Teleportation scheme of *S*-level quantum pure states by two-level Einstein-Podolsky-Rosen states

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Unknown quantum pure states of an *S*-level particle can be transferred onto a group of remote two-level particles with the aid of two-level Einstein-Podolsky-Rosen states. We present a scheme for such kind of teleportation. The unitary transformation to more than two particles, which is needed in the scheme, is written in the product form of two-body unitary transformations.

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I. INTRODUCTION

Quantum mechanics offers us the capabilities of transferring quantum information (quite different from the classical one), for the use of either computation or communication. Bennett *et al.* [1] suggested a quantum method of teleportation, through which an unknown quantum pure state of a spin-$\frac{1}{2}$ particle (we may call it a "qubit" [2,3]) is teleported from the sender "*Alice*" at the sending terminal; onto the qubit at the receiving terminal, after that, the receiver "*Bob*" needs to perform a unitary transformation on his qubit. At first it is necessary to prepare two spin-$\frac{1}{2}$ particles in an Einstein-Podolsky-Rosen (EPR) entangled state [4] or so-called a "*Bell* state," and send them to the two different places to establish a quantum channel between *Alice* and *Bob*. The second step is for *Alice* to perform a *Bell* operator measurement [5] to the quantum system involving her share of the two entangled particles, together with the particle carrying the information to be transferred. Then through classical channels, for example, broadcasting, *Alice* let *Bob* know her result of the *Bell* operator measurement. After *Bob* performs on his share of the two formerly entangled particles one of four unitary transformations determined by *Alice*'s result, this particle will be carrying the original information state. In this way, the unknown state is teleported from one place to another.

The new method of teleportation has interested a lot of research groups. Research work on quantum teleportation was soon widely started up, and has got great development, theoretical and experimental as well. It was generalized to the case of continuous variables [6,7]. Yu *et al.* have investigated canonical quantum teleportation of finite-level unknown states by introducing a canonically conjugated pair of quantum phase and number [8]. The successful experimental realization of quantum teleportation of unknown polarization states carried on a photon [9] and the succedent experiments on finite-level quantum system teleportation [10,11] have aroused a series of discussions [12–14] and further research of this topic from various aspects [15–17]. Possible applications have been considered in Refs. [18,19], and the method of teleportation in the case of continuous variables [6] got its experimental realization in 1998 [20].

From a general point of view, no matter what form it is, there are four steps to realize the quantum teleportation, which can be seen clearly in *Bennett*'s initial scheme [1]: (a) EPR entangled states preparing; (b) *Bell* operator measurements by the sender; (c) the sender informing the receiver of his outcomes through classical channels; (d) the receiver performing unitary transformation according to the classical information. However, here we can substitute step (b) by a two-particle unitary transformation along with local measurements (here "local" means to single particles). More specifically, the unitary transformation is performed on the sender's potion of the EPR pair and the state-unknown particle to form some sort of entangled state involving the latter together with both two particles of the EPR pair, while the local measurements are performed one by one on *Alice*'s particles. These measurements will result in the random collapse of all the sender's particles onto definite states. At the other end of communication, the receiver will got the same results as in the case of performing *Bell* operator measurements. In other words, the unitary transformation and local measurements is equivalent to a *Bell* operator measurement. More discussions about step (b) can be found in Ref. [21], in which Brassard *et al.* indicated the possibility of realizing teleportation by controlled NOT gates and single qubit operations used in quantum networks.

II. TELEPORTATION SCHEME FOR *S*-LEVEL STATE

Here we supposed that the unknown state to be transferred is an arbitrary but definite *S*-level pure quantum state carried by one particle labeled *C*. Here, different from Ref. [8], in which the shared state is a maximally entangled EPR states of *S*-level, a multichannel made up of *L* two-level EPR's is used instead. It means that at first *Alice* and *Bob* have to prepare this group of EPR's and share each of them, with one particle of each EPR controllable to the sender and the other to the receiver. We shall see how the unknown state of *S*-level is teleported from *C* at *Alice*'s hand, to *Bob*'s portions of the group of EPR states. It is necessary here to indicate that the two Hilbert spaces are not the same, one is for a single particle while the other for L particles, but from the Hilbert space with more dimensions (the bigger one) we can always select a subspace equivalent to the other (the smaller one). In our case, $2^L \geqslant S$ is required and therefore we can select S normalized orthogonal vectors as the bases of the subspace from the Hilbert space for L two-level particles to make them mapping one by one to the S eigenvectors of C. Two states respectively in the two sorts of Hilbert space will be regarded as the same if the coefficients are the same when expressed as linear superposition of their own bases. Only in this means can we say that the state on C is teleported onto the L particles.

We label all the EPR's with serial numbers $0,1,\ldots,L-1$, while the corresponding particles at *Alice* and *Bob*'s places are labeled $A_0,A_1,\ldots,A_{L-1}$ and $B_0,B_1,\ldots,B_{L-1}$ respectively. The EPR entangled state of each pair of particles A_k and B_k ($k=0,1,\ldots,L-1$) can be chosen as follows

$$|\Phi\rangle_{A_kB_k}=\frac{1}{\sqrt{2}}(|0\rangle_{A_k}|0\rangle_{B_k}+|1\rangle_{A_k}|1\rangle_{B_k}), \tag{1}$$

where we express the eigenvectors of the two-level particles as $|0\rangle,|1\rangle$ which in the case of $\frac{1}{2}$-spin particles, for example, refer to spin-up state and spin-down state, respectively. Moreover, the state of C is generally written as

$$|\psi\rangle_C=\sum_{m=0}^{S-1}\alpha_m|m\rangle_C \tag{2}$$

in which α_m ($m=0,1,\ldots,S-1$) is a complex number satisfying $\Sigma_{m=0}^{S-1}|\alpha_m|^2=1$, and $|0\rangle,|1\rangle,\ldots,|S-1\rangle$ denote the S eigenvectors of the S-level particle. It is convenient for us to distinguish $|0\rangle$ and $|1\rangle$ only with subscript, i.e., $|0\rangle_{A_k}$ or $|0\rangle_{B_k}$ is not the same state as $|0\rangle_C$, and so is not $|1\rangle_{A_k}$, $|1\rangle_{B_k}$, $|1\rangle_C$. Further restriction $2^{L-1}<S$ is set on L, so that L EPR's is the least, but enough to realize teleportation.

Any number can be expressed in its binary form. We will mark the symbol ''$-$'' above the number if it is expressed in the binary form. For example, a number customarily in decimal form n is decomposed into L-bit number $n=2^{L-1}\cdot n_{L-1}+\cdots+2^1\cdot n_1+2^0\cdot n_0$, where $2^L\geqslant n$ and $n_k=0$ or 1 ($k=0,1,\ldots,L-1$), and is written as

$$n=\overline{n_{L-1}\cdots n_1n_0}. \tag{3}$$

On the other hand, any binary number has its decimal correspondence. If we regard the L particles $A_0,A_1,\ldots,A_{L-1}$ or $B_0,B_1,\ldots,B_{L-1}$ as ''qubits'' [2,3], each state $|n_{L-1}\rangle_{A_{L-1}}\ldots|n_1\rangle_{A_1}|n_0\rangle_{A_0}=|n_{L-1}\ldots n_1n_0\rangle_A$ or $|n_{L-1}\rangle_{B_{L-1}}\ldots|n_1\rangle_{B_1}|n_0\rangle_{B_0}=|n_{L-1}\ldots n_1n_0\rangle_B$ ($n_k=0$ or 1, $k=0,1,\ldots L-1$) will correspond to a binary number $\overline{n_{L-1}\cdots n_1n_0}$ and we introduce a symbol ''$|\rangle\rangle$'' to simplify the denotation of the state as

$$|n\rangle\rangle\equiv|n_{L-1}\cdots n_1n_0\rangle, \tag{4}$$

where n has the same meaning as in Eq. (3). The quantum state of the composite system consists of A, B, and C can thus be written as

$$\begin{aligned}|\Psi_0\rangle_{ABC}&=|\psi\rangle_C\prod_{k=0}^{L-1}|\Phi\rangle_{A_kB_k}\\&=\frac{1}{\sqrt{N}}\sum_{m=0}^{S-1}\sum_{n=0}^{N-1}\alpha_m|m\rangle_C|n\rangle\rangle_A|n\rangle\rangle_B,\end{aligned} \tag{5}$$

where $N=2^L$.

In principle, *Alice* is able to perform on the composite system AC any quantum operations, including unitary transformations and measurements. To realize the teleportation, a two-particle unitary transformation U_{AC} to all the bodies included in system AC is performed. U_{AC} will realize the following transformation

$$U_{AC}|m\rangle_C|n\rangle\rangle_A=\frac{1}{\sqrt{S}}\sum_{j=0}^{S-1}e^{i(2mj\pi/S)}|j\rangle_C|f^n(j,m)\rangle\rangle_A \tag{6}$$

in which $m=0,1,\ldots,S-1$, and $f^n(j,m)$ is a number of decimal form determined by j, m, and n so that $|f^n(j,m)\rangle\rangle$ is one of the N eigenstates. If we express j, m, and $f^n(j,m)$ in the binary form

$$j\equiv\overline{j_{L-1}\cdots j_1j_0}$$

$$m\equiv\overline{m_{L-1}\cdots m_1m_0}$$

$$f^n(j,m)\equiv\overline{f^n_{L-1}(j,m)\cdots f^n_1(j,m)f^n_0(j,m)}$$

$$j_k,m_k,f^n_k(j,m)=0,1(k=0,1,\ldots,L-1) \tag{7}$$

$f^n(j,m)$ will be determined by $f^n_k(j,m)$ satisfying

$$f^n_k(j,m)=n_k\oplus j_k\oplus m_k, \tag{8}$$

where ''$\oplus$'' denotes addition modulo 2. One can easily prove the unitarity of U_{AC} and the following orthogonal conditions

$$\langle\langle f^n(j,m)|f^{n'}(j,m)\rangle\rangle=\delta_{n'n}$$

$$\langle\langle f^n(j,m)|f^n(j',m)\rangle\rangle=\delta_{j'j}$$

$$\langle\langle f^n(j,m')|f^n(j,m)\rangle\rangle=\delta_{m'm}. \tag{9}$$

For example, Eq. (9) means that any two bases among $|f^n(j,m)\rangle\rangle$ with the same n and j, but different m will not be the same.

After the transformation of U_{AC}, due to Eqs. (6)–(8), the quantum state of system ABC will change to

$$|\Psi\rangle_{ABC}=U_{AC}|\Psi_0\rangle_{ABC}$$
$$=\frac{1}{\sqrt{S}}\sum_{j=0}^{S-1}\left\{|j\rangle_C\frac{1}{\sqrt{N}}\sum_{n=1}^{N-1}\left(|n\rangle\rangle_A\right.\right.$$
$$\left.\left.\times\sum_{m=0}^{S-1}\alpha_m e^{i(2mj\pi/S)}|f^n(j,m)\rangle\rangle_B\right)\right\}, \quad (10)$$

which is the entangled quantum state involving all the particles in system ABC.

If now *Alice* perform measurements on the single particles $C,A_0,A_1,\ldots,A_{L-1}$, with the same possibility of $1/NS$, she will acquire one of the outcomes, i.e., the collapse of the state of these particles to the possible eigenstate $|j\rangle_C|n\rangle\rangle_A$ ($j=0,1,\ldots,L-1$ and $n=0,1,\ldots,N-1$). Thus the entanglement among *A*, *B*, and *C* will be destroyed and *Bob* will acquire the state of *B*

$$|\psi^n(j)\rangle_B=\sum_{m=0}^{S-1}\alpha_m e^{i(2mj\pi/S)}|f^n(j,m)\rangle\rangle_B, \quad (11)$$

which is an entangled quantum state of particles $B_0,B_1,\ldots,B_{L-1}$. If *n* and *j* are definite, Eq. (9) ensures that we can define $|m\rangle'=e^{i2mj\pi/S}|f^n(j,m)\rangle\rangle, m=0,1,\ldots,S-1$, which form the bases of the subspace of the Hilbert space for system *B*. Therefore we get

$$|\psi^n(j)\rangle_B=\sum_{m=0}^{S-1}\alpha_m|m\rangle'_B. \quad (12)$$

According to the preceding discussion and the comparison of Eqs. (2) and (12), we can regard $|\psi^n(j)\rangle$ and $|\psi\rangle$ as the same. However, we need to indicate that $|m\rangle'$ relies on *j* and *n*, which makes it still necessary to setup the classical channels between *Alice* and *Bob* to transfer the information about *Alice*'s outcomes, or the information of *j* and *n* in the other words, since *Bob* could not know exactly what the $|m\rangle'$ means without the knowledge of *j* and *n*. It is just the necessity of classical information transferring that makes faster-than-light communication impossible. This type of teleporation that transfer the information carried on one *S*-level particle to *L* two-level particles can also be used as a method to store and express arbitrary signal state (need not be two-level) by standard qubit storages.

Furthermore, the receiver can also recover the initial signal state on an *S* -level particle at his hand that will makes the teleportation process thoroughly complete. The recovery procedure can be realized as follows.

Since *Bob* is told the results *j* and *n* that Alice got, he can prepare an *S*-level particle *D* on the state $|j\rangle_D$. The state of two particles (D and B) at his hand is

$$|\Phi^n(j)\rangle_{DB}=\sum_{m=0}^{S-1}\alpha_m e^{i(2mj\pi/S)}|j\rangle_D|f^n(j,m)\rangle\rangle_B.$$

First perform the following transformation [it is the inverse of *U* in Eq. (6)] to *D* and *B*,

$$U_{DB}|j\rangle_D|k\rangle\rangle_B=\frac{1}{\sqrt{S}}\sum_{l=0}^{S-1}e^{-i(2jl\pi/S)}|l\rangle_D|f^k(l,j)\rangle\rangle_B.$$

The result state is

$$U_{DB}|\Phi^n(j)\rangle_{DB}=\sum_{m=0}^{S-1}\alpha_m e^{i(2mj\pi/S)}\frac{1}{\sqrt{S}}$$
$$\times\sum_{l=0}^{S-1}e^{-i(2jl\pi/S)}|l\rangle_D|f^{f^n(j,m)}(l,j)\rangle\rangle_B$$
$$=\sum_{m=0}^{S-1}\alpha_m\frac{1}{\sqrt{S}}\sum_{l=0}^{S-1}e^{i[2j(m-l)\pi/S]}|l\rangle_D$$
$$\times|f^n(l,m)\rangle\rangle_B,$$

where the equality: $f^{f^n(j,m)}(l,j)=f^n(l,m)$ is utilized. Then perform measurements on the single particles $B_0,B_1,\ldots,B_{L-1}$, *Bob* will acquire one of the outcomes, i.e., the collapse of the state of these particles to the possible eigenstate $|q\rangle\rangle_B$ ($q=0,1,\ldots,N-1$). The state of particle *D* becomes

$$|\Psi_q^n(j)\rangle_D=\sum_{m=0}^{S-1}\alpha_m e^{i[2j(m-(m\oplus n\oplus q))\pi/S]}$$
$$\times|m\oplus n\oplus q\rangle_D$$

here "$\oplus$" means addition in binary form without carry (e.g., $0110\oplus1010=1100$). If we can construct the unitary transformation operator $V(j,n\oplus q)$:

$$V_D(j,n\oplus q)|r\rangle_D=e^{i[2j(r-(r\oplus n\oplus q))\pi/S]}|r\oplus n\oplus q\rangle_D,$$

and perform it on particle *D*. The original signal state is then recovered:

$$|\Psi_f\rangle_D=\sum_{m=0}^{S-1}\alpha_m|m\rangle_D.$$

III. DECOMPOSITION OF THE TRANSFORMATION

We have discussed above in principle the possibility of teleportation of any *S*-level quantum states by no less than $L\simeq\log_2 S$ two-level EPR's. In our discussion, we use the complicated unitary transformation U_{AC}, which means the evolution of the quantum state of system AC under the interaction of all those particles involved in AC. The complication of U_{AC} may lead to the complication of operation. It is even impossible for us to operate such a transformation unless further consideration is taken. The method of quantum computational networks has shown out the most feasible way of realizing the operation. The quantum computational networks have been much studied in Refs. [22–24]. Following their method, we make the transformation more practical by decomposing U_{AC}, which applies to $2L+1$ particles, into a sequence of two-body unitary transformations and a simple single-body unitary transformation. Only two classes of such transformations are used: (a) the discrete Fourier transform modulo S, denoted DFT_S, which is a unitary transformation in S dimensions. It is defined relative to the bases $|0\rangle_C, |1\rangle_C, \ldots, |S-1\rangle_C$ by

$$\mathrm{DFT}_S|m\rangle_C = \frac{1}{\sqrt{S}} \sum_{j=0}^{S-1} e^{i(2mj\pi/S)}|j\rangle_C, \tag{13}$$

(b) a combined unitary transformation U_{Ck} to the two particles C and A_k $(k=0,1,\ldots,L-1)$. U_{Ck} is defined by

$$U_{Ck}|m\rangle_C|n_k\rangle_{A_k} = |m\rangle_C|m_k \oplus n_k\rangle_{A_k}. \tag{14}$$

At last, U_{AC} can be decomposed into the product of these two classes of transformation

$$U_{AC} = \left(\prod_{k=0}^{L-1} U_{Ck}\right) \cdot \mathrm{DFT}_S \cdot \left(\prod_{k=0}^{L-1} U_{Ck}\right) \tag{15}$$

since $[U_{Ck'}, U_{Ck}] = 0$ for any $k, k' = 0,1,\ldots,L-1$, we need not distinguish their order. By Eq. (15) we simplify the problem in operation of U_{AC}, for the quantum operation on two bodies is far more feasible than that on a many bodies.

IV. CONCLUSION

In summary, we construct the scheme of transferring an arbitrary S-level quantum state by using two-level EPR's. The importance of this construction lies not only on the scheme itself, but also on the possibility of further research and application of teleportation. It leads us to more general, more feasible, and simultaneously more challenging considerations on the problem of teleportation. A lot of questions, such as probabilistic teleportation and teleportation of unknown quantum states by definite number of EPR's, are thus put forward as challenge to future research works.

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[1] C. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K.Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[2] B. Schumacher, Phys. Rev. A **51**, 2738 (1995).
[3] S. Lloyd and J. J. E. Slotine, Phys. Rev. Lett. **80**, 4088 (1998).
[4] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[5] S. L. Braunstein and A. Mann, Phys. Rev. A **51**, R1727 (1995).
[6] L. Vaidman, Phys. Rev. A **49**, 1473 (1994).
[7] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
[8] Sixia Yu and Chang-pu Sun, Phys. Rev. A **61**, 022310 (2000), LANL e-print quant-ph/0001052.
[9] D. Boumeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).
[10] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[11] M. A. Nielsen, E. Knill, and R. Laflamme, Nature (London) **396**, 52 (1998).
[12] S. L. Braunstein and H. J. Kimble, Nature (London) **394**, 840 (1997).
[13] D. Boumeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **394**, 841 (1997).
[14] L. Vaidman and N. Yoran, Phys. Rev. A **59**, 116 (1999).
[15] S. Stenholm and P. J. Bardroff, Phys. Rev. A **58**, 4373 (1998).
[16] A. Karlsson and M. Bourennane, Phys. Rev. A **58**, 4394 (1998).
[17] T. C. Ralph and P. K. Lam, Phys. Rev. Lett. **81**, 5668 (1998).
[18] M. S. Zubairy, Phys. Rev. A **58**, 4368 (1998).
[19] C. S. Maierle, D. A. Lidar, and R. A. Harris, Phys. Rev. Lett. **81**, 5928 (1998).
[20] A. Furasawa, J. L. Sorenson, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).
[21] G. Brassard, S. L. Braunstein, and R. Cleve, Physica D **120**, 43 (1998).
[22] D. Deutsch, Proc. R. Soc. London, Ser. A **400**, 97 (1985).
[23] D. Deutsch, Proc. R. Soc. London, Ser. A **425**, 73 (1989).
[24] D. P. DiVincenzo, Phys. Rev. A **51**, 1015 (1995).