modes 5 and 4 by 45° using λ/2 wave plates (R_{45}). These transformations are expressed by

$$|1\rangle_{k\mathrm{H}}\rightarrow\frac{1}{\sqrt{2}}(|1\rangle_{k'\mathrm{H}}+|1\rangle_{k'\mathrm{V}}), \quad (4)$$

$$|1\rangle_{k\mathrm{V}}\rightarrow\frac{1}{\sqrt{2}}(|1\rangle_{k'\mathrm{H}}-|1\rangle_{k'\mathrm{V}}), \quad (5)$$

and

$$|1\rangle_{k\mathrm{H}}|1\rangle_{k\mathrm{V}}\rightarrow\frac{1}{\sqrt{2}}(|2\rangle_{k'\mathrm{H}}-|2\rangle_{k'\mathrm{V}}), \quad (6)$$

where $k=4,5$. The state of Eq. (3) is then transformed into

$$\begin{aligned}|\Psi\rangle=&\frac{\alpha^2}{\sqrt{2}}|0\rangle_{5'}(|1\rangle_{4'\mathrm{H}}+|1\rangle_{4'\mathrm{V}})|1\rangle_{6\mathrm{H}}|1\rangle_{6\mathrm{V}}|1\rangle_{2\mathrm{H}}\\&+\frac{\beta^2}{2}(|2\rangle_{5'\mathrm{H}}|1\rangle_{4'\mathrm{H}}-|2\rangle_{5'\mathrm{H}}|1\rangle_{4'\mathrm{V}}-|2\rangle_{5'\mathrm{V}}|1\rangle_{4'\mathrm{H}}\\&+|2\rangle_{5'\mathrm{V}}|1\rangle_{4'\mathrm{V}})|0\rangle_{6}|1\rangle_{2\mathrm{V}}\\&+\frac{\alpha\beta}{\sqrt{2}}(|1\rangle_{5'\mathrm{H}}|1\rangle_{4'\mathrm{H}}|\Phi^{(+)}\rangle_{62}-|1\rangle_{5'\mathrm{H}}|1\rangle_{4'\mathrm{V}}|\Phi^{(-)}\rangle_{62}\\&+|1\rangle_{5'\mathrm{V}}|1\rangle_{4'\mathrm{H}}|\Phi^{(-)}\rangle_{62}-|1\rangle_{5'\mathrm{V}}|1\rangle_{4'\mathrm{V}}|\Phi^{(+)}\rangle_{62}),\end{aligned} \quad (7)$$

where $|\Phi^{(\pm)}\rangle_{62}\equiv 1/\sqrt{2}(|1\rangle_{6\mathrm{H}}|1\rangle_{2\mathrm{H}}\pm|1\rangle_{6\mathrm{V}}|1\rangle_{2\mathrm{V}})$ is the state of the maximally entangled photon pair. If Alice and Bob detect a single photon at $D_{5'\mathrm{H}}$ and $D_{4'\mathrm{H}}$ (or $D_{5'\mathrm{V}}$ and $D_{4'\mathrm{V}}$) and the state is projected to $|1\rangle_{5'\mathrm{H}}|1\rangle_{4'\mathrm{H}}|\Phi^{(+)}\rangle_{62}$ (or $|1\rangle_{5'\mathrm{V}}|1\rangle_{4'\mathrm{V}}|\Phi^{(+)}\rangle_{62}$), they can share a maximally entangled photon pair in the state $|\Phi^{(+)}\rangle_{62}$. If they detect a single photon at $D_{5'\mathrm{H}}$ and $D_{4'\mathrm{V}}$ (or $D_{5'\mathrm{V}}$ and $D_{4'\mathrm{H}}$), they receive a maximally entangled photon pair in the state $|\Phi^{(-)}\rangle_{62}$. In this case, they can easily transform it into the form of $|\Phi^{(+)}\rangle_{62}$. Therefore the probability of sharing a maximally entangled photon pair in the state $|\Phi^{(+)}\rangle_{62}$ is $2|\alpha\beta|^2$.

In this scheme, Alice and Bob need not know the values of α and β. Suppose that they receive the photons in a mixed state written as

$$\rho=\int P(\alpha,\beta)|\alpha,\beta\rangle_{12}\langle\alpha,\beta|\otimes|\alpha,\beta\rangle_{34}\langle\alpha,\beta|d^2\alpha d^2\beta, \quad (8)$$

where $P(\alpha,\beta)$ is the probability distribution of their receiving the photon pairs in the state $|\alpha,\beta\rangle_{12}|\alpha,\beta\rangle_{34}$. In this case, the state of the photons just before the detection becomes a mixture of Eq. (7) with various values of α and β. They can, nevertheless, obtain a maximally entangled photon pair with the probability $\int 2|\alpha\beta|^2P(\alpha,\beta)d^2\alpha d^2\beta$. Since they can share a maximally entangled photon pair from pairs in a mixed state, this scheme can be called entanglement purification.

III. PURIFICATION USING IMPERFECT DETECTION

In this section, we study the property of output states in modes 6 and 2 when detectors with quantum efficiency η are used. We consider two kinds of detector, conventional photon detectors and single photon detectors. Conventional photon detectors (e.g., EG&G SPCM) cannot distinguish a single photon from two or more photons. Single photon detectors, which were recently demonstrated experimentally, can distinguish a single photon from two or more photons [16]. In the following, we investigate the influence of the quantum efficiency on the output states in modes 6 and 2, and show that Alice and Bob receive a mixture of $|\Phi^{(+)}\rangle_{62}$ and $|0\rangle_6|1\rangle_{2\mathrm{V}}$ unless they use single photon detectors with unit quantum efficiency.

Consider a photon detector with quantum efficiency η, which can distinguish any number of photocounts. Positive-operator-valued-measure (POVM) elements [14] of finding n photocounts can be written as [12]

$$\Pi_n=\sum_{m=n}^{\infty}\eta^n(1-\eta)^{m-n}C_n^m|m\rangle\langle m|, \quad (9)$$

where C_n^m is the binomial coefficient and $\Sigma_{n=0}^{\infty}\Pi_n=1$. Using this POVM, we can obtain the expression for the POVM elements for a conventional photon detector and a single photon detector. The POVM elements for a conventional photon detector can be written as [13]

$$\Pi_{c0}=\Pi_0=\sum_{m=0}^{\infty}(1-\eta)^m|m\rangle\langle m| \quad (10)$$

and

$$\Pi_{c1}=1-\Pi_0=\sum_{m=1}^{\infty}[1-(1-\eta)^m]|m\rangle\langle m|. \quad (11)$$

Here Π_{c0} is the POVM element for no photocounts, and Π_{c1} is that for photocounts. The POVM elements for a single photon detector can be written as

$$\Pi_{s0}=\Pi_0=\sum_{m=0}^{\infty}(1-\eta)^m|m\rangle\langle m|, \tag{12}$$

$$\Pi_{s1}=\Pi_1=\sum_{m=1}^{\infty}m\eta(1-\eta)^{m-1}|m\rangle\langle m|, \tag{13}$$

and

$$\Pi_{s2}=1-\Pi_0-\Pi_1=\sum_{m=2}^{\infty}[1-(1-\eta+m\eta)(1-\eta)^{m-1}]|m\rangle\langle m|. \tag{14}$$

Here Π_{s0} is the POVM element for no photocounts, Π_{s1} is that for single photocounts, and Π_{s2} is that for multiple photocounts. Using these POVM elements, we can calculate the output states after the detection at imperfect detectors $D_{5'\mathrm{H}}$, $D_{4'\mathrm{H}}$, $D_{5'\mathrm{V}}$, and $D_{4'\mathrm{V}}$.

Let us first consider the case where Alice and Bob use conventional photon detectors. Suppose that a coincidence detection is obtained at detectors $D_{5'\mathrm{H}}$ and $D_{4'\mathrm{H}}$. In this case photons are not detected at the detector $D_{5'\mathrm{V}}$ nor $D_{4'\mathrm{V}}$. The output state in modes 6 and 2 after this detection is calculated as

$$\rho_{\mathrm{out}}^{\mathrm{c}}=\frac{\mathrm{Tr}_{5',4'}[\Pi_{\mathrm{c1}}^{5'\mathrm{H}}\Pi_{\mathrm{c1}}^{4'\mathrm{H}}|\Psi\rangle\langle\Psi|]}{\mathrm{Tr}[\Pi_{\mathrm{c1}}^{5'\mathrm{H}}\Pi_{\mathrm{c1}}^{4'\mathrm{H}}|\Psi\rangle\langle\Psi|]}=\frac{|\alpha|^2|\Phi^{(+)}\rangle_{62}\langle\Phi^{(+)}|+[1-(\eta/2)]|\beta|^2|0\rangle_6\langle 0|\otimes|1\rangle_{2\mathrm{V}}\langle 1|}{1-(\eta/2)|\beta|^2}, \tag{15}$$

where the superscripts of the POVM elements represent the modes. Note that Eq. (15) is a classical mixture of the desired state $|\Phi^{(+)}\rangle_{62}$ and an error state $|0\rangle_6|1\rangle_{2\mathrm{V}}$. The probability of the coincidence detection P can thus be regarded as the sum of two probabilities P_{s} and P_{e}, where P_{s} is the probability of obtaining a photon pair in the state $|\Phi^{(+)}\rangle_{62}$ and P_{e} is the probability of obtaining a single photon in the state $|0\rangle_6|1\rangle_{2\mathrm{V}}$. These probabilities are calculated as $P=\mathrm{Tr}[\Pi_{\mathrm{c1}}^{5'\mathrm{H}}\Pi_{\mathrm{c1}}^{4'\mathrm{H}}|\Psi\rangle\langle\Psi|]=\eta^2|\beta|^2[2|\alpha|^2+(2-\eta)|\beta|^2]/4$, $P_{\mathrm{s}}=\eta^2|\alpha\beta|^2/2$, and $P_{\mathrm{e}}=\eta^2(2-\eta)|\beta|^4/4$. The minimum value of P_{e} is $|\beta|^4/4$. Alice and Bob can also obtain the output state $\rho_{\mathrm{out}}^{\mathrm{c}}$ when they obtain the other three combinations of coincidence, namely, $(D_{5'\mathrm{V}}, D_{4'\mathrm{V}})$, $(D_{5'\mathrm{H}}, D_{4'\mathrm{V}})$, and $(D_{5'\mathrm{V}}, D_{4'\mathrm{H}})$. Therefore the probability of obtaining the output state $\rho_{\mathrm{out}}^{\mathrm{c}}$ is $4P$.

Similarly, in the case where Alice and Bob use single photon detectors, the output state in modes 6 and 2 after the detection is calculated as

$$\rho_{\mathrm{out}}^{\mathrm{s}}=\frac{\mathrm{Tr}_{5',4'}[\Pi_{\mathrm{s1}}^{5'\mathrm{H}}\Pi_{\mathrm{s1}}^{4'\mathrm{H}}|\Psi\rangle\langle\Psi|]}{\mathrm{Tr}[\Pi_{\mathrm{s1}}^{5'\mathrm{H}}\Pi_{\mathrm{s1}}^{4'\mathrm{H}}|\Psi\rangle\langle\Psi|]}=\frac{|\alpha|^2|\Phi^{(+)}\rangle_{62}\langle\Phi^{(+)}|+(1-\eta)|\beta|^2|0\rangle_6\langle 0|\otimes|1\rangle_{2\mathrm{V}}\langle 1|}{1-\eta|\beta|^2}. \tag{16}$$

Note that Eq. (16) is also a classical mixture of $|\Phi^{(+)}\rangle_{62}$ and $|0\rangle_6|1\rangle_{2\mathrm{V}}$. The probabilities P, P_{s}, and P_{e} are calculated as $P=\mathrm{Tr}[\Pi_{\mathrm{s1}}^{5'\mathrm{H}}\Pi_{\mathrm{s1}}^{4'\mathrm{H}}|\Psi\rangle\langle\Psi|]=\eta^2|\beta|^2[|\alpha|^2+(1-\eta)|\beta|^2]/2$, $P_{\mathrm{s}}=\eta^2|\alpha\beta|^2/2$, and $P_{\mathrm{e}}=\eta^2(1-\eta)|\beta|^4/2$. Note that P_{s} is the same as in the case using the conventional photon detectors, but P_{e} is different and its minimum value is 0.

The error in the output state $\rho_{\mathrm{out}}^{\mathrm{c}}$ or $\rho_{\mathrm{out}}^{\mathrm{s}}$ stems from the state $|0\rangle_6|1\rangle_{2\mathrm{V}}$ containing only one photon. Therefore, if Alice and Bob are allowed to perform postselection, in which they select the events of the photocounts in modes 6 and 2, they can discard the events of error. In this situation, the type of detector is not relevant, and the success probability is solely determined by the quantum efficiency η.

IV. IMPLEMENTATION WITH A PDC SOURCE

In this section, we consider the use of spontaneous parametric down-conversion as a photon source of the input states for the proposed purification scheme, and discuss the property of the output state. We also discuss the effect of the dark counts of the detectors.

A. Entangled photon pairs from PDC

The partially entangled photon pair $|\alpha,\beta\rangle_{12}$ can be generated by pumping combined crystals, as shown in Fig. 2 [15]. The degree of entanglement can be continuously changed by rotating the polarization of the pump beam. The generated state $|\Psi\rangle_{12}$ can be written as $|\Psi\rangle_{12}=|\Psi\rangle_{12\mathrm{H}}|\Psi\rangle_{12\mathrm{V}}$, where $|\Psi\rangle_{12\mathrm{H}}$ and $|\Psi\rangle_{12\mathrm{V}}$ are the down-converted states generated from crystals C_{H} and C_{V}, respectively, and are written as [17]

$$|\Psi\rangle_{12\mathrm{H}}=\mathrm{sech}|\gamma_{\mathrm{H}}|\sum_{n=0}^{\infty}\left(\frac{\gamma_{\mathrm{H}}}{|\gamma_{\mathrm{H}}|}\tanh|\gamma_{\mathrm{H}}|\right)^n|n\rangle_{1\mathrm{H}}|n\rangle_{2\mathrm{H}} \tag{17}$$

and

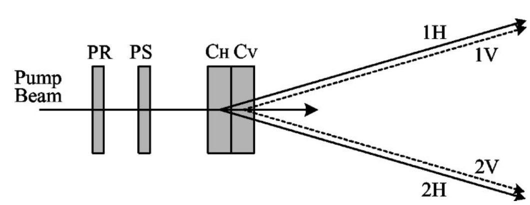

FIG. 2. Partially entangled photon source. A photon pair in modes H and V is generated at nonlinear crystals C_{H} and C_{V}, respectively. PR is a polarization rotator and PS is a phase shifter.

$$|\Psi\rangle_{12\mathrm{V}}=\mathrm{sech}|\gamma_{\mathrm{V}}|\sum_{n=0}^{\infty}\left(\frac{\gamma_{\mathrm{V}}}{|\gamma_{\mathrm{V}}|}\tanh|\gamma_{\mathrm{V}}|\right)^{n}|n\rangle_{1\mathrm{V}}|n\rangle_{2\mathrm{V}} \quad (18)$$

with $\gamma_{\mathrm{H}}\equiv|\gamma_{\mathrm{H}}|e^{i(\phi_{\mathrm{p}}+\Delta\phi_{\mathrm{p}}/2)}$ and $\gamma_{\mathrm{V}}\equiv|\gamma_{\mathrm{V}}|e^{i(\phi_{\mathrm{p}}-\Delta\phi_{\mathrm{p}}/2)}$. Here, γ_{H} (γ_{V}) is proportional to the complex amplitude of the classical pump beam for C_{H} (C_{V}). The phases of the pump beams for C_{H} and C_{V} are expressed by $\phi_{\mathrm{p}}+\Delta\phi_{\mathrm{p}}/2$ and $\phi_{\mathrm{p}}-\Delta\phi_{\mathrm{p}}/2$, respectively, where $\Delta\phi_{\mathrm{p}}$ is the phase difference between the two pump beams. The ratio of $|\gamma_{\mathrm{H}}|$ and $|\gamma_{\mathrm{V}}|$ can be controlled by rotating the polarization of the pump beam by the polarization rotator PR, and $\Delta\phi_{\mathrm{p}}$ can be controlled by the phase shifter PS. Using the expressions

$$\gamma\equiv\sqrt{\tanh^{2}|\gamma_{\mathrm{H}}|+\tanh^{2}|\gamma_{\mathrm{V}}|},$$

$$\alpha e^{i\phi_{\mathrm{p}}}\equiv\frac{\gamma_{\mathrm{H}}}{|\gamma_{\mathrm{H}}|}\,\frac{\tanh|\gamma_{\mathrm{H}}|}{\gamma},$$

$$\beta e^{i\phi_{\mathrm{p}}}\equiv\frac{\gamma_{\mathrm{V}}}{|\gamma_{\mathrm{V}}|}\,\frac{\tanh|\gamma_{\mathrm{V}}|}{\gamma},$$

and

$$g\equiv\mathrm{sech}^{2}|\gamma_{\mathrm{H}}|\mathrm{sech}^{2}|\gamma_{\mathrm{V}}|=(1-\gamma^{2}|\alpha|^{2})(1-\gamma^{2}|\beta|^{2}), \quad (19)$$

we can write the state of the down-converted field as

$$|\Psi\rangle_{12}=\sqrt{g}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(\gamma\alpha e^{i\phi_{\mathrm{p}}})^{n}\times(\gamma\beta e^{i\phi_{\mathrm{p}}})^{m}|n\rangle_{1\mathrm{H}}|n\rangle_{2\mathrm{H}}|m\rangle_{1\mathrm{V}}|m\rangle_{2\mathrm{V}}. \quad (20)$$

Collecting the terms of the same total photon number, we can rewrite the state $|\Psi\rangle_{12}$ in the form

$$|\Psi\rangle_{12}=\sqrt{g}(|\Psi^{(0)}\rangle_{12}+\gamma e^{i\phi_{\mathrm{p}}}|\Psi^{(1)}\rangle_{12}+\gamma^{2}e^{2i\phi_{\mathrm{p}}}|\Psi^{(2)}\rangle_{12}+\cdots), \quad (21)$$

where

$$|\Psi^{(0)}\rangle_{12}\equiv|0\rangle_{1\mathrm{H}}|0\rangle_{2\mathrm{H}}|0\rangle_{1\mathrm{V}}|0\rangle_{2\mathrm{V}}, \quad (22)$$

$$|\Psi^{(1)}\rangle_{12}\equiv\alpha|1\rangle_{1\mathrm{H}}|1\rangle_{2\mathrm{H}}|0\rangle_{1\mathrm{V}}|0\rangle_{2\mathrm{V}}+\beta|0\rangle_{1\mathrm{H}}|0\rangle_{2\mathrm{H}}|1\rangle_{1\mathrm{V}}|1\rangle_{2\mathrm{V}}=|\alpha,\beta\rangle_{12}, \quad (23)$$

and

$$|\Psi^{(2)}\rangle_{12}\equiv\alpha\beta|1\rangle_{1\mathrm{H}}|1\rangle_{2\mathrm{H}}|1\rangle_{1\mathrm{V}}|1\rangle_{2\mathrm{V}}+\alpha^{2}|2\rangle_{1\mathrm{H}}|2\rangle_{2\mathrm{H}}|0\rangle_{1\mathrm{V}}|0\rangle_{2\mathrm{V}}+\beta^{2}|0\rangle_{1\mathrm{H}}|0\rangle_{2\mathrm{H}}|2\rangle_{1\mathrm{V}}|2\rangle_{2\mathrm{V}}. \quad (24)$$

Note that $|\Psi^{(0)}\rangle_{12}$ and $|\Psi^{(1)}\rangle_{12}$ are normalized, but $|\Psi^{(2)}\rangle_{12}$ is not normalized.

We will be able to obtain two photon pairs by pumping a nonlinear crystal twice with a short pulse as in Fig. 3, as was done in several experiments [2,18]. The state $|\Psi\rangle_{1234}$ generated from this source can be expressed as

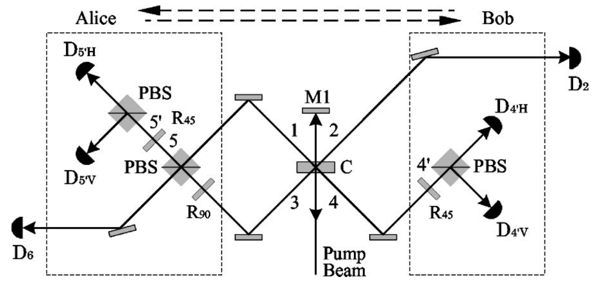

FIG. 3. Schematic of the purification procedure using spontaneous parametric down-conversion as a photon source.

$$\begin{aligned}|\Psi\rangle_{1234}=|\Psi\rangle_{12}|\Psi\rangle_{34}=g[&|\Psi^{(0)}\rangle_{12}|\Psi^{(0)}\rangle_{34}\\&+\gamma e^{i\phi_{\mathrm{p}}}(|\Psi^{(1)}\rangle_{12}|\Psi^{(0)}\rangle_{34}+|\Psi^{(0)}\rangle_{12}|\Psi^{(1)}\rangle_{34})\\&+\gamma^{2}e^{2i\phi_{\mathrm{p}}}(|\Psi^{(1)}\rangle_{12}|\Psi^{(1)}\rangle_{34}+|\Psi^{(2)}\rangle_{12}|\Psi^{(0)}\rangle_{34}\\&+|\Psi^{(0)}\rangle_{12}|\Psi^{(2)}\rangle_{34})+\cdots]\\=g(&|\Psi^{(0)}\rangle_{1234}+\gamma e^{i\phi_{\mathrm{p}}}|\Psi^{(1)}\rangle_{1234}\\&+\gamma^{2}e^{2i\phi_{\mathrm{p}}}|\Psi^{(2)}\rangle_{1234}+\cdots),\end{aligned} \quad (25)$$

where $|\Psi^{(0)}\rangle_{1234}\equiv|\Psi^{(0)}\rangle_{12}|\Psi^{(0)}\rangle_{34}$, $|\Psi^{(1)}\rangle_{1234}\equiv|\Psi^{(1)}\rangle_{12}|\Psi^{(0)}\rangle_{34}+|\Psi^{(0)}\rangle_{12}|\Psi^{(1)}\rangle_{34}$, and $|\Psi^{(2)}\rangle_{1234}\equiv|\Psi^{(1)}\rangle_{12}|\Psi^{(1)}\rangle_{34}+|\Psi^{(2)}\rangle_{12}|\Psi^{(0)}\rangle_{34}+|\Psi^{(0)}\rangle_{12}|\Psi^{(2)}\rangle_{34}$. In our scheme, Alice and Bob do not know the phase ϕ_{p}, so that the state received by them is the mixed state $\rho_{1234}^{\mathrm{PDC}}$ that is obtained by averaging Eq. (25) over ϕ_{p} as

$$\rho_{1234}^{\mathrm{PDC}}=g^{2}(|\Psi^{(0)}\rangle_{1234}\langle\Psi^{(0)}|+\gamma^{2}|\Psi^{(1)}\rangle_{1234}\langle\Psi^{(1)}|+\gamma^{4}|\Psi^{(2)}\rangle_{1234}\langle\Psi^{(2)}|+\cdots). \quad (26)$$

In the following, we assume that γ is small, so that we restrict the analysis up to $O(\gamma^{4})$.

B. Purification using imperfect detection

As shown in Fig. 3, the state $\rho_{1234}^{\mathrm{PDC}}$ is transformed by the same operations described in Sec. II. The term $|\Psi^{(1)}\rangle_{12}|\Psi^{(1)}\rangle_{34}$ becomes Eq. (7) and the other terms are calculated as

$$|\Psi^{(1)}\rangle_{12}|\Psi^{(0)}\rangle_{34}\to\alpha|1\rangle_{6\mathrm{H}}|1\rangle_{2\mathrm{H}}+\frac{\beta}{\sqrt{2}}(|1\rangle_{5'\mathrm{H}}-|1\rangle_{5'\mathrm{V}})|1\rangle_{2\mathrm{V}}, \quad (27)$$

$$|\Psi^{(0)}\rangle_{12}|\Psi^{(1)}\rangle_{34}\to\frac{\alpha}{\sqrt{2}}|1\rangle_{6\mathrm{V}}(|1\rangle_{4'\mathrm{H}}+|1\rangle_{4'\mathrm{V}})+\frac{\beta}{2}(|1\rangle_{5'\mathrm{H}}+|1\rangle_{5'\mathrm{V}})(|1\rangle_{4'\mathrm{H}}-|1\rangle_{4'\mathrm{V}}), \quad (28)$$

$$|\Psi^{(2)}\rangle_{12}|\Psi^{(0)}\rangle_{34}\rightarrow \alpha^2|2\rangle_{6\mathrm{H}}|2\rangle_{2\mathrm{H}}+\frac{\beta^2}{2}(|2\rangle_{5'\mathrm{H}} - \sqrt{2}|1\rangle_{5'\mathrm{H}}|1\rangle_{5'\mathrm{V}}+|2\rangle_{5'\mathrm{V}})|2\rangle_{2\mathrm{V}} + \frac{\alpha\beta}{\sqrt{2}}(|1\rangle_{5'\mathrm{H}}-|1\rangle_{5'\mathrm{V}})|1\rangle_{6\mathrm{H}}|1\rangle_{2\mathrm{H}}|1\rangle_{2\mathrm{V}}, \tag{29}$$

and

$$|\Psi^{(0)}\rangle_{12}|\Psi^{(2)}\rangle_{34}\rightarrow \frac{\alpha^2}{2}(|2\rangle_{4'\mathrm{H}}+\sqrt{2}|1\rangle_{4'\mathrm{H}}|1\rangle_{4'\mathrm{V}} +|2\rangle_{4'\mathrm{V}})|2\rangle_{6\mathrm{V}}+\frac{\beta^2}{4}(|2\rangle_{5'\mathrm{H}} + \sqrt{2}|1\rangle_{5'\mathrm{H}}|1\rangle_{5'\mathrm{V}}+|2\rangle_{5'\mathrm{V}})(|2\rangle_{4'\mathrm{H}} - \sqrt{2}|1\rangle_{4'\mathrm{H}}|1\rangle_{4'\mathrm{V}}+|2\rangle_{4'\mathrm{V}}) + \frac{\alpha\beta}{2}(|1\rangle_{5'\mathrm{H}}+|1\rangle_{5'\mathrm{V}})(|2\rangle_{4'\mathrm{H}} -|2\rangle_{4'\mathrm{V}})|1\rangle_{6\mathrm{V}}. \tag{30}$$

Using these expressions, we can obtain the state $\rho^{\mathrm{PDC}}_{24'5'6}$ after transforming $\rho^{\mathrm{PDC}}_{1234}$.

We calculate the output state in modes 6 and 2 by using a similar method as in Sec. III. Let us consider the case where Alice and Bob use conventional photon detectors. Suppose that a coincidence detection is obtained at detectors $D_{5'\mathrm{H}}$ and $D_{4'\mathrm{H}}$. In contrast to the case in Sec. III, the modes $5'V$ and $4'V$ are not always in the vacuum. If photocounts are recorded at detector $D_{5'\mathrm{V}}$ or $D_{4'\mathrm{V}}$, the vacuum appears in modes 6 and 2. It is thus better to discard such events in order to reduce errors. When the detectors $D_{5'\mathrm{V}}$ and $D_{4'\mathrm{V}}$ record no photocounts, the output state in modes 6 and 2 after the detection is calculated as

$$\rho^{\mathrm{c}}_{\mathrm{out}}=\frac{\mathrm{Tr}_{5',4'}[\Pi^{5'\mathrm{H}}_{\mathrm{c1}}\Pi^{4'\mathrm{H}}_{\mathrm{c1}}\Pi^{5'\mathrm{V}}_{\mathrm{c0}}\Pi^{4'\mathrm{V}}_{\mathrm{c0}}\rho^{\mathrm{PDC}}_{24'5'6}]}{\mathrm{Tr}[\Pi^{5'\mathrm{H}}_{\mathrm{c1}}\Pi^{4'\mathrm{H}}_{\mathrm{c1}}\Pi^{5'\mathrm{V}}_{\mathrm{c0}}\Pi^{4'\mathrm{V}}_{\mathrm{c0}}\rho^{\mathrm{PDC}}_{24'5'6}]} = \frac{1}{C^{\mathrm{c}}}\{8\gamma^2|\alpha|^2|\Phi^{(+)}\rangle_{62}\langle\Phi^{(+)}| +[4+(4-3\eta)^2\gamma^2|\beta|^2]|0\rangle_6\langle 0|\otimes|0\rangle_2\langle 0| +4(2-\eta)\gamma^2|\beta|^2|0\rangle_6\langle 0|\otimes|1\rangle_{2\mathrm{V}}\langle 1| +4(2-\eta)\gamma^2|\alpha|^2|1\rangle_{6\mathrm{V}}\langle 1|\otimes|0\rangle_2\langle 0|\}, \tag{31}$$

where $C^{\mathrm{c}}=4+4(4-\eta)\gamma^2|\alpha|^2+(24-28\eta+9\eta^2)\gamma^2|\beta|^2$. Note that Eq. (31) is also a classical mixture of $|\Phi^{(+)}\rangle_{62}$ and error states containing a smaller number of photons. As in Sec. III, we use the probabilities P, P_{s}, and P_{e}, but here we further decompose P_{e} as $P_{\mathrm{e}}=P^{(0)}_{\mathrm{e}}+P^{(1)}_{\mathrm{e}}$, where $P^{(0)}_{\mathrm{e}}$ is the probability of having the vacuum in modes 6 and 2, and $P^{(1)}_{\mathrm{e}}$ is that of having a photon in either mode 6 or 2. Each probability is expressed as

$$P=\eta^2 g^2\gamma^2|\beta|^2C^{\mathrm{c}}/16,$$

$$P_{\mathrm{s}}=\eta^2 g^2\gamma^4|\alpha\beta|^2/2,$$

$$P^{(0)}_{\mathrm{e}}=\eta^2 g^2\gamma^2|\beta|^2[4+(4-3\eta)^2\gamma^2|\beta|^2]/16,$$

and

$$P^{(1)}_{\mathrm{e}}=\eta^2(2-\eta)g^2\gamma^4|\beta|^2/4. \tag{32}$$

In this case the minimum values of $P^{(0)}_{\mathrm{e}}$ and $P^{(1)}_{\mathrm{e}}$ are $g^2\gamma^2|\beta|^2[4+\gamma^2|\beta|^2]/16$ and $g^2\gamma^4|\beta|^2/4$, respectively. If Alice and Bob do not discard the events when photocounts are recorded at detector $D_{5'\mathrm{V}}$ or $D_{4'\mathrm{V}}$, $P^{(0)}_{\mathrm{e}}$ increases to $\eta^2g^2\gamma^2|\beta|^2[4+(4-\eta)^2\gamma^2|\beta|^2]/16$ and the minimum value of $P^{(0)}_{\mathrm{e}}$ increases to $g^2\gamma^2|\beta|^2[4+9\gamma^2|\beta|^2]/16$.

Similarly, in the case where Alice and Bob use single photon detectors, the output state in modes 6 and 2 after the detection is calculated as

$$\rho^{\mathrm{s}}_{\mathrm{out}}=\frac{\mathrm{Tr}_{5',4'}[\Pi^{5'\mathrm{H}}_{\mathrm{s1}}\Pi^{4'\mathrm{H}}_{\mathrm{s1}}\Pi^{5'\mathrm{V}}_{\mathrm{s0}}\Pi^{4'\mathrm{V}}_{\mathrm{s0}}\rho^{\mathrm{PDC}}_{24'5'6}]}{\mathrm{Tr}[\Pi^{5'\mathrm{H}}_{\mathrm{s1}}\Pi^{4'\mathrm{H}}_{\mathrm{s1}}\Pi^{5'\mathrm{V}}_{\mathrm{s0}}\Pi^{4'\mathrm{V}}_{\mathrm{s0}}\rho^{\mathrm{PDC}}_{24'5'6}]} = \frac{1}{C^{\mathrm{s}}}\{2\gamma^2|\alpha|^2|\Phi^{(+)}\rangle_{62}\langle\Phi^{(+)}| +[1+4(1-\eta)^2\gamma^2|\beta|^2]|0\rangle_6\langle 0|\otimes|0\rangle_2\langle 0| +2(1-\eta)\gamma^2|\beta|^2|0\rangle_6\langle 0|\otimes|1\rangle_{2\mathrm{V}}\langle 1| +2(1-\eta)\gamma^2|\alpha|^2|1\rangle_{6\mathrm{V}}\langle 1|\otimes|0\rangle_2\langle 0|\}, \tag{33}$$

where $C^{\mathrm{s}}=1+2(2-\eta)\gamma^2|\alpha|^2+2(3-2\eta)(1-\eta)\gamma^2|\beta|^2$. Each probability is expressed as

$$P=\eta^2 g^2\gamma^2|\beta|^2C^{\mathrm{s}}/4,$$

$$P_{\mathrm{s}}=\eta^2 g^2\gamma^4|\alpha\beta|^2/2,$$

$$P^{(0)}_{\mathrm{e}}=\eta^2 g^2\gamma^2|\beta|^2[1+4(1-\eta)^2\gamma^2|\beta|^2]/4,$$

and

$$P^{(1)}_{\mathrm{e}}=\eta^2(1-\eta)g^2\gamma^4|\beta|^2/2. \tag{34}$$

In this case the minimum values of $P^{(0)}_{\mathrm{e}}$ and $P^{(1)}_{\mathrm{e}}$ are $g^2\gamma^2|\beta|^2/4$ and 0, respectively. Note that, in comparison with the case using conventional photon detectors, P_{s} is the same, but $P^{(0)}_{\mathrm{e}}$ and $P^{(1)}_{\mathrm{e}}$ are different. If Alice and Bob do not discard the events when photocounts are recorded at detector $D_{5'\mathrm{V}}$ or $D_{4'\mathrm{V}}$, $P^{(0)}_{\mathrm{e}}$ increases to $\eta^2g^2\gamma^2|\beta|^2[1+(2-\eta)^2\gamma^2|\beta|^2]/4$ and the minimum value of $P^{(0)}_{\mathrm{e}}$ increases to $g^2\gamma^2|\beta|^2[1+\gamma^2|\beta|^2]/4$.

The error in the output state $\rho_{\rm out}^{\rm c}$ or $\rho_{\rm out}^{\rm s}$ stems from the states with smaller numbers of photons. Therefore, if Alice and Bob are allowed to perform postselection, they can discard the error events as in the case of the ideal photon pair source. In this situation, again, the type of detector is not relevant and the success probability is solely determined by the quantum efficiency η. Moreover, they need not refer to the detectors $D_{5'{\rm V}}$ and $D_{4'{\rm V}}$ because the vacuum is removed by the postselection.

C. The effect of dark counts

When photon detectors have dark counts, the probability of error $P_{\rm e}$ increases, and the error cannot always be discarded even by postselection. In the following, we derive the conditions where we can neglect the effect of dark counts.

We assume that the dark counts are random detection events, namely, each event is uncorrelated to other dark or real counts. Let the mean number of dark counts during each run of the purification scheme be ν for each detector. We assume $\nu \ll 1$. Consider the case where Alice and Bob obtain a fourfold coincidence detection at detectors $D_{5'{\rm H}}$, $D_{4'{\rm H}}$, D_6, and D_2. The probability that all four counts are caused by real photons is $P_0 = O(\gamma^4)$. γ^2 is the generation probability of a photon pair. The probabilities P_i that the fourfold coincidence detection includes i dark counts are of the order $P_1 = O(\gamma^4 \nu)$, $P_2 = O(\gamma^2 \nu^2)$, $P_3 = O(\gamma^2 \nu^3)$, and $P_4 = O(\nu^4)$. To satisfy $P_0 \gg P_i$ $(i=1,2,3,4)$, ν must satisfy $\nu^2/\gamma^2 \ll 1$. Therefore, the condition for the effect of dark counts to be negligible is $\nu \ll 1$ and $\nu^2/\gamma^2 \ll 1$.

In a teleportation experiment [2,19], where a nonlinear crystal is pumped twice by a short pulse, γ^2 is of order $\sim 10^{-4}$. Conventional photon detectors (e.g., EG&G SPCM) typically have dark count rates of the order of 100 s^{-1}, which gives a value of $\nu \sim 10^{-6}$ for the coincidence time ~ 10 ns. Single photon detectors [16] have dark count rates of the order of 10^4 s^{-1}, which gives a value of $\nu \sim 10^{-4}$ for the coincidence time ~ 10 ns. In both cases the effect of dark counts is negligible.

V. DISCUSSION AND CONCLUSION

In the following we consider the required properties of quantum channels for the proposed purification scheme to be applicable. Assume that two photon pairs are initially prepared in the state $|\Phi^{(+)}\rangle_{12}|\Phi^{(+)}\rangle_{34}$ and sent to Alice and Bob through noisy quantum channels. The quantum channels are assumed to have polarization-dependent transmissivities and are modeled by the state transformation

$$|1\rangle_{kL} \rightarrow (\mu_{kL}|1\rangle_{kL} + \sqrt{1-|\mu_{kL}|^2}|1\rangle_{\overline{kL}}), \tag{35}$$

where $k=1,2,3,4$, $L=H,V$, and μ_{kL} is the complex transmission coefficient. We introduce modes $\overline{kL}$ to model lossy channels. The coefficients μ_{kL} are fluctuating and we denote the average over the fluctuations as $\langle \cdots \rangle_\mu$. The state of photon pairs received by Alice and Bob is

$$\langle (1-P)\rho_{1234}^{n\leqslant 3} + P\rho_{1234}\rangle_\mu , \tag{36}$$

where $\rho_{1234}^{n\leqslant 3}$ is the state containing less than four photons in total and

$$P \equiv \frac{1}{4}(|\mu_{1{\rm H}}\mu_{2{\rm H}}|^2 + |\mu_{1{\rm V}}\mu_{2{\rm V}}|^2)(|\mu_{3{\rm H}}\mu_{4{\rm H}}|^2 + |\mu_{3{\rm V}}\mu_{4{\rm V}}|^2),$$

$$\rho_{1234} \equiv |\alpha_{12},\beta_{12}\rangle_{12}\langle \alpha_{12},\beta_{12}| \otimes |\alpha_{34},\beta_{34}\rangle_{34}\langle \alpha_{34},\beta_{34}|,$$

$$\alpha_{12} \equiv \frac{\mu_{1{\rm H}}\mu_{2{\rm H}}}{\sqrt{|\mu_{1{\rm H}}\mu_{2{\rm H}}|^2 + |\mu_{1{\rm V}}\mu_{2{\rm V}}|^2}},$$

$$\beta_{12} \equiv \frac{\mu_{1{\rm V}}\mu_{2{\rm V}}}{\sqrt{|\mu_{1{\rm H}}\mu_{2{\rm H}}|^2 + |\mu_{1{\rm V}}\mu_{2{\rm V}}|^2}},$$

$$\alpha_{34} \equiv \frac{\mu_{3{\rm H}}\mu_{4{\rm H}}}{\sqrt{|\mu_{3{\rm H}}\mu_{4{\rm H}}|^2 + |\mu_{2{\rm V}}\mu_{4{\rm V}}|^2}},$$

and

$$\beta_{34} \equiv \frac{\mu_{3{\rm V}}\mu_{4{\rm V}}}{\sqrt{|\mu_{3{\rm H}}\mu_{4{\rm H}}|^2 + |\mu_{2{\rm V}}\mu_{4{\rm V}}|^2}}. \tag{37}$$

If postselection is allowed, Alice and Bob can select two photon pairs in the state $\langle P\rho_{1234}\rangle_\mu / \langle P\rangle_\mu$. To purify the mixed state $\langle P\rho_{1234}\rangle_\mu / \langle P\rangle_\mu$, it must be written in the form of Eq. (8). By comparing the matrix elements of these expressions, we obtain the condition for the purification as $\langle P|\alpha_{12}\beta_{34} - \beta_{12}\alpha_{34}|^2\rangle_\mu = 0$. Using the complex variable

$$F \equiv \frac{\alpha_{12}\beta_{34}}{\beta_{12}\alpha_{34}} = \frac{\mu_{1{\rm H}}\mu_{2{\rm H}}\mu_{3{\rm V}}\mu_{4{\rm V}}}{\mu_{1{\rm V}}\mu_{2{\rm V}}\mu_{3{\rm H}}\mu_{4{\rm H}}}, \tag{38}$$

the condition for the purification becomes $F=1$. Even if $F \neq 1$, Alice and Bob can transform F into 1 by introducing an additional attenuation and a phase shift as long as the value of F is constant. The fluctuations in the transmissivities of the quantum channels may be assumed to be independent for Alice's side and Bob's side. In this case we can introduce complex variables

$$F_{\rm A} \equiv \frac{\mu_{1{\rm H}}\mu_{3{\rm V}}}{\mu_{1{\rm V}}\mu_{3{\rm H}}} \tag{39}$$

and

$$F_{\rm B} \equiv \frac{\mu_{2{\rm H}}\mu_{4{\rm V}}}{\mu_{2{\rm V}}\mu_{4{\rm H}}}, \tag{40}$$

where $F = F_{\rm A}F_{\rm B}$. Since $F_{\rm A}$ and $F_{\rm B}$ are independent, the condition for purification is that $F_{\rm A}$ and $F_{\rm B}$ are constant.

In the special cases where each pair is received as a known pure state $|\alpha_{12},\beta_{12}\rangle \otimes |\alpha_{34},\beta_{34}\rangle$, the Procrustean method [6] can be applied to each pair. In this method, Alice and Bob perform an additional polarization-dependent transformation to discard the extra probability of the larger term in the state $|\alpha_{12},\beta_{12}\rangle_{12}$. Since they manipulate one photon pair, this method is simpler than the proposed scheme to share a maximally entangled state. To simplify our explanation, we consider the situations where Bob prepares the photon pairs and sends one member of each photon pair to Alice through quantum channels 1 and 3, namely, $\mu_{2H}=\mu_{2V}=\mu_{4H}=\mu_{4V}=1$. The Procrustean method is then applicable when the fluctuations μ_{1H}, μ_{1V}, μ_{3H}, and μ_{3V} are correlated pairwise—if the values $F_{A1}\equiv\mu_{1H}/\mu_{1V}$ and $F_{A3}\equiv\mu_{3H}/\mu_{3V}$ are constant, Alice and Bob receive the two pairs in a pure state $|\alpha_{12},\beta_{12}\rangle\otimes|\alpha_{34},\beta_{34}\rangle$ with $\alpha_{12}/\beta_{12}=F_{A1}$ and $\alpha_{34}/\beta_{34}=F_{A3}$. If the values μ_{1H}/μ_{3H} and μ_{1V}/μ_{3V} are constant, Bob exchanges modes $1V$ and $3H$ before transmission and Alice exchanges the modes back to obtain each pair in a pure state. The situation is similar for the case where the values μ_{1H}/μ_{3V} and μ_{3H}/μ_{1V} are constant.

Let us consider an example in which Bob sends one member of a pair (mode 1) to Alice through a polarization maintaining fiber and one member of the other pair (mode 3) through the same fiber after a time delay Δt. Alice compensates the time delay Δt after receiving the photons. The states $|1\rangle_{1H}$, $|1\rangle_{3H}$, $|1\rangle_{1V}$, and $|1\rangle_{3V}$ are transformed into $e^{i\phi_H(t)}|1\rangle_{1H}$, $e^{i\phi_H(t+\Delta t)}|1\rangle_{3H}$, $e^{i\phi_V(t)}|1\rangle_{1V}$, and $e^{i\phi_V(t+\Delta t)}|1\rangle_{3V}$, where $\phi_H(t)$ and $\phi_V(t)$ represent phase shifts in modes H and V induced by the fiber for photons input at time t. Since Bob initially has the photon pairs in the state $|\Phi^{(+)}\rangle_{12}|\Phi^{(+)}\rangle_{34}$, Alice and Bob share the photon pairs in the states

$$\frac{e^{i[\varphi_+(t)+\varphi_-(t)]}}{\sqrt{2}}(|1\rangle_{1H}|1\rangle_{2H}+e^{-2i\varphi_-(t)}|1\rangle_{1V}|1\rangle_{2V})$$
$$\otimes\frac{e^{i[\varphi_+(t+\Delta t)+\varphi_-(t+\Delta t)]}}{\sqrt{2}}(|1\rangle_{3H}|1\rangle_{4H}$$
$$+e^{-2i\varphi_-(t+\Delta t)}|1\rangle_{3V}|1\rangle_{4V}), \tag{41}$$

where $\varphi_+(t)\equiv[\phi_H(t)+\phi_V(t)]/2$ and $\varphi_-(t)\equiv[\phi_H(t)-\phi_V(t)]/2$. Assuming that $\phi_H(t)$ and $\phi_V(t)$ are temporally fluctuating, this state becomes a mixed state. For simplicity, we assume the channel is symmetric about H and V. The fluctuations of $\varphi_+(t)$ and $\varphi_-(t)$ are then independent, and we denote the correlation times of $\varphi_+(t)$ and $\varphi_-(t)$ by τ_+ and τ_-, respectively.

We classify the situation into four cases: (a) $\Delta t\ll\tau_+,\tau_-$, (b) $\tau_+\ll\Delta t\ll\tau_-$, (c) $\tau_-\ll\Delta t\ll\tau_+$, and (d) $\tau_+,\tau_-\ll\Delta t$. In case (a), since we can use the approximations $\varphi_+(t)=\varphi_+(t+\Delta t)$ and $\varphi_-(t)=\varphi_-(t+\Delta t)$, if Bob exchanges modes $1V$ and $3H$ before transmission and Alice exchanges the modes back, they can share the maximally entangled photon pairs. Therefore it is not necessary to use the proposed purification scheme. In case (b), where $\varphi_+(t)\neq\varphi_+(t+\Delta t)$ and $\varphi_-(t)=\varphi_-(t+\Delta t)$, the above method does not work. But the proposed scheme works in this case as the mixture of Eq. (41) has the form of Eq. (8). In case (c), where $\varphi_+(t)=\varphi_+(t+\Delta t)$ and $\varphi_-(t)\neq\varphi_-(t+\Delta t)$, if Bob exchanges modes $1V$ and $3V$ before transmission and Alice exchanges the modes back, the situation is the same as in case (b). In case (d), where $\varphi_+(t)\neq\varphi_+(t+\Delta t)$ and $\varphi_-(t)\neq\varphi_-(t+\Delta t)$, Alice and Bob cannot obtain the photon pairs in the state of Eq. (8), and they cannot purify the output even if the proposed scheme is used.

In summary, we have proposed a purification scheme using linear optical elements and photon detectors. We have investigated errors in the output state when down-converted photons and imperfect detectors are used. We have shown that the errors can be discarded by postselection because the error states contain fewer photons than the maximally entangled state. It became clear that the effect of dark counts is negligible. We have also discussed the required properties of quantum channels for the proposed purification scheme.

Notes added. A proposal based on essentially the same idea has been independently made by Zhao *et al*. [20]. Recently, another type of purification scheme for photon pairs was proposed by Pan *et al*. [21].

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